# Systemic Risk-Taking:

### Amplification Effects, Externalities, and Regulatory Responses

### Anton Korinek\* University of Maryland

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Work-in-Progress - Comments are Welcome

#### Abstract

This paper develops a simple model of systemic risk in the form of financial accelerator effects whereby adverse developments in financial markets and in the real economy mutually reinforce each other and lead to a feedback cycle of falling asset prices, deteriorating balance sheets and tightening financing conditions. The paper shows that the free market equilibrium in such an environment is generically inefficient because constrained market participants do not internalize that their actions entail amplification effects. Therefore they undervalue the social benefits of liquidity during crises and take on too much systemic risk.

We use our framework to shed light on a number of current policy issues. First, we draw lessons for the ex-ante regulation of systemic risk: we develop a new analytical framework of macro-prudential capital adequacy requirements that take into account systemic risk. Second, we show that banks face socially insufficient incentives to raise more capital during systemic crises. We also analyze channels of financial contagion and explain why private agents will take insufficient precautions against contagion from other sectors in the economy.

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### 1 Introduction

The current financial crisis has powerfully demonstrated how prone modern economies are to systemic risk, i.e. the risk that a shock of sufficient magnitude impairs the financial system to the point that it can no longer perform its function of allocating capital to the most efficient use. This paper develops a simple model of systemic risk stemming from financial accelerator effects, whereby declining asset prices and deteriorating balance sheets mutually reinforce each other and thereby magnify the effects of shocks to the financial system.

Accelerator effects typically involve the following four elements:<sup>1</sup> First, a negative shock tightens financial constraints on entrepreneurs and limits their economic activity. Second, the decline in aggregate economic activity reduces the price of productive assets in the economy. Third, the price decline decreases the net worth of entrepreneurs who own the assets. Fourth, the decline in net worth reduces the creditworthiness of entrepreneurs and tightens their financial constraints further. This feeds back into the first element, leading to a self-reinforcing cycle of tightening financing conditions, declining economic activity, falling asset prices and shrinking net worth (see figure 1).

Every crisis therefore brings up the question of whether exisiting regulations are sufficient or whether new regulations to limit risk-taking by financial market participants are desirable. For government regulations to enhance social welfare, they must correct a market imperfection. Otherwise, if markets functioned well and rational market participants knowingly took on extensive risk, then crises would be a socially desirable outcome (see e.g. Allen and Gale, 1998), and government regulations to limit risk-taking would reduce social welfare.

This leads to the central question that we pose in this paper: Are the financing and investment decisions of decentralized agents in an economy that is prone to systemic risk socially optimal? We find that the answer is a clear no. We show that rational atomistic agents do not internalize that their actions give rise to amplification effects when financing constraints in the economy are binding. They balance off the private benefits and costs of their financing and investment decisions, including the private costs of potential future constraints, while taking aggregate prices as given. However, when a significant number of agents are forced to reduce their economic activity in response to an aggregate shock, general equilibrium effects imply that asset prices have to decline. In perfect markets, this would constitute a purely pecuniary externality, which has no efficiency implications. However, in an economy with binding financing constraints, this pecuniary externality has real effects: asset price declines tighten financing constraints and trigger amplification effects.

Since atomistic agents do not internalize this, they undervalue the social benefits of liquidity in crisis states. This leads them to take on a socially excessive level of systemic

<sup>&</sup>lt;sup>1</sup>See for example Bernanke and Gertler (1989); Kiyotaki and Moore (1997); Bernanke et al. (1999).

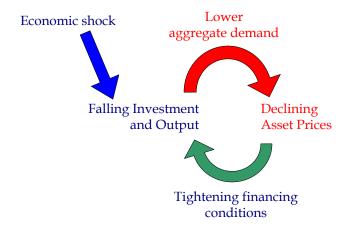


Figure 1: A simple schematic model of financial amplification

risk in their financing and investment decisions. We call the distortion that arises from individual agents' failure to internalize the amplification effects that they give rise to a systemic externality.<sup>2,3</sup> A social planner would internalize that a lower level of risk-taking or a higher level of insurance would mitigate financing constraints and amplification effects in low states of nature. This would lead to lower volatility in aggregate economic activity, in asset prices and in financing constraints, benefitting all agents in the economy.

Translating our theoretical results into practical policy advice, we develop a comprehensive theoretical framework of macro-prudential financial regulation. We derive a social pricing kernel and an externality kernel as a guide for how capital adequacy requirements can be adjusted for systemic risk. The social pricing kernel quantitatively captures a social planner's state-contingent valuation of liquidity, accounting for the social costs of financial amplification effects. The externality kernel is the difference between social and private valuation of liquidity in each state of nature, i.e. it reflects the state-contingent magnitude of systemic externalities created by the excessive risk-taking of decentralized agents. In states when financing constraints are loose, the externality kernel is zero as no amplification effects arise; in constrained states the externality kernel captures the social costs of amplification

<sup>&</sup>lt;sup>2</sup>In recent policy discussions, the term *systemic risk* has been used to describe risk that endangers the stability of the entire financial system. Note that this definition contrasts with the definition of systemic risk (or systematic risk, aggregate risk, market risk) in the asset pricing literature as risk that cannot be diversified. The view proposed in this paper captures both definitions, as the excessive exposure of individual agents to undiversifiable market risk can give rise to large amplification effects that destabilize the financial system when financial constraints in the economy are binding.

<sup>&</sup>lt;sup>3</sup>The analogy to more traditional forms of externalities should be clear: for example, when a polluter ceases to pollute, he bears all the costs, but society at large reaps the benefits. In our example, a financial institution that limits its risk-taking bears all the costs in terms of foregone profits, but society at large benefits from the mitigation of amplification effects and greater financial stability.

effects created by payoffs. Just as pricing kernels are used to calculate the decentralized market price of risky assets, the externality kernel can be used by regulators to price the systemic externalities created by assets and liabilities with state-contingent payoffs. The externality can be corrected by imposing a Pigovian tax of equal magnitude, or any policy measure that has tax-like effects, such as e.g. increased capital adequacy requirements.

We find that from our macro-prudential perspective, the optimal tax on an asset that offsets the externality is given by the expected magnitude of payoffs in constrained states times the social cost of such payoffs in terms of inducing amplification effects, as measured by the externality kernel. This would induce market participants to internalize their systemic externalities. Such regulations should also apply to the so-called "shadow financial system," which contributes strongly to amplification effects in modern financial systems.

We believe that the systemic externalities laid out in this paper should be a cornerstone of financial regulation. The main motivation behind current banking regulations (see e.g. the discussion in Borio, 2003; Brunnermeier et al., 2009) is to limit the risk of failure of financial institutions, originally with a view towards protecting their depositors. The framework is largely based on a partial equilibrium view that analyzes each single institution in isolation and cannot adequately account for the systemic feedback effects and externalities that are the topic of our paper. However, note that strong amplification effects can arise even in the absence of individual bank failures, and that the social costs of a bank failure consist largely of the resulting shockwaves in the financial system (i.e. of amplification effects) rather than the direct losses that accrue to the failed bank's creditors.

It has been argued that risk-sensitive capital adequacy regulations with a purely microe-conomic focus can contribute to pro-cyclicality (see e.g. Catarineu-Rabell et al., 2005): when financial institutions suffer losses or when the riskiness of their assets rises, they have to set aside more capital and are often forced to engage in fire sales, which can lead to financial amplification effects and magnify the increase in risk. Our analysis indicates that banks will not internalize the social costs of the resulting pro-cyclicality, and that it is privately optimal for them to take on excessive systemic risk in the presence of such regulations. It is often argued that market discipline would induce transparent banks to adopt rules that smooth their capital holdings throughout the business cycle (Gordy and Howells, 2006). However, our analysis implies that markets would punish financial institutions that behave socially responsibly and reward those that behave irresponsibly, since maximizing shareholder value involves socially excessive risk-taking.

The paper also discusses a number of additional results. A consequence of our finding that decentralized agents undervalue liquidity in crisis states is that they also undervalue the social benefits of raising new capital in constrained states of nature: any capital injection mitigates financial amplification effects and moderates the decline in asset prices; again,

atomistic agents do not internalize the social value of stabilizing asset prices since they take prices as given.

We show that the largest systemic externalities arise when financial market participants that are prone to financing constraints are close to risk-neutral (e.g. hedge funds) but trade with risk averse creditors: they do not face an insurance motive as a result of their utility function and are willing to take on large amounts of risk, which exposes them to financial constraints in bad states of nature and leads to systemic amplification effects.

We analyze the effectiveness of bailouts to constrained market participants so as to avert amplification effects, and we find that any anticipated bailouts will be undone by market participants and will therefore have no effects. However, transfers that are unanticipated or that are made to agents that do not have sufficient access to financial markets to undo them can be be effective.

We also discuss the importance for market participants of having correct expectations about future prices. In crisis times when constraints are binding, the welfare costs of expectational errors are by an order of magnitude larger than in normal times, since financial amplification effects magnify the impact of any unexpected change to the liquidity position of market participants. There is therefore a role for financial regulators to conduct "systemic stress tests," which serve to ensure that market participants are better informed about the potential magnitude of declines in asset prices during crises.

While our benchmark model captures a situation where only one constrained sector is affected by an aggregate shock, we also analyze the potential for contagion among different sectors in an extension. We describe two channels of contagion, through asset prices and through contingent lines of liquidity. A sector can suffer from contagion through asset prices if is financially constrained in some states of the world and it uses as collateral assets that experience price fluctuations because of the pecuniary externalities of other sectors. This channel was of great importance for hedge funds and financial institutions in the subprime crisis (Adrian and Brunnermeier, 2008). Similarly, a sector can experience contagion through contingent lines of liquidity if it offers credit lines to other sectors that, if drawn upon in case of crises, are sufficiently large to make the lending sector itself financially constrained. Examples include the credit lines from their parent banks that many SIVs and conduits drew upon in the subprime crisis (Brunnermeier, 2008). In both cases, decentralized agents in sectors that are subject to binding financing constraints in some states generally underinsure against contagion from other sectors.

Our work fundamentally builds on the literature on financial amplification effects, which started with Fisher (1933)'s work on the debt deflation theory of the Great Depression. We described the basic mechanism above in figure 1. More recent seminal contributions to this literature are for example Bernanke and Gertler (1990), Kiyotaki and Moore (1997) and

Bernanke et al. (1999). While these papers analyze the mechanism of financial amplification effects, we focus on the implications for ex ante decentralized financing decisions.

In this vein, Krishnamurthy (2003) shows that if entrepreneurs have access to risk-neutral financial markets (so that insurance against adverse shocks is actuarially fair), they would always fully insure against systemic risks that lead to amplification effects, and a social planner could not improve on this allocation. This result is a special case that only holds when lenders are perfectly risk-neutral and ex-ante insurance markets are perfect. In the real world, entrepreneurs are clearly not fully insured against shocks. Instances of binding financing constraints and financial amplification effects are a recurring feature of modern market economies. Krishnamurthy (2003) captures this by assuming that limited aggregate supply of collateral prevents full insurance.

By a similar token, Lorenzoni (2008) analyzes the case where two-sided limited commitment constrains the amount of insurance that entrepreneurs can obtain from lenders and shows that this leads entrepreneurs to over-invest, since they do not internalize that higher investment increases the amount of fire-sales that they need to engage in in bad states of nature. Whereas he examines the socially optimal amount of investment, our main focus is on the optimal degree of risk-taking in the financing and investment decisions of decentralized agents. We show that even when entrepreneurs have access to unconstrained insurance markets, they take on excessive risk if we deviate from the benchmark of risk-neutrality. In this respect, the paper is related to Korinek (2008), who shows that decentralized agents in an emerging market economy borrow excessively in dollars from international lenders because they do not internalize that risky financing decisions contribute to the financial amplification effects that are triggered during emerging market crises.

More generally, the externality result in our paper is an application of the proposition that the market equilibrium in economies with constraints that endogenously depend on market prices is not constrained efficient (Arnott et al., 1992): decentralized agents do not internalize that their pecuniary externalities affect the tightness of the constraint. In our case, asset prices determine the tightness of financing constraints, and decentralized agents do not internalize that changes in their net worth affect asset prices.

A number of recent papers document the importance of amplification effects empirically. For example, Adrian and Shin (2008) find that leverage among investment banks is strongly pro-cyclical, implying that they take on more risk in good times and sell off risky assets in bad times. Adrian and Brunnermeier (2008) show that VaR – a measure for the riskiness of a financial institution's assets – rises strongly when another financial is in distress. They also document that financial institutions that increase their exposure to systemic risk raise their expected return, consistent with our theoretical model.

The rest of the paper is structured as follows. The following section 2 develops a simple

model of financial amplification effects, which are triggered if bankersface binding financing constraints and need to engage in fire sales. We show that in states with binding constraints, a social planner always values liquidity more highly than decentralized bankers. In section 3 we analyze their financing decision and show that the undervaluation of liquidity induces them to take on too much systemic risk. Section 4 shows a number of extensions to our baseline model, for example that decentralized agents also take on too much systemic risk in their investment decisions. In section 5 we discuss the implications of our findings for banking regulation and other government policies. Section 6 concludes.

## 2 A Simple Model of Financial Amplification

In this section we set up a simple model of financial amplification effects. Assume a production economy that consists of two time periods 1 and 2 and is inhabited by two types of agents, bankers and households, which we each analyze below. The bankers in our model can alternatively be interpreted as entrepreneurs – the important characteristic is that they make financing decisions and are subject to business risk. At the beginning of time the outcome  $\omega$  of an aggregate productivity shock is observed. In the analysis of this section we can regard the realization of  $\omega$  as given since it is realized before any decisions are made. (In the next section we will introduce a financing decision that allows bankers to insure against this shock.)

#### 2.1 Bankers

We model bankers as agents in a simplified version of Kiyotaki and Moore (1997): they are risk-neutral<sup>4</sup> and value consumption (or profits) according to the utility function

$$u = c_1^{\omega} + c_2^{\omega} \tag{1}$$

where we restrict  $c_1^{\omega}$ ,  $c_2^{\omega} \geq 0$ . They are born with an amount  $t_1$  of a productive asset (loans, land, machines etc.) and an initial amount  $b_1$  of debt. The asset yields a level of production of  $A_1^{\omega}t_1$  units, which depends on the productivity shock to the economy. At the end of period 1, bankers need to invest a fixed fraction  $\alpha$  per unit of the productive asset in order to obtain a return in the next period. In case they need to raise additional finance, they sell some of the asset, i.e. they reduce their holdings from  $t_1$  to  $t_2^{\omega}$  at a market price  $q_1^{\omega}$ , which is taken as given by decentralized agents. Furthermore, there is a bond market in which bonds that

<sup>&</sup>lt;sup>4</sup>However, despite of being risk-neutral, we can show in the following section that bankers have an incentive to hedge against binding financing constraints, as emphasized by Froot et al. (1993). The assumption of risk-neutrality was made for analytical simplicity; our results continue to hold if bankers are assumed to be risk-averse.

pay off one unit in period 2 are traded at a price  $m_1^{\omega}$ . We denote the quantity of bonds sold by the banker as  $b_2^{\omega}$ . The resulting budget constraint in period 1 is

$$c_1^{\omega} + \alpha t_2^{\omega} + b_1^{\omega} = A_1^{\omega} t_1 + q_1^{\omega} \cdot (t_1 - t_2^{\omega}) + m_2^{\omega} b_2^{\omega}$$

We follow Kiyotaki and Moore (1997) in assuming that the total quantity of the productive asset is fixed at  $t_1$  and that bankers can pledge the value of their asset holdings but not of their period production. Since the economy ends after period 2 the asset is worthless at that time, i.e.  $q_2^{\omega} \equiv 0$ . As a result, bankers have nothing to pledge and cannot borrow in period 1, i.e.  $b_2^{\omega} \leq 0$ . Furthermore, in unconstrained states it can be shown that it is never optimal for bankers to shift resources from period 1 into period 2 for future consumption, since this is costly as households require compensation to deviate from their perfect consumption smoothing plan.<sup>5</sup> We can therefore set  $b_2^{\omega} = 0$  in the budget constraint (3). However, note that when period 1 liquidity is tight, the constraint  $c_1^{\omega} \geq 0$  will be binding.

In period 2 bankers obtain their production of  $\bar{A}t_2^{\omega}$ , where we have kept the productivity parameter  $\bar{A}$  constant for simplicity. We can express the banker's decentralized optimization problem as

$$V(b_1^{\omega}) = \max_{\{c_1^{\omega}, t_2^{\omega}\}} c_1^{\omega} + \bar{A}t_2^{\omega} \quad \text{s.t.} \quad c_1^{\omega} \ge 0$$
 (2)

$$c_1^{\omega} + \alpha t_2^{\omega} \le A_1^{\omega} t_1 - b_1^{\omega} + q_1^{\omega} \cdot (t_1 - t_2^{\omega}) \tag{3}$$

This results in a Lagrangian of

$$\mathcal{L} = c_1^{\omega} + \bar{A}t_2^{\omega} + \lambda^{\omega}c_1^{\omega} - \mu^{\omega} \left[ c_1^{\omega} + \alpha t_2^{\omega} - A_1^{\omega}t_1 + b_1^{\omega} - q_1^{\omega} \cdot (t_1 - t_2^{\omega}) \right]$$

where  $\lambda^{\omega}$  is the shadow price of the banker's non-negativity constraint on consumption and  $\mu^{\omega}$  is the shadow value on the period 1 budget constraint, i.e. the banker's valuation of period 1 liquidity. We will refer to the banker as constrained when the constraint on consumption is binding, and unconstrained otherwise.

#### 2.2 Households

We assume that households are risk averse and derive utility from consumption according to the function<sup>6</sup>

$$U^{\omega} = u(C_1^{\omega}) + u(C_2^{\omega}) \tag{4}$$

<sup>&</sup>lt;sup>5</sup>To see this analytically would require adding a lending decision to households' maximization problem in equation (5). However, under our assumptions the equilibrium amount of lending would always be zero.

<sup>&</sup>lt;sup>6</sup>If households were risk-neutral, they would offer to fully insure bankers against the incidence of binding borrowing constraints. If constraints never bind, the externality result of our paper would disappear. However, evidence clearly shows that bankers are not fully insured in practice (Brunnermeier, 2008).

Households receive an endowment e every period. Furthermore, they buy  $T_2^{\omega}$  assets when bankers engage in fire sales and employ them in an alternative (though less productive) function. Specifically, we assume that households can employ productive assets using a decreasing returns-to-scale production technology  $F(T_2^{\omega})$  with  $F'(0) = \bar{A} - \alpha$  and F'' < 0. The resulting optimization problem is

$$\max_{T_2^{\omega}} u(e - q_1^{\omega} \cdot T_2^{\omega}) + u(e + F(T_2^{\omega}))$$
 (5)

The first-order condition implies an aggregate demand function for assets by households given by

$$q_1^{\omega} = \frac{u'(C_2^{\omega})}{u'(C_1^{\omega})} \cdot F'(T_2^{\omega}) \tag{6}$$

where we denote period 1 and 2 consumption of the household as  $C_1^{\omega}$  and  $C_2^{\omega}$ . It can easily be seen that our parameter assumption imply that the demand for land is zero at a price of  $q_1^{\omega} = F'(0) = \bar{A} - \alpha$ , and that the price falls the more assets bankers sell, i.e.  $dq_1^{\omega}/dT_2^{\omega} < 0$ .

#### 2.3 Decentralized Equilibrium

An equilibrium in the economy consists of a set of allocations  $(c_1^{\omega}, c_2^{\omega}, t_2^{\omega}, T_2^{\omega})$  which satisfy the maximization problem (2) and the first-order condition (6) as well as the market clearing conditions, in particular that  $t_2^{\omega} + T_2^{\omega} = t_1$ .

The first-order conditions for the decentralized bankers can be expressed as

$$FOC(c_1^{\omega}):$$
  $\mu^{\omega} = 1 + \lambda^{\omega}$   $\bar{A} = \mu_1^{\omega} \left[ \alpha + q_1^{\omega} \right]$ 

When the banker is unconstrained, then  $\lambda^{\omega} = 0$  and the banker's valuation of liquidity is  $\mu^{\omega} = 1$ . As a result asset prices satisfy  $q_1^{\omega} = \bar{A} - \alpha$  and there are no fire-sales, i.e.  $t_2^{\omega} = t_1$ .

On the other hand, when liquidity is tight and the consumption non-negativity constraint is binding, i.e.  $\lambda^{\omega} > 0$ , then the valuation of liquidity by decentralized (DE) agents is

$$\mu_{DE}^{\omega} = \frac{\bar{A}}{\alpha + q_1^{\omega}} \tag{7}$$

The bankerwill set  $c_1^{\omega} = 0$ , but its liquidity position is still insufficient to cover the required investment needs of  $\alpha t_1$ . Therefore the banker needs to raise liquidity by selling some of its asset holdings. However, the more the bank sells, the more asset prices in the economy decline and so the less it obtains per unit of asset sold. This is our simple version of financial amplification. We can obtain the equilibrium quantity of assets sold by setting in the budget constraint (3) of bankers and solving that equation jointly with equation (6) determining the equilibrium asset price  $q_1^{\omega}$ .

Note that the effects of any shock under this constrained regime are magnified by financial amplification effects: suppose e.g. that the representative banker is constrained and experiences a small negative shock dx to its liquidity position. Then the partial equilibrium effect is that the banker is forced to sell an amount  $\frac{dx}{q_1^{\omega}}$  of its assets and it loses future production of  $\frac{\bar{A}dx}{q_1^{\omega}}$ . However, in general equilibrium the additional asset sale depresses the price  $q_1^{\omega}$  further, by an amount of  $\frac{dx}{q_1^{\omega}} \cdot \frac{dq_1^{\omega}}{dt_2^{\omega}}$ . By implication the banker receives less for all the asset sales  $t_1 - t_2^{\omega}$  that were already planned. It needs to increase its fire sales even further, and so on.

### 2.4 Social Planner's Optimum

Let us next investigate the behavior of a social planner who optimizes bankers' allocations. The social planner's objective (2) is the same as that of decentralized agents. However, whereas decentralized bankers take asset prices  $q_1^{\omega}$  as given, the social planner internalizes that the valuation of assets declines the more she sells. This changes the social planner's first-order condition on land  $t_2^{\omega}$  to

$$FOC(t_2^{\omega}): \ \bar{A} = \mu_1^{\omega} \left[ \alpha + q_1^{\omega} - \frac{dq_1^{\omega}}{dt_2^{\omega}} \cdot (t_1 - t_2^{\omega}) \right]$$

As long as bankers are unconstrained and  $\lambda^{\omega} = 0$ , we can see that  $t_2^{\omega} = t_1$  and the planner's first-order condition coincides with that of the decentralized banker. However, when period 1 liquidity declines to the point that the constraint on  $c_1^{\omega}$  becomes binding, then the social planner's valuation of liquidity becomes

$$\mu_{SP}^{\omega} = \frac{\bar{A}}{\alpha + q_1^{\omega} - dq_1^{\omega}/dt_2^{\omega} \cdot (t_1 - t_2^{\omega})} > \mu_{DE}^{\omega}$$
 (8)

The asset price  $q_1^{\omega}$  declines the more of the asset is sold from bankers to households (i.e. the smaller  $t_2^{\omega}$ ), since households will put the asset to a less productive use than bankers. Therefore the derivative  $dq_1^{\omega}/dt_2^{\omega}$  is clearly positive. We can summarize the result in the following proposition:

**Proposition 1** When financing constraints on bankers are binding, a social planner values liquidity more since he internalizes that higher liquidity would reduce the quantity of firesales required and would therefore mitigate the decline in asset prices and the tightness of economy-wide financing constraints.

This is the basis of the externality result in our paper: when financing constraints are binding, a decline in asset prices hurts all bankers since it reduces the amount of liquidity that they can raise from the sale of each unit of assets. Bankers take asset prices as given

since they realize that their individual behavior has only an infinitesimal effect on asset prices. However, the behavior of all bankers together can cause strong movements in asset prices.

### 3 Optimal Financing Decisions

In the previous section we demonstrated that decentralized bankers and a social planner value liquidity differently when constraints in the described economy become binding. However, in the simple model we discussed, both equilibria coincide since bankers and the social planner agree that the optimum under binding constraints is to sell the least amount of assets possible to raise the liquidity necessary to cover investment  $\alpha$  on the remaining assets. This section demonstrates the implications of the discussed undervaluation for the optimality of private financing decisions. For this purpose, we introduce an additional time period 0 in which bankers need to make a financing decision.

#### 3.1 Households' Demand for Bonds

We assume that there is a full set of Arrow-Debreu securities contingent on the state  $\omega$ , which is realized at the beginning of period 1. Each bond pays out one unit in the assigned state  $\omega$  and is bought by households at a state-contingent price  $M_1^{\omega}$  in period 0.

As in the previous section we assume that there is a generation of investors living from period 0 to period 1 that can buy the bonds of bankers. Their maximization problem is to decide how much of the bonds to buy in each period, given a market price  $M_1^{\omega}$ . We can describe this as

$$\max_{\{B_1\}} u(e - E[M_1^{\omega} B_1^{\omega}]) + E[u(e + B_1^{\omega})]$$

The resulting demand curve for bonds is

$$M_1^{\omega} = \frac{u'(C_2^{\omega})}{u'(C_1)} \tag{9}$$

The more bonds contingent on a state  $\omega$  the household buys, the higher his consumption  $C_2^{\omega}$  in period 2, the smaller the value of additional payoffs in that state, and hence the lower the price  $M_1^{\omega}$  that he is willing to pay. This results in a standard downward-sloping demand curve for bonds.

### 3.2 Bankers' Financing Decision

We assume that bankers need to finance an investment  $\alpha t_1$  in order to produce  $A_1^{\omega}t_1$  in period 1. Since their assets have a positive value  $q_1^{\omega} > 0$  in period 1, bankers have collateral

available to secure their loans.<sup>7</sup> Denoting the quantity of bonds contingent on state  $\omega$  that is sold by the banker as  $b_1^{\omega}$ , we can express their period 0 budget constraint as

$$\alpha t_1 = E\left[M_1^{\omega} b_1^{\omega}\right] \tag{10}$$

The Lagrangian associated with the resulting optimization problem can be expressed as

$$\mathcal{L}^{\mathcal{DE}} = E\{V(b_1^{\omega})\} - \nu\{\alpha t_1 - E[M_1^{\omega} b_1^{\omega}]\}$$

It is easy to see that the resulting first-order condition can be expressed as

$$-\frac{dV}{db_1^{\omega}} = \mu^{\omega} = \nu \cdot M_1^{\omega} \quad \text{or} \quad \frac{\mu^{\omega}}{E[\mu^{\omega}]} = M_1^{\omega}$$
 (11)

For decentralized bankers, the relevant shadow price is  $\mu_{DE}^{\omega}$ ; for the social planner it is  $\mu_{SP}^{\omega}$ .

#### 3.3 Equilibrium

We can denote the economy's decentralized (private) pricing kernel  $D^{\omega} = \frac{\mu_{DE}^{\omega}}{E[\mu_{DE}^{\omega}]}$ . A decentralized equilibrium in the economy in period 0 then requires that the bond market clears, i.e.  $b_1^{\omega} = B_1^{\omega} \ \forall \omega$  and that bankers equalize their pricing kernel  $D^{\omega}$  to households' pricing kernel  $M_1^{\omega}$  in every state, as captured by the right-hand condition of equation (11). This latter condition is common in optimal insurance problems.

Since bankers are risk-neutral, their marginal utility of payoffs is constant  $\mu^{\omega} = 1$  as long as they do not experience binding constraints. This implies that the optimal risk-sharing arrangement between bankers and households entails that bankers keep all production risk on their books and promise a constant payoff to households, which is equivalent to issuing a bond in the amount of their period 0 investment  $\alpha t_1$ . The gross interest rate R that households demand on such a risk-free bond would be defined by the equation  $R = u'(C_1)/u'(C_2) = u'(e - \alpha t_1)/u'(e + R\alpha t_1)$ . Bankers can commit to repaying a bond so long as their lowest possible return is sufficient to cover the period 0 investment with interest plus the period 1 investment, i.e.

$$A_1^{\omega} \ge \alpha (1+R) \ \forall \omega \tag{12}$$

If condition (12) is not satisfied for a given  $\omega$ , then bankers do not have the means to repay households the full amount of  $\alpha t_1 R$  in that state without selling off some of their asset holdings. The privately optimal risk-sharing contract between households and bankers then entails that bankers sell some of their assets while both parties agree to reduce the repayment  $b_1^{\omega}$  in that particular state and uniformly increase the repayments in unconstrained states.

<sup>&</sup>lt;sup>7</sup>Without loss of generality, we assume here that bankers can raise the optimum amount of finance in period 0 without experiencing binding borrowing constraints.

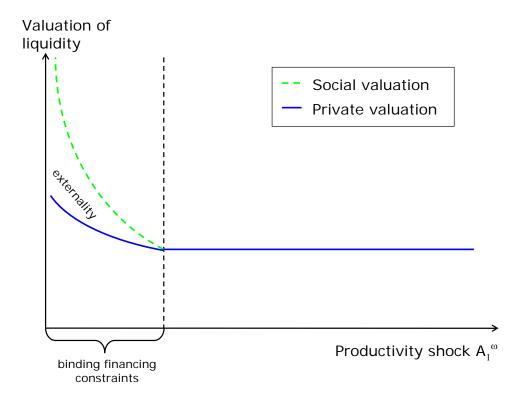


Figure 2: Private and Social Pricing Kernel

Both actions are costly for bankers: First, when they engage in fire-sales, asset prices decline so that their proceedings are less than the marginal product that they could have earned on the assets – this makes them act as if they were risk-averse. Second, when they repay more contingent bonds in some states than in other states, their total interest bill rises since the price  $M_1^{\omega}$  at which households are willing to buy bonds, as captured by equation (9), is a declining function of the quantity of bonds sold in a given state – this is because households are risk-averse.

Next we define the planner's social pricing kernel  $S^{\omega} = \frac{\mu_{SP}^{\omega}}{E[\mu_{SP}^{\omega}]}$ . As shown in figure 2, the decentralized and the planner's valuation of liquidity are identical in normal times when constraints are loose. However, when financing constraints are binding,  $\mu_{SP}^{\omega} > \mu_{DE}^{\omega}$  since the social planner internalizes that higher liquidity would relax the financing constraint and would mitigate the downward spiral in asset prices and economic activity.

Given the demand curves of households, the social planner's higher valuation of liquidity implies that he takes on fewer liabilities contingent on constrained states than decentralized bankers. At the same time, the denominator  $E[\mu^{\omega}]$  in equation (11) for the social planner will be higher, implying that he would sell more bonds contingent on unconstrained states. In other words, the social planner would contract more liquidity in bad states, i.e. insure better against the systemic risk of binding financing constraints than the decentralized agent.

Furthermore, note that the expected valuation of liquidity for the social planner is higher  $E[\mu_{SP}^{\omega}] > E[\mu_{DE}^{\omega}]$  if bankers are constrained in some states of the world. Since these expressions appear in the denominators of the pricing kernels  $S^{\omega}$  and  $D^{\omega}$ , we can conclude from expression (11) that the social planner would contract less liquidity in unconstrained states, i.e. he would pay for the lower repayments in bad states by issuing more debt contingent on good states of the world.

**Proposition 2** The social pricing kernel  $S^{\omega}$  is higher than the decentralized pricing kernel  $D^{\omega}$  in constrained states, but lower in unconstrained states. As a result, a social planner would preserve more liquidity than decentralized bankers in states in which financing constraints are binding, and repay more in states when constraints are loose.

#### 3.4 Comparative Statics

In the described economy, the productivity shock  $\omega$  constitutes systemic risk. In states of the world when productivity  $A_1^{\omega}$  is low, bankers' liquidity position will be strained. Since households are risk-averse, they require compensation for taking on some of this risk, and the decentralized equilibrium is characterized by the privately optimal trade-off between the cost of consumption volatility for households and the efficiency cost for bankers of having to sell assets at fire-sale prices when constrained. However, since decentralized bankers internalize only part of the social benefit of insuring against such fire-sales, they will take on too much systemic liquidity risk. As a result, financial amplification effects are magnified and the economy exhibits excessive volatility.

The magnitude of the externality is greater the higher the degree of risk aversion of households, since risk aversion makes them more reluctant to provide socially beneficial insurance. In the case that households are perfectly risk-neutral, they would be willing to fully insure bankers; as a result constraints would never be binding. Note also that households can perfectly diversify idiosyncratic risk; therefore they can insure bankers against this form of risk at no cost, and idiosyncratic risk would never lead to crises as long as risk markets are complete.

In our model above we assumed that bankers are risk neutral. While this was mainly a simplifying assumption, it can be shown that the externality declines if bankers become risk averse, since the risk aversion makes it privately optimal for them to take on more insurance, with the side effect of mitigating socially costly fire-sales. However, note that even if bankers were more risk averse than households, privately optimal risk-sharing would still entail that both parties hold some risk (as long as households are not perfectly risk-netural) and that binding constraints may arise and result in the described externality. We summarize our findings in the following.

**Proposition 3** The systemic externality is stronger the more risk averse households and the less risk averse bankers.

This may explain for example why hedge funds routinely expose themselves to large amounts of systemic risk that result in the liquididation of large positions and strong fire sale effects when adverse shocks are realized.

A decentralized financial system always allocates risks into the hands of those who are privately most willing to bear it, even if this leaves the financial system excessively exposed to systemic risk.

### 4 Applications

Having analytically characterized the systemic externality that is the subject of this paper, we now turn our attention to a number of applications, including the effects of anticipated government bailouts, the suboptimal incentives for raising new capital in the midst of crises, the possibility of contagion and bankers' excessive exposure to it and the role of rational expectations during crises.

#### 4.1 Anticipated Bailouts

The externality in this paper arises because decentralized bankers make their privately optimal insurance decisions without regard for systemic feedback effects. When a financial crises in such an economy is triggered, government authorities are usually tempted to intervene by providing bailouts to constrained agents so as to mitigate the downward spiral into which the economy is plunged. However, we can show that if such liquidity assistance is anticipated, then decentralized agents will fully undo it.

Assume that the government is committed to a transfer  $T^{\omega}$  that is in expectation revenueneutral, i.e. which is positive  $T^{\omega} > 0$  in case of binding constraints and negative  $T^{\omega} < 0$ in normal times so as to raise revenue for the transfers in crisis times. Assume that the government buys the respective bonds from households at time 0 and distributes the transfers at time 1 after the productivity shock is realized. The assumption of revenue neutrality implies that the total outlays of cash in period 0 are

$$E\left[M_1^{\omega}T^{\omega}\right] = 0$$

However, note that if we add these transfers to the problem described in the previous section, the first order conditions of all agents are unaffected: decentralized bankers chose their equilibrium allocations on the basis of an optimal tradeoff of risk versus return. If they receive one more dollar in period 1 of a given state  $\omega$ , they will sell one more bond contingent

on that state so as to restore their privately optimal equilibrium. This leads to the following proposition:

**Proposition 4** Any form of anticipated liquidity transfer to bankers is undone and will be ineffective.

This result is reminiscent of the common claim that government bailouts induce moral hazard. However, our result is even stronger than that: moral hazard is a phenomenon that occurs under asymmetric information when a principal (the government) cannot observe the inefficient actions of an agent (the banker). In our example, by contrast, it is common knowledge that any bailout will be undone, and that anticipated government transfer are therefore ineffective. Our finding is therefore closely related to Ricardian equivalence (Barro, 1974).

On the other hand, liquidity assistance can be effective if it is either unanticipated or if it is directed at agents who do not have access to financial markets and cannot undo the effects of the transfer, such as e.g. unemployed workers who might be forced to fire-sell assets.

Furthermore, note that if a bailout was expected in a particular state of the world and does not take place, the negative effects on the economy will be severe: The expectation of a bailout leads bankers to take on even larger risks than what is privately optimal in the absence of government intervention; their liquidity position after the shock is therefore strongly impaired in the absence of a bailout, and amplification effects magnify the impact of this unexpected shock to their liquidity position even further.

### 4.2 Raising New Capital

The undervaluation of liquidity that we analyzed also implies that bankers will undervalue the benefits of raising new capital during crises: any increase in liquidity would mitigate bankers' need to engage in fire sales or would enable them to buy assets from the fire-sales of other bankers. This would moderate the decline in aggregate asset prices and reduce the pressure on the balance sheets of other bankers. As a result the social value of raising new capital is higher than the private value – individual bankers have socially insufficient incentives to issue new equity.

Analytically, let us denote the cost of new equity in period one of a given state  $\omega$  by  $\gamma^{\omega}$ .<sup>8</sup> Given the concavity of  $V(\cdot)$ , bankers in our model that have access to equity markets raise capital until  $V'_{DE} = \mu^{\omega}_{DE} = \gamma^{\omega}$  in the given state  $\omega$ , i.e. until the private value of adding one more dollar of capital is equal to the cost of it.

<sup>&</sup>lt;sup>8</sup>In order to focus our analysis on the social efficiency of bankers' incentives to raise new capital, we take the cost of capital as given here. We could easily endogenize this, e.g. by assuming that households can buy an equity stake in bankers so as to infuse liquidity.

We demonstrated earlier that for any given level of firm liquidity, the social planner's marginal valuation of liquidity  $\mu_{SP}^{\omega} > \mu_{DE}^{\omega}$  is higher than that of decentralized agents in constrained states, since he internalizes that capital in the hands of bankers entails amplification effects through the effects on the valuation of collateral. By implication, when a decentralized banker has raised sufficient capital so that  $V_{DE}' = \gamma^{\omega}$ , the social planner's valuation of liquidity is higher and he would raise further capital until  $V_{SP}' = \mu_{SP}^{\omega} = \gamma^{\omega}$ . This implies that the social planner is willing to give up a larger share of the banker's ownership to new equity holders.

**Proposition 5** A social planner would raise more new capital in crisis states than decentralized bankers.

### 4.3 Contagion

Our framework also provides a natural environment to analyze financial contagion between various sectors in the economy. For the purposes of our analysis, we define contagion as a situation when shocks in a sector of the economy that has no direct relationship with bankers nonetheless spills over to bankers' balance sheets and causes financing constraints on bankers to bind.

Since we assumed a complete set of Arrow securities in period 0 of our model, the market equilibrium in that period is the outcome of the privately optimal risk-sharing decisions of all agents in the economy. In our benchmark model, bankers were the only sector that was subject to aggregate productivity shocks. However, if there is another sector in the economy that is subject to some aggregate shock, contagion between that sector and bankers can occur. This will typically take place through different channels, e.g. contagion through asset prices or contagion through liquidity flows. The first form occurs when another sector in the economy experiences an adverse shock and sells off the same productive asset t that bankers are holding. In that case, bankers will be willing to buy up assets at the efficient market price  $q_1^{\omega} = \bar{A} - \alpha$  as long as they have sufficient liquidity. In case their liquidity is exhausted, households will buy the remainder and asset prices will decline, which will reduce the value of bankers' collateral and set in motion amplification effects. The second form occurs if households are subject to additional sources of uncertainty (either directly or indirectly through bankruptcy risk) and transfer some of their risk through contingent financial contracts to bankers.

<sup>&</sup>lt;sup>9</sup>Strictly speaking, our benchmark model does not capture these effects, since we assumed the collateral value of assets in period 1 to be zero. However, a model with more than three time periods where collateral values are affected by adverse shocks, such as Kiyotaki and Moore (1997), could easily be modified to account for this form of contagion.

As the contagion effects of all the discussed cases will be similar, we focus on the second channel. We assume for simplicity that there is no uncertainty in bankers' asset holdings, i.e. that  $A_1^{\omega} \equiv \bar{A}$ , but that households are holding shares in a risky business operation (in addition to their loans to bankers) that pays off  $Z_1^{\omega}$  in period 1 with  $E[Z_1^{\omega}] = 0$ . Consider a state  $\omega$  in which  $Z_1^{\omega} < 0$ , i.e. in which households experience a loss on the risky asset holdings. By implication they will value payoffs from bankers in that state more highly, i.e. they are willing to buy Arrow securities that pay off in that state at a higher price  $M_1^{\omega}$ , as captured by their pricing kernel (9). As long as bankers are unconstrained in all states, their risk-neutrality implies that they are willing to promise additional payoffs in states with low realizations of  $Z_1^{\omega}$  and correspondingly reduce their payoffs in states with high realizations of  $Z_1^{\omega}$ . If bankers are unconstrained, households can in fact transfer the entire risky payoff  $Z_1^{\omega}$  onto households' balance sheet at no cost. This will be the case as long as

$$Z_1^{\omega} + \left[ \bar{A} - \alpha (1+R) \right] t_1 \ge 0 \ \forall \omega$$

However, if households' losses on  $Z_1^{\omega}$  are sufficiently large that the above condition is violated, then bankers will become constrained in those states. However, households' risk aversion will make it optimal for bankers to sell off some of their assets so as to provide insurance to households, i.e. the risk is borne partly by bankers, for which it is costly because they face binding constraints and need to engage in inefficient fire sales, and partly by households, for whom it is costly because they are risk-averse.

We noted earlier in proposition 1 that decentralized bankers do not internalize the social benefits of liquidity in mitigating asset price declines when they are constrained. This insight also applies in the example we just discussed: the privately optimal insurance arrangement between households and bankers provides so much liquidity from bankers to households that bankers end up constrained. Again, they do not internalize the full social costs of these constraints and therefore provide more contingent liquidity to households than what is socially optimal.

While we have used Arrow securities to describe contingent payoffs in our abstract model, contracts that provide contingent liquidity services are widespread in the real world. Let us discuss three concrete examples: First, bankers provide contingent liquidity services by borrowing in the form of short-term debt, which is routinely rolled over for long periods of time. However, as soon as households experience special liquidity needs, e.g. because of a shock in another sector in the economy, they can refuse to roll over and the debt is immediately due. For commercial banks, one of the most common forms of short-term debt is sight deposits. Secondly, bankers offer contingent liquidity by extending credit lines, which can be drawn upon by households whenever they experience liquidity needs. Thirdly, modern bankers increasingly participate in futures markets and take on obligations that are

explicitly contingent. Our analysis shows that whenever contingent liquidity services make constraints on bankers binding, they undervalue the social cost of these liquidity services. Therefore they take on too much liquidity risk and are excessively vulnerable to contagion:

**Proposition 6** Contagion from other sectors of the economy to bankers can arise through asset price channels or through contingent liquidity flows. Since bankers undervalue the social cost of constraints in crisis times, they do not sufficiently insure against financial contagion.

### 4.4 Role of Rational Expectations

It is often argued that many financial crises are so severe because market participants did not expect that some asset prices would decline so strongly, i.e. because of a failure of rational expectations. While our model is formulated in a rational expectations framework, we can show in our setup that the social costs of expectational errors are by an order of magnitude higher in states of the world when constraints are binding than when they are loose.

When constraints are loose, a marginal (unexpected) change in bankers' liquidity position arising from an expectational error can be absorbed by reducing consumption (or, in a model of risk-averse bankers, by smoothing the shock over time). On the other hand, if financing constraints are binding, an unexpected shock affects the amount of fire sales of assets that bankers engage in; this feeds back into asset prices and into what they receive on their previous fire-sales. In other words, the impact of expectational errors in constrained states of the world on social welfare is amplified.

Assume for example that bankers in period 0 are over-optimistic about future productivity  $A_2^{\omega}$  in a given state  $\omega$  and expect asset prices in that states to be  $q_1^{\omega+} = q_1^{\omega} + \Delta$ , where  $q_1^{\omega}$  is the correct value. In period 1 they find out about the true realizations of  $A_2^{\omega}$  and  $q_1^{\omega}$ , implying an unexpected negative shock to their net worth of  $t_1\Delta$ . If borrowing constraints in period 1 are loose, then the unexpected negative shock is simply absorbed by a reduction in consumption and there are no efficiency implications for the production side.

By contrast, if borrowing constraints are binding, then the unanticipated shock not only affects consumption, but also reduces the amount of capital that can be raised through fire sales by  $\Delta(t_2^{\omega} - t_1)$ . As a result, more land needs to be sold, i.e.  $\frac{dt_2^{\omega}}{d\Delta} = \frac{t_2^{\omega} - t_1}{q_1^{\omega}}$ , which in turn depreciates asset prices further, amplifies the financial loss and so forth.

### 5 Policy Implications

### 5.1 First-Best Policy Measures

The externality in our setup stemmed from the failure of decentralized bankers to internalize the effects of their risk-taking decisions on asset prices and by implication the effects on the financial constraints faced by other bankers. First-best policy measures against the described systemic externalities would attempt to break the feedback cycle underlying the financial amplification effects. In our example there are two ways of doing so, by alleviating financial constraints and by preventing asset prices from falling.

However, both of these measures are extremely problematic in practice: First, let us discuss the effects of relaxing financial constraints. In an ideal Walrasian capital market financing and investment decisions can be made separately from each other, and capital is always allocated to the sector that can earn the highest return on it, i.e. to bankers in our analytical example. If this first-best equilibrium can be reached, relaxing constraints would be optimal. However, in practice market participants are always subject to some equity requirement, and relaxing constraints usually takes the form of allowing for increased leverage. This makes the potential for financial amplification effects even more severe, exacerbating the externality that we identified above. Secondly, providing price guarantees on asset prices is fraught with strong moral hazard problems and should therefore be used only with extreme caution, or not at all.

Given the problems associated with first-best policy measures, financial regulators have routinely resorted to second-best measures that induce market participants to take precautions against some of the risk they are holding on their balance sheets.

### 5.2 Procyclicality and Basel II

Following the principle primum non nocere an important implication of our findings is that capital market regulations should not introduce additional sources of pro-cyclicality and financial amplification (see e.g. Catarineu-Rabell et al., 2005): when the riskiness of the assets held by a financial institution rises, current regulations require they need to set aside more capital. In times of systemic crisis this can make it necessary for them to engage in fire sales of some of their holdings, which in turn entails financial amplification effects. Our analysis indicates that banks will not fully internalize the systemic cost of the resulting asset price declines, and that it is privately optimal for them to take on excessive systemic risk.

It has been argued that transparency requirements in conjunction with the market discipline embodied by pillar 3 of the Basel accord would induce transparent banks to adopt rules that smooth their capital holdings throughout the business cycle (see e.g. Gordy and

Howells, 2006, for a discussion of this argument). In fact, our analysis suggests that, in the absence of new regulations against systemic risk-taking, markets would punish banks that behave socially responsibly and would reward banks that take on socially excessive risks, since maximizing shareholder value involves excessive risk-taking.

However, the systemic externalities analyzed in this paper are more general and extend far beyond any pro-cyclicality introduced by the Basel regulations; they stem from market imperfections that exist even in the absence of any financial regulations, i.e. from the inherent pro-cyclicality of capital markets. This creates a case for active government intervention to discourage excessive systemic risk-taking. Current banking regulations require banks to limit their individual risk-taking, but do not adequately distinguish between risk that is specific to an individual institution and systemic risk. In the following subsection we will develop a theoretical framework that motivates such regulations from micro-foundations, i.e. from the discussed externality.

#### 5.3 Second-Best Measures: Regulating Systemic Risk-Taking

In light of the systemic externality that we have identified in this paper, the task for a financial regulator who employs second-best policy measures is to induce market participants to internalize the potential social costs that they impose on others by taking on excessive systemic risk. A natural way of doing this is to impose regulations that increase the cost of holding systemic risks to banks. For simplicity we will formulate our policy measures below in the form of traditional taxes. In practice, regulations of the banking system usually take the form of capital adequacy ratios (which have tax-like effects since bank capital is costly).

Let us define the difference between the private and social valuation of liquidity in period 1 as the externality kernel  $\tau^{\omega}$ :

$$\tau^{\omega} = \mu_{SP}^{\omega} - \mu_{DE}^{\omega} \tag{13}$$

This  $\tau^{\omega}$  measures the un-internalized social cost of making a payment of one dollar in state  $\omega$ . It is straightforward to see that a standard Pigovian tax in the amount of  $\tau^{\omega}$  on any payoff made in state  $\omega$  could restore constrained social efficiency.

To relate our discussion from Arrow-Debreu assets to real world financial assets, assume that bankers in our model have sold a financial asset with a state-contingent profile of payoffs  $X^{\omega}$  in period 1. The optimal period 0 tax that makes bankers internalize the social cost of selling this claim would be

$$\tau^* = E[\tau^\omega X^\omega]$$

To gain some intuition for this, let us compare the optimal tax on a risk-free one-dollar bond and on an asset with a face value of one dollar and payoffs that are indexed to the systemic risk factor  $A_1^{\omega}$ . For the risk-free bond,  $X^{\omega} \equiv 1$  across all states of nature<sup>10</sup>, including constrained states. The optimal tax on such a bond is therefore  $E[\tau^{\omega}]$ .

On the other hand, the return on the indexed security moves in parallel with the state of productivity  $A_1^{\omega}$ . To guarantee that the expected payoff is unity we normalize the payoffs of one unit of the security to  $\frac{A_1^{\omega}}{E[A_1^{\omega}]}$ . Selling such indexed securities diversifies systemic risk away from bankers. Using the formula above we can therefore see that the optimal tax (or capital adequacy requirement) on selling such a claim is

$$E\left[\frac{\tau^{\omega}A_{1}^{\omega}}{E[A_{1}^{\omega}]}\right] = E[\tau^{\omega}] + Cov\left(\tau^{\omega}, \frac{A_{1}^{\omega}}{E[A_{1}^{\omega}]}\right) < E[\tau^{\omega}]$$

since the covariance between the two terms is negative (the social valuation of liquidity is high when the state of productivity is low). In a regulatory framework that addresses the systemic externalities arising during financial crises, what matters for the determination of taxes/capital adequacy requirements is not the risk inherent in a given asset, but the correlated systemic risk that the bank takes on that has the potential of leading to systemwide fire sales and financial amplification effects.

To whom shall such a tax or capital adequacy requirement apply? Any financial market participant who might potentially be forced to engage in fire-sales, including hedge funds and other actors in the so-called "shadow financial system," is prone to imposing an externality on other market participants, because he does not internalize the price effects of his fire sale and the consequences on the financing constraints of other market participants. Therefore any institution that might be forced to engage in fire-sales during systemic crises should be covered by the discussed regulations.

In our theoretical model above, bankers needed liquidity because of the requirement to invest  $\alpha t$  every period. In the real world, financial amplification effects often arise when leveraged market participants suffer losses and engage in fire sales so as to unwind their leverage and e.g. meet margin calls. Institutions with high leverage are therefore particularly prone to creating systemic externalities.

### 5.4 Socially Risk-Neutral Probabilities

It is a standard result in finance that pricing kernels can alternatively be represented as a risk-neutral probability measure that weighs states against which agents are risk-averse more highly. We can apply a similar transformation to the social planner's social pricing kernel. If regulators can instruct banks to employ the regulator's risk-neutral probabilities in their risk management systems, the systemic externality that is the topic of this paper would be alleviated.

 $<sup>^{10}</sup>$ For simplicity we abstract from the possibility of default here.

Analytically the socially risk-neutral probabilities can be obtained from the standard formula

$$\nu^{\omega} = \frac{\pi^{\omega} \mu_{SP}^{\omega}}{E[\mu_{SP}^{\omega}]}$$

and the true social value of an asset or cost of a liability with payoffs  $X^{\omega}$  can be expressed as  $E_{\nu}[X^{\omega}]$ , where  $E_{\nu}[\cdot]$  represents the expectations operator under the socially risk-neutral probability measure  $\nu$ . Note that this socially risk-neutral probability measure weighs states of the world in which constraints are binding and changes in liquidity entail amplification effects more highly than what would be indicated by a traditional risk-neutral probability measure. The latter in turn assigns more weight to such states than the objective probability of the state.

### 5.5 Systemic Stress Tests

Rational decentralized agents should find it optimal to invest in risk models that analyze systemic feedback effects in order to form rational expectations about future asset prices. However, such systemic risk models are to an important extent public goods, and hence that it is likely that there is under-investment in them from a social point of view. Furthermore, some of the information to make informed forecasts, e.g. the risk exposure of agents such as hedge funds, may simply not be publicly available.<sup>11</sup>

In such an environment, there is a role for central banks (or, even better, an international consortium of central banks) to conduct systemic stress test that simulate the aggregate behavior of all market participants in case of financial market turmoil, including actors in the shadow financial system. Technically speaking, the role of a regulator in such a setup would be to compile the asset demand/supply curves of individual financial sector participants and provide the resulting probability distribution of asset prices (or, better, a range of scenarios), taking into account systemic feedback effects. The private sector can in turn use these prices to "stress test" their private risk models, and re-derive their asset demand/supply curves given the new information. In the end, the probability distribution of asset prices is a fixed point of the two operations. In equilibrium this point will coincide with the pricing function give by equation (6).

<sup>&</sup>lt;sup>11</sup>In fact, financial market participants might even have incentives to keep as much of their risk models as possible private, since building a good risk model is one of the dimensions along which financial institutions compete with each other.

### 6 Conclusions

It is well known that financial markets are inherently pro-cyclical, i.e. that endogenous financing constraints loosen in good times and tighten in bad times. It is just as well known that this phenomenon can entail financial amplification effects: in case of strong negative shocks, for example, many bankers experience binding borrowing constraints, which require them to cut back on investment. This depresses asset prices further, deteriorates their balance sheets, leads to tighter financing conditions etc.

The contribution of this paper is to demonstrate that such financial amplification effects introduce an externality into the economy that leads individual bankers to undervalue liquidity in crisis states. Small agents take asset prices – and the tightness of financing conditions – as given and do not internalize the general equilibrium effects of their actions on prices and constraints. They do not realize e.g. that fire sales during crises depress asset prices, which trigger amplification effects that hurt other bankers in the economy.

The undervaluation of liquidity in crisis times in turn leads to a number of distortions: bankers take on too much risk in both their financing and investment decisions; more generally they over-borrow and over-invest; they also under-value the benefits of raising new capital during crises. Our paper develops a simple model that allows us to analytically examine these inefficiencies and investigate several other related questions, such as the effectiveness of anticipated government bailouts and the welfare costs of expectational errors.

Finally, our paper provides clear analytical guidelines for a new regulatory framework of macro-prudential capital adequacy requirements that account for systemic risk and systemic externalities, with the goal of reducing financial instability and avoiding future systemic financial crises.

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