On the Sources and Value of Information:
Public Announcements and Macroeconomic Performance

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Abstract. In the context of macroeconomic coordination, studies of the social value of information distinguish sharply between private and public information. However, no information is truly public (that is, common knowledge) or truly private in the established sense. Building upon a Lucas-Phelps island-economy model, this paper allows for many informative signals each of which incorporates elements of both public and private information. A measure of relative publicity determines a signal's equilibrium use and its impact on macroeconomic performance: output gaps open up when signals differ in their publicity, because such differences drive a wedge between price-formation and expectations-formation processes. Turning to the effect of public announcements, it is never optimal to withhold information completely, nor is it optimal to release perfectly public (or, indeed, perfectly private) information. Instead, when perfect communication is feasible, limited clarity helps to stabilise output. **JEL classification codes**: C72, D83, and E5.

1. PUBLIC ANNOUNCEMENTS AND TRANSPARENCY

"Since I have become a central banker I've learned to mumble with great incoherence. If I seem unduly clear to you, you must have misunderstood what I said." (Alan Greenspan, quoted by the *Wall Street Journal*, 1987)

This paper asks whether, as the oft-cited quotation above suggests, it is ever in the interests of a central banker (or some other macroeconomic policy-maker) to release relevant information in a less than maximally transparent manner. Briefly, the answer is this: it is never optimal to withhold such information entirely, but nor is it ever optimal to release perfectly transparent information. Even if central bankers were able to communicate with perfect clarity, they would never wish to do so.

The context for this conclusion is a simple Lucas-Phelps island economy (Section 2). There is supply-side uncertainty over economy-wide prices, and demand-side uncertainty over an underlying fundamental. (A known fundamental would serve as a nominal anchor for prices and so would prevent deviations of output away from the natural rate.) Each island sector receives (possibly many) noisy signals of the true fundamental and a local price equilibrates aggregate supply and demand. In so doing, each island's inhabitants evaluate not only their expectations of the fundamental, but also the expectations held by other sectors which determine economy-wide prices and so influence aggregate supply.

Morris and Shin (2002) considered a related model with two signals of this sort: one private (an independent signal realisation for each sector) and one public (common knowledge to all). They argued that a social planner would sometimes (but not always) wish to suppress the public signal. Their insight was that a public signal exerts a disproportionately large influence: such a signal is particularly useful for the formation of higher-order expectations, since agents know that the inhabitants of other sectors have seen the same signal realisation. This influential idea prompted a growing literature which emphasises the sharp distinction between public and private information.

Here the information structure blunts this distinction (Section 3). Each signal is characterised not only by its variance but also by its cross-sectoral correlation. Equivalently, a signal has both accuracy (signal quality) and transparency (signal clarity). A signal with imperfect quality (it imperfectly identifies the fundamental) but with perfect clarity (everyone sees the same thing) is "purely public" and is perfectly correlated across the economy. A signal with perfect quality but with imperfect clarity (islands observe different signal realisations, but the average is correct) is "purely private" and so (conditional on the fundamental) is uncorrelated. The model allows for a general (conditional) correlation coefficient for each of many signals and so intermediate values of "publicity" are feasible; equivalently, arbitrary mixes of signal quality and signal clarity are permitted.

The move to a more general information structure is important for (at least) three reasons. Firstly, when existing comparative-static exercises have varied the precisions of public and private signals they have conflated two distinct properties: the signal's quality in the first case, and its clarity in the second. Secondly, reducing the transparency of a public signal in a two-signal world results in an imperfectly correlated signal: this is a step outside the boundaries of a "public and private" model. Thirdly, assessments of transparency must envisage at least three sources of information: agents' prior beliefs, any independent island-specific information, and the announcement itself.

Despite the step forward in generality, the equilibrium is characterised easily: the price in a sector is a weighted average of the signals received by its inhabitants (Section 4). The influence of a signal is increasing in its precision and its publicity, where a signal's

publicity is a monotonic transformation of its cross-sectoral correlation; at the ends of the publicity spectrum lie purely private and purely public signals.

A central claim of the extant literature is that public signals can be detrimental. Evaluating this claim requires an appropriate performance criterion. In a Lucas-Phelps island economy a measure of macroeconomic performance can be derived from the output gap: the distance between equilibrium output and natural output in each sector (Section 5). An appropriate aggregation of output gaps across the island economy yields a performance measure, which coincides with a measure of output stability. The reactions of macroeconomic performance to signal precision and publicity are readily found.

It is true that relatively public signals exert a disproportionate influence (given their precisions) and so can increase the variability of prices. Nevertheless, an output gap is opened only by any separation between a sector's price and its inhabitants' expectations of economy-wide prices. No matter how noisy signals (and hence prices) are, a gap will not open so long as the price-setting and expectation-formation processes remain synchronised. For instance, in a world with just a single signal (or many identically correlated signals) prices and expectations move together. However, gaps arise when the correlation coefficients of signals differ since expectations react more strongly to relatively public signals than do prices. An insight is that what matters is differences in the publicity of information sources available to agents. Output gaps are shrunk by "averagely public" signals but are opened by both very public and very private ones (Section 6).

Applying this insight, the optimal information-release policy of a planner is studied (Section 7), where this planner wishes to stabilise output by minimising output gaps. In addition to any existing information, the central bank (for instance) can choose whether or not, and how publicly, to release a further signal. A signal can be characterised in terms of its variance (or precision) and correlation (or publicity) and if these could be chosen separately then the bank would release a precise but averagely public signal. However, it seems more appropriate to describe a signal in terms of its quality and clarity. The idea is that the central bank sees a signal of the fundamental with a given precision (its quality) and then communicates it to the sectors with a given transparency (its clarity). The bank may observe less carefully and so degrade the signal quality, and it can communicate less clearly and so degrade the clarity. Intuitively, however, it cannot increase the quality or clarity beyond some exogenously given bounds (the information source's "technology").

The notion of publicity is critical for the optimal information-release policy. When the central bank has access to an information source whose technology is more public than average, its announcement policy involves maximal quality but may involve reduced clarity. Indeed, if the technology is sufficiently public (this is true when, for instance, the bank is able to make commonly understood announcements), then the bank will certainly

degrade the clarity of its signal. On the other hand, if the information source's technology is less public than average then the central bank will communicate with maximal clarity, but may degrade the quality (and will do so if the technology is sufficiently private).

The intuition is straightforward. If the bank speaks clearly then its announcement will be highly correlated and so relatively public. Since it is heterogeneity in the publicity of signals which drives apart the price-formation and expectation-formation processes, performance can be enhanced by making the announcement less public. One way of doing this is to muddy the communication process by adding noise. It is never optimal to suppress communication completely; once enough noise is added to the communication process then the bank's signal becomes averagely public and so helps to unify prices and expectations. A symmetric logic applies to a relatively private information source.

A detailed discussion of related literature is postponed to Section 9. However, a few contributions are highlighted here. The analysis applies to the island-economy (Phelps, 1970; Lucas, 1973) and beauty-contest (Keynes, 1936) parables that were developed formally in recent years by Morris and Shin (2002, 2005), Angeletos and Pavan (2004, 2007, 2009), and Hellwig (2005), amongst others. Beyond the study of macroeconomic coordination, similar models have been applied to financial markets (Allen, Morris, and Shin, 2006), the leadership of political parties (Dewan and Myatt, 2008), and other problems. Most studies have specified a public-and-private information structure, although some recent work (Baeriswyl and Cornand, 2006, 2007; Baeriswyl, 2007; Hellwig and Veldkamp, 2009; Angeletos and Pavan, 2009; Dewan and Myatt, 2008) has incorporated partial correlation.

This paper brings three contributions to the literature. Firstly, it differs in focus from many earlier papers: it examines a performance measure (the output gap) derived from the Lucas-Phelps island economy, rather than assessing the welfare of the players of a beauty-contest game. Secondly, it abandons the public-and-private signal taxonomy and develops the idea of a signal's publicity: what creates real output variation is differences in the publicities of different signals. Thirdly, the paper offers a re-examination of public announcements: it illustrates how such announcements ought to vary with the nature of economy's extant information. Contrary to Morris and Shin (2002), it is never optimal to release a purely public (or, for that matter, a purely private) signal; moreover, and again contrary to Morris and Shin (2002), it is never optimal to withhold the signal completely.

This paper proceeds by describing the island economy (Section 2), the information available to each island sector (Section 3), and the equilibrium response of prices to signal realisations (Section 4). The main focus of the paper is then the response of macroeconomic performance (Section 5) to the signals' properties (Section 6) and the optimal announcement policy of a central bank (Sections 7 and 8). The paper concludes by further exploring the relationship of the results to those of the established literature (Section 9).

2. ISLAND ECONOMIES AND BEAUTY CONTESTS

The island-economy parable was discussed by Phelps (1970) and explored more formally by Lucas (1973); Blanchard and Fischer (1989, Chapter 7, pp. 356–61) offered a textbook treatment. The economy in question consists of a unit mass archipelago of "island" sectors indexed by $\ell \in [0, 1]$. The (natural logarithm of) nominal price in sector ℓ is p_ℓ and the economy-wide aggregate price level is $\bar{p} = \int_0^1 p_l \, dl$. The natural level of economic activity is normalised so that its logarithm is zero. Activity in sector ℓ is y_ℓ which, given the normalisation, is also the gap between output and capacity.

An economy-wide fundamental $\theta \in \mathbb{R}$ drives aggregate demand. As will become clear, if this fundamental were common knowledge then (in equilibrium) all prices would satisfy $p_\ell = \theta$ and output gaps would be eliminated. However, the inhabitants of each sector are uncertain of the fundamental and of the aggregate price level. They form expectations $\mathrm{E}_\ell[\theta]$ and $\mathrm{E}_\ell[\bar{p}]$, where the subscripts indicate expectations taken with the respect to the (common) beliefs held in sector ℓ. Aggregate supply and demand in sector ℓ satisfy

$$y_{\ell S} = \alpha_S(p_\ell - \mathrm{E}_\ell[\bar{p}]) \quad \text{and} \quad y_{\ell D} = \alpha_D(\mathrm{E}_\ell[\theta] - p_\ell).$$

Equating supply and demand yields the market-clearing price p_ℓ in sector ℓ:

$$p_\ell = \pi \, \mathrm{E}_\ell[\theta] + (1 - \pi) \, \mathrm{E}_\ell[\bar{p}], \quad \text{where} \quad \pi = \frac{\alpha_D}{\alpha_S + \alpha_D}. \tag{1}$$

The market-clearing nominal price in sector ℓ combines expectations of the fundamental and of the economy-wide price level; the relative weight placed on these expectations depends on the relative slope of aggregate supply and demand.

A natural macroeconomic performance measure is based upon output gaps. If θ were known then setting $p_\ell = \theta$ for all sectors would lead to $y_\ell = 0$, and hence no deviation from the natural level of economic activity. However, uncertainties over the fundamental and the aggregate price level allow gaps to open; indeed, it will be shown that $y_\ell \neq 0$ except in special circumstances. Aggregating across the islands and treating positive and negative gaps symmetrically suggests the use of $\int_0^1 y_l^2 \, dl$. Ex ante this measure becomes $\mathrm{E}[y_\ell^2]$. The unique equilibrium characterised later in the paper has the feature that $\mathrm{E}[y_\ell] = 0$, and so $\mathrm{E}[y_\ell^2] = \mathrm{var}[y_\ell]$. Given that the model is specified in log terms, $\mathrm{var}[y_\ell]$ is an appropriate unit-free measure of the variability of real output, and so $\mathrm{E}[y_\ell^2]$ can readily be used to assess output stability. Thus, maximisation of macroeconomic performance via minimisation of this output-gap measure is equivalent to output stabilisation.

As Morris and Shin (2002, 2005) have noted, there is a close connection between the island-economy model and a "beauty contest" coordination game reminiscent of a story

told by Keynes (1936, Chapter 12). Morris and Shin (2002) described a continuum of players engaged in a simultaneous-move game. Player $\ell \in [0, 1]$ chooses an action $p_\ell \in \mathbb{R}$ and receives a payoff u_ℓ. This payoff depends upon the distance of the player's action from some underlying fundamental $\theta \in \mathbb{R}$ (yielding a fundamental motive for each player) and from the aggregate action taken across the set of players $\bar{p} = \int_0^1 p_l \, dl$ (which generates a coordination motive). Combining these two elements of players' preferences and adopting a quadratic-loss specification yields

$$u_\ell = \bar{u} - \pi(p_\ell - \theta)^2 - (1 - \pi)(p_\ell - \bar{p})^2. \tag{2}$$

Taking expectations of u_ℓ with respect to the beliefs held by player ℓ, the optimal choice of action is $p_\ell = \pi \, \mathrm{E}_\ell[\theta] + (1 - \pi) \, \mathrm{E}_\ell[\bar{p}]$. This, of course, is the market clearing price from (1), and so the beauty-contest and island-economy models are isomorphic.

Dealing with the beauty-contest game is particularly convenient because the specification of (2) describes a potential game (Monderer and Shapley, 1996). The strategic interaction in a potential game is captured concisely by a function which maps the action profile to a single real value. Payoff-motivated players then act as if they are jointly maximising this single function. Writing p as a strategy profile, it is straightforward to check that

$$\phi(p) = \bar{u} - \pi \int_0^1 (p_l - \theta)^2 dl - (1 - \pi) \int_0^1 (p_l - \bar{p})^2 dl \tag{3}$$

is an exact potential function for the payoff specification (2).[2] This feature of the beauty-contest game proves useful for equilibrium characterisation.[3]

3. INFORMATION

Each island's price depends upon the beliefs of its inhabitants and hence upon any informative signals at their disposal. Here attention turns to the specification of such signals.

It is assumed that all islands share a common prior over θ. Without loss of generality this is assumed to be an improper prior; any substantive prior belief can be accommodated via the specification of informative signals described here.

Island sector ℓ receives a vector $x_\ell \in \mathbb{R}^n$ of informative signals. Signals are independent across the n information sources. Fixing an information source $j \in \{1, \ldots, n\}$, however,

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[2] Notice that $\phi(p) = \int_0^1 u_l \, dl$ and so the potential function aggregates the players' payoffs; a consequence is that the equilibrium is efficient. This does not apply more widely to other strategically equivalent beauty-contest specifications. Morris and Shin (2002), for instance, added a strategically irrelevant but welfare-relevant term of the form $(1 - \pi) \int_0^1 (p_l - \bar{p})^2 \, dl$ to the payoff of player ℓ.

[3] This feature appears to have received relatively little attention in the literature; notable exceptions include work by Ui (2008) and by Calvó-Armengol and de Martí Beltran (2009). Unlike in Monderer and Shapley (1996), there is a continuum of players, as in Sandholm (2001). The corresponding potential of (3) is the limit of the potential of a finite player game as the player set grows large; see (22) in Appendix B.

the observations of different sectors are correlated. Conditional on θ, the signals observed are jointly distributed according to the normal, with common variance σ_j^2. Any pair of sectors ℓ and $\ell' \neq \ell$ have a correlation coefficient of ρ_j, so that

$$x_{j\ell} \,|\, \theta \sim N(\theta, \sigma_j^2) \quad \text{and} \quad \text{cov}[x_{j\ell}, x_{j\ell'} \,|\, \theta] = \rho_j \sigma_j^2. \tag{4}$$

The quality of the jth informative source is indexed by its precision $\psi_j \equiv 1/\sigma_j^2$. Given the improper prior, the conditional expectation of the fundamental θ satisfies

$$\text{E}[\theta \,|\, x_\ell] = \frac{\sum_{i=1}^{n} \psi_i x_{i\ell}}{\sum_{i=1}^{n} \psi_i}. \tag{5}$$

If prices were determined only by this then only the precision ψ_j of a signal would be relevant. However, the price p_ℓ in sector ℓ depends upon expectations of economy-wide prices and so, implicitly, upon beliefs about the beliefs held in other sectors. This means that the commonality of signals, indexed by the correlation coefficient ρ_j, is also relevant.

The specification considered here was proposed in a political-science context by Dewan and Myatt (2008). It encompasses the information structure used by Morris and Shin (2002, 2005), Angeletos and Pavan (2004, 2007), Hellwig (2005), and others. Most authors have considered beauty-contest games in which each player receives a public signal and a private signal of the fundamental. Here, a public signal is obtained by setting $\rho_j = 1$; the same realisation is seen in every sector. A private signal, in contrast, corresponds to $\rho_j = 0$; conditional on the fundamental, it says nothing about the signals received in other sectors. Only in recent research (Hellwig and Veldkamp, 2009; Angeletos and Pavan, 2009; Dewan and Myatt, 2008) have authors begun to use imperfectly correlated signals. The earlier focus on the public-versus-private taxonomy proved useful, since it highlighted the role played by higher-order expectations in determining players' behaviour (in beauty contests) or islands' prices (in a Lucas-Phelps economy). However, it rules out a class of interesting scenarios in which signals are correlated but imperfectly so.

Consider the following interpretation of the information sources. Suppose that the jth signal is provided by a sender of information; a central bank or a financial newspaper for example. The sender observes a noisy signal of the true value of the fundamental θ,

$$\tilde{x}_j = \theta + \eta_j \quad \text{where} \quad \eta_j \sim N(0, \kappa_j^2), \tag{6}$$

so that η_j is "sender noise" attributable to the information acquisition of the sender; the precision $1/\kappa_j^2$ measures the ability of the sender to identify θ. The sender then communicates the signal to the agents in the various sectors of the island economy. However, the agents in sector ℓ observe the signal imperfectly:

$$x_{j\ell} = \tilde{x}_j + \epsilon_{j\ell} \quad \text{where} \quad \epsilon_{j\ell} \sim N(0, \xi_j^2), \tag{7}$$

so that $\epsilon_{j\ell}$ is "receiver noise" from errors in the communication process; the precision $1/\xi_j^2$ measures the clarity of communication between sender and receiver. The various noise terms (η_j and $\epsilon_{j\ell}$) are assumed to be independently distributed. Combining sender noise and receive noise yields the specification of (4), where

$$\sigma_j^2 = \kappa_j^2 + \xi_j^2 \quad \text{and} \quad \rho_j = \frac{\kappa_j^2}{\kappa_j^2 + \xi_j^2}. \tag{8}$$

The quality of information provided by a signal for identification of the fundamental depends only upon the total noise. However, the balance between sender noise and receiver noise influences the commonality of the views held in different island sectors.

The sender-receiver specification is recovered via $\kappa_j^2 = \rho_j \sigma_j^2$ and $\xi_j^2 = (1 - \rho_j)\sigma_j^2$; similarly, public and (pure) private signals are easily obtained by setting either $\xi_j^2 = 0$ or $\kappa_j^2 = 0$, respectively. However, the sender-receiver model proves useful by illustrating the somewhat restrictive nature of the public-private classification.

To see this, begin with a central bank that communicates its information perfectly to the economy, so that $\xi_j^2 = 0$ and $\rho_j = 1$. Suppose now that the central bank muddles its communications by transmitting via an imperfect channel. Adopting the model proposed here, this change corresponds to an increase in ξ_j^2, which in turn leads to $\rho_j \in (0, 1)$. The signal received by the various island sectors is partially private, and partially public; the variance parameters κ_j^2 and ξ_j^2 indexing sender and receiver noise might equivalently be labelled as public and private noise. This parameter change seems to represent an interesting thought experiment, and yet it is excluded by (most) existing models.

4. EQUILIBRIUM

Attention now turns to finding the signal-contingent prices which equilibrate aggregate supply and demand on each island. A pricing rule for a sector maps signal realisations to market-clearing prices, so that $p_\ell = P_\ell(x_\ell) : \mathbb{R}^n \mapsto \mathbb{R}$. Since sectors are symmetric and each sector is negligible, it is without loss of generality to restrict attention to symmetric pricing rules so that $p_\ell = P(x_\ell)$ for all $\ell \in [0, 1]$. The equilibrium condition (1) reduces to

$$P(x_\ell) = \pi \, \mathrm{E}[\theta \, | \, x_\ell] + (1 - \pi) \, \mathrm{E}[P(x_{\ell'}) \, | \, x_\ell]. \tag{9}$$

For general signal specifications an equilibrium pricing rule takes an arbitrary form. However, as is now well known, the adoption of normal distributions for signals ensures that there is a unique linear equilibrium. That is, for some set of weights $w \in \mathbb{R}_+^n$,

$$P(x_\ell) = \sum_{i=1}^{n} w_i x_{i\ell}, \quad \text{where} \quad \sum_{i=1}^{n} w_i = 1, \tag{10}$$

so that the price in a sector is a weighted average of the signals seen by its inhabitants. This is natural since the regressions $\mathrm{E}[\theta \,|\, x_\ell]$ and $\mathrm{E}[x_{\ell'} \,|\, x_\ell]$ are both linear in their conditioning arguments. That is, $\mathrm{E}[\theta \,|\, x_\ell] = a \cdot x_\ell$ for some vector a where "$\cdot$" indicates the usual vector product, and $\mathrm{E}[x_{\ell'} \,|\, x_\ell] = B x_\ell$ for the $n \times n$ inference matrix B. Similarly, writing w for the vector of weights with jth element w_j, and restricting attention to the class of linear equilibria, the market-clearing condition (9) for each sector reduces to

$$w \cdot x_\ell = \pi a \cdot x_\ell + (1 - \pi) w \cdot B x_\ell \quad \Leftrightarrow \quad w = \pi \left[I - (1 - \pi) B' \right]^{-1} a. \tag{11}$$

The restriction of attention to linear equilibria in the (now familiar) derivation of (11) is not quite without loss of generality. Recursive application of (9) as proposed by Morris and Shin (2002) does not successfully demonstrate uniqueness, as explained by Angeletos and Pavan (2007, footnote 5) amongst others. On the other hand, it is possible to show that the linear equilibrium is unique among a class of appropriately bounded equilibrium pricing rules.[4] The linear equilibrium is, in any case, rather natural, and, following in the footsteps of the earlier literature, will be the focus of the subsequent analysis.

The linear equilibrium may be characterised by solving (11) directly. However, it is rather more straightforward to utilise the fact (Section 2) that the isomorphic beauty contest is an exact potential game. This means that the equilibrium weights w successfully maximise the ex ante expectation of $\phi(p)$ in (3).[5] Writing expectations operators in place of integrals and with $p_\ell = w \cdot x_\ell$, the equilibrium weights minimise

$$\pi \, \mathrm{E} \left[(p_\ell - \theta)^2 \right] + (1 - \pi) \, \mathrm{E} \left[(p_\ell - \bar{p})^2 \right] \tag{12}$$

subject to $\sum_{i=1}^{n} w_i = 1$. Note that $\sum_{i=1}^{n} w_i = 1$ is not an exogenous restriction; the set of coefficients must satisfy this equality if it is to successfully maximise the expected potential.[6] Consider the first element in (12). Since the weights add to one,

$$\mathrm{E} \left[(p_\ell - \theta)^2 \right] = \sum_{i=1}^{n} w_i^2 \, \mathrm{E} \left[(x_{i\ell} - \theta)^2 \right] = \sum_{i=1}^{n} w_i^2 \sigma_i^2 = \sum_{i=1}^{n} w_i^2 (\kappa_i^2 + \xi_i^2),$$

where the second equality follows from (4) and the third from (8). The second element may be expressed in a similar way. First, note that $\bar{p} \equiv \int_0^1 p_l \, dl = \int_0^1 \sum_i w_i x_{il} \, dl = \sum_i w_i \tilde{x}_i$,

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[4]Consider a pricing rule $P(x_\ell)$ for which there exists a set of coefficients w such that $|P(x_\ell) - w \cdot x_\ell|$ remains bounded for all x_ℓ. The unique linear equilibrium is unique within the class of pricing rules which satisfy this criterion. The appendix to the paper by Dewan and Myatt (2008) explains more fully. More recently, Calvó-Armengol, de Martí Beltran, and Prat (2009) described another way to establish uniqueness. Roughly speaking, they imposed bounds upon both signal realisations and action choices in a game with beauty-contest features. They established the existence of a unique equilibrium, and investigated the properties of that equilibrium as the bounds are loosened. In the limit, as the bounds are removed, the equilibrium converges to the unique linear equilibrium of an unbounded game.

[5]Beauty contests are Bayesian potential games in the sense of van Heuman, Peleg, Tijs, and Borm (1996).

[6]The ex ante expectation of $\phi(p)$ remains finite if and only if $\sum_{i=1}^{n} w_i = 1$. In any case, this equality emerges readily from the familiar method of matching coefficients which yields (11).

so that the expected price level across sectors is the weighted sum of the senders' observations as defined in (6). Now the second key element of (12) is

$$\mathrm{E}\left[(p_\ell - \bar{p})^2\right] = \sum_{i=1}^{n} w_i^2 \, \mathrm{E}\left[(x_{i\ell} - \tilde{x}_i)^2\right] = \sum_{i=1}^{n} w_i^2 \xi_i^2,$$

where the second equality follows directly from the specification given in (7). Collecting these two elements together again, the minimisation programme can be restated as

$$\min_{w} \sum_{i=1}^{n} w_i^2 \left(\pi \kappa_i^2 + \xi_i^2\right) \quad \text{such that} \quad \sum_{i=1}^{n} w_i = 1. \tag{13}$$

Solving this latter problem is straightforward and yields the weights w for the linear equilibrium pricing rule of (10). Note that the weight attached to each information source j depends on the correlation of signals received by different sectors of the island economy. More emphasis is placed on receiver noise (or errors in communication) than on sender noise (errors in the senders' observation of the fundamental).

A simple intuition for this is obtained by recalling that the market-clearing prices must satisfy (1), so that $p_\ell = \pi \, \mathrm{E}_\ell[\theta] + (1 - \pi) \, \mathrm{E}_\ell[\bar{p}]$. (Equivalently, the player of the beauty-contest game aims to be close to both the fundamental and the aggregate action of others.) Any receiver noise frustrates both of these objectives. However, sender noise moves a sector away from the fundamental, but does not play a role in moving p_ℓ away from the economy-wide average $\bar{p}$. Since adherence to the fundamental carries a reduced weight of π, so too does the corresponding sender-noise term κ_j^2.

The linear equilibrium pricing rule may now be formulated.

Proposition 1. *There is a unique equilibrium pricing rule $P(x_\ell) = \sum_{i=1}^{n} w_i x_{i\ell}$, satisfying*

$$w_j = \frac{\psi_j \beta_j}{\sum_{i=1}^{n} \psi_i \beta_i}, \quad \text{where} \quad \beta_j = \frac{1}{1 - \rho_j(1 - \pi)}.$$

The relative influence of an information source increases with the quality of information it provides and the correlation of signals that different sectors receive. Equivalently, since

$$\psi_j \beta_j = \frac{1}{\pi \kappa_j^2 + \xi_j^2},$$

the influence of an information source decreases with both sender noise and receiver noise. Across information sources, if $\rho_j > \rho_{j'}$, the influence of j relative to j' increases as π falls.

β_j is a natural measure of the "publicity" of a signal j. As discussed earlier, a purely public signal of the sort considered in the literature corresponds to $\rho_j = 1$, which means $\beta_j = 1/\pi$. On the other hand, purely private information yields $\rho_j = 0$ and $\beta_j = 1$. Between these bounds, publicity increases with correlation. Moreover, β_j depends upon the importance

of coordination, measured by $(1 - \pi)$. As greater emphasis is placed on coordination (so that π falls; this happens when the Lucas supply function becomes shallower) the influence of the most public signals rises at the expense of the least public signals.

A signal's publicity is central to its influence, and the signals' publicities are also critical for the analysis of macroeconomic performance considered in the next section. For the results that follow, two different notions of average publicity are important.

$$\bar{\beta} \equiv \sum_{i=1}^{n} w_i \beta_i = \frac{\sum_{i=1}^{n} \psi_i \beta_i^2}{\sum_{i=1}^{n} \psi_i \beta_i} \quad \text{and} \quad \hat{\beta} \equiv \frac{\sum_{i=1}^{n} \psi_i \beta_i}{\sum_{i=1}^{n} \psi_i}.$$

Here $\bar{\beta}$ is the equilibrium-weighted average publicity, in which the importance of a signal is determined by its influence in equilibrium. In contast, $\hat{\beta}$ is the precision-weighted average publicity, where the weights used are those which corresponds to the formation of the conditional expectation of θ. It is straightforward to confirm that $\bar{\beta} \geq \hat{\beta}$.

5. MACROECONOMIC PERFORMANCE AND THE OUTPUT GAP

Attention now turns to assessing the relationship between macroeconomic performance and the properties of the various informative signals available to the island sectors.

Returning to the aggregate demand equation for island ℓ, note that in equilibrium

$$y_{\ell D} = \alpha_D(E_\ell[\theta] - p_\ell) \quad \Rightarrow \quad y_\ell \propto (E[\theta \,|\, x_\ell] - P(x_\ell)).$$

Recall that y_ℓ is the output gap in sector ℓ, and that the proposed macroeconomic performance index is determined by $E[y_\ell^2]$, which aggregates the output gap across the economy and also serves to measure output stability. Taking appropriate expectations,

$$E[y_\ell^2 \,|\, \theta] \propto \underbrace{E\left[(E[\theta \,|\, x_\ell] - P(x_\ell))^2 \,|\, \theta\right]}_{L_Y}.$$

Using (5) and Proposition 1, and following some algebraic manipulation, it is straightforward to show that the expectation L_Y is independent of θ and satisfies

$$L_Y = \frac{1}{\sum_{i=1}^{n} \psi_i} \left[\frac{\bar{\beta}}{\hat{\beta}} - 1\right], \tag{14}$$

where $\hat{\beta}$ is the benchmark precision-weighted average publicity, and $\bar{\beta}$ is the equilibrium-weighted average publicity, and where $\hat{\beta} = \bar{\beta}$ if and only if $\beta_j = \beta_{j'}$ for all j and j'.

Note immediately that (14) must be weakly positive (since $\bar{\beta} \geq \hat{\beta}$). Real output variation is eliminated (that is, $L_Y = 0$) if all signals share the same publicity or, equivalently, the same (conditional) correlation coefficient: if all information is identically correlated (or in effect, there is a single signal) then there are no output gaps.

Proposition 2. *The output gap is identically zero (and so macroeconomic performance is maximised) when all signals share the same correlation coefficient (so that $\rho_j = \rho_{j'}$ for all j and j').*

Output gaps arise from the divergence of the price-setting process and the expectation-formation process. When there is one signal (or many identically correlated signals) this cannot happen. An immediate corollary arises: in the absence of any substantive prior and with a single signal or with multiple signals sharing the same correlation coefficient there is no role whatsoever for further announcements. That is, if a central bank (for example) provides an additional $(n+1)$th signal, because the output gap is identically zero to start with in these instances, things can only be made worse.

Corollary to Proposition 2. *When there is a single signal (or many identically correlated signals) public announcements can only reduce macroeconomic performance.*

Nevertheless, announcements (that is, the release of additional informative signals) can be helpful when the conditions of Proposition 2 fail; that is, when islands receive multiple signals with different publicities or when there is a substantive prior. Such announcements are considered in subsequent sections of the paper.

Before moving on, however, and stepping outside the island-economy model, other performance criteria are briefly considered. In the context of a related investment game, Angeletos and Pavan (2004) defined heterogeneity as $\mathrm{var}[p_\ell \,|\, \theta]$ and volatility as $\mathrm{var}[\bar{p} \,|\, \theta]$. In an island economy these loss functions measure the extent to which (nominal) prices coincide with the underlying fundamental. A third loss function of this kind is $\mathrm{var}[p_\ell \,|\, \bar{p}]$, which measures the dispersion of prices across the economy.

The three loss functions are simply related: $\mathrm{var}[p_\ell \,|\, \theta] = \mathrm{var}[p_\ell \,|\, \bar{p}] + \mathrm{var}[\bar{p} \,|\, \theta]$. Furthermore, they are all determined by three factors: the total precision of the signals; the equilibrium-weighted average publicity $\bar{\beta}$; and the precision-weighted average publicity $\hat{\beta}$. For instance, straightforward calculations reveal that heterogeneity satisfies

$$L_H \equiv \mathrm{var}[p_\ell \,|\, \theta] = \frac{1}{\sum_{i=1}^{n} \psi_i} \frac{\bar{\beta}}{\hat{\beta}}.$$

This is the loss measure used by Morris and Shin (2002).[7] It is possible to characterise dispersion (written L_D) and volatility (written L_V) in terms of L_H and $\bar{\beta}$. They are

$$L_D \equiv \mathrm{var}[p_\ell \,|\, \bar{p}] = \frac{1/\bar{\beta} - \pi}{1 - \pi} \times L_H \quad \text{and} \quad L_V \equiv \mathrm{var}[\bar{p} \,|\, \theta] = \frac{1 - 1/\bar{\beta}}{1 - \pi} \times L_H.$$

Further results focus on the measure L_Y derived from the output gap, although it is possible to perform similar exercises with the other measures described here.

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[7]Further loss functions also depend on the same three factors. Were the beauty contest itself a game being played (rather than a convenient isomorphism for the island-economy model) the expected potential would be a natural welfare measure. From (3), the associated social loss is $L_B \equiv \pi L_H + (1 - \pi)L_D = L_H/\bar{\beta}$.

The importance of using $\mathrm{E}[y_\ell^2] \propto L_Y$ as a performance measure rather than the other loss functions described above is that the former tracks real deviations in the economy away from the natural level. The latter measures focus rather on nominal deviations. The nominal values of the fundamental θ and of aggregate prices $\bar{p}$ do not in themselves matter for pricing decisions; it is expectations of the fundamental and expectations of the economy-wide price level that determine aggregate demand and supply respectively. Therefore, the deviation of prices from expectations is what matters. Expectations of the fundamental are formed according to (5), for instance; prices are set using the weights described in Proposition 1. Only when these weights differ will there be any real-output implication. The expression in (14) precisely reflects the different weights that come into play for expectations formation and price-setting through its emphasis on the divergence between $\hat{\beta}$ and $\bar{\beta}$. When there is no divergence, there is no output variation.

6. PUBLICITY AND PRECISION

An output-stabilising planner considering whether to release an informative signal j might not reasonably be expected to manipulate ψ_j (precision) and β_j (publicity) directly; more sensible would be to consider the (partial) control of $1/\xi_j^2$ or $1/\kappa_j^2$. Under the interpretation of Section 3 the former can be seen as the clarity of an announcement, over which a planner might reasonably have some influence, whilst the latter is the precision with which the planner itself sees the fundamental. Of course ψ_j and β_j can be written in terms of ξ_j^2 and κ_j^2, yielding indirect control over the publicity and precision of the signal; but one cannot be changed without an impact upon the other. Nevertheless, a starting point for an analysis of performance (using L_Y) is provided in Propositions 3 and 4.

Proposition 3. *Macroeconomic performance is quasi-concave in each β_j: it is increasing for all $\beta_j < \bar{\beta}$ and decreasing for all $\beta_j > \bar{\beta}$. Hence performance is maximised by setting $\beta_j = \bar{\beta}$.*

Put succinctly, this says that performance is enhanced by "averagely public" signals; for instance, a planner would like to reduce the publicity of a relatively public signal. Informative signals with extreme publicity (whether extremely private or extremely public) drive a wedge between prices and expectations, so opening output gaps.

Proposition 4. *Fixing the publicity β_j of a signal, macroeconomic performance is increasing in the signal's precision ψ_j if and only if its publicity is relatively average. More formally,*

$$\frac{\partial L_Y}{\partial \psi_j} < 0 \quad \Leftrightarrow \quad (\beta_j - \bar{\beta})^2 < \bar{\beta}^2 - \hat{\beta}^2.$$

Moreover, performance is quasi-convex in the signal's precision. Fixing the publicity, it is optimal to either (i) withhold a signal entirely or (ii) release it with as much precision as possible.

More information is good if and only the signal's publicity is "relatively average" in the sense of Proposition 4; both "very public" and "very private" information can be harmful. It has been noted that signals with publicities that differ markedly from the average drive apart the expectations-formation and price-formation processes; as they become more precise (locally) the problem is exacerbated as they gain influence.

It is instructive to consider these insights in the context of a two-signal world with a purely public signal ($\rho_1 = 1$, so that $\beta_1 = 1/\pi$) and a purely private signal ($\rho_2 = 0$, so that $\beta_2 = 1$) respectively. This is the standard model found in the preceding literature; it also corresponds to a single perfectly private signal coupled with a substantive common prior. Clearly $\beta_1 > \bar{\beta} > \beta_2$. In such a world there is always a lower range of ψ_1 for which L_Y is increasing in ψ_1. Making purely public information more precise reduces performance for this range. Confirming the Corollary to Proposition 2, performance is maximised by setting $\psi_1 = 0$, which is equivalent to releasing no information.

For ψ_1 sufficiently large, L_Y is decreasing in ψ_1 and so performance is improved locally by increasing the precision of the public signal. In this world $\partial L_Y / \partial \psi_1 < 0$ if and only if $\psi_1 > \psi^*$ where, by substituting in the expression in Proposition 4 for β_1 and β_2,

$$\psi^* \equiv \frac{\psi_2}{4} \left[\sqrt{1 + 8\pi} - 1 \right].$$

Note that ψ^* is increasing in π and ψ_2. When π is higher the pricing rule places less weight on $\mathrm{E}_\ell[\bar{p}]$ and so the equilibrium price-formation process is less biased toward the public signal (β_1 is lower) in equilibrium. As a result the public signal does not serve as an effective coordination device until higher values of its precision are reached: it is for these higher values of ψ_1 that more information is better (locally). ψ^* is also increasing in ψ_2. A similar intuition applies: the higher ψ_2 the higher the quality of the purely private signal, and the less relatively useful the purely public signal is in equilibrium.

However, since output gaps are eliminated when $L_Y = 0$, a zero-precision signal is always better than $\psi_1 > 0$ no matter how large is ψ_1. Only if a "perfectly precise" public signal ($\sigma_1^2 = 0$) is available will it do as well as a zero-precision signal. This is because performance losses are the result of a divergence between price setting and expectations. If each island receives one signal only then this divergence cannot arise. It does not matter which signal is heard, just that a sector's price does not deviate from the perception of the price level; such a deviation can happen only when more than one source of information receives attention. (Of course, the presence of a prior means that more than one source is typically present.)

Returning to the point made just prior to Proposition 3, the focus of the analysis here is on a situation in which there is no strong distinction between public and private information as assumed in the preceding paragraphs. Rather, the publicity of a signal in equilibrium is

indexed by $\beta_j \in [1, 1/\pi]$; and there are many such signals. Nevertheless a result analogous to the preceding discussion is available as a corollary of Proposition 4.

Corollary to Proposition 4. *For each signal, there exists ψ_j^* such that for all $\psi_j \geq \psi_j^*$ performance is increasing in precision. If the signal is neither too public nor too private then $\psi_j^* = 0$.*

This is reinforces a central message: it is differences in the publicities (equivalently, correlations) of signals that can separate price formation and expectations formation.

7. OPTIMAL ANNOUNCEMENTS

Moving on from the analysis in terms of β_j and ψ_j, if a planner (or a central bank, for instance) has to decide whether to release information, it seems unlikely that it would have full control over the publicity and the precision of the signal. The approach favoured here is to allow the planner limited control over ξ_j^2 and κ_j^2 for some information source j. This fits with the interpretation of these variances given in Section 3: the planner may manipulate the clarity with which the information is communicated and the precision with which the information source is observed respectively.

From (8) and Proposition 1, ψ_j and β_j may be written

$$\psi_j = \frac{1}{\kappa_j^2 + \xi_j^2} \quad \text{and} \quad \beta_j = \frac{\kappa_j^2 + \xi_j^2}{\pi \kappa_j^2 + \xi_j^2}. \tag{15}$$

Thus, any change in ξ_j^2 (or in κ_j^2) will change both ψ_j and β_j. In particular, an increase in clarity (ξ_j^2 falls) increases both precision and publicity, whereas an increase in quality (κ_j^2 falls) increases precision, but decreases publicity. Fixing the values of these parameters across the n signals, it is possible to characterise the overall impact upon performance that a local change in any ξ_j^2 or κ_j^2 would have in terms of the publicity term β_j.

Proposition 5. *Fix $n \geq 2$ signals with clarities ξ_j^2 and qualities κ_j^2. For each signal j,*

$$\frac{\partial L_Y}{\partial \xi_j^2} > 0 \quad \Leftrightarrow \quad \beta_j \in (\beta_\xi, \beta^\xi) \quad \text{and} \quad \frac{\partial L_Y}{\partial \kappa_j^2} > 0 \quad \Leftrightarrow \quad \beta_j \in (\beta_\kappa, \beta^\kappa),$$

where these intervals' boundaries satisfy: $\max\{1, \beta_\xi\} < \beta_\kappa < \bar{\beta} < \beta^\xi < \min\{\frac{1}{\pi}, \beta^\kappa\}$. *The four interval boundaries (β_ξ, β^ξ, β_κ, and β^κ) depend only on the two measures of average publicity.*

This reinforces earlier results. If a signal is neither too public nor too private (formally, if $\beta_\kappa < \beta_j < \beta^\xi$) then output variation can be reduced (locally) by enhancing the signal's quality (a reduction in κ_j^2) or its clarity (a reduction in ξ_j^2). If the information source is sufficiently private or public, however, at least one of these claims will fail. For instance, if $\beta > \beta^\xi$ (a sufficiently public signal) then increasing ξ^2 is helpful, as it reduces the publicity

of the signal. More generally, performance benefits (locally) from less clarity of communication (or more "receiver noise") when an information source is very public, and from less underlying quality (or more "sender noise") when a source is very private.

Suppose now that a planner has at its disposal a signal with underlying quality $\tilde{\kappa}^2$ and clarity $\tilde{\xi}^2$. The planner may choose to reduce the quality of the signal (a central bank might commit to doing so by reducing the size of its research staff) or by communicating with less than maximal clarity (perhaps by obfuscating in its announcements), but cannot improve on the underlying quality and clarity so easily. This is represented by a choice of κ^2 and ξ^2 for this signal such that $\kappa^2 \geq \tilde{\kappa}^2$ and $\xi^2 \geq \tilde{\xi}^2$. Altering these parameters will have an impact upon both the signal's precision and its publicity. The lower bounds for clarity and quality, $\tilde{\xi}^2 \geq 0$ and $\tilde{\kappa}^2 \geq 0$ will be referred to as the "technology" available to the planner, and it is assumed that $\max\{\tilde{\xi}^2, \tilde{\kappa}^2\} > 0$, so that the technology never admits a perfectly revealing signal. Hence the publicity of a planner's technology is given by

$$\tilde{\beta} = \frac{\tilde{\kappa}^2 + \tilde{\xi}^2}{\pi\tilde{\kappa}^2 + \tilde{\xi}^2}.$$

Notions of relative publicity for the planner's technology are straightforward. Beginning in a world without the planner, suppose that there are $n \geq 2$ distinct signals so that the equilibrium-weighted average publicity satisfies $\bar{\beta} \in (1, \frac{1}{\pi})$. The planner's technology is relatively public if $\tilde{\beta} > \bar{\beta}$ and relatively private if $\tilde{\beta} < \bar{\beta}$. In choosing the actual characteristics of any announcement made, the planner degrades the signal's quality if $\kappa^2 > \tilde{\kappa}^2$, and degrades its clarity if $\xi^2 > \tilde{\xi}^2$. With this terminology in hand, Proposition 6 shows how these parameters ought to be chosen to maximise performance.

Proposition 6. *Fix $n \geq 2$ distinct signals, so that $\bar{\beta} \in (1, \frac{1}{\pi})$. A planner has an extra information source at its disposal, and chooses ξ^2 and κ^2 to maximise performance.*

(i) If the technology is relatively public then it maximises its signal's quality; it degrades clarity if its technology is sufficiently public ($\tilde{\beta} > \beta^\xi$); its signal's publicity satisfies $\bar{\beta} \leq \beta \leq \max\{\beta^\xi, \tilde{\beta}\}$.

(ii) If the technology is relatively private then it maximises its signal's clarity; it degrades quality if its technology is sufficiently private ($\tilde{\beta} < \beta_\kappa$); its signal's publicity satisfies $\min\{\beta_\kappa, \tilde{\beta}\} \leq \beta \leq \bar{\beta}$.

(iii) If the planner's technology is neither relatively public nor relatively private ($\tilde{\beta} = \bar{\beta}$), the optimal choice satisfies $\xi^2 = \tilde{\xi}^2$ and $\kappa^2 = \tilde{\kappa}^2$; its signal's publicity satisfies $\beta = \tilde{\beta} = \bar{\beta}$.

Hence a planner may wish to degrade its signal's quality or its clarity, but never both. Furthermore, a careful inspection of the claims reveals that it never degrades a signal completely.[8] Next, consider a perfectly public signal technology; this is when it is possible

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[8]For instance, in case (i) the planner may choose to degrade its signal's clarity. If it did so completely, by allowing $\xi^2 \to \infty$, then the signal would become relatively private. However, the optimal clarity satisfies $\beta \geq \bar{\beta}$, and so the signal is only partially degraded, if at all. A similar argument applies to case (ii).

to release information with perfect clarity. If $\tilde{\xi}^2 = 0$ then $\tilde{\beta} = \frac{1}{\pi}$, and so case (i) applies; κ^2 is chosen as small as possible. Now note that $\tilde{\beta} > \beta^\xi$ since $\beta^\xi < \frac{1}{\pi}$ (from Proposition 5). It follows, from Proposition 6, that clarity will certainly be degraded. Finally, consider a perfectly private signal technology; this is when the planner is able to identify perfectly the fundamental. Case (ii) applies: it is optimal to communicate with maximal clarity. However, it is optimal to damage the signal's quality, by choosing $\kappa^2 > 0$, since $\tilde{\beta} < \beta_\kappa$.

Corollary to Proposition 6. *So long as there are $n \geq 2$ existing signals with distinct correlations, a planner would never wish to withhold its information completely. When it is possible to release a purely public signal, the planner never wishes to do so: it degrades the clarity of its communication. Similarly, if the planner can identify perfectly the underlying fundamental, it never wishes to do so: it degrades the quality of its information acquisition.*

Driving this corollary is the planner's desire for averagely public information; this reduces the undesirable wedge between price formation and expectations formation.

8. OPTIMAL OBFUSCATION

Proposition 6 and its corollary confirm that an output-stabilising central bank would never wish to communicate perfectly its information, and so there is always a role for what might be called obfuscation. This naturally leads to a reduction in the informativeness of the bank's announcement. This section studies the response of optimal obfuscation (equivalently, optimal clarity) to (i) the fundamental-versus-coordination parameter π, (ii) the properties of the island sectors' existing information sources, and (iii) the central bank's ability to observe the underlying fundamental.

Before considering further the second and third parameters, recall that the fundamental-versus-coordination parameter π is the relative weight placed on the fundamental by players of a beauty-contest coordination game. Interest here, however, lies in the associated Lucas-Phelps island economy. From (1) recall that

$$\pi = \frac{\alpha_D}{\alpha_S + \alpha_D},$$

where α_D is the slope of the aggregate demand curve $y_{\ell D} = \alpha_D(\mathrm{E}_\ell[\theta] - p_\ell)$ and α_S is the slope of the corresponding aggregate supply $y_{\ell S} = \alpha_S(p_\ell - \mathrm{E}_\ell[\bar{p}])$. It follows that π is large when (for instance) the Lucas supply function is relatively unresponsive.

The background to the central bank's announcement is a world in which there are two existing information sources available to each island sector: a purely public signal with precision $\psi_1 = 1 - \omega$ and a purely private signal with precision $\psi_2 = \omega$; the corresponding publicities of these signals are $\beta_1 = 1/\pi$ and $\beta_2 = 1$ respectively. The fact that $\psi_1 + \psi_2 = 1$ is a normalisation which is made without the loss of any generality, and so, fixing the

total amount of information available to an island, the parameter ω reflects the relative importance of purely private versus purely public information.

It is worth noting that the model specification prescribes a diffuse (improper) prior over θ, and so any substantive prior beliefs held by the economy's inhabitants must be incorporated as one of the signals available to them. This means that $\psi_1 = 1 - \omega$ can be appropriately interpreted as the precision of a common prior, and hence $\psi_2 = \omega$ represents the relative precision of any new (private) information available. It follows that allowing ω to become small can be interpreted as a situation in which the identity of the economy's nominal anchor is already well established, and allowing ω to become large reflects a situation in which islands are readily influenced by the arrival of new information.

A central bank has at its disposal a third information source (suppressing subscripts) which it may release (if it wishes) with perfect clarity ($\tilde{\xi}^2 = 0$) but which imperfectly reveals the fundamental ($\tilde{\kappa}^2 > 0$). The precision of the bank's information is $\tilde{\psi} \equiv 1/\tilde{\kappa}^2$, which in turn represents (given the normalisation $\psi_1 + \psi_2 = 1$) the precision of its information relative to the precision of the interim beliefs held on an island.

With these parameter values Proposition 6 reveals that a central bank always chooses $\kappa^2 = \tilde{\kappa}^2$ and an optimal (and obfuscatory) $\xi^2 > 0$, or equivalently an optimal publicity satisfying $1 < \beta < \tilde{\beta} = 1/\pi$. The precision of the released signal satisfies $\psi = 1/(\tilde{\kappa}^2 + \xi^2)$, and its correlation coefficient is

$$\rho = \frac{\tilde{\kappa}^2}{\tilde{\kappa}^2 + \xi^2} = \frac{\psi}{\tilde{\psi}}.$$

Hence ρ is the precision of the released signal relative to the quality of the information on which it is based; equivalently, it captures the transparency of the bank's announcement. When $\rho = 1$ the bank openly releases its information and so generates a perfectly public signal, whereas it can only achieve a perfectly private signal satisfying $\rho = 0$ by babbling (allowing $\xi^2 \to \infty$) and so (effectively) throwing its information away.

Summarising, the exogenous parameters for the comparative-static exercises which follow are ω (the importance of private signals relative to the prior), π (the responsiveness of aggregate demand relative to aggregate supply), and $\tilde{\psi}$ (the precision of the central bank's information source relative to agents' interim beliefs). Figures 1 and 2 plot the optimal transparency against ω and π for two different values of $\tilde{\psi}$.

When private signals are swamped by prior beliefs (corresponding to $\omega \approx 0$) the average publicity of the information available to the agents is very high (close to $1/\pi$). In order to release "averagely public" information, the bank releases its signal transparently, so that $\rho \approx 1$. As the relative importance of prior beliefs fall, so that the nominal anchor is highly uncertain ex ante, the central bank responds by obfuscating; it becomes less transparent in communicating its own information about the fundamental.
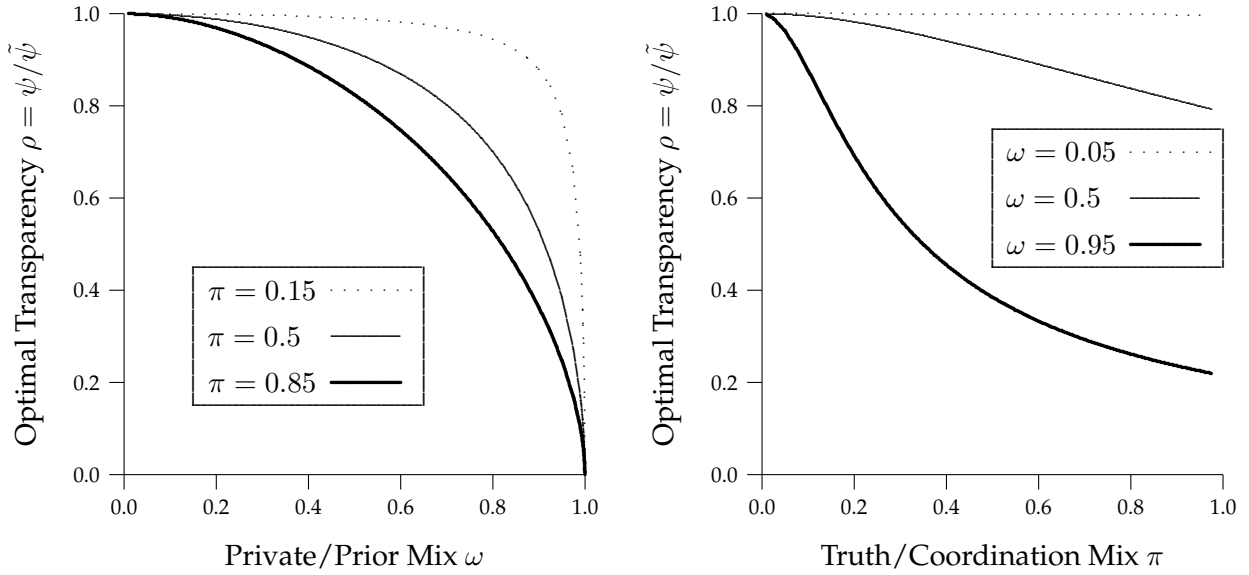
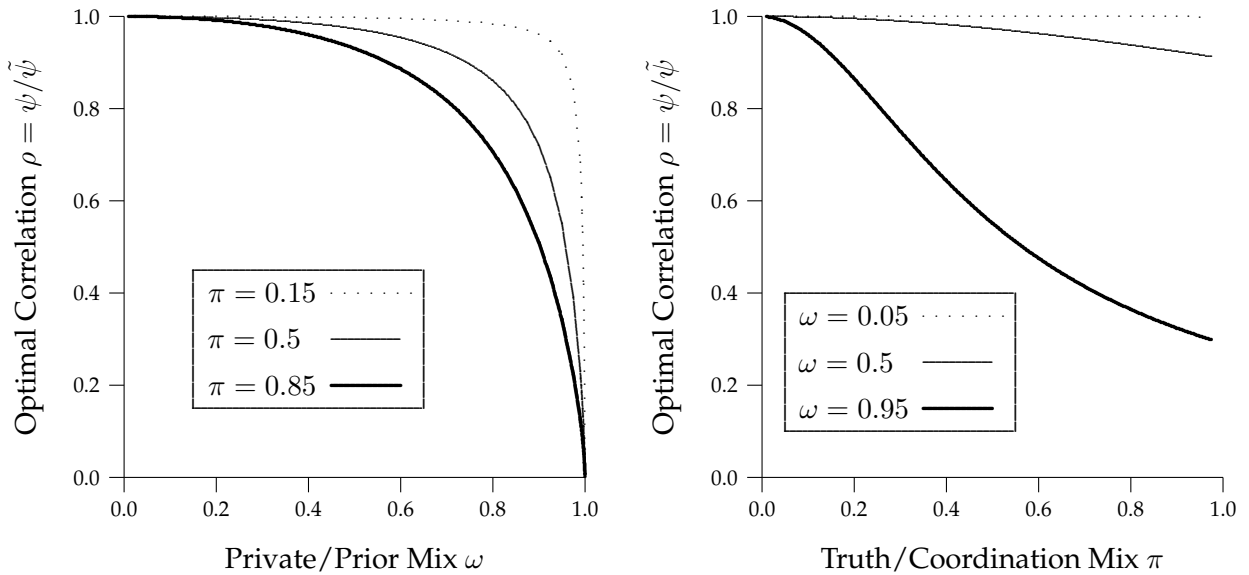FIGURE 1. Optimal Transparency: Low-Quality Information ($\tilde{\psi} = 0.2$)

FIGURE 2. Optimal Transparency: High-Quality Information ($\tilde{\psi} = 5.0$)

The quality of the signal released by the bank is always degraded ($\psi < \tilde{\psi}$): its clarity is imperfect. However, comparing Figures 1 and 2, when the bank's own information improves it also enhances its transparency: a planner with a relatively informative signal ought to speak with relative clarity.

Finally, consider the changes in the optimally chosen ρ with respect to the parameter π. As π increases, the clarity with which the planner ought to release its information falls. For higher values of ω, where the purely private signal is very precise relative to the prior, ρ falls very rapidly indeed as π increases. It is here that, to a great extent, the private signal drowns out the prior in the equilibrium price-setting weights. Any very public signal

released by the planner would inevitably drive a wedge between the price-formation process and the expectations-formation process. To avoid doing so, and thereby generating losses, the planner must obscure its message and release an uncorrelated and relatively uninformative signal. For low values of π when the prior is relatively strong, the planner releases a correlated and very public signal to reinforce the already strong connection between expectations and price setting.

9. RELATED LITERATURE

A recent literature has explored formally the beauty-contest and island-economy parables. The former parable emerges from Keynes (1936, Chapter 12) who documented newspaper competitions in which entrants were asked to choose the prettiest face from a collection of photographs, but where the winner was based on the popularity of choices; players become concerned with second-guessing others rather than focusing on the declared target. The Morris and Shin (2002) game captures the spirit of the story. For them it was natural to consider a performance criterion of the form $\mathrm{E}[(p_\ell - \theta)^2]$; higher-order beliefs lead players to abandon highly informative (but not commonly known) private signals in favour of focal public ones and so deviate further from the fundamental.

Beauty-contest play is closely related to the prices in an island economy (Phelps, 1970, 1983; Lucas, 1972, 1973). This interpretation was central to work (Amato, Morris, and Shin, 2002; Morris and Shin, 2005) which emphasised the so-called double-edged nature of public information: Morris and Shin (2005) explained that "[w]hen there is the potential for a strong consensus to prevail [...] incentives may become distorted in such a way as to reduce the informational value of economic outcomes." A problem, recognised by Woodford (2005) and others, is the use of $\mathrm{E}[(p_\ell - \theta)^2]$ as a performance criterion: the deviation of nominal prices from the nominal anchor. A real measure (such as the output-gap-derived $\mathrm{E}[y_\ell^2]$) may be more appropriate.

Other performance criteria were considered by Angeletos and Pavan (2004, 2007).[9] They studied investment games in which a player ℓ chooses an investment k_ℓ to maximise a payoff $\mathrm{E}[\pi\theta + (1 - \pi)\bar{k}]k_\ell - \frac{1}{2}k_\ell^2$, where $\bar{k}$ is aggregate investment.[10] Optimal actions are linear in players' expectations of the unknown productivity fundamental and the choices of others, so mimicking (1). One natural measure of welfare is then the sum of payoffs; this might be appropriate for the beauty contest itself (although not for the isomorphic

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[9]Some of these criteria were considered briefly in the web-distributed appendix to Morris and Shin (2002).

[10]In their later paper Angeletos and Pavan (2007) developed applications of the basic game to production externalities, beauty contests, business cycles, and large Cournot and Bertrand games. Other authors have also used beauty-contest games of this kind to investigate endogenous communication in networks (Calvó-Armengol and de Martí Beltran, 2007, 2009; Calvó-Armengol, de Martí Beltran, and Prat, 2009).

island economy). [11] In other prominent work, Hellwig (2005) followed in the spirit of earlier literature (Blanchard and Kiyotaki, 1987; Kiyotaki, 1988; Woodford, 2003) by studying the incomplete nominal adjustment of monopolistic firms via a linear price-setting rule. In his model, price dispersion drives welfare effects and the implications of the comparative-static exercises differ markedly from those of Morris and Shin (2002, 2005).

A distinctive feature of the literature is the focus on a two-signal public-and-private information structure in which each agent (or island sector) sees a public (commonly observed) signal and a private (independently distributed when conditioned on the fundamental) signal.[12] There have been some steps away from this benchmark: in their appendix Morris and Shin (2002) considered a two-player model in which one player sees two private correlated signals; Angeletos and Pavan (2009) and Baeriswyl (2007) specified frameworks in which different players' private signal realisations are partially correlated; and Baeriswyl and Cornand (2006, 2007) considered a multiple-fundamental specification. Only recently have models emerged in which agents receive many informative signals: papers by Dewan and Myatt (2008) and Hellwig and Veldkamp (2009) both allow for this.

Relative to the literature, this paper sharply focuses on the island-economy interpretation of the beauty contest model, derives its performance measure from the output gap, and exploits a far richer information structure. This structure allows for an arbitrarily large number of signals with arbitrary conditional correlation coefficients, and moves away from the public-and-private taxonomy: instead, a signal is characterised by its quality and its clarity. The literature has considered changes in the precisions of public and private signals. These amount to changes in a signal's quality and clarity respectively. This paper allows a single signal to possess both properties, and leads naturally to the consideration of a signal's publicity. It is the difference in the publicity of signals, and not the presence of a perfectly public signal per se, which undesirably separates the price-formation and expectation-formation processes.[13]

The paper also provides a reassessment of the transparency of public announcements. The important insight is that output gaps are closed by averagely public signals. For instance, a relatively public signal becomes more average by muddying its clarity, whereas a

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[11]As has been noted, for the beauty contest considered here players maximise welfare and so there is no role for the public-information suppression which has been a theme of the literature. This point was made by Woodford (2005), who wrote "[...] if one were to rank the outcomes on the basis of [...] the population average of the individual loss function [...] then this alternative social loss function is *necessarily* reduced by increasing the precision of the public signal" (emphasis in original).

[12]Morris and Shin (1998) used a similar distinction in a global-game model of regime change and triggered a large subsequent literature (Hellwig, 2002; Corsetti, Dasgupta, Morris, and Shin, 2004; Hellwig, Mukherji, and Tsyvinski, 2006; Angeletos, Hellwig, and Pavan, 2006, 2007; Angeletos and Werning, 2006).

[13]Some insights (such as the desirability of so-called "averagely public" signals) carry over to the other performance indices, including the Morris-Shin heterogeneity measure L_H discussed in Section 5.

relatively private signal becomes more average by worsening its underlying quality. Contrary to Morris and Shin (2002), it is never optimal to release a purely public or a purely private signal; moreover, it is never optimal to withhold the signal completely.[14] This implies that partial transparency, via obfuscatory communication, is an optimal feature of policy announcements. This resonates with the conclusions of Cornand and Heinemann (2008) who argued that the partial disclosure of a public signal is better than reducing the precision of a fully disclosed signal; hence this paper and theirs offer complementary notions of partial publicity. Another strand of literature (Eijffinger, Hoeberichts, and Schaling, 2000; Faust and Svensson, 2001, 2002; Jensen, 2002; Beetsma and Jensen, 2003; Geraats, 2002, 2007) has argued that uncertainty over a central bank's objectives may be beneficial. Here the role of "central bank mystique" is to ensure that a public announcement has similar characteristics to agents' extant information.

APPENDIX A. OMITTED PROOFS

Proof of Proposition 1. The programming problem may be solved using traditional methods, resulting in first-order conditions of the form $2w_j(\pi\kappa_j^2 + \xi_j^2) = \lambda + \mu_j$, where λ is the Lagrange multiplier on the constraint in (13) and μ_j is the multiplier on the implicit non-negativity constraint for w_j. Since $\sum_{i=1}^n w_i = 1$, there is at least one j such that $w_j > 0$. For this j, $\mu_j = 0$ by complementary slackness, and hence $\lambda > 0$. But then, by non-negativity of μ_j, $w_j > 0$ for all j. Thus

$$w_j = \text{const} \times \frac{1}{\pi\kappa_j^2 + \xi_j^2}, \quad \text{where} \quad \text{const} = 1 \Big/ \sum_{i=1}^n \frac{1}{\pi\kappa_i^2 + \xi_i^2},$$

so that the w_j's sum to one. Rewriting in terms of β_j as defined in the text and $\psi_j \equiv 1/\sigma_j^2$ leads to the first expression. The remainder of the proposition follows by inspection. □

Proof of Proposition 2. Follows by inspection of (14) (as does the corollary). □

Proof of Proposition 3. First-order conditions may be obtained from (14):

$$\frac{\partial L_Y}{\partial \beta_j} = \frac{1}{\hat{\beta}\sum_{i=1}^n \psi_i} \left[\frac{\partial\bar{\beta}}{\partial\beta_j} - \frac{\bar{\beta}}{\hat{\beta}}\frac{\partial\hat{\beta}}{\partial\beta_j} \right].$$

Differentiating $\bar{\beta}$ with respect to β_j yields

$$\frac{\partial\bar{\beta}}{\partial\beta_j} = \frac{\partial}{\partial\beta_j}\left[\frac{\sum_{i=1}^n \psi_i\beta_i^2}{\sum_{i=1}^n \psi_i\beta_i}\right] = \frac{2\beta_j\psi_j}{\sum_{i=1}^n \psi_i\beta_i} - \frac{\sum_{i=1}^n \psi_i\beta_i^2}{(\sum_{i=1}^n \psi_i\beta_i)^2}\psi_j = \frac{\psi_j}{\hat{\beta}\sum_{i=1}^n \psi_i}\left[2\beta_j - \bar{\beta}\right].$$

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[14]The policy implications of Morris and Shin (2002, 2005) paper have been debated. For instance, Svensson (2006) argued that better public information is harmful only for extreme parameter values, and that the release of a public signal is always welfare-enhancing if its precision exceeds that of individual private signals; this was in the context of a deviation-from-the-fundamental performance criterion. The response by Morris, Shin, and Tong (2006) was broadly accommodating, although it suggested that the presence of partially correlated signals might restore the conclusion that transparency might be harmful.

$\partial \hat{\beta} / \partial \beta_j = \psi_j / \sum_{i=1}^{n} \psi_i$. Hence, substituting back into the original expression gives

$$\frac{\partial L_Y}{\partial \beta_j} = \frac{2\psi_j}{(\hat{\beta} \sum_{i=1}^{n} \psi_i)^2} \left(\beta_j - \bar{\beta} \right). \tag{16}$$

This is zero, and hence there is a stationary point, at $\beta_j = \bar{\beta}$. It is positive for all $\beta_j > \bar{\beta}$ and negative for all $\beta_j < \bar{\beta}$. The function is therefore quasi-convex and has a unique minimum. $\qquad \square$

Proof of Proposition 4. First, partially differentiate L_Y with respect to ψ_j:

$$\frac{\partial L_Y}{\partial \psi_j} = \frac{-1}{(\sum_{i=1}^{n} \psi_i)^2} \left[\frac{\bar{\beta}}{\hat{\beta}} - 1 \right] + \frac{1}{\hat{\beta} \sum_{i=1}^{n} \psi_i} \frac{\partial \bar{\beta}}{\partial \psi_j} - \frac{\bar{\beta}}{\hat{\beta}^2 \sum_{i=1}^{n} \psi_i} \frac{\partial \hat{\beta}}{\partial \psi_j},$$

where $\partial \bar{\beta} / \partial \psi_j = \beta_j (\beta_j - \bar{\beta}) / \sum_{i=1}^{n} \psi_i \beta_i$ and $\partial \hat{\beta} / \partial \psi_j = (\beta_j - \hat{\beta}) / \sum_{i=1}^{n} \psi_i$, so

$$\frac{\partial L_Y}{\partial \psi_j} = \frac{1}{(\sum_{i=1}^{n} \psi_i)^2} \left[\frac{\hat{\beta} - \bar{\beta}}{\hat{\beta}} + \frac{\sum_{i=1}^{n} \psi_i}{\hat{\beta}} \frac{\beta_j (\beta_j - \bar{\beta})}{\sum_{i=1}^{n} \psi_i \beta_i} - \frac{\bar{\beta} \sum_{i=1}^{n} \psi_i}{\hat{\beta}^2} \frac{(\beta_j - \hat{\beta})}{\sum_{i=1}^{n} \psi_i} \right]$$

$$= \frac{1}{(\hat{\beta} \sum_{i=1}^{n} \psi_i)^2} \left[\hat{\beta}(\hat{\beta} - \bar{\beta}) + \beta_j (\beta_j - \bar{\beta}) - \bar{\beta}(\beta_j - \hat{\beta}) \right]$$

$$= \frac{1}{(\hat{\beta} \sum_{i=1}^{n} \psi_i)^2} \left[\hat{\beta}^2 + \beta_j (\beta_j - 2\bar{\beta}) \right] \tag{17}$$

$$= \frac{1}{(\hat{\beta} \sum_{i=1}^{n} \psi_i)^2} \left[(\beta_j - \bar{\beta})^2 - (\bar{\beta}^2 - \hat{\beta}^2) \right].$$

Hence $\partial L_Y / \partial \psi_j < 0$ if and only if the second term in the above expression is negative, as required. To establish the quasi-concavity of L_Y, partially differentiate L_Y again and evaluate at a stationary point (where $\partial L_Y / \partial \psi_j = 0$). First note that from the penultimate expression above:

$$\left. \frac{\partial^2 L_Y}{\partial \psi_j^2} \right|_{\frac{\partial L_Y}{\partial \psi_j} = 0} = \frac{1}{(\hat{\beta} \sum_{i=1}^{n} \psi_i)^2} \left[\hat{\beta} \frac{\partial \bar{\beta}}{\partial \psi_j} - \beta_j \frac{\partial \hat{\beta}}{\partial \psi_j} \right].$$

The sign of this second differential is therefore determined by the second term of this expression.

$$\left. \frac{\partial^2 L_Y}{\partial \psi_j^2} \right|_{\frac{\partial L_Y}{\partial \psi_j} = 0} < 0 \qquad \Leftrightarrow \qquad \frac{\hat{\beta}(\beta_j - \hat{\beta})}{\sum_{i=1}^{n} \psi_i} - \frac{\beta_j^2 (\beta_j - \bar{\beta})}{\sum_{i=1}^{n} \psi_i \beta_i} < 0$$

$$\Leftrightarrow \qquad \hat{\beta}^2 (\beta_j - \hat{\beta}) - \beta_j^2 (\beta_j - \bar{\beta}) < 0.$$

Using the first-order condition $\hat{\beta}^2 = -\beta_j (\beta_j - 2\bar{\beta})$ from above and dividing by β_j,

$$\left. \frac{\partial^2 L_Y}{\partial \psi_j^2} \right|_{\frac{\partial L_Y}{\partial \psi_j} = 0} < 0 \quad \Leftrightarrow \quad (2\bar{\beta} - \beta_j)(\beta_j - \hat{\beta}) - \beta_j (\beta_j - \bar{\beta}) < 0$$

$$\Leftrightarrow \quad \left[\bar{\beta} \mp \sqrt{\bar{\beta}^2 - \hat{\beta}^2} \right] \left[\bar{\beta} \pm \sqrt{\bar{\beta}^2 - \hat{\beta}^2} - \hat{\beta} \right] \mp \left[\bar{\beta} \pm \sqrt{\bar{\beta}^2 - \hat{\beta}^2} \right] \sqrt{\bar{\beta}^2 - \hat{\beta}^2} < 0,$$

where the second line follows from the first-order condition substitution $\beta_j = \bar{\beta} \pm \sqrt{\bar{\beta}^2 - \hat{\beta}^2}$. After some algebraic simplification the final inequality above can be shown to be equivalent to

$(2\hat{\beta} + \bar{\beta})^2 > \bar{\beta}^2 - \hat{\beta}^2$, which is certainly true since $\bar{\beta} \geq \hat{\beta} \geq 1$. Thus L_Y is quasi-concave in ψ_j. The remaining statement then follows immediately from quasi-concavity (as does the corollary). $\quad\square$

Proof of Proposition 5. To examine the loss function of (14) with respect to ξ_j^2, first note that

$$\frac{\partial L_Y}{\partial \xi_j^2} = \frac{\partial L_Y}{\partial \beta_j} \frac{\partial \beta_j}{\partial \xi_j^2} + \frac{\partial L_Y}{\partial \psi_j} \frac{\partial \psi_j}{\partial \xi_j^2}.$$

Using the definitions of ψ_j and β_j in (15), the following can be derived:

$$\frac{\partial \beta_j}{\partial \xi_j^2} = \psi_j \beta_j (1 - \beta_j) \leq 0 \quad \text{and} \quad \frac{\partial \psi_j}{\partial \xi_j^2} = -\psi_j^2 \leq 0.$$

The remaining elements of the expression are derived in (16) and (17), to yield

$$\frac{\partial L_Y}{\partial \xi_j^2} = \frac{\psi_j^2}{(\hat{\beta} \sum_{i=1}^n \psi_i)^2} \left[\beta_j^2 (1 + 2\bar{\beta} - 2\beta_j) - \hat{\beta}^2 \right]. \tag{18}$$

Performing the same operation for $\partial L_Y / \partial \kappa_j^2$, and noting from (15) that

$$\frac{\partial \beta_j}{\partial \kappa_j^2} = \psi_j \beta_j (1 - \pi \beta_j) \geq 0 \quad \text{and} \quad \frac{\partial \psi_j}{\partial \kappa_j^2} = -\psi_j^2 \leq 0,$$

gives an analogous expression:

$$\frac{\partial L_Y}{\partial \kappa_j^2} = \frac{\psi_j^2}{(\hat{\beta} \sum_{i=1}^n \psi_i)^2} \left[\beta_j^2 (1 + 2\pi\bar{\beta} - 2\pi\beta_j) - \hat{\beta}^2 \right]. \tag{19}$$

Loss is increasing in ξ_j^2 and κ_j^2 if and only if the expressions in (18) and (19), respectively, exceed zero. Thus, loss is increasing in ξ_j^2 if $\beta_j^2 (1 + 2\bar{\beta} - 2\beta_j) > \hat{\beta}^2$, which is certainly satisfied at $\beta_j = \bar{\beta}$ since $\bar{\beta} > \hat{\beta}$. Treating $\hat{\beta}$ and $\bar{\beta}$ as constants, this expression is cubic in β_j. Its leading term is negative, and therefore it reaches a local minimum at 0 and a local maximum at $(1 + 2\bar{\beta})/3 < \bar{\beta}$. Therefore there are two positive roots, $\beta_\xi < \bar{\beta}$ and $\beta^\xi > \bar{\beta}$; and $\partial L_Y / \partial \xi_j^2$ is positive if $\beta_j \in (\beta_\xi, \beta^\xi)$.

Now, loss is increasing in κ_j^2 if $\beta_j^2 (1 + 2\pi\bar{\beta} - 2\pi\beta_j) > \hat{\beta}^2$ (again, this is certainly satisfied at $\beta_j = \bar{\beta}$). This is another cubic (when $\bar{\beta}$ and $\hat{\beta}$ are treated as constants) in β_j, with its local minimum at 0 and its local maximum at $(\frac{1}{\pi} + 2\bar{\beta})/3 > \bar{\beta}$. Therefore there are two positive roots, $\beta_\kappa < \bar{\beta}$ and $\beta^\kappa > \bar{\beta}$; and $\partial L_Y / \partial \kappa_j^2$ is positive if $\beta_j \in (\beta_\kappa, \beta^\kappa)$. This latter cubic lies below the previous cubic when $\beta_j < \bar{\beta}$ and above elsewhere, thus confirming the ordering of these two pairs of roots.

It remains to show that $\beta^\xi < \frac{1}{\pi}$ and $\beta_\kappa > 1$. This is equivalent to showing that, for any j, $\partial L_Y / \partial \xi_j^2 < 0$ at $\xi_j^2 = 0$ and $\partial L_Y / \partial \kappa_j^2 < 0$ at $\kappa_j^2 = 0$. Setting $\xi_j^2 = 0$ yields $\beta_j = \frac{1}{\pi}$, and hence, from (18), the derivative of loss with respect to ξ_j^2 at $\xi_j^2 = 0$ is negative if and only if

$$\frac{1}{\pi^2} \left(1 + 2\bar{\beta} - \frac{2}{\pi} \right) < \hat{\beta}^2 \quad \Leftrightarrow \quad 2\bar{\beta} - \pi^2 \hat{\beta}^2 < \frac{2}{\pi} - 1. \tag{20}$$

The proof proceeds by showing that the maximum possible value of the left-hand side of this latter expression given any constellation of β_js and ψ_js is less than the constant on the right-hand side. First, maximise the left-hand side by choosing a constellation of β_js to

$$\max_{(\beta_1, \ldots, \beta_n)} \left[2\bar{\beta} - \pi^2 \hat{\beta}^2 \right] \quad \text{subject to} \quad 1 \leq \beta_j \leq \frac{1}{\pi} \quad \text{for } j = (1, \ldots, n):$$

a standard constrained optimisation problem with $2n$ constraints. The first order conditions are

$$\frac{2\psi_j}{\hat{\beta}\sum_{i=1}^{n}\psi_i}\left(2\beta_j - \bar{\beta} - \pi^2\hat{\beta}^2\right) + \lambda_j - \mu_j = 0,$$

where λ_j is the Lagrange multiplier associated with the constraint $\beta_j \geq 1$, μ_j is the Lagrange multiplier associated with the constraint $\beta_j \leq \frac{1}{\pi}$, and the first part of the expression follows from substitution for $\partial\bar{\beta}/\partial\beta_j$ and $\partial\hat{\beta}/\partial\beta_j$ found in the proof to Proposition 3.

At any strictly interior β_j such that $1 < \beta_j < \frac{1}{\pi}$, $\lambda_j = \mu_j = 0$ by complementary slackness. Hence $\beta_j = \frac{1}{2}(\bar{\beta} + \pi^2\hat{\beta}^2) \equiv \beta_*$. So the maximising constellation of β_is involves at most three values: $\beta_j \in \{1, \beta_*, \frac{1}{\pi}\}$. (Note that not all β_j may be interior therefore, since then $\beta_* = \bar{\beta} = \hat{\beta} = \frac{1}{\pi^2} > \frac{1}{\pi}$).

Suppose some (non-empty) subset of signals $m \subset (1, \ldots, n)$ has publicity values $\beta_j = 1$ for all $j \in m$, some (non-empty) subset m' has values $\beta_j = \beta_*$ for $j \in m'$ and some (non-empty) subset m'' has values $\beta_j = \frac{1}{\pi}$ for $j \in m''$. Let $\Psi = \sum_{i\in m}\psi_i/\sum_{i=1}^{n}\psi_i$, $\Psi' = \sum_{i\in m'}\psi_i/\sum_{i=1}^{n}\psi_i$, and $\Psi'' = \sum_{i\in m''}\psi_i/\sum_{i=1}^{n}\psi_i$. Now $\hat{\beta}$ may be rewritten $\hat{\beta} = \Psi + \beta_*\Psi' + \frac{1}{\pi}\Psi''$. Note that, since $\beta_* \in (1, \frac{1}{\pi})$, it may be rewritten as $\beta_* = \gamma\frac{1}{\pi} + (1-\gamma)$ for some $\gamma \in (0,1)$.

Now consider an alternative constellation of ψ_js (subscripted with a) such that $\Psi_a = \Psi + (1-\gamma)\varepsilon$, $\Psi'_a = \Psi' - \varepsilon$, and $\Psi''_a = \Psi'' + \gamma\varepsilon$ but where each β_j remains the same. Note that $\hat{\beta}_a = \hat{\beta}$. Furthermore,

$$\bar{\beta}_a = \frac{\Psi_a + \beta_*^2\Psi'_a + (\frac{1}{\pi})^2\Psi''_a}{\Psi_a + \beta_*\Psi'_a + \frac{1}{\pi}\Psi''_a} = \frac{(\Psi + (1-\gamma)\varepsilon) + \beta_*^2(\Psi' - \varepsilon) + (\frac{1}{\pi})^2(\Psi'' + \gamma\varepsilon)}{(\Psi + (1-\gamma)\varepsilon) + \beta_*(\Psi' - \varepsilon) + \frac{1}{\pi}(\Psi'' + \gamma\varepsilon)} \equiv \frac{N}{D}.$$

Consider $\partial\bar{\beta}_a/\partial\varepsilon \equiv \bar{\beta}'_a$. This is strictly positive if and only if $(DN' - ND')/D^2 > 0$. But $D = \hat{\beta}$, so $D' = 0$. Hence $\bar{\beta}_a$ is strictly increasing in ε if and only $N' > 0$. Now

$$N' = (1-\gamma) - \beta_*^2 + \gamma\frac{1}{\pi^2} = (1-\gamma) - \left[\gamma\frac{1}{\pi} + (1-\gamma)\right]^2 + \gamma\frac{1}{\pi^2} = \gamma(1-\gamma)\left[\frac{1}{\pi} - 1\right]^2 > 0.$$

Therefore, $\bar{\beta}_a$ is strictly increasing in ε. Since $\hat{\beta}_a$ is constant with respect to ε, the expression of interest, $2\bar{\beta} - \pi^2\hat{\beta}^2$ from (20), is also strictly increasing in ε. As a result, it is maximised when ε takes its maximal value: $\varepsilon = \Psi'$, so that $\Psi'_a = 0$. Thus the expression in (20) is maximised by a constellation of β_js and ψ_js such that $\beta_j \in \{1, \beta_*, \frac{1}{\pi}\}$ for all j, where $\psi_j = 0$ for any j such that $\beta_j = \beta_*$. It remains to show that $2\bar{\beta} - \pi^2\hat{\beta}^2 < \frac{2}{\pi} - 1$ for any such constellation.

Since $\Psi' = 0$ for any such constellation, $\Psi'' = (1 - \Psi)$, so

$$\bar{\beta} = \frac{\Psi + (1-\Psi)(\frac{1}{\pi})^2}{\Psi + (1-\Psi)\frac{1}{\pi}} \quad \text{and} \quad \hat{\beta} = \Psi + (1-\Psi)\frac{1}{\pi}.$$

Substituting, the inequality in (20) holds if and only if

$$2\left[\Psi + (1-\Psi)\frac{1}{\pi^2}\right] - \pi^2\left[\Psi + (1-\Psi)\frac{1}{\pi}\right]^3 < \left(\frac{2}{\pi} - 1\right)\left[\Psi + (1-\Psi)\frac{1}{\pi}\right].$$

Cancelling terms, multiplying out, and rearranging yields the following equivalent expression:

$$-3\pi\Psi\left[\frac{1}{\pi} - 1\right]^2 + \pi^2\Psi^2\left[\frac{1}{\pi} - 1\right]^3 < \frac{2}{\pi} \quad \Leftrightarrow \quad -3 + (1-\pi)\Psi < \frac{2}{\pi^2\Psi}\left[\frac{1}{\pi} - 1\right]^{-2}.$$

The left-hand side is negative since $\pi \in (0,1)$ and $\Psi \in [0,1]$; the right-hand side is positive. Thus the largest that $2\bar{\beta} - \pi^2 \hat{\beta}^2$ can be is still smaller than $\frac{2}{\pi} - 1$ and so $\partial L_Y / \partial \xi_j^2 < 0$ at $\xi_j^2 = 0$ as required. For $\beta_\kappa > 1$, note that $\kappa_j^2 = 0$ implies $\beta_j = 1$. The proof then proceeds in precisely the same way, but using the discriminant in (19) with $\beta_j = 1$ as its starting point. □

Proof of Proposition 6. Suppose that κ^2 and ξ^2 are set optimally. Admitting the possibility that the planner may choose to withhold the signal altogether is equivalent to allowing either of the values of κ^2 and ξ^2 to become unboundedly large, so that $\psi = 0$ (and hence loss is invariant to β). This cannot be optimal: choose κ^2 and ξ^2 so that $\beta = \bar{\beta}$, where, by Proposition 4, loss is decreasing in ψ. Increasing ψ away from 0 by proportionally reducing κ^2 or ξ^2 reduces loss.

Suppose now that the planner chooses $\kappa^2 > \tilde{\kappa}^2$ and $\xi^2 > \tilde{\xi}^2$. Then, if these are optimal, $\partial L_Y / \partial \kappa^2 = 0$ and $\partial L_Y / \partial \xi^2 = 0$. However, the former implies $\beta = \beta_\kappa$ or $\beta = \beta^\kappa$. The latter implies $\beta = \beta_\xi$ or $\beta = \beta^\xi$. These implications are mutually exclusive, by the ordering given in Proposition 5.

Therefore, one of the two optimally chosen parameters must be at its lower bound. Suppose $\kappa^2 > \tilde{\kappa}^2$. Then $\xi^2 = \tilde{\xi}^2$ and $\partial L_Y / \partial \kappa^2 = 0$. β must equal either β_κ or β^κ. Consider the latter: here, $\partial L_Y / \partial \xi^2 < 0$ and so an increase in ξ^2 would reduce loss. This contradicts the fact that the parameters were chosen optimally. Hence $\beta = \beta_\kappa$. It follows that $\beta < \bar{\beta}$. Since $\kappa^2 > \tilde{\kappa}^2$ and $\xi^2 = \tilde{\xi}^2$, $\beta > \tilde{\beta}$. Suppose, on the other hand that $\xi^2 > \tilde{\xi}^2$. Then $\kappa^2 = \tilde{\kappa}^2$ and $\partial L_Y / \partial \xi^2 = 0$. β must equal either β_ξ or β^ξ. Consider the former: here, $\partial L_Y / \partial \kappa^2 < 0$ and so an increase in κ^2 would reduce loss. This contradicts the fact that the parameters were chosen optimally. Hence $\beta = \beta^\xi$. It follows that $\beta > \bar{\beta}$. Since $\xi^2 > \tilde{\xi}^2$ and $\kappa^2 = \tilde{\kappa}^2$, $\beta < \tilde{\beta}$. Finally, suppose that the optimal parameter combination resulted in a β outside the range $(\beta_\kappa, \beta^\xi)$. To the right of this range, loss is decreasing in ξ^2 and to the left loss is decreasing in κ^2. Hence this could not have been optimal. Collecting these facts together proves statements (i), (ii), and (iii) in the proposition, as required. □

APPENDIX B. SUPPLEMENTARY MATERIAL

Potential and Welfare. For the beauty contest considered in the paper, the potential $\phi(p)$ is also a natural welfare measure. Note that a player exerts externalities only via the average action $\bar{p}$.[15] Locally raising $\bar{p}$ helps the "above average" players satisfying $p_l > \bar{p}$ while harming the "below average" players satisfying $p_l < \bar{p}$. However, since $\bar{p}$ is the average action, these spillovers cancel:

$$\frac{\partial}{\partial \bar{p}} \int_0^1 u_\ell \, d\ell = 2(1-\pi) \int_0^1 (p_\ell - \bar{p}) \, d\ell = 0. \tag{21}$$

This means that equilibrium play is efficient. Once all of the various information structures have been put into place, finding equilibria reduces to looking for efficient strategy profiles.

This is not true when there is a finite number of players. Nevertheless, finite-player contests are potential games, and equilibria can be found by seeking potential maximisers.

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[15]Given the continuum-of-players assumption, each individual player exerts a negligible effect on $\bar{p}$. Nevertheless, this effect is felt by a large mass of others, and so the spillover effects are indeed present. The important observation is that positive and negative spillovers cancel out, as confirmed formally by (21).

To see this more formally, consider an L-player beauty contest by retaining the specification (2) but with $\bar{p} \equiv \frac{1}{L} \sum_{l=1}^{L} p_l$. The optimal action of player ℓ differs from the continuum-of-players case since a change in p_ℓ exerts a non-negligible effect on the average action $\bar{p}$. Straightforward calculations confirm that the optimal action choice satisfies

$$p_\ell = \hat{\pi} \, \mathrm{E}_\ell[\theta] + (1 - \hat{\pi}) \, \mathrm{E}_\ell \left[\frac{\sum_{l \neq \ell} p_l}{L - 1} \right], \quad \text{where} \quad \hat{\pi} \equiv \frac{\pi L^2}{\pi L^2 + (1 - \pi)(L - 1)^2}.$$

Clearly, taking $L \to \infty$ leads back to $p_\ell = \pi \, \mathrm{E}_\ell[\theta] + (1 - \pi) \, \mathrm{E}_\ell[\bar{p}]$. Similarly, the aggregate externality exerted by player ℓ on all other players $l \neq \ell$ is readily calculated and satisfies

$$\frac{\partial \sum_{l \neq \ell} u_l}{\partial p_\ell} = \frac{2(1 - \pi)(\bar{p} - p_\ell)}{L}.$$

This aggregate externality is non-zero and actions are efficient only in the limit as $L \to \infty$. This reinforces the message that a maximise-welfare-to-find-equilibria technique only works in special cases. Nevertheless, the finite-player beauty contest is a potential game:

$$\phi(p) = \bar{u} - \frac{\tilde{\pi} \sum_l (p_l - \theta)^2}{L} - \frac{(1 - \tilde{\pi}) \sum_l (p_l - \bar{p})^2}{L}, \quad \text{where} \quad \tilde{\pi} = \frac{\pi L}{\pi L + (1 - \pi)(L - 1)} \tag{22}$$

is an exact potential function. It is straightforward to verify that this real-valued function captures the strategic incentives in the game, since $\partial \phi / \partial p_\ell = \partial u_\ell / \partial p_\ell$ for any player ℓ. This means that a maximise-potential-to-find-equilibria technique always works for general L. $\qquad \square$

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