Egalitarianism Under Earmark Constraints

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Abstract

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1 Introduction

Egalitarianism is the central principle of fair division that may conflict with a number of incentives, feasibility or efficiency constraints. Maximizing the leximin ordering over profiles of relevant characteristics (a. k. a. the Rawlsian approach) is the most common method to implement egalitarianism under constraints. It is however a controversial method, because it justifies the sacrifice of large amounts of resources for the "rich", if it allows to raise by a tiny amount the lot of the "poor". The only case where egalitarianism eschews this devastating critique is when we can find a Lorenz dominant distribution of welfare, or resources: at the Lorenz dominant outcome, we simultaneously maximize the share of the k poorest individuals, for *any number* k of agents[1]. However, unlike the leximin ordering that always reaches a unique maximum in any closed convex set, a Lorenz dominant outcome may not exist in such sets. We know in fact very few fair division models admitting Lorenz dominating solutions over a reasonably rich domain of problems. The two main exceptions follow.

Dutta and Ray (1989) observed that the core of a supermodular (convex) cooperative game is one very general instance where a Lorenz dominant solution exists; this solution has been known after their work as the *egalitarian* selection in the core. The second model is the fair division of a single commodity under single-peaked preferences and no free disposal (Benassy, 1982, Sprumont, 1991). The *uniform solution* selects for each agent either his peak, or a common share in such a way that the resource is fully distributed. Although the original motivation of the uniform solution was its incentive properties (Benassy, 1982), its most compelling fairness property, and its shortest definition, is to be Lorenz dominant among all Pareto optimal allocations of the resource (De Frutos and Massó, 1995).

Building on the techniques of submodular optimization as in Dutta and Ray (1989), we propose a considerable generalization of the Sprumont model, where a homogeneous commodity (the resource) is still shared by several agents with single-peaked preferences, but the resource is coming from any number of different suppliers, under arbitrary bilateral feasibility constraints: each supplier can only deliver to a certain subset of agents. Examples where such constraints are critical include:

- Balancing the workloads of several machines, when each machine can only process certain jobs, but the processing speed is uniform. Assigning customers to service persons when language constraints limits the set of customers each service person can handle.
- Sharing earmarked resources when the earmarking is not one-to-one[2], for instance dividing funds between different research projects, when the foundations, agencies, or

[1] Reference on the lorenz optimum: Sen's 1970; Moulin 1988; Foster and Sen

[2] As opposed to the usual meaning of an earmark as "*a provision in Congressional legislation that allocates a specified amount of money for a specific project, program, or organization*" (Merriam-Webster dictionary).

private donors put overlapping constraints on the use of their gift. For instance one donor funds projects relevant to global warming, another donor looks for projects with a Latin American component, a third one for those involving minorities, etc..

- Assigning students to schools when each school can only handle a certain subset of students –e.g., those coming from certain neighborhoods–, and.these subsets overlap.

Assuming that each recipient of the resources want to maximize her share if the resource is a "good" (money), or minimize it if it is a "bad" (workload) is a reasonable first approximation. But in most concrete examples the situation is more nuanced. Under the widespread buraeucratic constraint that funds must be spent in a given calendar year, and the belief that returning funds has a negative impact on future funding, a project manager does not want her budget to be too large, lest it becomes difficult to find justifiable ways to spend it. If the workload of a worker is too small, his machine or his job may soon be deemed redundant, so his most preferred workload is not zero. Similarly a school principal has in mind an ideal amount of students she would like to handle, not too large so that classes will not be crowded, not too small lest some of his faculty or staff becomes idle. And so on.

Single-peaked preferences provide a rich model of the agents' goals, from always increasing (more commodity is always better) to always decreasing (less is always better), and much in between. But the target share of an agent depends upon many subjective, privately known factors, therefore any division rule, fair or otherwise, to allocate the resources must worry about the agents' incentives to truthfully reveal their true target. We take *strategyproofness* (truthful report is a dominant strategy) as our incentive compatibility design constraint. We identify a canonical division rule that simultaneously aligns incentives with efficiency (is strategyproof and selects a Pareto optimal allocation), and is egalitarian-fair, in the sense that it selects the Lorenz dominant Pareto optimal allocation.

A simple blood division example will help develop intuition for our *egalitarian* solution. A blood bank must divide the (objective) blood needs of a group of patients between a set of donors; patients and donors are partitioned by blood type and transfusions must respect the familiar compatibility constraints: (i) type O are universal donors (ii) type AB are universal receivers (iii) type A can also give to A , (iii) type B can also give to B (iv) type AB can only give to AB (for simplicity we ignore the Rhesus factor).

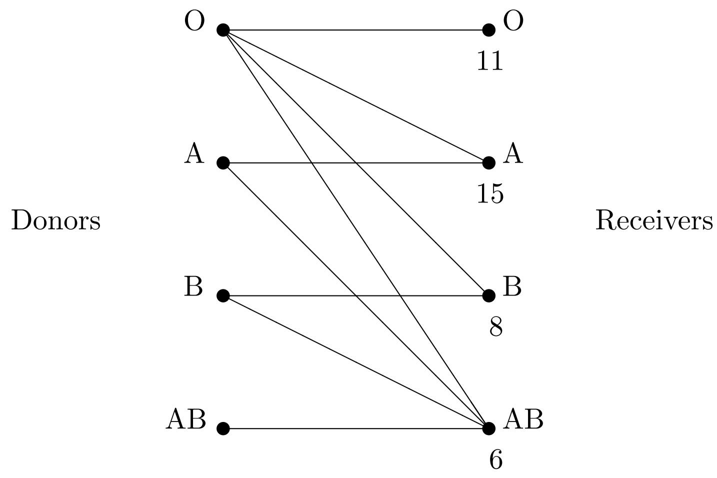
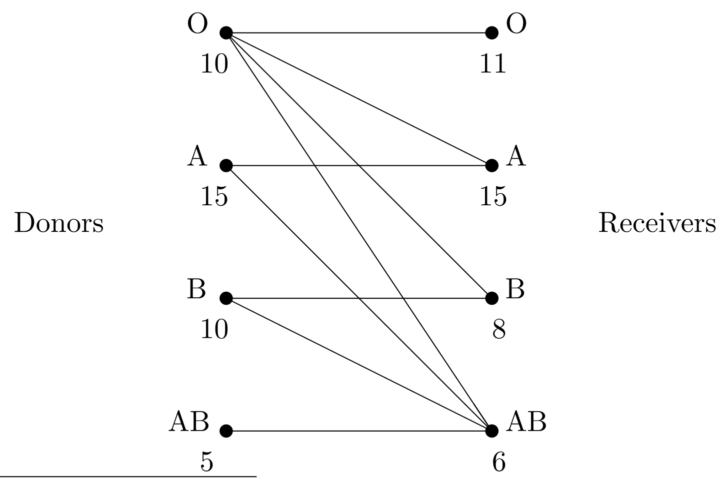


Figure 1: The donor-receiver example

Figure 1 shows on the right side the the quantity of blood needed by each group of receivers of the same type. On the left side a node is a group of donors of a given type. The total demand of 40 units must be served, and we assume first that the bank wishes to share the burden equally among the blood types. Taking 10 units from each group of donors is clearly not feasible. The most egalitarian division of the burden compatible with Figure 1 is: $AB : 6$, $B : 8$, $A : 13$, $O : 13$. Type AB donors cannot contribute more than 6 units toward the patients' demand. Given AB's share, type B donors give as much as type AB and B receivers can still accept. Finally, types A and O donors supply equal amounts to cover the rest of the demand.

A more refined version of the model takes into account the fact that the distribution of blood types is not homogeneous[3]. Ideally the blood bank wishes to spend the blood it receives from various groups of donors in proportion to the representation of these groups in the population. We assume that those proportions are $AB : 12.5\%$, $B : 25\%$, $A : 37.5\%$, $O : 25\%$, so the ideal distribution of the demand in the example of Figure 1 is as follows:



[3]For instance, in Norway A type represent 50% of the population, while in Iceland O type represent 56% of the population (source Wikipedia).

Consider the egalitarian allocation $AB : 6$, $B : 8$, $A : 13$, $O : 13$ identified above. It uses more type AB blood than the target of 5, and less of B blood than the target of 10. Similarly type A is giving more than its target 10, and type O is giving less than 15. Within the compatibility constraints, we can rearrange the shares of type AB and type B as $AB : 5$, $B : 9$, which will bring the contribution of both types closer to their target; similarly we can modify the shares of types A and O as $A : 15$, $O : 11$. When each node on the left of the graph is interpreted as a different agent with her own single-peaked preferences, we will speak of Pareto improving reallocations. There are other Pareto improving moves in the example such as $AB : 4$, $B : 10$, but there the difference between AB's and B's shares is larger, implying that such a move would lead to an allocation Lorenz dominated by our egalitarian solution $AB : 5$, $B : 9$, $A : 15$, $O : 11$.

Our egalitarian solution shares many properties of of the familiar Uniform solution in Sprumont's one supplier model, but there are important differences as well. We start with the latter.

Feasibility of allocations (divisions of the resources) imposes a set of linear constraints, interpretable as the core of a concave or a convex game and analyzed by means of the sub and supermodularity techniques as in (Dutta Ray 1989);

Pareto optimality identifies an over-demanded and an under-demanded side of the market. In the example of Figure 2 $\{AB, B\}$ is the underdemanded side because these donors cannot meet more than 14 units of demand, but they would like to give 15 units; and $\{O\}$ is the overdemanded side because it must supply at least 11 units but would like to contribute only 10. Over-demanded (resp. under-demanded) agents never get less (resp. more) than their peak in a Pareto optimal allocation; finally A always get exactly its peak 15 in any Pareto optimal allocation. From a technical standpoint, the key tool in analysing Pareto optimality is a version of a well-known result in graph theory, the Gallai-Edmonds (henceforth, GE) decomposition for bipartite graphs, (see Ore (1962) for a formal treatment, or Bogomolnaia and Moulin (2004) for a matching application).

The two equity tests *No Envy* and the less demanding *Equal Treatment of Equals*, cannot be formulated as easily as in the one supplier model. We postulate that if Ann envies Bob's share, her claim is legitimate only if it is feasible to give her a share she likes more at the expense only of Bob, i.e., while preserving the shares of every agent other than Bob. Similarly Equal Treatment of Equals is violated if we can bring Ann's and Bob's shares closer together without altering other agents' shares.

Finally our model accomodates capacity constraints, i.e., arbitrary exogenous lower and upper bounds on each agent's allocation. This is important in all examples we discussed, where shares cannot be arbitrarily large, or small. This feature by itself is already a generalization of the standard model.

We turn to the features common to our and the one supplier model.

Our egalitarian solution is a Lorenz dominant allocation of the resources among Pareto optimal allocations; it coincides with the uniform solution if there is a single supplier.

The egalitarian solution is incentive compatible in the strong sense of strategyproofness; moreover it is characterized, –as the uniform solution in (Sprumont, 1991; Ching, 1994)–, by the combination of *strategy-proofness*, *Pareto optimality* and *equal treatment of equals.*

References

Aumann, R.J. and Maschler, M. (1985) "Game Theoretic Analysis of a Bankruptcy Problem from the Talmud," *Journal of Economic Theory*, 36, 195-213.

Benassy, J-P. (1982) *The Economics of Market Disequilibrium*, New York Academic Press.

Bogomolnaia A., Moulin H., (2004), "Random Matching Under Dichotomous Preferences." *Econometrica*, 257-279.

Ching, S. (1994) "An Alternative Characterization of the Uniform Rule," *Social Choice and Welfare*, 40, 57-60.

De Frutos A., Massó J., (1995), "More on the Uniform Allocation Rule: Equality and Consistency." WP 288.95 UAB.

Dutta B., Ray D., (1989), "A Concept of Egalitarianism Under Participation Constraints." *Econometrica*, 57, 615-635.

Ore O., (1962). "Theory of Graphs." American Mathematical Society Colloquium Publications, VOL XXXVIII. Providence R.I.: American Mathematical Society.

Sprumont, Y. (1991) "The Division Problem with Single-Peaked Preferences: A Characterization of the Uniform Allocation Rule," *Econometrica*, 59, 509-519.

Thomson, W. (1994a) "Consistent Solutions to the Problem of Fair Division when Preferences are Single-Peaked," *Journal of Economic Theory*, 63, 219-245.

Thomson, W. (1994b) "Resource Monotonic Solutions to the Problem of Fair Division when Preferences are Single-Peaked," *Social Choice and Welfare*, 63, 205-224.

Thomson, W. (1995) "Population Monotonic Solutions to the Problem of Fair Division when Preferences are Single-Peaked," *Economic Theory*, 5, 229-246.

Thomson, W. (1997) "The Replacement Principle in Economies with Single-Peaked Preferences," *Journal of Economic Theory*, 24, 145-168.

Thomson, W. (2003) "Axiomatic and Game-theoretic Analysis of Bankruptcy and Taxation Problems: a survey," *Mathematical Social Sciences*, 45, 249-297.

Thomson, W. (2005) *The Theory of Fair Allocation*, Ch. 11, unpublished book manuscript, University of Rochester.