Optimal Redistributive Taxation with both Extensive and Intensive Responses*

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Abstract

This paper characterizes the optimal income taxation when individuals respond along both the intensive and extensive margins. Individuals are heterogeneous in two dimensions: their skill and their disutility of participation. Preferences over consumption and work effort can differ with the skill level, only the Spence-Mirrlees condition being imposed. We derive an optimal tax formula thanks to a tax perturbation approach. This formula generalizes previous results by allowing for income effects and extensive margin responses. We provide a sufficient condition for optimal marginal tax rates to be non-negative everywhere. The relevance of this condition is discussed with analytical examples and numerical simulations on U.S. data.

JEL Classification: H21, H23.

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I Introduction

This paper provides an optimal nonlinear income tax formula that solves the redistribution problem when individuals respond along both the intensive (in-work effort) and extensive (participation) margins. For that purpose, we consider an economy where individuals are heterogeneously endowed with two unobserved characteristics: their skill level and their disutility of participation. Because of the first heterogeneity, employed workers typically choose different earnings levels. Because of the second heterogeneity, at any skill level, only some individuals choose to work. The government can only condition taxation on endogenous earnings and not on the exogenous characteristics whose heterogeneity in the population are at the origin of the redistribution problem.[1] Therefore, positive marginal tax rates are necessary to transfer income from rich to poor individuals, while inherently distorting intensive labor supply decisions. Moreover, when individuals of a given skill level experience a rise either in the tax level they paid when employed or in the benefit for the non-employed, some of them leave the labor force. Such a rise of the so-called *participation tax*[2] thereby generates distortions along the extensive margin of the labor supply.

Since Mirrlees (1971), the optimal tax problem is usually solved by searching for the best incentive-compatible allocation using optimal control. While this method has been proved successful, it lacks economic intuitions. We instead derive the optimal tax formula by measuring the effects of a change in marginal tax rates on a small interval of income levels.[3] This "tax perturbation approach" emphasizes the economic mechanisms at work but faces the following difficulty: because of the non-linearity of the tax schedule, when an individual responds to a tax perturbation by a change in her labor supply, the induced change in her gross income affects in turn her marginal tax rate, thereby inducing a further labor supply response. To take this "circular process" into account, we define behavioral elasticities along the optimal nonlinear tax schedule. Thanks to this redefinition, we can intuitively express optimal marginal rates as a function of the social welfare weights, the skill distribution and the behavioral elasticities. This formula generalizes previous results by allowing for income effects and extensive margin responses.

We also provide a sufficient condition under which optimal marginal tax rates are non-negative. Clarifying the restrictions that ensure this result is an issue in the optimal income tax

[1] Because the second heterogeneity matters only for the participation decisions, the government faces a multi-dimensional screening problem that is reduced to the "random participation" model introduced by Rochet and Stole (2002).

[2] Which equals the tax level plus the benefit for the non-employed, so that each additional worker increases the government's revenue by the level of the participation tax.

[3] We verify in Appendix B that the solution derived thanks to the tax perturbation approach is consistent with the Mirrleesian approach in terms of incentive-compatible allocations.

literature with only intensive responses.[4] Intuitively, the optimality of non-negative marginal tax rates holds whenever social welfare weights are decreasing along the skill distribution, so that the distortions induced by positive marginal tax rates are compensated by the equity gains of transferring income from high to low-skilled workers. Adding an extensive margin response, we find a condition on the ratio one minus the social welfare weights over the extensive behavioral response. Strikingly, the optimal participation tax equals this ratio when individuals respond only along the extensive margin. When both margins are included, we show that optimal marginal tax rates are non-negative whenever this ratio decreases along the skill distribution. While our sufficient condition is expressed in terms of endogenous variables, we discuss its relevance in practice and give examples of specifications on primitives where this condition holds. For instance, when the government has a Maximin objective, we argue that the additional restrictions are fairly weak.

Using U.S. data, we also calibrate the model to illustrate the quantitative implications of our optimal tax formula. These simulations suggest that a more responsive extensive margin reduces marginal tax rates by a significant amount without changing qualitatively its profile. In our sensitivity analysis, marginal tax rates are always positive. However, for the least skilled workers, participation taxes are typically negative under a Benthamite criterion, while they are always positive under Maximin. The literature on optimal taxation in the pure extensive model has typically found the latter results. The optimality of the negative participation tax at the bottom is interpreted as a case for an Earned Income Tax Credit (EITC) instead of a Negative Income Tax (NIT) (see Saez 2002). We provide numerical examples with a strictly positive lower bound for the earnings distribution,[5] a negative participation tax at this minimum (as for the EITC) and non-negative marginal tax rates above this minimum (as for the NIT).

Our paper contributes to the literature that aims at making the literature on optimal income taxation useful for applied thinking in public finance. For many years after the seminal paper of Mirrlees (1971), the numerous theoretical developments focused on useful technical refinements but provided little economic intuitions. A first important progress was made when, in the absence of income effects, Atkinson (1990), Piketty (1997) and Diamond (1998) re-expressed optimality conditions derived from the Mirrlees model in terms of behavioral elasticities. Saez (2001) made a second important step forward by deriving an optimal tax formula thanks to a tax perturbation approach.[6] He took into account the above-mentioned "circular process" by

[4] See e.g. Mirrlees (1971), Sadka (1976), Seade (1982), Werning (2000) or Hellwig (2007), or the counterexamples given by Choné and Laroque (2009b).

[5] We assume a strictly positive minimum for the skill distribution.

[6] Christiansen (1981) introduced the tax perturbation approach. However, he did not derive any implication for the optimal income tax, his focus being on the optimal provision of public goods and the structure of commodity taxation. Revecz (1989) proposed a method to derive an optimal income tax formula in terms of elasticities but did not consider the above-mentioned circular process. Hence, his solution was not consistent with the Mirrlees one (see Revecz 2003 and Saez 2003). Using a tax perturbation method, Piketty (1997) derived the optimal

expressing his optimal tax formula in terms of the unappealing notion of "virtual"[7] earnings distribution and verified the consistency of his solution to the Mirrlees one. He furthermore allowed for income effects. We avoid the use of virtual densities thanks to our redefinition of behavioral elasticities.

The aforementioned papers neglected labor supply responses along the extensive margin, while the empirical labor supply literature emphasizes that labor supply responses along the extensive margin are much more important (see e.g. Heckman 1993). Saez (2002) derived an optimal tax formula in an economy with both intensive and extensive margins. For that purpose, he developed a model where agents can choose among a finite set of occupations, each of them being associated to an exogenous level of earnings. However, he had no analytical result for the mixed case where both the extensive and intensive margins matter. Moreover, he focused essentially on the EITC/NIT debate about whether the working poor should receive more transfers than the non-employed individuals, while we discuss the conditions under which marginal tax rates should be non-negative. In addition, our formula allows for income effects.[8] Finally, our treatment of the intensive margin is more standard and it allows considering a continuous earnings distribution. This seems to us more appropriate for studying marginal tax rates than the discrete occupation setting of Saez (2002).[9]

The paper is organized as follows. The model is presented in Section II. Section III derives the optimal tax formula in terms of behavioral elasticities thanks to a tax perturbation method. This section also compares this tax formula to the literature. Section IV provides a condition sufficient to get optimal non-negative marginal tax rates and examples where this condition is satisfied. Section V presents simulations for the U.S. In the appendix, we develop the formal model. In particular, we solve it for the optimal allocations thanks to the usual optimal control approach. We verify that this solution is consistent with the one we derive in the main text.

nonlinear income tax schedule under Maximin. He too neglected to take into account the circular process but this has no consequence since he assumes away income effects. Roberts (2000) derived an optimal tax formula also under Benthamite preferences.

[7] Saez (2001, p.215) defines the virtual density at earnings level z as "the density of incomes that would take place at z if the tax schedule $T(.)$ were replaced by the linear tax schedule tangent to $T(.)$ at level z".

[8] The formal model in the Appendix of Saez (2002) allows for the possibility of income effects. Moreover, the appendix of the NBER version of Saez (2002) extends his optimal tax formula with both extensive and intensive responses to the case of a continuum of earnings but without income effects.

[9] Boone and Bovenberg (2004) introduce search decisions in the Mirrlees model. This additional margin has a similar flavor as a participation decision. However, their specification of the search technology implies that any individual with a skill level above (below) an endogenous threshold searches at the maximum intensity (does not search).

II The model

II.1 Individuals

Each individual derives utility from consumption C and disutility from labor supply or effort L. More effort implies higher earnings Y, the relationship between the two depending also on the individual's skill endowment w. The literature typically assumes that $Y = w \times L$. To avoid this unnecessary restriction on the technology, we express individuals' preferences in terms of the observables (C and Y) and the individuals' exogenous characteristics (in particular w). This in addition enables us to consider cases where the preferences over consumption C and effort L are skill-dependent. Skill endowments are exogenous, heterogeneous and unobserved by the government. Hence, consumption C is related to earnings Y through the tax function $C = Y - T(Y)$.

The empirical literature has emphasized that a significant part of labor supply responses to tax reforms are concentrated along the extensive margin. We integrate this feature by considering a specific disutility of participation which makes a difference in the level of utility only between *workers* (for whom $Y > 0$) and the *non-employed* (for whom $Y = 0$). This disutility may be due to commuting, job-search effort, or a reduced amount of time available for home production. However, for some people, employment has a value *per se*. Some people enjoy to work (see e.g. Polachek and Siebert (1993, Page 101)). Some would even feel stigmatized if they had no job. Let χ denote an individual's disutility of participation net of such an intrinsic value of a job. We assume that people are endowed with different positive or negative (net) disutility of participation χ. As for the skill endowment, χ is exogenous and the government cannot observe it. Because of this additional heterogeneity, individuals with the same skill level may take different participation decisions. This is consistent with the observation that in all OECD countries, skill-specific employment rates always lie inside $(0, 1)$.

For tractability, we need that labor supply decisions Y among employed individuals depend only on their skill and not on their net disutility of participation. To get this simplification, we need to impose some separability in individuals' preferences. We specify the utility function of an individual of type (w, χ) as:

$$\mathcal{U}(C, Y, w) - \mathbb{I}_{Y>0} \cdot \chi \tag{1}$$

where $\mathbb{I}_{Y>0}$ is an indicator variable equal to one if the individual works and zero otherwise. The *gross* utility function $\mathcal{U}(.,.,.)$ is twice-continuously differentiable and is concave with respect to (C, Y). Individuals derive utility from consumption C and disutility from labor supply, so $\mathcal{U}'_C > 0 > \mathcal{U}'_Y$. Last, we impose the strict-single crossing (Spence-Mirrlees) condition. We assume that, starting from any positive level of consumption and earnings, more skilled workers need to be compensated by a smaller increase in their consumption to accept a unit rise in their earnings.

This implies that the marginal rate of substitution $-\mathcal{U}'_Y(C,Y,w)/\mathcal{U}'_C(C,Y,w)$ decreases in the skill level. Hence we have:

$$\mathcal{U}''_{Yw}(C,Y,w)\cdot\mathcal{U}'_C(C,Y,w)-\mathcal{U}''_{Cw}(C,Y,w)\cdot\mathcal{U}'_Y(C,Y,w)>0 \tag{2}$$

The distribution of skills is described by the density $f(.)$, which is continuous and positive over the support $[w_0,w_1]$, with $0<w_0<w_1\leq+\infty$. It is worth noting that the lowest skill is positive. The size of the total population is normalized to 1 so $\int_{w_0}^{w_1}f(w)\,dw=1$. The distribution of χ conditional on the skill level w is described by the conditional density $k(.,w)$ and the cumulated density function $K(.,w)$, with $k(\chi,w)\stackrel{\text{def}}{\equiv}\partial K(\chi,w)/\partial\chi$. The density is continuously differentiable. It is worth noting that w and χ can be distributed independently or can be correlated. The support of the distribution is $(-\infty,\chi^{\max}]$, with $\chi^{\max}\leq+\infty$. The assumption about the lower bound is made for tractability since it ensures a positive mass of employed individuals at each skill level.

Each agent solves the following maximization problem

$$\max_Y \quad \mathcal{U}(Y-T(Y),Y,w)-\mathbb{I}_{Y>0}\cdot\chi$$

where the choice of Y can be decomposed into a participation decision (i.e. $Y=0$ or $Y>0$) and an intensive choice (i.e. the value of Y when $Y>0$). For a worker of type (w,χ), choosing a positive earnings level Y to maximize $\mathcal{U}(C,Y,w)$ subject to $C=Y-T(Y)$ amounts to solve

$$U_w\stackrel{\text{def}}{\equiv}\max_Y \quad \mathcal{U}(Y-T(Y),Y,w) \tag{3}$$

In particular, two workers with the same skill level but with a different disutility of participation χ face the same intensive choice program, thereby taking the same decisions along the intensive margin.[10] Let Y_w be the intensive choice of a worker of skill w and let C_w be the corresponding consumption level, so $C_w=Y_w-T(Y_w)$. The gross utility of workers of skill w therefore equals $U_w=\mathcal{U}(C_w,Y_w,w)$. We ignore the non-negativity constraint on Y_w when solving the intensive choice program. We verify in our simulations that the minimum of the

[10]The key assumption for this result is that preferences over consumption and earnings for employed agents vary only with skills and do not depend on the net disutility of participation χ. Such property is obtained under weakly separable preferences of the form

$$W(C,Y,w,\chi)=\begin{cases} V(\mathcal{U}(C,Y,w),w,\chi) & if \quad Y>0 \\ \mathcal{U}^0(C,w,\chi) & \quad\quad Y=0\end{cases}$$

where W is discontinuous at $Y=0$. $V(.,.,.)$ is an aggregator that is increasing in its first argument. Function $\mathcal{U}(.,.,.)$ verifies $\mathcal{U}'_C>0>\mathcal{U}'_Y$ and (2). $\mathcal{U}^0(.,.,.)$ describes the preference of the non-employed and increases in its first argument. Functions $\mathcal{U}(.,.,.)$, $\mathcal{U}^0(.,.,.)$ and $V(.,.,.)$ are twice-continuously differentiable over respectively $\mathbb{R}^+\times\mathbb{R}^+\times[w_0,w_1]$, $\mathbb{R}^+\times[w_0,w_1]\times\mathbb{R}^+$ and $\mathbb{R}\times[w_0,w_1]\times\mathbb{R}^+$. Finally, we assume that for given levels of C, Y, w and b, the function $\chi\mapsto V(\mathcal{U}(C,Y,w),w,\chi)-\mathcal{U}^0(b,w,\chi)$ is decreasing and tends to $+\infty$ whenever χ tends to the lowest bound of its support. All results derived in this paper can be obtained under this more general specification, the additional difficulty being only notational.

earnings distribution is always positive (since we assume $w_0 > 0$). So, we are right to neglect the possibility of bunching due to the non-negativity constraint.

We now turn to the participation decisions. Let $b = -T(0)$ denote the consumption level for individuals out of the labor force. We call b the welfare benefit. If an individual of type (w, χ) chooses to work, she gets utility $U_w - \chi$. If she chooses not to participate she obtains $\mathcal{U}(b, 0, w)$. An individual of type (w, χ) chooses to work if $U_w - \chi \geq \mathcal{U}(b, 0, w) \Leftrightarrow \chi \leq U_w - \mathcal{U}(b, 0, w)$. Therefore, the density of workers of skill w is given by $h(w)$ defined as:

$$h(w) \stackrel{\text{def}}{\equiv} K(U_w - \mathcal{U}(b, 0, w), w) \cdot f(w) \tag{4}$$

with some abuse of notation since $h(w)$ does not make explicit the dependence of $h(.)$ with respect to b and to U_w. The function $h(w)$ is twice-continuously differentiable, increasing in U_w and decreasing in b, with respective derivatives $h'_U(w)$ and $h'_b(w)$. The cumulative distribution is $H(w) = \int_{w_0}^{w} h(n) \cdot dn$. There are $H(w_1)$ employed individuals and $1 - H(w_1)$ non-employed.

II.2 Behavioral elasticities

We define the behavioral elasticities from the intensive choice program (3) and the extensive margin decision (4). When the tax function is differentiable, the first-order condition associated to the intensive choice (3) implies:

$$1 - T'(Y_w) = -\frac{\mathcal{U}'_Y}{\mathcal{U}'_C} \tag{5}$$

where the derivatives of $\mathcal{U}(.)$ are evaluated at (C_w, Y_w, w). When, in addition, the tax function is twice differentiable, the second-order condition writes:[11]

$$\mathcal{U}''_{YY} - 2\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_C}\right)\mathcal{U}''_{CY} + \left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_C}\right)^2 \mathcal{U}''_{CC} - T''(Y_w) \cdot \mathcal{U}'_C \leq 0 \tag{6}$$

Whenever the second-order condition (6) holds strictly, which we henceforth assume through the rest of this section, the first-order condition (5) defines implicitly[12] earnings Y_w as a function of the skill level and of the tax function. The elasticity α_w of earnings with respect to the skill level equals:[13]

$$\alpha_w \stackrel{\text{def}}{\equiv} \frac{w}{Y_w} \cdot \dot{Y}_w = -\frac{\frac{w}{Y_w} \cdot \left[\mathcal{U}''_{Yw} \cdot \mathcal{U}'_C - \mathcal{U}''_{Cw} \cdot \mathcal{U}'_Y\right]}{\left[\mathcal{U}''_{YY} - 2\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)\mathcal{U}''_{CY} + \left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)^2 \mathcal{U}''_{CC} - T''(Y_w) \cdot \mathcal{U}'_C\right] \cdot \mathcal{U}'_C} \tag{7}$$

[11] By the concavity of $U(.,.,.)$ on (C, Y), the second-order condition is satisfied if the tax schedule is locally linear or convex (so that $T''(.) \geq 0$), or is not "too concave".

[12] In addition, one has to assume that among the possible multiple local maxima of $Y \mapsto U(Y - T(Y), Y, w)$, a single one corresponds to the global maximum. If program $Y \mapsto U(Y - T(Y), Y, w^*)$ admits two global maxima for a skill level w^*, workers of a skill level w slightly above (below) w^* would strictly prefer the higher (lower) maximum due to the strict single-crossing condition (see Equation (2)). Hence, function $w \mapsto Y_w$ exhibits a discontinuity at skill w^*. Moreover, again by the strict single-crossing condition, function $w \mapsto Y_w$ is non-decreasing. So, it is discontinuous on a set of skill levels that is at worst countable (and at best empty). Because the skill distribution is assumed continuous without any mass point, the latter set is of zero measure.

[13] See Appendix A.

Let $\hat{h}(Y)$ and $\hat{H}(Y)$ denote respectively the density and cumulated density function of the earnings distribution among employed individuals, with $\partial\hat{H}(Y)/\partial Y = \hat{h}(Y)$. For all skill level, one has that $\hat{H}(Y_w) \equiv H(w)$. From Equation (7), $h(w)$ and $\hat{h}(Y_w)$ are related by:

$$\frac{Y_w}{w} \cdot \alpha_w \cdot \hat{h}(Y_w) \equiv h(w) \tag{8}$$

If the left-hand side of (6) is nil, then the function $Y \mapsto \mathcal{U}(Y - T(Y), Y, w)$ becomes typically constant around w. Therefore, individuals of type w are indifferent between a range of earnings levels, so the function $n \mapsto Y_n$ becomes discontinuous at skill $n = w$. The same phenomenon also occurs when the tax function is downward discontinuous at Y_w ($T''(Y)$ tends to minus infinity, so (6) is violated). Conversely, bunching of types occurs when $\alpha_w = 0$ (i.e. $T''(Y)$ tends to plus infinity). This corresponds to a kink of the tax function. From now on, we assume that $T(.)$ is differentiable. Hence, we rule out bunching. However, this assumption is relaxed in the appendix where we solve the model in terms of incentive-compatible allocations and study what happens when bunching occurs.

We now consider different elementary tax reforms and compute how they affect the intensive (3) and extensive (4) choices. The first elementary tax reform captures the *substitution* effect around the actual tax schedule. The marginal tax rate $T'(Y)$ is decreased by a small amount τ over the range of earnings $[Y_w - \delta, Y_w + \delta]$. So doing, the level of tax at earnings level Y_w is kept constant, and so is C_w. The reform is illustrated in the left panel of Figure 1.

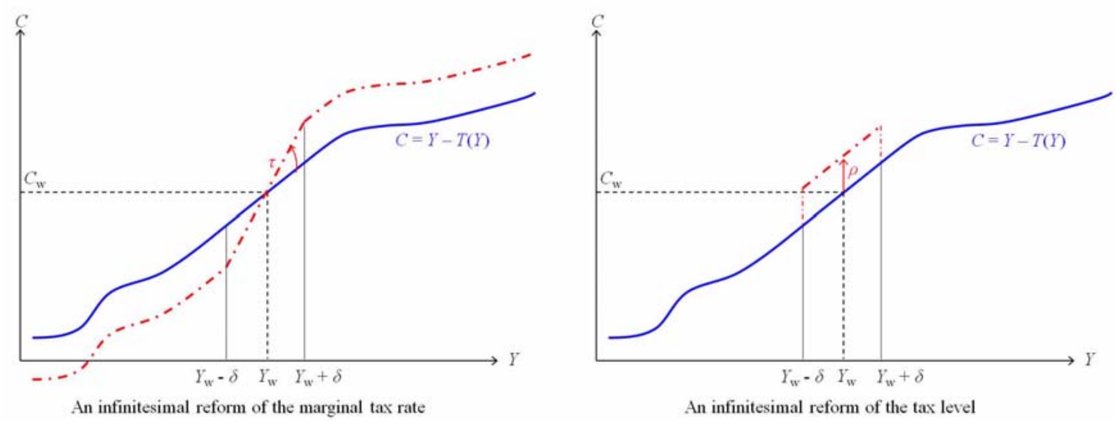

Figure 1: Tax reforms around Y_w defining behavioral responses ε_w and α_w

The behavioral response to such a reform for a worker of skill w is captured by the *compensated* elasticity of earnings with respect to $1 - T'(Y)$:[14]

$$\varepsilon_w \stackrel{\text{def}}{\equiv} \frac{1 - T'(Y_w)}{Y_w} \cdot \frac{\partial Y}{\partial \tau} = \frac{\mathcal{U}'_Y}{Y_w \cdot \left[\mathcal{U}''_{YY} - 2\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)\mathcal{U}''_{CY} + \left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)^2 \mathcal{U}''_{CC} - T''(Y_w)\cdot\mathcal{U}'_C\right]} > 0 \tag{9}$$

[14]The elasticity ε_w is called *compensated* since the tax level is kept unchanged at earnings level Y_w.

When the marginal tax rate is decreased by τ, a unit rise ΔY_w in earnings generates a higher gain $\Delta C_w = (1 - T'(Y_w) + \tau)\Delta Y_w$ of consumption. Therefore, the workers substitute earnings for leisure. Finally, this reform only has a second-order effect on U_w, thereby on the participation decisions.[15]

The next elementary tax reform captures the *income* effect around the actual tax schedule. The level of tax is decreased by a small lump-sum amount ρ over a range of earnings $[Y_w - \delta, Y_w + \delta]$. This reform is illustrated in the right panel of Figure 1. Along the intensive margin, the behavioral response for a worker of skill w to this reform is captured by the income effect:

$$\eta_w \stackrel{\text{def}}{\equiv} \frac{\partial Y}{\partial \rho} = \frac{\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_C}\right)\mathcal{U}''_{CC} - \mathcal{U}''_{CY}}{\mathcal{U}''_{YY} - 2\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)\mathcal{U}''_{CY} + \left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)^2 \mathcal{U}''_{CC} - T''(Y_w)\cdot\mathcal{U}'_C} \tag{10}$$

This term can be either positive or negative. However, when leisure is a normal good, the numerator is positive, hence the income effect (10) is negative.

The "ρ-reform" illustrated in the right panel of Figure 1 also induces some individuals of skill w to enter the labor market. We capture this extensive response for individuals of skill w by:

$$\kappa_w \stackrel{\text{def}}{\equiv} \frac{1}{h(w)}\cdot\frac{\partial h(w)}{\partial \rho} = \frac{h'_U(w)}{h(w)}\cdot\mathcal{U}'_C = \frac{k(U_w - \mathcal{U}(b,0,w))}{K(U_w - \mathcal{U}(b,0,w))}\cdot\mathcal{U}'_C > 0 \tag{11}$$

which stands for the percentage of variation in the number of workers with a skill level w. Finally, we measure the elasticity of participation when, together with a uniform decrease of the tax level by ρ, the welfare benefit b rises by ρ (i.e. when $T(Y) + b$ is kept constant). This reform captures income effects along the extensive margin. The (endogenous) semi-elasticity of the number of employed individuals of skill w with respect to such a reform equals:

$$\nu_w \stackrel{\text{def}}{\equiv} \kappa_w + \frac{h'_b(w)}{h(w)} = \frac{k(U_w - \mathcal{U}(b,0,w))}{K(U_w - \mathcal{U}(b,0,w))}\cdot\left[\mathcal{U}'_C(C_w, Y_w, w) - \mathcal{U}'_C(b,0,w)\right] \tag{12}$$

When the utility function $\mathcal{U}(.,.,.)$ is additively separable and concave in consumption and if $C_w > b$, $\mathcal{U}'_C(C_w, Y_w, w)$ is lower than $\mathcal{U}'_C(b,0,w)$. So, income effects along the extensive margin are negative, which corresponds to the "normal" case.

The behavioral responses given in (7), (9), (10), (11) and (12) are endogenous. They depend on the skill level w, the earnings level Y and the tax function $T(.)$. In particular, the various responses along the intensive margin given in (7), (9) and (10) are standard (see e.g. Saez (2001)), except for the presence of $T''(.)$ in their denominators. An exogenous increase in either w, τ, or ρ induces a direct change in earnings $\Delta_1 Y_w$. However, this change in turn modifies the marginal tax rate by $\Delta_1 T' = T''(Y_w)\times\Delta_1 Y_w$, inducing a second change in earnings $\Delta_2 Y_w$. Therefore,

[15] Decreasing $T'(.)$ by τ implies a rise ΔY_w of earnings, which itself increases C_w by $\Delta C_w = (1 - T'(Y_w) + \tau)\Delta Y_w$. Therefore the impact on U_w is given by $\Delta U_w = \Delta\mathcal{U}(C_w, Y_w, w) = [(1 - T'(Y_w) + \tau)\mathcal{U}'_C + \mathcal{U}'_Y]\Delta Y_w = \mathcal{U}'_C\cdot(\varepsilon_w Y_w/(1 - T'(Y_w)))\,\tau^2$ where the second equality follows (5) and (9) through $\Delta Y_w = (\varepsilon_w Y_w/(1 - T'(Y_w)))\,\tau$.

a "circular process" takes place: The earnings level determines the marginal tax rate through the tax function and the marginal tax rate affects the earnings level through the substitution effect. The term $T''(Y_w) \cdot \mathcal{U}'_C$ captures the indirect effects due to this circular process (in the words of Saez (2001), see also Saez (2003) p. 483 and Appendix A). Unlike Saez (2001), we do not define the behavioral responses along an hypothetical linear tax function, but along the actual (or later optimal) tax schedule, that we allow to be nonlinear. Therefore, our behavioral responses parameters (7), (9) and (10) take into account the circular process and exhibit a term $T''(.)$ in their denominator.[16]

II.3 The Government

The government's budget constraint takes the form

$$b = \int_{w_0}^{w_1} (T(Y_w) + b) \cdot h(w) \cdot dw - E \tag{13}$$

where E is an exogenous amount of public expenditures. For each additional worker of skill w, the government collects taxes $T(Y_w)$ and saves the welfare benefit b.

Turning now to the government's objective, we adopt a welfarist criterion that sums over all types of individuals a transformation $G(v, w, \chi)$ of individuals' utility v, with $G(., ., .)$ twice-continuously differentiable and $G'_v > 0$. Given the labor supply decisions, the government's objective is

$$\begin{aligned} \Omega \quad = \quad & \int_{w_0}^{w_1} \left\{ \int_{-\infty}^{U_w - \mathcal{U}(b,0,w)} G(U_w - \chi, w, \chi) \cdot k(\chi, w)\, d\chi \right. \\ & \left. + \int_{U_w - \mathcal{U}(b,0,w)}^{\chi^{\max}} G(\mathcal{U}(b, 0, \chi), w, \chi) \cdot k(\chi, w)\, d\chi \right\} f(w)\, dw \end{aligned} \tag{14}$$

The social transformation function $G(., ., .)$ depends not only on the utility levels v of individuals, but also on their exogenous type (w, χ). Our social welfare function generalizes the Bergson-Samuelson social objective which does not depend on the individuals' type. With the latter criterion, preferences for redistribution would be induced by the concavity of $G(.)$, that is by $G''_{vv} < 0$. Our specification also encompasses the case where function $G(., ., .)$ equals a type-specific exogenous weight times the individuals' level of utility. The government's desire to compensate for heterogeneous skill endowments would require $G''_{vw} < 0$.

Let λ denote the marginal social cost of the public funds E. For a given tax function $T(.)$,we denote g_w (respectively g_0) the (average and endogenous) marginal social weight associated to

[16] See also Blumquist and Simula (2010).

employed individuals of skill w (to the non-employed), expressed in terms of public funds by:

$$g_w \stackrel{\text{def}}{\equiv} \mathbb{E}_\chi \left[\frac{G'_v(U_w - \chi, w, \chi) \cdot \mathcal{U}'_C(C_w, Y_w, w)}{\lambda} | w, \chi \leq U_w - \mathcal{U}(b, 0, w) \right] \tag{15}$$

$$g_0 \stackrel{\text{def}}{\equiv} \mathbb{E}_{w,\chi} \left[\frac{G'_v(\mathcal{U}(b, 0, w), w, \chi) \cdot \mathcal{U}'_C(b, 0, w)}{\lambda} | \chi > U_w - \mathcal{U}(b, 0, w) \right] \tag{16}$$

The government values an additional dollar to the $h(w)$ employed individuals of skill w (to the $1 - H(w_1)$ non-employed) as g_w times $h(w)$ dollars (g_0 times $1 - H(w_1)$ dollars). The government wishes to transfer income from individuals whose social weight is below 1 to those for which the social weight is above 1. As will be clear below, g_0 and the shape of the marginal social weights $w \mapsto g_w$ entirely summarize how the government's preferences influence the optimal tax policy. The only properties we have is that g_0 and g_w are positive. In particular, the shape of $w \mapsto g_w$ can be non-monotonic, decreasing or increasing and we can have g_0 above or below g_{w_0}. However, a government that has a redistributive motive would typically adopt a decreasing shape $w \mapsto g_w$ of social welfare weights, as it will be discussed in Section IV.

III Optimal marginal tax rates

III.1 Derivation of the optimal marginal tax formula

The government's problem consists in finding a nonlinear income tax schedule $T(.)$ and welfare benefit b to maximize the social objective (14), subject to the budget constraint (13) and to the labor supply decisions along both margins. In this section we directly derive the optimal tax formula through a small perturbation of the optimal tax function. Following Mirrlees (1971), Appendix B solves the government's problem in terms of incentive-compatible allocations, using optimal control techniques and verifies that both methods lead to the same optimal tax formulae:

Proposition 1 *The optimal tax policy has to verify*

$$\frac{T'(Y_w)}{1 - T'(Y_w)} = \mathcal{A}(w) \cdot \mathcal{B}(w) \cdot \mathcal{C}(w) \tag{17}$$

$$0 = \mathcal{C}(w_0) \tag{18}$$

$$\begin{aligned} 1 - g_0 \left(1 - \int_{w_0}^{w_1} h(n) \cdot dn\right) - \int_{w_0}^{w_1} g_n \cdot h(n) \cdot dn = \\ \int_{w_0}^{w_1} \left\{\eta_n \cdot T'(Y_n) + \nu_n \cdot (T(Y_n) + b)\right\} \cdot h(n) \cdot dn \end{aligned} \tag{19}$$

where

$$\mathcal{A}(w) \stackrel{def}{\equiv} \frac{\alpha_w}{\varepsilon_w} \qquad \mathcal{B}(w) \stackrel{def}{\equiv} \frac{H(w_1) - H(w)}{w \cdot h(w)}$$

$$\mathcal{C}(w) \stackrel{def}{\equiv} \frac{\int_w^{w_1} \{1 - g_n - \eta_n \cdot T'(Y_n) - \kappa_n (T(Y_n) + b)\} \cdot h(n) \cdot dn}{H(w_1) - H(w)}$$

Equation (17) summarizes the trade-off behind the choice of the marginal tax rate at earnings level Y_w. We consider the effects of the infinitesimal perturbation of the tax function depicted in the left panel of Figure 2. Marginal tax rates are uniformly decreased by an amount τ over a range of earnings $[Y_w - \delta, Y_w]$. Therefore, the tax levels are uniformly decreased by an amount $\rho = \tau \times \delta$ for all skill levels n above w. This tax reform has four effects: a *substitution* effect for tax payers whose earnings before the reform are in $[Y_w - \delta, Y_w]$, and some *mechanical*, *income* and *participation* response effects for tax payers with skill n above w.

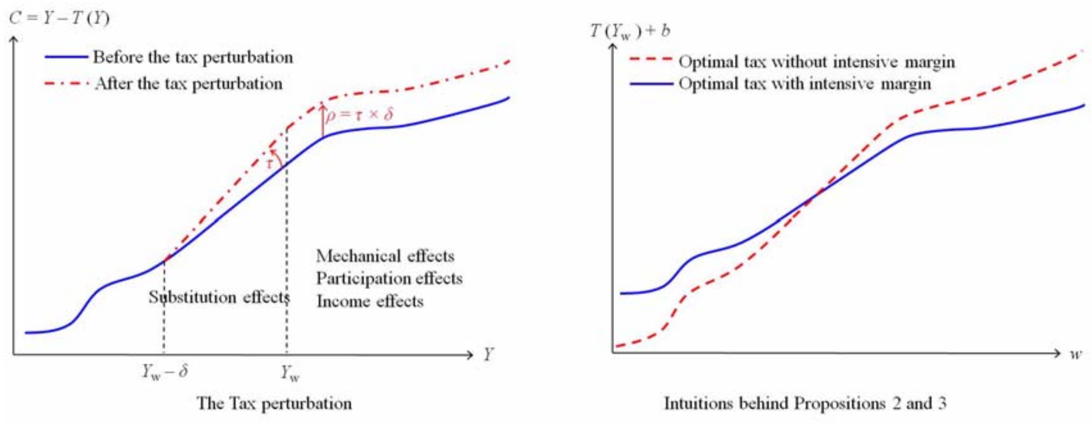

Figure 2: The optimal tax schedule

Substitution effect The substitution effect takes place on the range of gross earnings $[Y_w - \delta, Y_w]$. The mass of workers affected by the substitution effect is $\hat{h}(Y_w) \cdot \delta$. For these workers, according to Equation (9), the decrease by τ of the marginal tax rate induces a rise ΔY_w of their earnings, with

$$\Delta Y_w = \frac{\varepsilon_w \cdot Y_w}{1 - T'(Y_w)} \cdot \tau$$

The tax reform has only second-order effect on U_w, thereby on the participation decisions and on their contribution to the government objective. However, the rise in their earnings increases the government's tax receipt by $T'(Y_w) \cdot \Delta Y_w$. Hence, given that $\tau \times \delta = \rho$, the total substitution effect equals

$$\mathcal{S}_w = \frac{T'(Y_w)}{1 - T'(Y_w)} \cdot \varepsilon_w \cdot Y_w \cdot \hat{h}(Y_w) \cdot \rho \tag{20}$$

Workers of skill n above w face a reduction ρ in their tax level with no change in their marginal tax rate. This has three consequences.

Mechanical effects First, absent any behavioral response for these workers, the government gets ρ units of tax receipts less from each of the $h(n)$ workers of skill n. However, the tax reduction induces a higher consumption level C_n, which is valued g_n by the government. Hence the total mechanical effect at skill w is:

$$\mathcal{M}_w = -\int_w^{w_1} (1 - g_n) \cdot h(n) \cdot dn \cdot \rho \tag{21}$$

Income effects Second, the tax reduction induces each of the workers of skill n to change their intensive choice by $\Delta Y_n = \eta_n \cdot \rho$ (see Equation (10)). This income response has only a first-order effect on the government's budget: each of the $h(n)$ workers of skill n pays $T'(Y_n) \cdot \Delta Y_n$ additional tax. Hence, the total income effect at skill n equals:

$$\mathcal{I}_w = \int_w^{w_1} \eta_n \cdot T'(Y_n) \cdot h(n) \cdot dn \cdot \rho \tag{22}$$

Participation effects Finally, the reduction in tax levels induces $\kappa_n \cdot h(n) \cdot \rho$ individuals of skill n to enter employment (see Equation (11)). The change in participation decisions has only a first-order effect on the government's budget. Each additional worker of skill n pays $T(n)$ taxes and the government saves the welfare benefit b. Hence, the total participation effect at skill w equals:

$$\mathcal{P}_w = \int_w^{w_1} \kappa_n \cdot (T(Y_n) + b) \cdot h(n) \cdot dn \cdot \rho \tag{23}$$

The sum of $\mathcal{S}_w$, $\mathcal{M}_w$, $\mathcal{I}_w$ and $\mathcal{P}_w$ should be zero if the original tax function is optimal. Rearranging terms then gives

$$\frac{T'(Y_w)}{1 - T'(Y_w)} = \frac{1}{\varepsilon_w} \times \frac{\int_w^{w_1} \{1 - g_n - \eta_n \cdot T'(Y_n) - \kappa_n (T(Y_n) + b)\} \cdot h(n) \cdot dn}{Y_w \cdot \hat{h}(Y_w)} \tag{24}$$

which gives (17) thanks to (8).

Equation (18) describes the effects of giving a uniform transfer ρ to all employed individuals. This tax perturbation does not affect marginal tax rates, so it only induces mechanical, income and participation effects. The sum of (21), (22) and (23) evaluated for $w = w_0$ should be nil at the optimum, which leads to (18). Equations (17) and (18) imply that the optimal marginal tax rate is nil at the minimum earnings level.[17]

To grasp the intuition behind Equation (19), consider a unit increase in welfare benefit b and a unit lump-sum decrease in the tax function for all skill levels. This reform does neither change marginal nor participation tax rates. Hence, it has only mechanical and income effects along the intensive and extensive margins. This reform induces a (mechanical) loss of the tax revenues valued 1 by the government and a gain in the social objective. The latter amounts

[17] Intuitively, increasing the marginal tax rate at a skill level w' improves equity when the extra tax revenue can be redistributed towards a positive mass of people with skills equal or lower to w'. Since the mass of agents with a skill level lower or equal to w_0 is nil, a positive marginal tax rate at w_0 does not improve equity. It does however distort the labor supply. The optimal marginal tax rate at the lowest skill level then equals zero. This result does not longer hold if there is bunching at the bottom of skill distribution (Seade (1977)).

to $g_0 \cdot \left(1 - \int_{w_0}^{w_1} h(n) \cdot dn\right)$ for non-employed people and to $\int_{w_0}^{w_1} g_n \cdot h(n) \cdot dn$ for the employed individuals. Therefore, the mechanical effect corresponds to the left-hand side of (19). The right-hand side captures the income effects along both margins.[18] First, through the income response along the intensive margin, earnings change by $\Delta Y_n = \eta_n$. This affects tax revenues by the weighted integral of $\Delta Y_n \cdot T'(Y_n) = \eta_n \cdot T'(Y_n)$. Second, participation decisions change through the income effect by $\Delta h(n) = \nu_n \cdot h(n)$. Since for each additional worker of skill n, tax revenues increase by $T(Y_n) + b$, the total impact is the weighted integral of $\nu_n \cdot (T(Y_n) + b)$. In the normal case, $\eta_n < 0$ and $\nu_n < 0$. Therefore, since $T(Y_n) + b$ is typically positive for most workers, we expect that larger income effects along both margins increase the average of social welfare weights (g_0 and g_n's) above 1.

III.2 Comparison with the optimal tax literature

Equation (17) decomposes the determinants of the optimal marginal tax rates into three components. $\mathcal{A}(w)$ is the *efficiency term.* $\mathcal{B}(w)$ captures the role of the *skill distribution* among employed individuals. Finally, $\mathcal{C}(w)$ stands for the *social preferences for income redistribution*, taking into account the induced responses through income effects and along the participation margin.

There are two apparent differences between our formulation of the *efficiency term* $\mathcal{A}(w)$ and the ones in the literature. The first is the presence of $T''(Y_w)$ in the definitions (7) and (9) of α_w and ε_w. This is due to our definitions of behavioral responses along a potentially nonlinear income tax schedule and the induced endogeneity of marginal tax rates. However, in the ratio α_w/ε_w, these additional terms cancel out. So, the term $\mathcal{A}(w)$ is the same whether we define behavioral elasticities α_w and ε_w along the optimal tax schedule (as we do in the present paper) or along a "virtual" linear tax schedule (as usually done in the literature, see e.g. Piketty 1997, Diamond 1998 and Saez 2001). The second difference is induced by our assumption on preferences (1). The literature typically restricts to the case where preferences over consumption and in-work effort do not vary with skill levels, and are described by $\mathfrak{U}(C, Y/w)$. Then, it happens that the numerator of $\mathcal{A}(w)$ coincides with one plus the *uncompensated* elasticity of the labor supply. This is counter-intuitive, since it suggests that *ceteris paribus* marginal tax rates increase with the latter elasticity. Our more general assumption on preferences enables us to stress that what matters is the elasticity α_w of earnings with respect to skill levels. Marginal tax rates are then inversely related to the compensated elasticity in the vein of the"inverse elasticity"rule of Ramsey.

The term $\mathcal{B}(w)$ captures the role of the *skill distribution.* Consider an increase in the marginal

[18] Diamond (1975), Sandmo (1998) and Jacobs (2009) emphasize that the social value of public funds should only take into account behavioral responses due to income effects. Equation (19) shows that only income effects along the intensive η_w and extensive ν_w margins matter.

tax rate around the earnings level Y_w (the left part of Figure 2). The induced distortions along the intensive margin are larger, the higher is the skill w times the number of workers at that skill level, $w \cdot h(w)$ (Atkinson 1990). However, the gain in tax revenues is proportional to the number $H(w_1) - H(w)$ of employed individuals of skill n above w. Two differences with the literature are worth noting. First, because of the extensive margin responses, what matters is the distribution of skills among employed individuals, and not within the entire population. Since $h(w)/f(w)$ equals the employment rate of workers of skill w and $(H(w_1) - H(w))/(1 - F(w))$ equals the aggregate employment rate above skill w, one can further decompose $\mathcal{B}(w)$ into its exogenous and endogenous components through:

$$\mathcal{B}(w) = \frac{1 - F(w)}{w \cdot f(w)} \cdot \frac{\frac{H(w_1) - H(w)}{1 - F(w)}}{\frac{h(w)}{f(w)}}$$

The first term on the right-hand side equals the exogenous skill distribution term of Diamond (1998).[19] Second, the distribution term in (Saez 2001, Equation (19)) concerns the (virtual) distribution of *earnings* and not the *skill* distribution. This is how he gets rid of the counter-intuitive presence of the uncompensated labor supply elasticity in the numerator of his efficiency term. Using (7), one then gets that $\alpha_w \mathcal{B}(w) = \left(\hat{H}(Y_{w_1}) - \hat{H}(Y_w)\right) / \left(Y_w \cdot \hat{h}(Y_w)\right)$, so our optimal tax formula can also be expressed in terms of the earnings distribution, as in (24). Both formulations have their advantage. The earnings distribution has the advantage to be directly observable. However, earnings are endogenous, and hence, the observed and optimal earnings distributions might be different. To compute optimal tax rates, one has then to specify the utility function. Once this is done, the individuals' first-order condition (5) enables to recover the individual's skill level w from her observed earnings Y (and from the knowledge of the tax function). So, the advantage of the formulation in term of the earnings distribution vanishes. We present both formulations and let the reader choose which of the two she/he prefers.

The term $\mathcal{C}(w)$ captures the influence of *social preferences for income redistribution*, taking into account the induced responses through income effects and along the participation margin. It equals the average of mechanical, income and participation effects for all workers of skill n above w. Diamond (1998) considers the case where participation is exogenous and there is no income effect.[20] Introducing income effects or participation responses in the analysis amounts to modifying the social weight to

$$\breve{g}_n \stackrel{\text{def}}{\equiv} g_n + \kappa_n \cdot (T(Y_n) + b) + \eta_n \cdot T'(Y_n)$$

[19] Diamond (1998)'s $\mathcal{C}(w)$ corresponds to our $\mathcal{B}(w)$ and vice-versa.

[20] Under redistributive preferences, marginal social weights g_w are decreasing in skill levels w. Then, $\mathcal{C}(w)$ is increasing, but remains below 1. When in addition preferences are Maximin (see Atkinson 1975, Piketty 1997, Salanié 2005, Boadway and Jacquet 2008 among others), then the marginal social weights for workers g_w are nil, so $\mathcal{C}(w)$ is constant and equals 1.

Saez (2002, p. 1055) has explained why the government is more willing to transfer income to groups of employed individuals for which the participation response κ_n or the participation tax $T(Y_n)+b$ is larger. The behavioral parameter κ_n is positive, so a decrease in the level of tax paid by workers of skill n induces more of them to work. Whenever the participation tax $T(Y_n)+b$ is positive, tax revenues increase, which is beneficial. We argue that a similar interpretation can be made for the income effect. Typically, leisure is a normal good (hence $\eta_n < 0$). Then, a decrease in the level of tax paid by workers of skill n induces them to work less through the income effect. Whenever they face a positive marginal tax rate, this response decreases the tax they pay, which is detrimental to the government. Therefore, the government is more willing to transfer income to groups of employed individuals for which income effects are lower (i.e. higher η_n) and marginal tax rates are lower (Saez 2001).

IV Properties of the second-best optimum

IV.1 Sufficient condition for non-negative marginal tax rates

We first consider the special case where labor supply decisions take place only along the extensive margin, as assumed in Diamond (1980) and Choné and Laroque (2005, 2009a), so $\varepsilon_w = \eta_w = 0$. The optimal tax formula then verifies:[21]

$$T(Y_w) = \frac{1-g_w}{\kappa_w} - b \tag{25}$$

The optimal level of tax then trades off the mechanical effect (captured by the social weight g_w) and the participation response effect (captured by the participation response κ_w) of a rise in the level of tax. Marginal tax rates are then everywhere non-negative if along the optimal allocation, the function $Y \mapsto (1-g_w)/\kappa_w$ is increasing. The following Proposition shows that this result remains valid in the presence of responses along the intensive margin.

Proposition 2 *If along the optimal allocation, $w \mapsto \frac{1-g_w}{\kappa_w}$ is increasing, marginal tax rates are always non-negative. Furthermore, they are almost everywhere positive, except at the two extremities Y_{w_0} and Y_{w_1}.*

This Proposition is proved in Appendix C. The intuition is illustrated in the right panel of Figure 2. This figure depicts the level of tax $T(Y_w)$ paid by a worker of skill w, as a function of her skill level. When labor supply responses are only along the extensive margin, the optimal tax schedule is represented by the dashed curve. It corresponds to the optimal trade-off between mechanical and participation effects. If $w \mapsto (1-g_w)/\kappa_w$ is increasing in w, this function

[21] In the absence of response along the intensive margin, substitution effects $\mathcal{S}_w$ in (20) and income effects $\mathcal{I}_w$ in (22) are nil at each skill level. Therefore, the sum of mechanical $\mathcal{M}_w$ and participation $\mathcal{P}_w$ effects have to be nil at each skill level, which gives (25).

is increasing in the skill level. However, when workers can also decide along their intensive margin, such an increasing tax function and its positive marginal tax rates induce distortions of the intensive choices. Hence, the optimal tax function, which is depicted by the solid curve, is flatter than the optimal curve without intensive margin to limit the distortions along the intensive margin. It also has to be as close as possible to the optimal curve without intensive margin to limit departures from the optimal trade-off between participation and mechanical effects.

Proposition 3 *If along the optimal allocation,* $w \mapsto \frac{1-g_w}{\kappa_w}$ *is increasing in* w *and if* $g_w \leq 1$ *for all skill levels, then in work benefits (if any) are smaller than the welfare benefit* b*.*

This Proposition is proved in Appendix D. The assumption that $g_w \leq 1$ for all skills is restrictive. It implies that in the case without intensive responses the optimal tax is characterized by leaving to the least skilled workers lower benefit than to the non-employed (hence a Negative Income Tax is optimal). This result remains valid in the presence of intensive responses since the optimal tax function under unobserved skills is flatter than the one under observed skills. Proposition 3 emphasizes this result.

In the absence of behavioral responses along the intensive margin, in-work benefits for the working poor (of skill w_0) are larger than welfare benefits if and only if $g_{w_0} > 1$. By continuity, as long as the compensated elasticity (along the intensive margin) ε_{w_0} is small enough, in-work benefits should remain higher than welfare benefits hence an EITC is optimal. This has already been emphasized by Saez (2002).

IV.2 Examples

The sufficient condition in Propositions 2 and 3 depends on the patterns of social weights g_w and extensive behavioral responses κ_w which are endogenous. This subsection provides examples where the primitives of the model guarantee the sufficient conditions in Propositions 2 and 3.

Our first example specifies the primitives of the model in such a way that g_w and κ_w become exogenous. For this purpose, individuals' preferences are quasilinear: $\mathcal{U}(C, Y, w) = C - \mathcal{V}(Y, w)$ with $\mathcal{V}'_Y, \mathcal{V}''_{YY} > 0 > \mathcal{V}''_{Yw}$. The marginal utility of consumption $\mathcal{U}'_C(C, Y, w)$ is then always equal to one. Moreover, we specify the distribution of the disutility of participation χ conditional on any skill level w as $K(\chi, w) = \exp(a_w + \kappa \cdot \chi)$, where a_w is a skill-specific parameter adjusted to keep some individuals out-of-the labor force at the optimum. Then, according to Equation (11), κ_w is always equal to parameter κ and is thereby constant along the skill distribution. Finally, the social objective is linear in utility levels with skill-specific weights γ_w. Since the specification of individuals' utility rules out income effects, we have that $g_w = \gamma_w / \int_{w_0}^{w_1} \gamma_w dw$ (see (15), (16) and (19)). Therefore, under redistributive social preferences, $w \mapsto \gamma_w$ is decreasing, so $(1 - g_w)/\kappa_w$

is decreasing. Marginal Tax rates are then non-negative according to Proposition 2. Note that in this example g_{w_0} is necessarily strictly higher than one. So, the optimal participation tax might be negative at the bottom. A negative participation tax at the bottom is nevertheless consistent with non-negative marginal tax rates over the whole income distribution since we assume a positive lower bound for the skill distribution. Hence, the lowest earnings level is positive and the tax function can jump between $Y = 0$ and Y_{w_0}.

This first example is very specific. In general, we think it is very plausible that $w \mapsto 1 - g_w$ is non-increasing and $w \mapsto \kappa_w$ is strictly decreasing. First, a redistributive government typically puts a higher social welfare weight on the consumption of the least-skilled workers. Second, there is some empirical evidence that the elasticity of participation, which equals $(Y_w - T(Y_w) - b)\,\kappa_w$ is typically a non-increasing function (see e.g. Juhn *et alii* 1991, Immervoll *et alii* 2007 or Meghir and Phillips 2008). Since consumption $Y_w - T(Y_w)$ is an increasing function, one can expect κ_w to decrease along the skill distribution.

We now provide more general specifications on primitives where these two properties hold. Assume that the utility function is additively-separable, i.e.

$$\mathcal{U}(C, Y, w) = u(C) - \mathcal{V}(Y, w) \tag{26}$$

with $u'_C, \mathcal{V}'_Y, \mathcal{V}''_{YY} > 0 > u''_{CC}, \mathcal{V}''_{Yw}$. The additive separability restriction is only made for technical convenience. However, showing within the pure intensive model that marginal tax rates are positive without imposing the additive separability assumption (26) was a real issue (see e.g. Sadka 1976, Seade 1982, Werning 2000). We add another restriction on preferences. For an employed individual, a given earnings level is obtained thanks to lower effort, the more skilled the worker is. However, for a non-employed, no effort is supplied. Hence, a larger skill does not improve utility. We therefore assume:

$$\mathcal{V}'_w(Y, w) \underset{=}{<} 0 \quad \text{if} \quad Y \underset{=}{>} 0 \tag{27}$$

So, the skill-specific threshold $U_w - \mathcal{U}(b, 0, w)$ of χ is constrained to be an increasing function of the skill level. The following properties are shown in Appendix E.

Property 1 *If $K(\chi, w)$ is strictly log-concave with respect to χ, $w \mapsto k(\chi, w)/K(\chi, w)$ is non-increasing in w and (26)-(27) hold, the function $w \mapsto \kappa_w$ is strictly decreasing.*

The log-concavity of $K(., w)$ is a property verified by most distributions commonly used. It is equivalent to assuming that $k(\chi, w)/K(\chi, w)$ is decreasing in χ. That $k(\chi, w)/K(\chi, w)$ is non-increasing in w encompasses the specific case where w and χ are independently distributed.

Property 2 *Under either Maximin or Benthamite social preferences and (26)-(27), the function $w \mapsto g_w$ is non-increasing*

Maximin (i.e. maximizing $u(b)$) and Benthamite (i.e. $G(U_w - \chi, w, \chi) = U_w - \chi$) social preferences are polar specifications. Combining Properties 1 and 2, the relation $w \mapsto (1 - g_w)/\kappa_w$ is increasing provided that g_w remains below 1. Therefore, Propositions 2 and 3 hold under the Maximin, utility functions verifying (26) and (27), $K(\chi, w)$ strictly log-concave with respect to χ and $k(\chi, w)/K(\chi, w)$ non-increasing in w. Moreover, if the government is instead Benthamite and if $g_{w_0} \leq 1$, then Propositions 2 and 3 are again ensured.

V Numerical simulations for the U.S.

This section implements our optimal tax formula with U.S data to analyze if and to what extent optimal schedules resemble real-world schedules and if not, how to reform them. This exercise also allows checking whether our sufficient condition for non-negative marginal tax rates is empirically reasonable.

V.1 Calibration

To calibrate the model we need to specify social and individual preferences and the distribution of characteristics (w, χ). We consider Benthamite and Maximin social preferences. We choose a specification of individual preferences that enables to control behavioral responses along the intensive margin. Following Diamond (1998), we assume away income effects along the intensive margin (hence $\eta_w \equiv 0$) and assume the compensated elasticities to be constantly equal to ε along a linear tax schedule. Moreover, individuals' preferences are concave so that a Benthamite government has a preference to transfer income from high to low income earners. Hence, we specify

$$\mathcal{U}(C, Y, w) = \frac{\left(C - \left(\frac{Y}{w}\right)^{1+\frac{1}{\varepsilon}} + 1\right)^{1-\sigma}}{1-\sigma}$$

The parameter ε corresponds to the compensated elasticity along a linear tax schedule (see Equation (9)) while parameter σ drives the redistributive preferences of a Benthamite government. Saez *et al.* (2009) survey the recent literature that estimates the elasticity of earnings to marginal tax rates. They conclude that "The most reliable longer-run estimates range from 0.12 to 0.4" in the U.S. We take a central value of $\varepsilon = 0.25$ for our benchmark. For the concavity of preferences, we take $\sigma = 0.8$ in the benchmark case. We conduct a sensitivity analysis with respect to these two parameters.

To calibrate the skill distribution, we take the earnings distribution from the Current Population Survey for May 2007. We use the first-order condition (5) of the intensive program to infer the skill level from each observation of earnings. We consider only single individuals to avoid the complexity of interrelated labor supply decisions within families. Using OECD tax database, the real tax schedule of singles without dependent children is well approximated by a linear tax function at rate 27.9% and an intercept at $-\$4,024.9$ on an annual basis.[22] We use a quadratic kernel with a bandwidth of $\$3,822$ to smooth $h(w)$. High-income earners are underrepresented in the CPS. Diamond (1998) and Saez (2001) argue that the skill distribution actually exhibits a fat upper-tail in the US, which has dramatic consequence for the shape of optimal marginal tax rates. We therefore expand (in a continuously differentiable way) our kernel estimation by taking a Pareto distribution, with an index[23] $a = 2$ for skill levels between $w = \$20,374$ and $w_1 = \$40,748$. This represents only the top 3.1% of our approximation of the skill distribution. The lower bound of the skill distribution is $w_0 = \$202$.

One finally needs to calibrate the conditional distribution of χ. For numerical convenience, we choose a logistic and skill-specific specification of the form

$$K(\chi, w) = \frac{\exp(-a_w + \beta_w \chi)}{1 + \exp(-a_w + \beta_w \chi)}$$

Parameters a_w and β_w are calibrated to obtain empirically plausible skill-specific employment rates, denoted by L_w, and elasticities of employment rates with respect to the difference in disposable incomes $C_w - b$, denoted π_w.[24] We take

$$L_w = 0.7 + 0.1 \left(\frac{w - w_0}{w_1 - w_0}\right)^{1/3} \qquad \pi_w = \pi_0 - \pi_1 \left(\frac{w - w_0}{w_1 - w_0}\right) \text{ with } \pi_0 = 0.5 \text{ and } \pi_1 = 0.1$$

These specifications are consistent with the empirical fact that the employment rate L_w is larger for the high-skilled than for the low-skilled. The average employment rate in the current economy equals 75.3%. The elasticity π_w is equal to 0.45 on average. Unreported simulations point out that the properties of the optimal tax schedule are robust to changes in the parameters of the above $w \to L_w$ relationship. A sensitivity analysis will illustrate how the calibration of π_w modifies the optimal tax profile.

We take $b = \$2,381$ since the net replacement ratio for a long term unemployed worker whose previous earnings equals 67% of average wage equals 9% in 2007 according to the OECD. Given this calibration of the current economy, we find that the budget constraint (13) is verified only when we set the exogenous revenue requirement to $E = \$6,110$ per capita.

V.2 Benchmark simulations

Figure 3 plots the optimal marginal tax rates (Panel (a)) and participation tax levels (Panel (b)) as functions of earnings, under the Benthamite (solid line) and Maximin (dotted line) criteria. We focus on annual earnings below $\$100,000$.[25] Consistent with Proposition 2, marginal

[22] We multiply by 52 the weakly earnings given by the CPS survey.

[23] An (untruncated) Pareto distribution with Pareto index $a > 1$ is such that $\Pr(w > \widehat{w}) = C/\widehat{w}^a$ with $a, C \in \mathbb{R}_0^+$.

[24] with $\pi_w = \kappa_w \ (Y_w - T(Y_w) - b)$ in the current economy.

[25] Income earners above $\$100,000$ correspond respectively to 4.65%, 3.73% and 5.66% of the population in the current economy, at the Benthamite optimum and at the Maximin optimum,.

tax rates are always positive, under both criteria. Moreover, there is no distortion at the lower end of the earnings distribution whose value is $Y_{w_0} = \$508$. Under the Maximin, the latter result contrasts with optimal positive marginal tax rate in a model with intensive margin only (Boadway and Jacquet 2008). In this case, the social objectives values only the utility of employed individuals at $Y = Y_{w_0}$. When both extensive and intensive margins are modeled, the Maximin objective values only the utility of the non-employed. Panel (a) illustrates that the absence of distortion at the bottom is a very local property: When $Y = \$2,150$, the marginal tax rate climbs to 60.5% (58.8%) under Benthamite (Maximin) preferences. Beyond, marginal tax rates follow the usual U-shaped profile (Salanié 2003), under both objective functions. Under the Maximin, marginal tax rates are higher than under the Benthamite criterion, except at the bottom end (for Y lower than $Y = \$5,900$). Remarkably, optimal marginal tax rates are significantly higher than the current 27.9%, except for the very low end of the earnings distribution. This is valid under both objectives. However, our optimal marginal tax rates are much lower than those found by Saez (2001).

Figure 3(b) illustrates that participation tax levels at the bottom of the earnings distribution are typically negative under a Benthamite criterion. The optimality of a negative participation tax on the poorest workers is usually interpreted as a case for an Earned Income Tax Credit (EITC) (Saez 2002). We find $b = \$2,665$ and $-T(Y_{w_0}) = \$9,345$. Contrastingly, Figure 3(b) also emphasizes that participation tax levels at the bottom of the earnings distribution are positive, under Maximin. A Negative Income Tax (NIT) then prevails. This is a standard result of the pure extensive margin model (Choné Laroque 2005) which is still valid here when considering both extensive and intensive margins together.[26] Intuitively, it is hardly desirable to transfer income to the least skilled workers, since their well-being does not matter under Maximin. At the Maximin optimum, we find $b = \$4,190$ and $-T(Y_{w_0}) = \$3,860$.

Figure 4(a) describes how the negative participation tax on least skilled workers enables to boost employment rates well above their values in the current economy. Moreover, Panel (b) illustrates how these negative participation tax rates (in the Bentham economy) increase the *gross* utility levels U_w of low-skilled workers significantly beyond their values in the current economy.

V.3 Sensitivity analysis

All our various sensitivity analyses point out that the U-shape profile is valid and none of them displays negative marginal tax rates. The only configuration where our sufficient condition for non-negative marginal tax rate is violated requires an extremely low σ. And, even then, the marginal tax rates are still positive. This section therefore focuses on the quantitative

[26] Saez (2002) suggests this result in his mixed model.

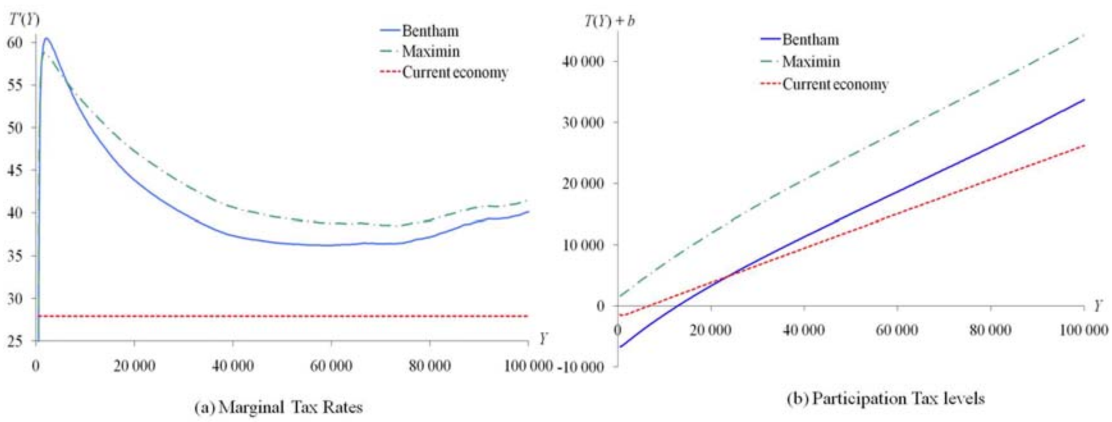

Figure 3: The simulation under the benchmark calibration

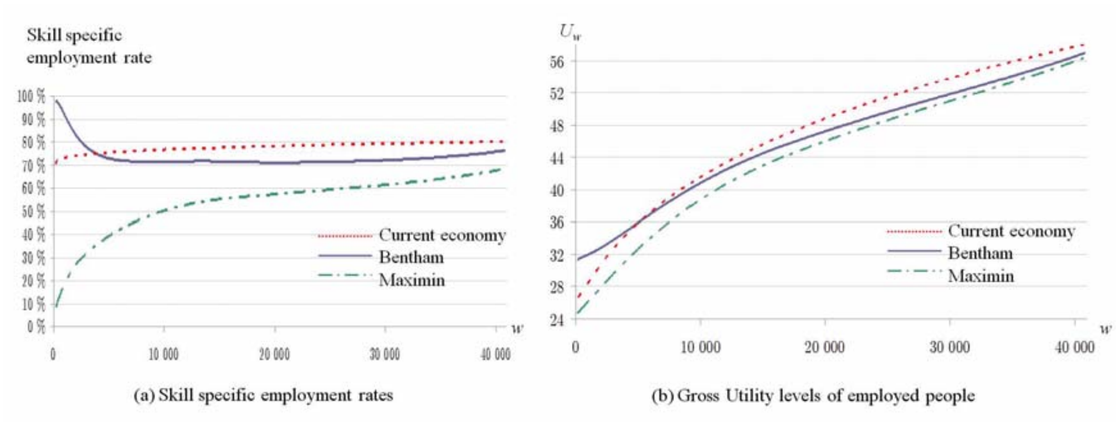

Figure 4: Optimal allocations

implications of parameters on the optimal tax rates.

As illustrated in Figure 5(a), the levels of marginal tax rates are quite sensitive to the parameter σ of the individual preferences. Any rise in σ increases the marginal tax profile by a substantial amount since the planner becomes more averse to inequality. The participation tax levels increase (decrease) with σ below (above) Y around \$20,000. Higher redistributive tastes increase the transfers towards the low-paid workers and the other workers pay more taxes (see Panel (b)).

Figure 6(a) illustrates that marginal tax rates decrease with the elasticity of earnings ε, as theoretically expected from the implied decrease of $\mathcal{A}(w)$ in Equation (17). Figure 6(a) illustrates this result with ε equal to 0.25 and 0.5, under Maximin and Benthamite preferences.[27] Figure 6(b) emphasizes that participation taxes decrease (increase) with ε for earnings above (below) roughly around \$30,000, under both criteria.

[27] Under Maximin, the marginal tax rates decrease with ε except for earnings below \$5,249.

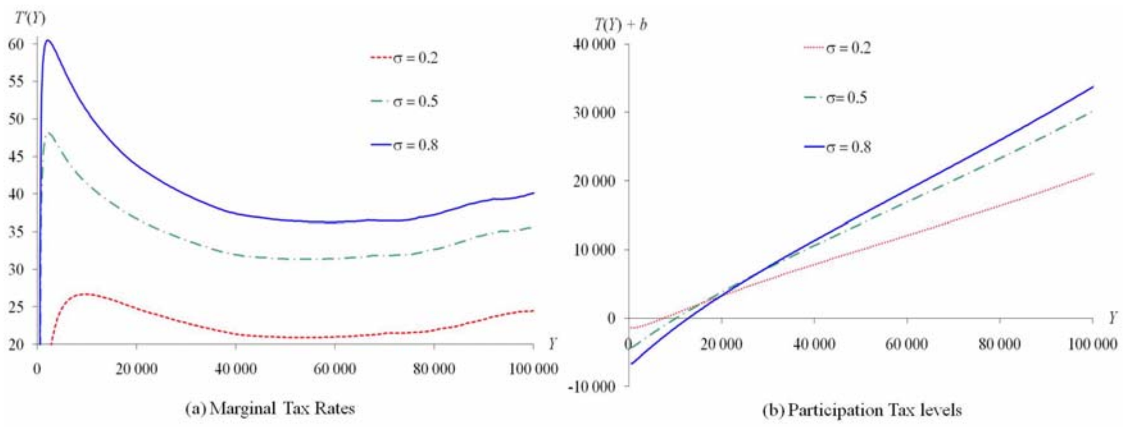

Figure 5: Sensitivity analysis with respect to σ for the Benthamite optimum

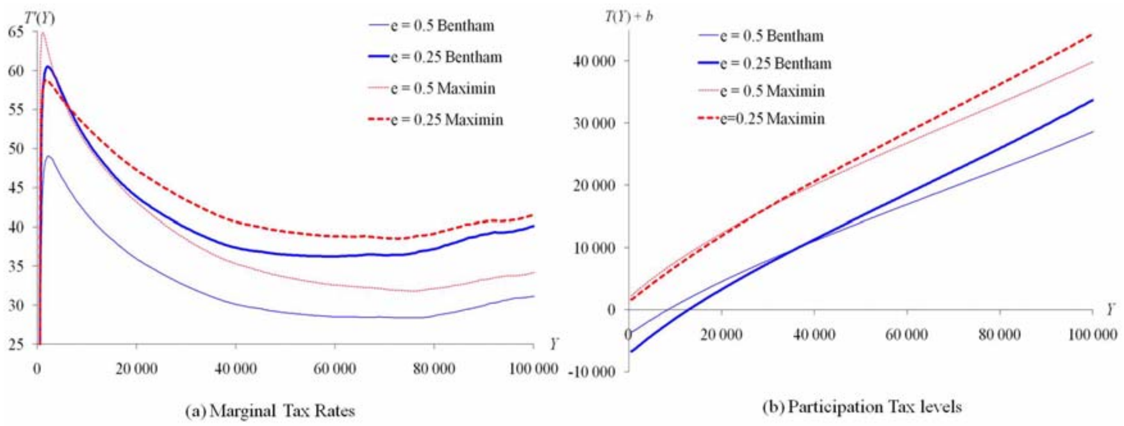

Figure 6: Sensitivity analysis with respect to ε

The next exercise studies the impact of reducing the participation response κ_w. Figure 7 plots the tax schedule when the parameter π_0 shrinks from 0.5 to 0.15. This reduction of the elasticities of employment rates $w \mapsto \pi_w$ (hence the reduction of κ_w) significantly increases the marginal tax rates (see Panel (a)), as expected from the implied decrease of $\mathcal{C}(w)$ in Equation (17). Moreover, as also expected from theory, the participation tax levels increase (Panel (b)). This exercise highlights the quantitative implications of introducing the extensive margin.

Another sensitivity analysis considers a more decreasing $w \mapsto \pi_w$ in the current economy hence a more decreasing $w \mapsto \kappa_w$. Figure 8 plots the tax rates when $(\pi_0, \pi_1) \equiv (0.75, 0.6)$ (solid curves) instead of $(\pi_0, \pi_1) \equiv (0.5, 0.1)$ (dashed curves). As expected from the $\mathcal{C}(w)$ term in Equation (17), the marginal tax rates then increase. Also, the participation tax curves become more increasing, under both criteria (Panel (b)), as expected from theory.

Our calibration abstracts from income effects. For consistency with the theoretical framework, we also focus on single households and so abstract from the interactions between labor

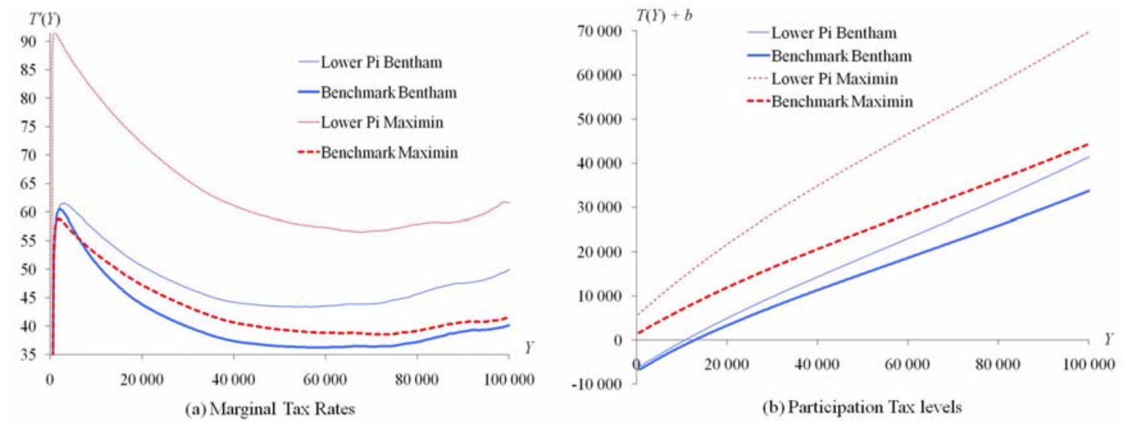

Figure 7: A lower $w \mapsto \pi_w$ in the calibration of the current economy

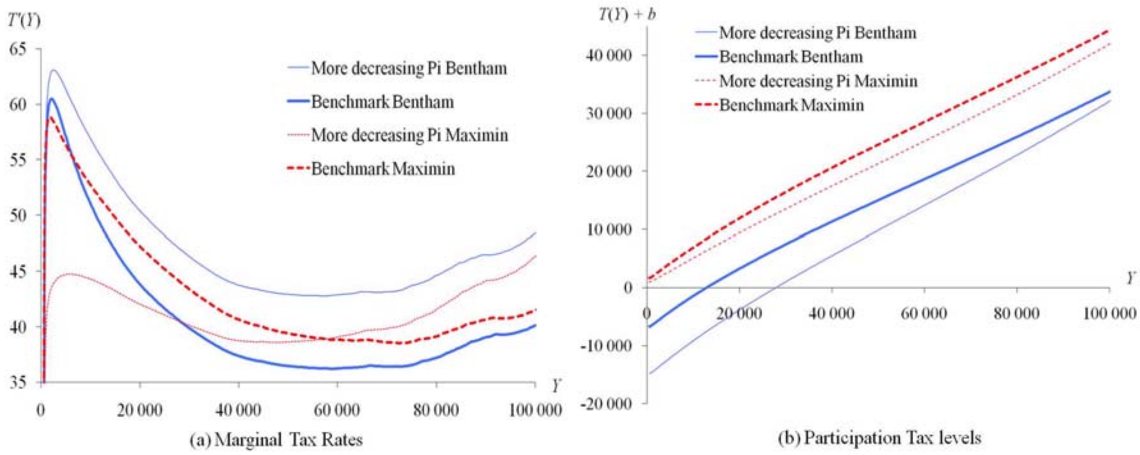

Figure 8: A more decreasing $w \mapsto \pi_w$ in the current economy

supply decisions within couples (see Kleven *et alii* 2009 for a theory of the optimal taxation of couples). However, those dimensions are not necessary to show how crucial it is to consider both labor supply margins to give tax policy recommendations.

VI Conclusion

This paper explored the optimal income tax schedule when labor supply responds simultaneously along both the extensive and the intensive margins. Individuals are heterogeneous in two dimensions: their skills and their disutility of participation. We derived a fairly mild sufficient condition for non-negative marginal tax rates over the entire skill distribution. This condition is derived thanks to a new method to sign distortions (along the intensive margin) in screening models with random participation. Our exercise illustrated that at the optimum, negative participation tax rates can coexist with positive marginal tax rates everywhere.

Using U.S. data, we implemented our optimal tax formula. This exercise emphasized that the U-shaped optimal tax schedule found in the literature with intensive margin only is still valid when both labor supply margins are considered. But introducing the extensive margin quite substantially reduces the marginal tax rates. Interestingly, the marginal tax rates are always positive in our simulations.

This paper also points to extensions. The method to sign distortion along the intensive margin can be applied to other contexts of nonlinear pricing theory where agents are characterized by a multi-dimensional parameter that is unobserved by the principal. It would also be interesting to extend the numerical simulations to other countries. Finally, we ignored the interactions of labor supply decisions within couples (Kreiner *et alii* (2009))

Appendices

A Behavioral Elasticities

We define

$$\begin{aligned}&\mathcal{Y}(Y,w,\tau,\rho) \stackrel{\text{def}}{\equiv} \left(1-T'(Y)+\tau\right)\cdot \mathcal{U}'_C\left(Y-T(Y)+\tau(Y-Y_w)+\rho,Y,w\right)\\&+\mathcal{U}'_Y\left(Y-T(Y)+\tau(Y-Y_w)+\rho,Y,w\right)\end{aligned}$$

The first-order condition (5) is equivalent to $\mathcal{Y}(Y_w,w,0,0)=0$. When $T(.)$ is twice-differentiable, one has (using (5)):

$$\mathcal{Y}'_Y(Y_w,w,0,0) = \mathcal{U}''_{YY}-2\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)\mathcal{U}''_{CY}+\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)^2\mathcal{U}''_{CC}-T''(Y_w)\cdot\mathcal{U}'_C \tag{28a}$$

$$\mathcal{Y}'_w(Y_w,w,0,0) = \left(1-T'\right)\cdot\mathcal{U}''_{Cw}+\mathcal{U}''_{Yw}=\frac{\mathcal{U}''_{Yw}\cdot\mathcal{U}'_C-\mathcal{U}''_{Cw}\cdot\mathcal{U}'_Y}{\mathcal{U}'_C} \tag{28b}$$

$$\mathcal{Y}'_\tau(Y_w,w,0,0) = \mathcal{U}'_C \tag{28c}$$

$$\mathcal{Y}'_\rho(Y_w,w,0,0) = \left(1-T'\right)\cdot\mathcal{U}''_{CC}+\mathcal{U}''_{CY}=\frac{\mathcal{U}''_{CY}\cdot\mathcal{U}'_C-\mathcal{U}''_{CC}\cdot\mathcal{U}'_Y}{\mathcal{U}'_C} \tag{28d}$$

The second-order condition writes $\mathcal{Y}'_Y(Y_w,w,0,0)\leq 0$, which gives (6). When this condition holds with strict inequality, and when the global maximum in Y of $\mathcal{U}(Y-T(Y),Y,w)$ is unique, we can apply the implicit function theorem to $\mathcal{Y}(Y_w,w,0,0)$. Provided that the sizes of the changes in w, τ and ρ are small enough for the maximum of $Y\mapsto\mathcal{U}(Y-T(Y),Y,w)$ to change only marginally, one has for $x=w,\tau,\rho$, that $\partial Y/\partial x=-\mathcal{Y}'_x/\mathcal{Y}'_Y$ evaluated at $(Y_w,w,0,0)$. This leads directly to (7), (9) and (10).

We now make the link between our definitions of behavioral elasticities and the elasticities along a linear tax schedule used in Saez (2001). We denote the latter with a tilde. Rewriting (9) and (10) with $T''(.)=0$ yields:

$$\tilde{\varepsilon}_w=\frac{\mathcal{U}'_Y}{Y_w\left[\mathcal{U}''_{YY}-2\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_C}\right)\mathcal{U}''_{CY}+\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_C}\right)^2\mathcal{U}''_{CC}\right]}\quad \tilde{\eta}_w=\frac{\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_C}\right)\mathcal{U}''_{CC}-\mathcal{U}''_{CY}}{\mathcal{U}''_{YY}-2\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)\mathcal{U}''_{CY}+\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)^2\mathcal{U}''_{CC}} \tag{29}$$

Consider now a uniform decrease τ of marginal tax rates (respectively a rise ρ of the level of tax). Such a reform has a *first* impact on earnings $\Delta_1 Y_w$ that equals

$$\Delta_1 Y_w = \tilde{\varepsilon}_w \times \frac{Y_w}{1 - T'(Y_w)} \times \tau \qquad \text{or} \qquad \Delta_1 Y_w = \tilde{\eta}_w \times \rho$$

which in turn implies a change in marginal tax rates of $-T''(Y_w) \times \Delta_1 Y_w$. Hence, the reform has a *second* impact on earnings that equals

$$\Delta_2 Y_w = -\tilde{\varepsilon}_w \times \frac{Y_w}{1 - T'(Y_w)} \times T''(Y_w) \times \Delta_1 Y_w$$

This "circular process" takes place infinitely, with the n^{th} impact on earnings being linked to the $(n-1)^{\text{th}}$ impact through

$$\Delta_n Y_w = -\tilde{\varepsilon}_w \times \frac{Y_w}{1 - T'(Y_w)} \times T''(Y_w) \times \Delta_{n-1} Y_w$$

Using the identity $1 - x + x^2 - x^3 ... = 1/(1+x)$, the total impact equals $\sum_{i=0}^{+\infty} \Delta_i Y_w = \Delta_1 Y_w / \left(1 + \tilde{\varepsilon}_w \times \frac{Y_w}{1-T'(Y_w)} \times T''(Y_w)\right)$. Hence ε_w, η_w, $\tilde{\varepsilon}_w$ and $\tilde{\eta}_w$ are linked through

$$\frac{\varepsilon_w}{\tilde{\varepsilon}_w} = \frac{\eta_w}{\tilde{\eta}_w} = \frac{\sum_{i=0}^{+\infty} \Delta_i Y_w}{\Delta_1 Y_w} = \frac{1}{1 + \tilde{\varepsilon}_w \times \frac{Y_w}{1-T'(Y_w)} \times T''(Y_w)}$$

Using (5) and (29), one retrieves (9) and (10).

B Government's optimum

This appendix solves the government's problem in terms of allocations, like in Mirrlees (1971) and studies what happens at bunching points. Using the obtained government's optimality conditions, we show the equivalence between this formulation and the optimal tax formula of Proposition 1.

According to the *taxation principle* (Hammond 1979, Rochet 1985 and Guesnerie 1995), the set of allocations induced by the tax function $T(.)$ corresponds to the set of incentive-compatible allocations $\{Y_w, C_w, U_w\}_{w \in [w_0, w_1]}$ that verify:

$$\forall (w, x) \in [w_0, w_1]^2 \qquad U_w \equiv \mathcal{U}(C_w, Y_w, w) \geq \mathcal{U}(C_x, Y_x, w) \tag{30}$$

The incentive-compatible restrictions (30) impose that, when taking their intensive decisions, workers of skill w prefer the bundle (C_w, Y_w) designed for them rather then the bundle (C_x, Y_x) designed for workers of any other skill level x. We assume that $w \mapsto Y_w$ is continuous on $[w_0, w_1]$ and differentiable everywhere, except for a finite number of skill levels. Finally, $w \mapsto U_w$ is differentiable. Hence, $w \mapsto C_w$ is also continuous everywhere and differentiable almost everywhere. These assumptions are made for tractability reasons and are standard since Guesnerie and Laffont (1984).[28]

[28] Hellwig (2008) explain how the same first-order conditions can be obtained under weaker assumptions on $w \mapsto Y_w$ and $w \mapsto U_w$.

From Equation (2), the strict single-crossing condition holds. Hence, constraints (30) are equivalent to imposing the differential equation:

$$\dot{U}_w \overset{\text{a.e.}}{=} \mathcal{U}'_w\left(C_w, Y_w, w\right) \tag{31}$$

(31) and the monotonicity requirement that the earnings level Y_w is a non-decreasing function of the skill level w. We get:

Lemma 1 *The necessary conditions for the government's problem are,*[29]

- *if there is no bunching at skill* w :

$$\left(1+\frac{\mathcal{U}'_Y}{\mathcal{U}'_C}\right)\cdot h\left(w\right) = Z_w \cdot \frac{\mathcal{U}''_{Yw}\mathcal{U}'_C - \mathcal{U}''_{Cw}\mathcal{U}'_Y}{\mathcal{U}'_C} \tag{32}$$

- *if there is bunching over* $[\underline{w}, \overline{w}]$:

$$\int_{\underline{w}}^{\overline{w}} \left(1+\frac{\mathcal{U}'_Y}{\mathcal{U}'_C}\right)\cdot h\left(w\right)\cdot dw = \int_{\underline{w}}^{\overline{w}} Z_w \cdot \frac{\mathcal{U}''_{Yw}\mathcal{U}'_C - \mathcal{U}''_{Cw}\mathcal{U}'_Y}{\mathcal{U}'_C}\cdot dw \tag{33}$$

For all skill levels

$$-\dot{Z}_w = \frac{\left(1-g_w\right)\cdot h\left(w\right) + Z_w\cdot \mathcal{U}''_{Cw}}{\mathcal{U}'_C} - \left(T\left(Y_w\right)+b\right)\cdot h'_U\left(w\right) \tag{34}$$

with $Z_{w_1} = Z_{w_0} = 0$ *and*

$$\left(1-\int_{w_0}^{w_1} h\left(w\right)\cdot dw\right)\left(1-g_0\right) = \int_{w_0}^{w_1} \left(Y_w - C_w + b\right)\cdot h'_b\left(w\right)\cdot dw \tag{35}$$

Proof. Since $\mathcal{U}\left(.,.,.\right)$ is increasing in C, we define C_w as function $\Gamma\left(U_w, Y_w, w\right)$ so that:

$$u = \mathcal{U}\left(C, Y, w\right) \qquad \Leftrightarrow \qquad C = \Gamma\left(u, Y, w\right)$$

We get

$$\Gamma'_u = \frac{1}{\mathcal{U}'_C} \qquad \Gamma'_Y = -\frac{\mathcal{U}'_Y}{\mathcal{U}'_C} \qquad \Gamma'_w = -\frac{\mathcal{U}'_w}{\mathcal{U}'_C} \tag{36}$$

where the functions are evaluated at $\left(w, C = \Gamma\left(u, Y, w\right), u = \mathcal{U}\left(C, Y, w\right), Y\right)$, Next, we rewrite (31) as $\dot{U}_w = \Psi\left(U_w, Y_w, w\right)$, where

$$\Psi\left(u, Y, w\right) \overset{\text{def}}{\equiv} \mathcal{U}'_w\left(\Gamma\left(u, Y, w\right), Y, w\right)$$

One has from (36)

$$\Psi'_Y = \frac{\mathcal{U}''_{Yw}\mathcal{U}'_C - \mathcal{U}''_{Cw}\mathcal{U}'_Y}{\mathcal{U}'_C} \qquad \Psi'_U = \frac{\mathcal{U}''_{Cw}}{\mathcal{U}'_C} \tag{37}$$

where the functions are evaluated at $\left(w, C_w, U_w, Y_w\right)$. We consider Y_w as the control variable and U_w as the state variable. Then λ equals the Lagrange multiplier associated to the budget

[29] where the various derivatives of $\mathcal{U}$ are evaluated at $\left(C_w, Y_w, w\right)$.

constraint (13). Let q_w be the co-state variable associated to (31) and let $Z_w = -q_w/\lambda$. The Hamiltonian writes:

$$
\begin{aligned}
&\mathcal{H}(Y,U,q,w,\lambda) \stackrel{\text{def}}{\equiv} \int_0^{U_w-\mathcal{U}(b,0,w)} G\left(V\left(U_w,w,\chi\right),w,\chi\right)\cdot k\left(\chi,w\right)\cdot d\chi\cdot f\left(w\right)\cdot dw \\
&+\int_{U_w-\mathcal{U}(b,0,w)}^{+\infty} G\left(\mathcal{U}^0\left(b,w,\chi\right),w,\chi\right)\cdot k\left(\chi,w\right)\cdot d\chi\cdot f\left(w\right)\cdot dw - \lambda\cdot b \\
&+\lambda\left[Y_w-\Gamma\left(U_w,Y_w,w\right)+b\right]\cdot h\left(w\right)+q_w\cdot\Psi\left(U_w,Y_w,w\right)
\end{aligned}
$$

The first-order conditions of the government's program are

- If there is no bunching at skill w, one must have

$$0=\frac{\partial\mathcal{H}}{\partial Y}\left(Y_w,U_w,q_w,w,\lambda\right)=\lambda\left[1-\Gamma'_Y\right]\cdot h+q_w\cdot\Psi'_Y$$

  Using $Z_w=-q_w/\lambda$, (36) and (37) leads to (32).

- If there is bunching over $[\underline{w},\overline{w}]$, one must have $\int_{\underline{w}}^{\overline{w}}\partial\mathcal{H}/\partial Y\left(Y_w,U_w,q_w,w,\lambda\right)\cdot dw=0$. Using again $Z_w=-q_w/\lambda$ (36) and (37) gives (33).

- The transversality conditions are $q_{w_0}=q_{w_1}=0$ and one gets for any skill level where $w\mapsto Y_w$ is continuous, $-\dot{q}_w=\partial\mathcal{H}/\partial U\left(Y_w,U_w,q_w,w,\lambda\right)$. Using $Z_w=-q_w/\lambda$ and (15) give (34).

- Finally, the first-order condition with respect to b gives (35).

■

We now show how to retrieve the formula in Proposition 1. Let

$$X_w=Z_w\cdot\exp\left[\int_{w_0}^{w}\Psi'_U\left(U_x,Y_x,x\right)\cdot dx\right]\qquad\text{and}\qquad J_w=Z_w\cdot\mathcal{U}'_C\left(C_w,Y_w,w\right)$$

Z_w and J_w have the same sign as X_w. As $w\mapsto Z_w$, $w\mapsto X_w$ is moreover differentiable with a derivative:

$$\dot{X}_w=\left[\dot{Z}_w+Z_w\cdot\Psi'_U\left(U_w,Y_w,w\right)\right]\cdot\exp\left[\int_{w_0}^{w}\Psi'_U\left(U_x,Y_x,x\right)\cdot dx\right]$$

Therefore, from (11), (34) and (37):

$$-\dot{X}_w=\left\{1-g_w-\kappa_w\cdot\left(T\left(Y_w\right)+b\right)\right\}\cdot\frac{h\left(w\right)}{\mathcal{U}'_C\left(C_w,Y_w,w\right)}\cdot\exp\left[\int_{w_0}^{w}\Psi'_U\left(U_x,Y_x,x\right)\cdot dx\right]\tag{38}$$

At skill levels for which there is no bunching, Equation (32) can be rewritten using (5), (28b) and (28c) as

$$T'\left(Y_w\right)\cdot h\left(w\right)=Z_w\cdot\mathcal{Y}'_w=J_w\cdot\frac{\mathcal{Y}'_w}{\mathcal{Y}'_\tau}$$

Using (7), (9) (28b) and (28c) we get

$$\frac{T'\left(Y_w\right)}{1-T'\left(Y_w\right)}\cdot h\left(w\right)=J_w\cdot\frac{\alpha_w}{\varepsilon_w\cdot w}\tag{39}$$

From (34) and (11) we get

$$\begin{aligned}\dot{J}_w &= -\left\{1-g_w-\kappa_w\cdot\left(T\left(Y_w\right)+b\right)\right\}\cdot h\left(w\right)-Z_w\cdot\mathcal{U}''_{Cw}\left(C_w,Y_w,w\right)\\&\quad+Z_w\left\{\mathcal{U}''_{CC}\left(C_w,Y_w,w\right)\dot{C}_w+\mathcal{U}''_{CY}\left(C_w,Y_w,w\right)\cdot\dot{Y}_w+\mathcal{U}''_{Cw}\left(C_w,Y_w,w\right)\right\}\end{aligned}$$

Assume now that the tax function is everywhere differentiable and there is no bunching. Differentiating $C_w = Y_w - T(Y_w)$ and using (5) gives:

$$\begin{aligned}\dot{J}_w &= -\left\{1-g_w-\kappa_w\cdot\left(T\left(Y_w\right)+b\right)\right\}\cdot h\left(w\right)\\&\quad+Z_w\left\{\mathcal{U}''_{CY}\left(C_w,Y_w,w\right)-\mathcal{U}''_{CC}\left(C_w,Y_w,w\right)\frac{\mathcal{U}'_{Y}\left(C_w,Y_w,w\right)}{\mathcal{U}'_{C}\left(C_w,Y_w,w\right)}\right\}\cdot\dot{Y}_w\end{aligned}$$

Using (28c), (28d) and again (5):

$$\dot{J}_w = -\left\{1-g_w-\kappa_w\cdot\left(T\left(Y_w\right)+b\right)\right\}\cdot\hat{h}\left(w\right)+J_w\cdot\frac{\mathcal{Y}'_\rho}{\mathcal{Y}'_\tau}\cdot\dot{Y}_w$$

With (7), (9), (10), (28c) and (28d) :

$$\dot{J}_w = -\left\{1-g_w-\kappa_w\cdot\left(T\left(Y_w\right)+b\right)\right\}\cdot h\left(w\right)+J_w\cdot\frac{\eta_w\cdot\alpha_w}{\varepsilon_w\cdot w}\left(1-T'\left(Y_w\right)\right)$$

Finally, using (39)

$$\dot{J}_w = -\left\{1-g_w-\kappa_w\cdot\left(T\left(Y_w\right)+b\right)\right\}\cdot h\left(w\right)+\eta_w\cdot T'\left(Y_w\right)\cdot h\left(w\right)$$

Since $Z_{w_1} = 0$, $J_{w_1} = 0$, so $J_w = \int_w^{w_1}\left(-\dot{J}_n\right)dn$. Using the last Equation and (39) gives (17). Equation (18) is obtained from the transversality condition $J_{w_1} = 0$. Equation (19) comes by adding (35) to (18).

C Proof of Proposition 2

We turn back to the case where $w \mapsto (C_w, Y_w)$ is continuous everywhere and differentiable everywhere except on a finite number of skill levels (so that bunching can occur on a finite number of skill intervals). Note that continuity of $w \mapsto Y_w$ implies that $w \mapsto U_w$ is continuously differentiable. We first show

Lemma 2 *X_w (thereby Z_w) is everywhere non-negative and almost everywhere positive within (w_0, w_1) whenever $w \mapsto \frac{1-g_w}{\kappa_w}$ is increasing.*

Proof. Assume by contradiction that $Z_{w'} \leq 0$ for some $w' \in (w_0, w_1)$. Then $X_{w'} \leq 0$. By continuity of $w \mapsto X_w$, and the transversality condition there exists a maximal interval $[w_2, w_3]$ where $X_w \leq 0$ for all $w \in [w_2, w_3]$ and $X_{w_2} = X_{w_3} = 0$. Moreover, since $w \mapsto C_w$ is also continuous everywhere and differentiable almost everywhere, X_w is everywhere differentiable with a derivative given by (38).

- Since $X_{w_2} = 0$ and $X_w \leq 0$ in the right neighborhood of w_2, one must have $\dot{X}_{w_2} \leq 0$. Hence, from (38)

$$\frac{1-g_{w_2}}{\kappa_{w_2}} \geq T\left(Y_{w_2}\right)+b \tag{40}$$

- Since $X_{w_3} = 0$ and $X_w \leq 0$ in the left neighborhood of w_3, one must have $\dot{X}_{w_3} \geq 0$. By a symmetric reasoning, this leads to

$$T\left(Y_{w_3}\right) + b \geq \frac{1 - g_{w_3}}{\kappa_{w_3}} \tag{41}$$

- One has

$$T\left(w\right) + b = Y_w - \Gamma\left(U_w, Y_w, w\right)$$

Function $w \mapsto Y_w - \Gamma\left(U_w, Y_w, w\right)$ is continuous and, except at a finite number of points, is differentiable with derivative

$$\frac{d\left(Y_w - \Gamma\left(U_w, Y_w, w\right)\right)}{dw} = \dot{Y}_w\left(1 - \Gamma'_Y\left(U_w, Y_w, w\right)\right) - \Gamma'_U\left(U_w, Y_w, w\right) \cdot \dot{U}_w - \Gamma'_w\left(U_w, Y_w, w\right)$$

$$= \dot{Y}_w\left(1 + \frac{\mathcal{U}'_Y\left(U_w, Y_w, w\right)}{\mathcal{U}'_C\left(U_w, Y_w, w\right)}\right) - \frac{\mathcal{U}'_w\left(C_w, Y_w, w\right)}{\mathcal{U}'_C\left(C_w, Y_w, w\right)} + \frac{\mathcal{U}'_w\left(C_w, Y_w, w\right)}{\mathcal{U}'_C\left(C_w, Y_w, w\right)} = \dot{Y}_w\left(1 + \frac{\mathcal{U}'_Y\left(U_w, Y_w, w\right)}{\mathcal{U}'_C\left(U_w, Y_w, w\right)}\right)$$

where the second equality follows (31) and (36). If there is bunching at w then $\dot{Y}_w = 0$. If there is no bunching at w, Equation (32) applies. Condition (2) and $Z_w \leq 0$ then induces that $w \mapsto Y_w - \Gamma\left(U_w, Y_w, w\right)$ admits a non-positive derivative. Hence, $w \mapsto Y_w - \Gamma\left(U_w, Y_w, w\right)$ is weakly decreasing over $[w_2, w_3]$, so

$$T\left(Y_{w_2}\right) + b \geq T\left(Y_{w_3}\right) + b \tag{42}$$

Inequalities (40), (41) and (42) imply:

$$\frac{1 - g_{w_2}}{\kappa_{w_2}} \geq \frac{1 - g_{w_3}}{\kappa_{w_3}}$$

This is consistent with the assumption that $w \mapsto \left(1 - g_w\right)/\kappa_w$ is increasing if and only if $w_2 = w_3$. Therefore $w' = w_2 = w_3$ and $X_{w'} \geq 0$ for all skill levels and $X_w = 0$ only pointwise. ■

Since X_w (hence Z_w) is non-negative everywhere and can be nil only pointwise, then, for skill levels where there is no bunching, according to (5) and (32) marginal tax rate is non-negative and can be nil only pointwise. Bunch of skills correspond to a mass point of the earnings distribution and to an upward discontinuity of marginal tax rates. However, the discontinuity is between two marginal tax rates that correspond to skill levels without bunching for which we have shown that marginal tax rates are non-negative.

D Proof of Proposition 3

Since $X_{w_0} = 0$ and for all w, $X_w \geq 0$ (from 2) then $\dot{X}_{w_0} \geq 0$. According to (38), this induces

$$\frac{1 - g_{w_0}}{\kappa_{w_0}} \leq T\left(Y_0\right) + b$$

Since $g_{w_0} \leq 1$, the left-hand side is positive, inducing that in work benefit (i.e. $-T\left(Y_0\right)$ when $T\left(Y_0\right) < 0$) is lower than welfare benefit b.

E Proofs of Properties 1 and 2

Under (27), U_w is increasing in skill w. Then, a Maximin government values only the welfare of non-employed and $g_w = 0$ for all skill levels, which ensures property 2 for a Maximin government.

Under (26), $\mathcal{U}'_C$ depends only on the consumption level. From (2), incentive compatible conditions (30) implies that $w \mapsto C_w$ is non-decreasing. Therefore, since $u''_{CC} < 0$, $w \mapsto \mathcal{U}'_C(C_w, Y_w, w)$ is non-decreasing, and is increasing without bunching.

Under (26) and a Benthamite government, g_w simply equals $\mathcal{U}'_C(C_w, Y_w, w)/\lambda$ according to (15), which ensures property 2 for a Benthamite government.

Under Assumption (27), one has that the threshold value $U_w - \mathcal{U}(b, 0, w)$ of χ below which individuals of type (w, χ) choose to work, is decreasing in skill level w. So, when $K(\chi, w)$ is strictly log-concave with respect to χ and $w \mapsto k(\chi, w)/K(\chi, w)$ is non-increasing in w then $w \mapsto k(U_w - \mathcal{U}(b, 0, w), w)/K(U_w - \mathcal{U}(b, 0, w), w)$ is decreasing. Together with $w \mapsto \mathcal{U}'_C(C_w, Y_w, w)$ being non-decreasing, using (11), insures that $w \mapsto \kappa_w$ is decreasing, even in the presence of bunching. So Property 1 is ensured.

E.1 Example 1

A Maximin government values only the welfare of non-employed so $g_w = 0$ for all skill levels and $(1 - g_w)/\kappa_w = 1/\kappa_w$. Since Property 1 holds, $(1 - g_w)/\kappa_w$ is therefore increasing in w and Proposition 2 applies. Moreover, as $g_w = 0$, Proposition 3 applies too.

E.2 Example 2

Combining Properties 1, 2 and $g_w \leq 1$ ensures that $(1 - g_w)/\kappa_w$ is increasing in w. So, Proposition 2 applies, thereby Proposition 3 since it has been assumed that $g_w \leq 1$.

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