Domestic or global imbalances? Rising inequality and the fall in the US current account

Tobias Broer*

First version: October 2008. This version: October 2009

Abstract

This paper shows how the rise in individual income risk in the US since the 1980s might help explain the fall in its foreign asset position. The key to this result is endogenous financial deepening in an open economy with participation-constrained domestic financial markets. More volatile income makes individuals less inclined to default on financial contracts as this triggers exclusion from future financial trade. Lower incentives to default, in turn, increase the insurability of income shocks, thus lowering the need for precautionary savings. My theoretical results show that, contrary to the case of unconstrained complete markets, individual participation-constraints guarantee a well-defined stationary equilibrium at a given world interest rate. Based on an analytical solution to the stationary consumption distribution, I show that higher income risk can lower mean consumption and aggregate asset holdings. Consumption inequality, on the other hand, is almost entirely determined by the level of world interest rates, and remains largely unaffected by changes in income risk. A quantitative exercise shows that the observed rise in individual income risk in the US since the 1980s can explain a significant fall in net foreign assets.

JEL Classification Codes: D31, D52, E21, F21, F41

Keywords: Current Account, Global Imbalances, Heterogeneous Agents, Inequality, Incomplete Markets, Participation Constraints, Default

---

*European University Institute, Florence, Italy. Contact: tobias.broer@eui.eu. I am indebted to Morten Ravn, Giancarlo Corsetti and Ramon Marimon for their support with this work. Also, I would like to thank Dirk Krueger, Assaf Razin, and seminar participants at the Econometric Society's European Winter Meetings 2008, the 2009 EER Young Talented Economists Clinic, Normac 2009, as well as seminars at the Board of Governors of the Federal Reserve, Cambridge University, the ECB, the Ente Einaudi, the Institute for International Economic Studies at Stockholm University, Queen Mary University of London, the Swiss National Bank, Universidad Carlos III, University of Bern, University of Bonn, and Warwick University, for helpful comments. All remaining errors are of course mine.

1 Introduction

Over the past 25 years, the US has experienced a significant rise in both cross-sectional income inequality and the uncertainty of individual incomes. Simple economic models suggest this should have increased individual savings at the same time as consumption inequality. But instead, during the same period, US savings fell, current account deficits accumulated to about 40 percent of 2004 GDP, while consumption inequality increased only little. Since 2007, while current account deficits narrowed, the declining value of the relatively risky US foreign investments increased the US net liability position further, thus reinforcing concerns about its sustainability. This paper shows how, in an open economy, a rise in individual income risk can actually lower the aggregate foreign asset position, while leaving consumption inequality largely unchanged. The crucial assumption is that individuals have access to complete domestic insurance markets, but also the option to default on contracts, at the price of permanent exclusion from financial trade. This restricts transfers under the insurance scheme to amounts that individuals find optimal to pay, rather than choose the outside option of default. Higher income risk increases individuals' incentives to remain insured and thus to honour contracts, which is equivalent to a financial deepening in the economy. Under these "debt-constraints" to complete domestic risk-sharing, I analyse the effect of changes in income risk on consumption volatility and aggregate savings in a small open economy. I analytically show that an increase in income risk can lower the mean of the stationary consumption distribution, thus decreasing the amount of stationary assets, while leaving relative consumption inequality unaffected. Also, I develop a new algorithm based on the associated planner's problem as in Marcet and Marimon (2009), to show quantitatively that the observed rise in individual income risk in the US between 1980 and 2003 can explain a significant fall in net foreign assets.

Figure 1 shows the large and, until recently, increasing US current account deficit since 1980. Understanding the reasons for the corresponding rise in foreign indebtedness is important, mainly because different explanations have different implications for its sustainability. For example, it has been argued that the fall in US net assets is a necessarily temporary phenomenon, linked to a strong rise in US house prices, that will eventually have to unwind (see e.g. Roubini et al 2004, Roubini 2005). Other authors, however, have attributed at least a part of this fall to changes in the structure of the world economy that imply a permanently lower US net asset position. Thus, Mendoza et al (2007) have focused on the impact of capital account liberalization in countries whose domestic financial markets are less developed relative to the US. In their model, once capital markets are liberalized, higher precautionary savings and lower appetite for risk in the rest of the world result in capital flows to the US concentrated in bonds, in line with the evidence. However, the underlying comparative advantage of deeper domestic financial markets in the US is exogenous to the model. In another contribution, Fogli and Perri (2006) show how the relatively more important reduction in US macro-volatility since 1980 implies a stronger reduction in the bufferstock savings of a representative US consumer than in other countries. But crucially, while international asset trade is limited to non-contingent bonds in their model, they assume domestic trade of a set of complete state-contingent assets that warrants the focus on representative national agents. This assumption, however, has been largely rejected by the data (see for example Zeldes (1989)). Moreover, as figure 1 shows, while US debt increased, cross-sectional domestic income inequality rose strongly, partly attributable to a rise in the uncertainty of individual incomes (see Krueger and Perri (2006), and more recently Heathcote et al (2008b)). And in the absence of perfect domestic risk-sharing, these changes in income risk will affect aggregate debt dynamics.

This paper analyses net asset positions in a simple open economy model that relaxes the assumption of a representative agent, and does not assume exogenous comparative financial advantage. Instead, it makes the depth of domestic financial markets depend endogenously on the riskyness of individual income. This allows me to look at the impact of changes in idiosyncratic income and consumption risk on aggregate savings and asset positions. But importantly, it also allows me to analyse the effect of international variables, such as interest rates, on individuals' decisions and, ultimately, the domestic consumption distribution.

If non-contingent debt was the main savings vehicle of the economy, as in Fogli et al (2006), an increase in individual income risk would yield a rise, not a fall, in equilibrium savings, together with higher consumption volatility. On the other hand, in an economy where domestic markets are complete, but individuals can default on contracts at the price of permanent exclusion from financial trade, the relationship between income risk and consumption volatility is known to be less simple. Krueger and Perri (2006) show that under this assumption of participation-constrained complete markets, a rise in income risk has two offsetting effects: first, it raises the income realizations of individuals who receive positive shocks, and thus, for a given upper limit to redistribution, increases the volatility of consumption. But higher income risk also makes the outside option of financial autarky, where it translates one-to-one into higher consumption volatility, less appealing. This second effect acts to increase the insurability of income shocks, and thus deepens financial markets and reduces consumption volatility. Krueger and Perri (2006) show that the latter, financial deepening effect becomes more important for high levels of income risk, causing consumption volatility to first rise and then fall as income risk increases. Aggregate savings mainly act as a precaution against this consumption volatility.

This paper shows analytically that the open economy setting breaks the closed economy link between consumption risk and precautionary savings. Particularly, relaxing individual debt constraints leaves relative consumption inequality largely unchanged. Rather, it can be interpreted as an increase in the country-wide borrowing capacity that leads to an increase in stationary debt holdings, or a fall in the net asset position. To derive these results, I first show that, unlike with unconstrained complete markets, the debt-constrained small open economy has a unique stationary equilibrium that does not depend on initial conditions. So individual participation-constraints "close small open economies" (Schmidt-Grohé et al 2003). The optimality conditions of an associated planner's problem, as in Marcet and Marimon (2009), allow me to solve analytically for the stationary consumption distribution even with standard, independent Markov processes for the incomes of a large number of individuals. The stationary equilibrium has the interesting feature that consumption follows a geometric distribution whose shape depends largely on the world interest rate, while its position is determined by participation constraints. Thus, looser participation constraints increase aggregate debt holdings and decrease aggregate consumption in stationary equilibrium. However, as mentioned above, the effect of higher income risk on participation-constraints depends on the initial level of income risk, and therefore the particular economy under analysis. A second part of the paper thus looks at the US example, and evaluates the effect of the observed rise in US income volatility on its net foreign asset position and the consumption distribution quantitatively. This analysis should ideally account for changes in income heterogeneity in both the US and its main economic partners during this period. Unfortunately, comprehensive cross-country data on the evolution of income risk are as yet unavailable, and in some cases unfeasible.[1] Comparative studies of simpler inequality measures have found that, apart from the United Kingdom, other OECD countries have experienced less important increases in income inequality since 1980 than the US (see e.g. Brandolini et al 2007). To focus on the open economy effect of the relatively large changes in income heterogeneity in the US, I first analyse their effect at an exogenously given world interest rate.[2] In a second exercise I analyse a two country general equilibrium model where the US trades bonds with a large developing country with less sophisticated domestic financial markets. To capture the change in income risk, I use the stochastic process of individual incomes in the US estimated by Krueger and Perri (2006) for the years 1980 and 2003. For the second country I choose a process in line with the observed change in inequality in China. To solve the model, I develop a new algorithm based on Marcet and Marimon (2009) to compute the stationary consumption

[1] Thus, in the UK, for example, household panel data have been collected only since the beginning of the 1990s. However, Heathcote et al (2008b) is one paper in a recent project to compare measures of individual inquality and income risk across countries. See http://www.econ.umn.edu/ fperri/Cross.html.

[2] The assumption of an exogenous interest rate has also been made in contributions concentrating entirely on the domestic consequences of increases in individual income volatility in the US. See for example Heathcote et al (2008a).

distributions and net asset positions. The results show that the increase in income risk in the US can indeed explain a significant part of the fall in the net foreign asset position, both at a given interest rate as well as in a general equilibrium exercise.

The rest of the paper is structured as follows: Section II describes the environment of a small open economy with debt-constrained domestic financial markets. Section III derives the analytical results on the basis of the associated planner's problem. Section IV reports the computational algorithm and quantitative results. An appendix contains most proofs.

2 A small open economy with debt-constrained domestic financial markets

This section presents a simple model of a small open economy where domestic financial markets are constrained by individual default, and defines the competitive equilibrium.

2.1 Agents, countries, time

The economy consists of a small country and a large rest of the world. The analysis focuses on the small country that takes prices of goods and assets traded with the rest of the world as given.
The small country is populated by a large number of individuals of unit mass. Individuals are indexed by i, located on a unit-interval $i \in \mathbb{I} = [0, 1]$. Time is discrete $t \in \{0, 1, 2, ..., \infty\}$ and a unique perishable endowment good is used for consumption.

2.2 The endowment process

The consumption endowment of agent i in period t, $z_{i,t}$, takes values in a finite set Z: $z_{i,t} \in Z = \{z_1 > z_2 > ... > z_N\}, N \geq 2$. Endowments follow a stochastic process described by a Markov transition matrix F. F has strictly positive entries, is identical across agents, monotone (in the sense that the conditional expectation of an increasing function of tomorrow's income is itself an increasing function of today's income), and has a unique ergodic distribution $\Phi_Z : \mathbb{Z} \rightarrow [0, 1]$, where $\mathbb{Z}$ is the power set of Z. Thus, in the long-run, aggregate income $Y = \int_{\mathbb{I}} z_i$ is constant, while individual income fluctuates.
Let s_t denote the state of the economy in period t, a vector containing individual incomes and asset holdings of all agents.

2.3 Preferences

Agents live forever and order consumption sequences according to the utility function

$$U = E_{s_0} \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \tag{1}$$

where E_{s_0} is the mathematical expectation conditional on s_0, $0 < \beta < 1$ discounts future utility, $c_{i,t}$ is consumption by agent i in period t, and $u : R^+ \rightarrow R$ is an increasing, strictly concave, continuously differentiable function that satifies Inada conditions and is identical for all agents in the economy.

2.4 Asset markets

I choose a specification of the economy similar to that by Alvarez and Jermann (2000), amended for the international setting. Agents engage in sequential trade of a complete set of state-contingent bonds domestically, but international asset trade is limited to non-contingent bonds.[3] Individual endowment realisations are verifiable and contractable, but asset contracts are not completelely enforceable: at any point, individuals can default on their contractual payments at the price of eternal exclusion from financial markets. Thus the total amount an agent can borrow today against any income state z_j tomorrow is bounded by the option to default into financial autarky. There, consumption is forever equal to income. Given the markov structure of income, the value of default as a function of the vector of current income z can be written as

$$W(z) = \sum_{t=0}^{\infty} (\beta F)^t U(z) = (I - \beta F)^{-1} U(z) \tag{2}$$

I denote holdings of bonds and Arrow-Debreu securities paying off in state s_t by b and $a(s_t)$ respectively. In any state s_t, $V(z(s_t), a(s_t), b_t)$ is the contract value as a function of income $z(s_t)$ and current asset holdings $\{a(s_t), b_t\}$.

As in Alvarez and Jermann (2000) individual i's participation constraint for any state s_{t+1} tomorrow can be written as a constraint on the claims she can issue against s_{t+1} income. This borrowing constraint is "not too tight" in the words of Alvarez and Jermann (2000) if it assures participation but does not constrain contracts otherwise

$$a_i(s_{t+1}) + Rb_{i,t+1} \geq A_i(s_{t+1}) = min\{\alpha(s_{t+1}) : V(z_i(s_{t+1}), \alpha(s_{t+1}, 0)) \geq W(z_i(s_{t+1}))\} \tag{3}$$

[3] This is non-restrictive as there is no aggregate risk and the law of large numbers holds. It requires, however, no default on foreign debt on a country level. In a previous version of this paper I show that Broner and Ventura's (2006) result applies to my setting. Thus, perfect secondary markets prevent governments from defaulting on agents' foreign liabilities.

Note that bonds are redundant in this setting, although including them facilitates the setup of the planner's problem in an open economy where aggregate bond holdings, denoted B, are potentially non-zero.

Importantly, the portfolio constraint (3) limits the issuance of assets that demand net repayments in high income periods, when the outside option of default is most attractive. On the one hand, this reduces transfers from high to low income individuals under insurance contracts. But on the other, it defines a maximum level of debt that individuals, and thus the country on aggregate, can sustain. The attractiveness of default during periods of high individual income, determined by the value of the outside option of financial autarky W, is thus the main determinant of the aggregate net asset position in stationary equilibrium. The next section briefly considers how W is affected by changes in income risk.

2.5 Income risk and the value of default

Under the assumption that default leads to exclusion from all financial transactions, this value of autarky equals the expected utility of individual income streams given by (2). The assumption of monotonicity of both utility and transitions ensures that these autarky values are increasing in the level of current income. However, the relationship between autarky values and income risk is more difficult to characterise. Particularly, a change in risk can come via changes in transition probabilities F, via a change in the support of endowments Z, or both. In this paper, I follow Kehoe and Levine (2001) and define a rise in risk as a mean-preserving spread to the income support Z. This, however, does not imply mean-preserving spreads to the conditional income distribution for all individuals. Rather, given persistence, it raises (lowers) current and expected future income for today's high (low) income earners. So for low levels of uncertainty, higher risk increases both expected income and autarky values for the income-rich. However, although their expected income continues to rise, as a consequence of concave utility the prospect of negative shocks weighs more heavily on expected utility as higher risk decreases income, and thus consumption, in low income states. Given Inada conditions, this effect necessarily outweighs the gain in expected income at some point. Thus, autarky values of high income individuals roughly follow an inverse U-shape relation with income risk. So we would expect portfolio constraints to first become tighter, and then loosen, as income risk rises. The analytical part of this paper shows that this is indeed the case. The quantitive section shows the location of a model calibrated to the US economy on this "Laffer curve" of default incentives.

2.6 The household's problem

In every period, households maximise their expected utility by choosing current consumption and assets subject to budget and borrowing constraints

$$V(z(s_t), a(s_t), b_t) = max_{c_t, \{a(s_{t+1})\}, b_{t+1}} \sum_{s=0}^{\infty} \beta^s u(c_{t+s})$$

$$s.t.\ c_t + \sum_{s_{t+1}} a(s_{t+1})q(s_{t+1}) + b_{t+1} \leq Rb_t + a(s_t) + z(s_t) \tag{4}$$

$$a(s_{t+1}) + Rb_{t+1} \geq A(s_{t+1}) \tag{5}$$

As shown in Alvarez and Jermann (2000) this problem has a recursive representation as

$$V(z(s), a(s), b) = max_{c, \{a(s')\}, b'} \{u(c) + \beta E_s V(z', a(s'), b')\}$$

$$s.t.\ c + \sum_{s'} a(s')q(s') + b' \leq Rb + a(s) + z(s)$$

$$a(s') + Rb' \geq A(s')$$

$$A(s') = min\{\alpha(s') : V(z(s'), \alpha(s'), 0) \geq W(z(s'))\}$$

where c, b', a' are policy functions of the state variables $(z(s), a(s), b)$.

2.7 Definition of competitive equilibrium

The competitive equilibrium in this economy is a set of asset prices $q(s'), R$, a set of individual decision rules $c, b', a'(s')$ with associated value functions $V(z, a, b)$
such that

1. $V(z, a, b)$ is the households maximum value function associated to the household problem given $q(s'), R$

2. $V(z, a, b)$ is attained by $c, b', a'(s')$

3. Markets for state-contingent assets clear
$\int_{\mathbb{I}} a_i(s_t) = 0,\ \forall s_t, t$

4. The interest rate on bonds is equal to the world interest rate R.

The competitive equilibrium is called "stationary" if prices and aggregate bond holdings are constant, and the distribution of individual consumption is stationary through time.

3 Analytical properties of the consumption distribution and aggregate savings in stationary equilibrium

In this section I show analytically how, unlike with unconstrained complete markets, individual participation constraints ensure the existence of a stationary equilibrium in a small open economy even when interest rates differ from the rate of time preference. I show how across stationary equilibria, a rise in income risk can leave consumption inequality unchanged, but decreases aggregate asset holdings if the initial level of income risk is high enough. Also, I show that market completeness does not help the most unfortunate individuals in this economy: both their current consumption and expected value from future consumption are the same as without any financial markets. Insurance, however, reduces the number of individuals in this situation significantly. To derive these results I exploit the constrained efficient nature of the economy that allows me to solve the associated planner's problem as in Marcet and Marimon (2009). I use this method to derive the closed form of the consumption distribution in the special case with two income values but a continuum of agents whose incomes follow identical independent Markov processes. Contrary to previous papers by Kehoe and Levine (2001) or Krueger and Perri (2006), this allows for potentially infinite history-dependence of individual consumption. Broer (2009b) generalises these results to an income process with $N > 2$.

3.1 The planner's problem and first order conditions

Alvarez and Jermann (2000) show that a version of the first welfare theorem applies to the closed economy version of this environment. The small open economy assumption changes aggregate feasibility constraints but, together with an appropriate No-Ponzi condition, leaves this result intact. This allows me to focus on participation-constrained efficient allocations. More particularly, I exploit the results in Marcet and Marimon (2009), and focus on the solution to the participation-constrained social planner's problem.

Marcet and Marimon (2009) show how the efficient competitive equilibrium allocation solves the following planners problem. For a given bounded measurable weighting function $\mu_{i,0} : \mathbb{I} \rightarrow R^+$ in a linear social welfare function $\Omega = \int_{\mathbb{I}} \mu_{i,0} E_0 \sum_0^\infty \beta^t u(c_{i,t})$ the problem of the planner is to distribute resources optimally subject to individuals' participation constraints and the aggregate resources of the economy

$$\mathbb{VV}(\Phi_{\mu_{i,0}}, B_0) = max_{\{c_{i,t}\}} \int_{\mathbb{I}} \mu_{i,0} \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \tag{6}$$

$$s.t. \int_{\mathbb{I}} c_{i,t} + B_{t+1} = \int_{\mathbb{I}} z_{i,t} + R_t B_t, \ \forall t$$

$$V_{i,t} \geq W(z_{i,t}), \ \forall t, i$$

$$B_t \geq -\frac{Y}{R-1}, \ \forall t$$

where the planner's maximum value $\mathbb{VV}$ is a function of $\Phi_{\mu_{i,0}}$, the initial distribution of multipliers induced by $\mu_{i,0}$, and aggregate bond holdings B_0. $V_{i,t}$ denotes the expected value of the consumption sequence the planner gives to agent i starting in period t, and the last line is a No-Ponzi condition on aggregate bonds B, which I assume to be 0 in period 0. Also, I assume that $\mu_{i,0}$ only takes a finite number of values.

Note that the problem in (6) is not recursive in the cross-sectional distribution of income. Intuitively, the planner optimally provides an increase in value $V_{i,t}$ to participation-constrained individual i by an increase in both current and future consumption. But this requires the planner to keep her consumption promise even if individual i receives a negative income shock tomorrow. The solution thus has potentially infinite history dependence. But Marcet and Marimon (2009) show how, based on the Lagrangian associated to the sequential planner's problem, this history-dependence can be encoded in a time varying value of individual welfare weights $\mu_{i,t}$. In particular, the assumptions on $\Phi_{\mu_{i,0}}$, utility and transition probabilities ensure that the problem is sufficiently well-behaved to have a saddle-point representation that is recursive in a time-varying distribution of weights $\Phi_{\mu_{i,t}}$ and aggregate bond holdings[4]

$$\mathbb{VV}(\Phi_{\mu_i}, B) = \inf_{\gamma_i \geq 0} \max_{\{c_i\}} \int_{\mathbb{I}} [(\mu_i + \gamma_i) u(c_i) - \gamma_i W_i] + \beta E[\mathbb{VV}((\Phi_{\mu_i'}, B')] \tag{7}$$

$$s.t. \int_{\mathbb{I}} c_i + B' = \int_{\mathbb{I}} z_i + RB$$

$$\mu_i' = \mu_i + \gamma_i \tag{8}$$

$$B_t \geq -\frac{Y}{R-1} \ \forall t \ \Phi_{\mu_i}$$

[4]To see this, note that the initial weighting function $\mu_{i,0}$ only takes a finite number of values, and that for every $t < \infty$ the set of possible income histories Z^t is finite and bounded. So the exogenous state space is the Euclidian Product of a countable number of compact sets, and thus, according to Tychonoff's theorem, compact. Also, given the No-Ponzi condition, aggregate bond holdings are bounded and thus lie in a convex compact set, implying that feasible consumption allocations are just a simplex, and thus a convex set, every period. With concave utility, the constraint set is therefore compact and convex, and non-empty since autarky is feasible and incentive-compatible. The Problem thus fulfills conditions A1 to A5 in Marcet and Marimon (2009), and therefore has a recursive saddle-point representation. For further detail, see the proof of uniqueness and existence in the Appendix.

where γ_i corresponds to the multiplier on i's participation constraint in the sequential problem (6). Note that the weights of individuals in the social welfare function are now updated every period to meet participation constraints.[5] And when γ_i is zero, so i is unconstrained, (8) ensures promise-keeping by the planner. Intuitively, by increasing multipliers the planner allocates a higher than expected consumption path to constrained individuals with positive income shocks, to keep them "happy" with the contract. The absolute weights of the remaining, unconstrained individuals are constant, but decline relative to those for individuals with positive income shocks. This leads to a gradual decline in consumption for these individuals until they either receive a positive income shock, or reach the level of constant consumption that, given prospects for future shocks, just meets the participation constraint corresponding to their income level. The solution of the planner's problem is a sharing rule $\Gamma : Z \times R^{+} \rightarrow R^{+2}$ that maps current weights μ_i and income shocks z_i into consumption c_i and new weights $\mu_i' = \mu_i + \gamma_i$.

The first order conditions[6] for individual consumption imply

$$\frac{U'(c_{i,t})}{U'(c_{j,t})} = \frac{\mu_{j,t} + \gamma_{j,t}}{\mu_{i,t} + \gamma_{i,t}} \tag{9}$$

Thus, since $U'(c)$ is decreasing, individuals with a higher weight receive higher consumption. Also, from the first order condition for aggregate bond holdings, the interest rate is tied to the ratio of the multipliers λ, associated to the aggregate feasibility constraint in (7)

$$R = \frac{\lambda}{\beta E[\lambda']} = \frac{\beta\lambda}{\lambda'} = \frac{U'(c_i)(\mu_i)}{\beta U'(c_i')(\mu_i + \gamma_i)} \tag{10}$$

where the second equality exploits the absence of aggregate uncertainty and the law of large numbers,[7] and the third uses the intratemporal optimality conditions for consumption. Importantly, the interest rate determines the slope of marginal utility for those consumers who are unconstrained tomorrow ($\gamma_j = 0$)

$$U'(c_i) = \beta R U'(c_i') \tag{11}$$

[5]Again, despite the continuum of agents, the values of multipliers remain countable, since $\mu_i' = \mu_i + \gamma_i$ is a function of current income and the past value of μ_i only. So, given my assumption of a countable support of $\Phi_{\mu_{i,0}}$, the number of individual multipliers remains countable.

[6]Note that continuously differentiable utility and a convex constraint set imply that the value function is differentiable. Also, Inada conditions and the concavity of the utility function imply that the first order conditions, together with participation constraints, are sufficient to characterise the optimum.

[7]Since the state space is finite every period, the assumption of independent shocks over a continuum of agents ensures that the law of large numbers applies. Formally, $\int_{\mathbb{I}} x(i,t) = \sum_{Z \times \{\mu_{i,t}\}} \int_{\mathbb{I}} I_{\mu,z} = \sum_{Z \times \{\mu_{i,t}\}} \amalg_{\mu_t,z}$ where $I_{\mu,z}$ is the indicator function of the set $\{i : \mu_i = \mu, z_i = z\}$ and $\amalg_{\mu_t,z} \in [0,1]$ is the mass of individuals with weight μ and income z in period t. So we can replace integrals with summation over countable sets. Given the continuum of agents $i \in \mathbb{I}$, this ensures that the law of large numbers applies. So the joint distribution of income and weights μ tomorrow is known today. On the law of large numbers in economies with a continuum of agents and independent idiosyncratic risk, see Uhlig (1996).

Given monotonicity of U', this provides a law of motion for the consumption of unconstrained agents. With CRRA preferences $u = \frac{c^{1-\sigma}}{1-\sigma}$, we can solve for c_i' as

$$c_i' = (\beta R)^{\frac{1}{\sigma}} c_i \tag{12}$$

So the lower R, the faster falls consumption of unconstrained agents. With CRRA preferences we can simplify equation (10) further by solving for c_i in terms of the multipliers, and integrating across agents, to get

$$R = \frac{1}{\beta}\left[\frac{C'}{C}\frac{\int_{\mathbb{I}}(\mu_i^{1/\sigma})}{\int_{\mathbb{I}}(\mu_i + \gamma')^{1/\sigma}}\right]^{\sigma} \tag{13}$$

Thus, a fall in the world interest rate either lowers aggregate consumption growth, or increases average growth in individual multipliers, or both. The first effect is standard and leads to non-existence of a stationary equilibrium in small open economies with unconstrained complete markets. The second effect comes from the participation-constrained nature of risk-sharing. It implies, for example, that unless there is perfect insurance ($\gamma_i = 0, \forall i$), the equilibrium closed economy interest rate is below the time preference rate, a result well-known from Alvarez and Jermann (2000). More generally, binding participation constraints increase the shadow value of future resources relative to today's. This is because current consumption only relaxes today's participation constraints. Future consumption relaxes all previous participation constraints, including today's, via the increase in continuation utility under the contract. So when more agents hit their participation-constraints every period, or when a given set of binding constraints becomes more binding, the planner reallocates aggregate consumption to the future. Below I show that this second effect ensures the existence of a stationary equilibrium in this economy.
Note that if $\frac{U'(z_1)}{\beta U'(z_N)} > 1$, (10) immediately yields a minimum interest rate $R_{min} > 1$ below which all individuals simply consume their endowments. This is because, whenever $1 < R < R_{min} = \frac{U'(z_1)}{\beta U'(z_N)}$, there are no participation-compatible unconstrained transitions in (11). So individual consumption is simply equal to individual income.

3.2 Existence, uniqueness and stationarity of equilibrium

The closed economy version of this economy is one of the examples discussed in Marcet and Marimon (2009). An appendix proves that the planner's problem has a unique solution also in this small open economy setting. However, in both cases, we do not know if this solution is stationary in terms of the long-run behaviour of aggregate consumption and its distribution across individuals.
For example, in a standard small open economy with complete domestic markets that are not participation-constrained, $R < 1/\beta$ implies that consumption levels are forever declining. So

no stationary solution exists. With participation constraints, however, this is not an equilibrium, as the total value that the planner can distribute to individuals declines with the level of aggregate resources. A permanently downward sloping path of aggregate consumption thus necessarily violates individual participation constraints at some point in the future. Instead, in an equilibrium with participation constraints, the aggregate consumption decline slows down as participation constraints become more binding. This is because for given weights $\mu_i + \gamma_i$, individual contract values decline with aggregate resources. This requires stronger increases in relative weights of participation-constrained indivdiuals γ_i. But more binding participation constraints increase the marginal value of future resources according to equation (10). This slows the decline in aggregate consumption until it settles down at a stationary level, with a corresponding stationary distribution of individual consumption and aggregate debt holdings. Equation (13) shows how the individual consumption volatility, expressed there as growth in average individual planner weights, effectively replaces the non-stationarity of aggregate consumption. In this way, individual participation constraints provide an additional way of "closing small open economies" (Schmidt-Grohé et al 2003).

In the resulting unique stationary equilibrium, consumption in all states is pinned down by participation constraints and the law of motion of unconstrained agents (11) given the exogenous interest rate R. Broer (2009b) uses this to show, by construction, the existence of equilibrium, and the characteristics of the consumption distribution, for an economy similar to that considered here. The following section uses a closed form example to illustrate the characteristics of the stationary distribution of consumption, and to show how aggregate foreign assets in this stationary equilibrium are effectively determined by individual income risks. To do this, I first show how, for a given interest rate R, the position of the consumption distribution moves up and down with autarky values. Then I show how the latter follow an inverse U-shaped relationship with income risk, and what this implies for foreign asset holdings.

3.3 A closed form example

Consider an economy in which the income process described in the previous section takes only two values $\{z_h, z_l\} = \{y_0 + \frac{1}{\nu}\epsilon, y_0 - \frac{1}{1-\nu}\epsilon\}$, $\epsilon \geq 0$, where $\nu = \frac{1-q}{2-q-p}$ is the stationary mass of high-income individuals, for transitions given by $F = [p, 1-p; 1-q, q]$. Monotonicity and absolute continuity require $0 < 1-q < p < 1$. Also, I assume income has persistence which is not too different in high and low income states:

$$p, q > 1/2 \tag{14}$$

$$\frac{\beta - 1}{\beta} < p - q < \frac{1-\beta}{\beta} \tag{15}$$

I define a "**marginal rise in income risk**" as a small widening of the income support $d\epsilon > 0$. The specification of Z ensures that this is a mean-preserving spread for all values of p, q, and thus leaves aggregate resources unchanged.
This example is a generalisation of that considered, in an economy with capital, by Kehoe and Levine (2001), or more recently by Krueger and Perri (2006), who, however, assume independent transitions.

3.3.1 The stationary consumption distribution

Remark 1 *There exists a unique stationary equilibrium with a distribution of consumption $\Phi_C : \mathbb{C} \subseteq R^+ \longrightarrow [0,1]$. If $1 < R < R_{min}$, the stationary distribution of consumption is equal to that of income, so $\Phi_C = \Phi_Z, \mathbb{C} = Z$. If $R_{min} < R < 1/\beta$, Φ_C is*

$$\Phi(c_1) = \frac{1-q}{2-q-p} = \nu \tag{16}$$

$$\Phi(c_i|_{1<i<m}) = \nu(1-p)q^{i-1} \tag{17}$$

$$\Phi(c_m) = (1-\nu)q^{m-1} \tag{18}$$

for

$$c_1 = \{\frac{(1-\sigma)(1-\beta q(\beta R)^{\frac{1-\sigma}{\sigma}})}{1+\beta(1-p-q)(\beta R)^{\frac{1-\sigma}{\sigma}} - (1-p)\beta^m q^{m-1}(\beta R)^{\frac{m(1-\sigma)}{\sigma}}}$$

$$[\frac{1-\beta(p+q)-\beta^2(1-p-q)}{1-\beta q}W_h - (1-p)\beta^m q^{m-2}(qW_l - \frac{(1-q)W_h}{1-\beta q})]\}^{\frac{1}{1-\sigma}}$$

$$c_i = c_1(\beta R)^{\frac{i}{\sigma}}, 1 < i < m$$

$$c_m = y_0 - \frac{1}{1-\nu}\epsilon \tag{19}$$

$$m = min\{x \in N : x > \frac{\sigma[ln(y_0 - \frac{1}{1-\nu}\epsilon) - ln(c_1)]}{ln(\beta R)}\} \tag{20}$$

Proof

This closed form of the consumption distribution is proved in detail in Broer (2009b). To see that it is bounded below by z_l, note that an individual at minimum consumption c_m is necessarily constrained today and tomorrow (from stationarity and minimality of c_m). So c_m is determined from her participation constraint

$$W_l = U(c_m) + \beta[(1-q)W_h + qW_l] \tag{21}$$

which is solved by $c_m = z_l = y_0 - \frac{1}{1-\nu}\epsilon$ by the definition of W_l.
An individual in the high income state is always constrained. To derive her consumption c_1,

express the expected value of her consumption stream under the contract as an infinite sum of lotteries with two outcomes: either, she receives value W_h. Or, in case of a low income realisation, she receives $(\beta R)^{\frac{i-1}{\sigma}} c_1,\ i = 1$, plus participation in the next lottery for $i = 2$, and so forth until hitting $c_m = z_l$, where she remains until a high income shock. The discounted sum of the values of these lotteries must be equal to $W_h - u(c_1)$ which defines c_1, and thus, by (12) the rest of the support.[8]
The stationary mass at c_1 is equal to that of high income individuals ν. The remaining mass function $\Phi(c_{1+i})$ is simply ν times the probability to move to low income and stay there for $i < m$ periods, which yields a geometric distribution with parameter q. The lower bound c_m has the remaining mass $\Phi(c_m) = \Phi(c_{m-1})\frac{q}{1-q} = \nu\frac{(1-p)q^{m-1}}{1-q} = (1-\nu)q^{m-1}$.■
This closed form solution of the distribution is a useful building block for characterise the relationship between aggregate debt and income risk in the following section.

3.3.2 Income risk and aggregate debt in stationary equilibrium

This section shows how an increase in the riskyness of incomes lowers aggregate assets in this economy, as long as the initial level of risk is high enough. Remark 1 shows that changes in income risk $d\epsilon$ affect the stationary consumption distribution only via shifts in its upper and lower bounds, through changes in autarky values W_h, W_l. Stationary assets, which finance the difference between the constant aggregate endowment and aggregate consumption, inherit these comparative statics of consumption with respect to ϵ. This yields the following proposition

Proposition 1 There is a value $\epsilon*$, such that for higher initial levels of income risk $\epsilon > \epsilon*$, a marginal increase $d\epsilon > 0$ decreases stationary asset holdings.

Proof
By summing over the distribution in remark 1, we can write aggregate consumption as[9]

$$C = \nu c_1[1 + (1-p)\sum_{i=1}^{m-1}(\beta R)^{\frac{i}{\sigma}} q^{i-1}] + (1-\nu)q^{m-1}(y_0 - \frac{1}{1-\nu}\epsilon) \tag{24}$$

[8]The corresponding equation is

$$W_h = \frac{c_1^{1-\sigma}}{1-\sigma} + pW_h +$$
$$(1-p)\{\sum_{i=1}^{\infty}\beta^i q^{i-2}[q\ max\{(\beta R)^{\frac{i(1-\sigma)}{\sigma}}\frac{c_1^{1-\sigma}}{1-\sigma}, \frac{(y_0 - \frac{1}{1-\nu}\epsilon)^{1-\sigma}}{1-\sigma}\} + (1-q)W_h]\}$$

[9]If $\Phi(c_m) \approx 0$, such that truncation of the geometric distribution is negligible (which is true necessarily as $R \longrightarrow 1/\beta$), we have

$$c_1 = \{\frac{(1-\beta(p+q) - \beta^2(1-p-q))(1-\beta q(\beta R)^{\frac{1-\sigma}{\sigma}})}{(1+\beta(1-p-q)(\beta R)^{\frac{1-\sigma}{\sigma}})(1-\beta q)}(1-\sigma)W_h\}^{\frac{1}{1-\sigma}} \tag{22}$$

Thus aggregate consumption is affected by income risk only via changes in the bounds of the consumption distribution. In particular, C is decreasing in income risk ϵ whenever c_1 is, which in turn, from remark 1 depends on autarky values W_h and W_l. These are

$$W_h = \frac{(1-\beta q)u(y_0 + \frac{1}{\nu}\epsilon) + \beta(1-p)u(y_0 - \frac{1}{1-\nu}\epsilon)}{1-\beta(q+p)-\beta^2(1-(q+p))} \tag{25}$$

$$W_l = \frac{\beta(1-q)u(y_0 + \frac{1}{\nu}\epsilon) + (1-\beta p)u(y_0 - \frac{1}{1-\nu}\epsilon)}{1-\beta(q+p)-\beta^2(1-(q+p))} \tag{26}$$

Given the assumptions on transition probabilities, W_l is always declining in ϵ, while the high income-autarky value W_h is concave in ϵ with a maximum at some $\epsilon* > 0$. It increases for $\epsilon < \epsilon*$, decreases for $\epsilon > \epsilon*$ and crosses the perfect insurance value at $\bar{\epsilon} > \epsilon*$.[10] Note that this result does not depend on CRRA preferences. So for $\epsilon > \epsilon*$ aggregate consumption declines with income risk ϵ. Stationary aggregate assets are monotonously increasing in aggregate consumption, so the result follows.■

3.3.3 The decoupling of income and consumption inequality in open economy

The following result shows that in an open economy facing a given world interest rate, the inequality of consumption can become completely independent from that of income.

Corollary 2: Variance of log-consumption
If $\Phi(c_m) \approx 0$, the variance of log-consumption is

$$Var_c = \Lambda[\frac{\log(\beta R)}{\sigma}]^2 \tag{29}$$

---

and aggregate consumption equals

$$C = \nu c_1[1 + \frac{(1-p)(\beta R)^{\frac{1}{\sigma}}}{1-(\beta R)^{\frac{1}{\sigma}} q}] \tag{23}$$

[10]To see this, take the first derivative of autarky values with respect to ϵ

$$\frac{dW}{d\epsilon} = (I - \beta F)^{-1}[\frac{1}{\nu}u'(y_0 + \frac{1}{\nu}\epsilon), -\frac{1}{1-\nu}u'(y_0 - \frac{1}{1-\nu}\epsilon)] \tag{27}$$

The persistence assumptions assures that for $\epsilon = 0$ the rise in current utility dominates the fall in future expected utility. With strictly positive entries of F, however, Inada conditions on u translate to W_h, so marginal utility goes to infinity as the low income realisation goes to zero: as $\epsilon \longrightarrow y_0$, $\frac{dW_l}{d\epsilon} \longrightarrow -\infty$. By the intermediate value theorem and continuity, there exists an $\epsilon*$ with $\frac{dW_h(\epsilon*)}{d\epsilon} = 0$, and $\bar{\epsilon} > \epsilon*$ with $W_h(\bar{\epsilon}) = 0$. Also, the concavity of the utility function translates to the concavity of autarky values as a function of ϵ

$$\frac{dW^2}{d\epsilon^2} = (I - \beta F)^{-1}[(\frac{1}{\nu})^2 u''(y_0 + \frac{1}{\nu}\epsilon), (\frac{1}{1-\nu})^2 u''(y_0 - \frac{1}{1-\nu}\epsilon)] < 0 \tag{28}$$

where $\Lambda > 0$ is a function of transition probabilities only. So (log) consumption inequality is entirely determined by world interest rates R, where a higher R lowers domestic consumption inequality. If there is a non-negligible mass at the truncation point, $\Phi(c_m) > 0$, this is an upper bound for the cross-sectional variance of individual consumption.

For the simple algebra that leads to the result see Broer (2009b). The intuition is straightforward: Income risk affects the stationary distribution of consumption mainly via the participation constraint at high income that determines its upper bound, and thus the position of the distribution. Apart from the truncation at z_l, the shape of this distribution, however, depends entirely on the value of interest rates R, via the law of motion (12). Therefore, international interest rates determine consumption inequality, while income risk determines mean consumption, and thus asset holdings.

3.4 Income risk, aggregate debt and consumption inequality with general uncertainty and preferences

Proposition 1 naturally generalises to the case $N > 2$ with well-behaved, non-CRRA preferences. To see this, note that in this case, the consumption distribution can be characterised by N minimum participation-compatible consumption levels, associated to N autarky values, that provide the upper bounds for geometric sub-distributions. Within these subdistributions, the support is entirely determined by the law of motion (11), and monotonously increasing in the upper bounds. So when a rise in income risk reduces all autarky values, the whole support of consumption declines, reducing aggregate consumption and asset holdings in stationary equilibrium (for detail see Broer 2009b).
The shape of the n sub-distributions is again independent of the upper bound, with variance that decreases in R. However, changes in income risk now change relative autarky values and thus do not move the subdistributions in parallel. So the shape of the overall consumption distribution is not independent of income risk. But it is easy to show that the width of the support $\mathbb{C}$ decreases with R.
Broer (2009b) also proves existence and uniqueness of stationary equilibrium in a closed economy version of the model. There, the results on the shape of the consumption distribution continue to hold, while the comparative static effect of changes in income risk does not. Consumption thus follows a geometric distribution, implying a significant left skew. Equilibrium interest rates are relatively low in the endowment version of the model, at about 2.5 percent.

4 Individual risk and global imbalances: income uncertainty and the US net foreign asset position 1980-2003

The previous section showed that in an open, debt-constrained economy, rises in income risk can lower aggregate savings and asset positions. But importantly, this only holds for an initial level of income risk that is sufficiently high. The sign and importance of the effect of changes in income risk on asset positions thus depends on the particular economy under analysis. Also, the independence of stationary consumption inequality from income risk only holds for the special case with two income values, at a given exogenous interest rate. Thus, this section first analyses a partial equilibrium version of the model that is calibrated to match some stylised features of the US economy in the years 1980 and 2003. Specifically, I use the stochastic process for US individual incomes estimated by Krueger and Perri (2006), and compare debt holdings and consumption inequality in stationary equilibria corresponding to the two endpoints of their sample, respectively 1980 and 2003. A second exercise analyses the General Equilibrium of a stylised 2-country economy, where the US trades bonds with a large developing country, calibrated to capture the evolution of individual income inequality in China. There, I assume domestic asset trade is limited to uncontingent assets, resulting in a rise of precautionary savings in response to an increase in individual income risk. Before turning to the results I briefly describe the calibration, and the algorithm I use to compute the stationary equilibria.

4.1 Calibration

I calibrate the income process following Krueger and Perri (2006), using their estimates for the years 1980 and 2003, the endpoints of their sample. The authors assume the log of post tax labour income plus transfers (LEA+) $log(z_t)$ to be the sum of a group specific component α_t and an idiosyncratic part y_t. The latter, in turn, is the sum of a persistent AR(1) process m_t, with persistence parameter ρ and variance σ_m^2, plus a completely transitory component ε_t which has mean zero and variance σ_ε^2.
The process for LEA+ is thus of the form

$$\begin{aligned} log(z_t) &= \alpha_t + y_t \\ y_t &= m_t + \varepsilon_t \\ m_t &= \rho m_{t-1} + \nu_t \\ \varepsilon &\sim N(0, \sigma_\varepsilon^2) \\ \nu_t &\sim N(0, \sigma_\nu^2) \end{aligned} \tag{30}$$

Using data from the Consumer Expenditure Survey (CEX), the authors first partial out the group-specific component α_t as a function of education and other variables, identifying the variance of the idiosyncratic part of income y_t, as well as (from the short panel dimension of the CEX) its first order autocorrelation. They then fix $\rho = 0.9989$, the value estimated by Storesletten et al (2004), which allows them to identify σ^2_ν and σ^2_ε.
The results show an increase in the variance of labour income of 18 percentage points between 1980 and 2003, the two periods I focus on. 11 percentage points are due to an increase in within-group inequality, out of which roughly two thirds are accounted for by an increase in the importance of persistent shocks, and one third by that of transitory shocks.
In my exercise I abstract from changes in the common wage rate and differences in the group specific component, which, in the present model as in that of Krueger and Perri, translate fully into consumption differences by construction.
As a baseline calibration, I choose a CRRA utility function with coefficient of relative risk aversion of 1 (log-preferences), a discount factor of 0.96, and a constant interest rate equal to the initial closed economy equilibrium rate of 3.4 percent. I then look at the sensitivity of the results to changes in parameters, and the world interest rate. And I look at the case when agents who default are excluded from all financial transactions in the current period, but allowed to invest in non-contingent bonds in the future to smooth income shocks over time. This reduces the impact of higher income risk on the value under default.

4.2 Model Solution

To solve the model, I first approximate the persistent process for m_t with a 7-state Markov chain using the standard Tauchen and Hussey (1999) method.[11] Following Krueger and Perri (2006) I choose a binary process for the transitory shock. The computational algorithm then follows Broer (2009b), who describes the recursions to derive the stationary consumption distribution in the general case. I amend this for the fact that, with purely transitory shocks ν_t, the monotonicity condition for F does not hold.

4.3 Partial equilibrium results

4.3.1 Income risk and net foreign assets

Table 1 shows the equilibrium asset positions for different specifications of the economy. In the baseline calibration (I), the rise in income risk between 1980 and 2003 leads to a fall in the

[11]Note that this method accords with my assumption of widening the support Z to increase risk, but leaving the transition probabilities unchanged.

stationary level of net foreign assets of more than 50 percent of annual GDP. However, this calibration features a relatively high world real interest rate, and very strong effects of income risk on the value of default, due to the assumption of permanent exclusion from all financial trade. Thus, a second calibration allows saving in non-contingent bonds starting from the period following default, and reduces the world interest rate to 2.5 percent.[12] The results are reported as calibration II in table 1. The fall in stationary assets from the observed rise in US income risk is now smaller, at 11 percent of GDP. This is because with saving after default, higher income risk has a smaller impact on autarky values. Calibration III in table 1 increases risk aversion in this second calibration to $\sigma = 2$. With more risk averse individuals, the income volatility under financial autarky provides stronger disincentives to default, even when agents are allowed to save in autarky. For a given level of income risk, this translates to lower stationary asset holdings. But as before, the increase in income risk between 1980 and 2003 decreases stationary assets further, by about 40 percent of GDP.

Figure 2 shows that this reduction in assets from a rise in income risk holds for all values of world interest rates in the base line calibration. But this monotonicity of stationary foreign assets in risk gets lost when agents are allowed to save under autarky, as figures 3 and 4 show. For high interest rates, the additional increase in risk now increases aggregate assets in stationary equilibrium.

4.3.2 Income and consumption risk

Figure 5 shows the consumption distributions in the baseline case, for low (1980) and high income risk (2003). The sub-distributions, of different colour in the graph, correspond to individuals that were last constrained in the same income state, and thus have a common starting value for their declining paths before the next positive shock. Importantly, these sub-distributions are geometric and their shape remains constant between 1980 and 2003 - this is because the interest rate is unchanged in the baseline case. Their positions, however, decline with the fall in autarky values caused by higher income risk. This fall is less pronounced in states that correspond to positive realisations of the binary transitory shock, such as state 1, as there, higher variance translates to an increase in current income, if not value. From table 1 we see that the corresponding change in the variance of log consumption is small, equal to 0.06 percentage points.

Figure 6 illustrates the relationship between interest rates and the consumption distribution. For the income process estimated for 2003, the figure shows how a lower interest rate widens the

[12] I choose the savings rate of 2.5 percent, which is close to the average ex-ante annualised real rate of 2.6 percent on 6 month US treasury bills between 1980 and 2003, deflated using University of Michigan 12 month inflation expectations.

consumption distribution significantly, as analytically shown for the special case above. Figure 7 confirms this finding: the change in consumption volatility due to a change in income risk is an order of magnitude smaller than the changes caused by movements in the world interest rate.

The rise in individual income risk observed in the US since the 1980s can thus potentially explain at least part of the fall in its net foreign asset position. And interestingly, for a given interest rate this rise in income risk leaves the distribution of consumption almost unaffected. But changes in world interest rates have an important effect on consumption inequality.

4.4 Endogenous financial deepening meets the savings glut: A world economy with rising idiosyncratic risk and differences in financial development

So far, the analysis was agnostic about the determinants of savings outside the US, taking as given a world interest rate. But of course, in a closed world economy, the fall in US savings caused by increased idiosyncratic risk affects the equilibrium interest rate. This section thus looks at the general equilibrium in a simple economy consisting of 2 countries that differ both in their domestic financial market structures and the evolution of idiosyncratic risk that their agents experience over time. In particular, I present a stylised world economy consisting of China and the US. Both countries experience a rise in idiosyncratic income uncertainty in line with their historical experience, but differ in their ability to insure against this risk through domestic financial trade. Specifically, US financial markets are assumed to be complete but subject to participation constraints as before, allowing individuals to save at the world interest rate after they default on contracts. Chinese consumers, on the other hand, do not have access to complete domestic financial markets. Rather, I assume that individuals there can only engage in self-insurance through trade in bonds subject to a borrowing limit. As before, I abstract from aggregate risk. International asset trade is limited to non-contingent bonds, whose prices all agents take as given. A stationary equilibrium of the world economy is thus a process for individual consumption in both countries, an aggregate net asset position between the two countries and a market clearing interest rate.

The analysis concentrates on the effect of changes in idiosyncratic risk on equilibrium net foreign asset positions over the last 25 years. The process of idiosyncratic risk in the US is unchanged from the previous section. Unfortunately, equivalent estimates of an income process with group-specific heterogeneity, as well as persistent and transitory within-group risk, is infeasible for China, where the necessary household panel survey is not available for the period of interest. We are thus left to estimates of cross-sectional income inequality. This is a problem, as we cannot identify the different components of individual income risk from cross-sectional data alone. But the calibrated model provides a mapping from a specific income process to the cross-sectional consumption inequality and a savings demand schedule. I thus calibrate the components of the income process to capture the Gini coefficients of consumption and income for Chinese urban regions reported in Perloff and Wu (2005) in 1985, plus a zero initial foreign asset position. Assuming that the income process in China has the same permanent-persistent-transitory structure as in the US, including the persistence parameter of 0.9989, this provides three targets for three parameters, namely the variances of the permanent, persistent and transitory component of the income process in (30).[13] The increase in idiosyncratic risk in China is then calibrated to capture the observed rise in both Gini coefficients until 2001. For this, I assume that the change in permanent income differences in China is entirely captured by the rise of Urban-Rural inequality. But I look at the sensitivity of the results to this assumption below. The results assume a relatively tight borrowing limit corresponding to average quarterly income. As country weights, I use relative GDP of both countries from the Penn World tables in 1980 and 2003.

The appendix reports the implied estimates of the income process in China. In line with the similar Gini coefficients for consumption and income, inequality in the 1980s is estimated to be mainly determined by permanent income differences: both the variance of persistent and transitory income shocks are small. But the observed rise in consumption and income inequality until the early 2000s, stronger for income than for consumption, is in line with a strong increase in both the variance of persistent and transitory shocks, by 5.4 and 7.0 percentage points respectively.

Figure 8 plots the resulting equilibria for the early 1980s and the early 2000s. Chinese assets are plotted with a negative sign, such that the intersections of the demand and supply schedules give equilibrium asset positions and interest rates. The initial net interest rate of 2.6 percent is low relative to the discount factor of 0.96, as in many models of imperfect insurance. The increase in risk in the US results in the familiar fall in the savings demand schedule as a result of financial deepening. But in China, the strong rise in idiosyncratic risk after the early 1980s results in a strong rise in precautionary savings. This is exactly as we would expect in a self-insurance economy, where the financial deepening effect of higher income risk is absent, and the precautionary savings effect is relatively strong. The corresponding net effect is a fall in the US net foreign asset position to minus 34 percent of GDP, and a fall in the world interest rate of about 25 basis points.

As it is impossible to distinguish the effect on cross-sectional inequality of increases in permanent income difference from those of the very persistent shocks in the model, Figure 8 was based

[13] For the permanent part of income risk, I choose a uniform distribution of log-income values with 5 support points, and calibrate the support width to capture the moments of the data. Also, for both countries the results reported below are based on a discretisation of the AR(1) component of the income process into a 5-state markov process.

on the assumption that increases in permanent income differences are entirely captured by the difference between urban and rural regions. Since precautionary savings are largely unaffected by changes in permanent inequality but rise with persistent shocks to income, this may overstate the equilibrium savings. Therefore, Figure 9 shows how the results change when I make the opposite assumption of unchanged persistent shocks (which requires some recalibration also of the variance for transitory shocks, to match both Gini coefficients). As expected, the rise in equilibrium US liabilities is lower, but at 25 percent is still sizeable.

5 Conclusion

This paper has looked at the link between domestic income uncertainty, consumption inequality and net foreign asset positions in an economy where financial markets suffer from enforcement constraints. Domestic financial markets were assumed to be complete, but constrained by individuals' option to default on contracts, at the price of permanent exclusion from insurance markets. I showed that, contrary to economies with unconstrained complete markets, this economy has a well-defined stationary equilibrium for any given world interest rate. An analytical solution to the cross-sectional consumption distribution showed that higher income risk can indeed lower aggregate savings by making the punishment of default, financial autarky, less attractive, thus endogenously "deepening" financial markets. However, changes in income risk have only a small effect on consumption inequality, which depends mainly on the international interest rate. A calibration of the model to the US case showed that the changes in income risk observed between 1980 and 2003 might indeed explain an important part of the fall in the net foreign asset position. This holds not only at a constant world interest rate, but also in the general equilibrium of a simple world economy where the US trades bonds with a country that has less sophisticated markets and experiences a strong increase in idiosyncratic risk similar to that seen in China. The "glut" in precautionary savings there and the endogenous financial deepening in the US, both caused by rising idiosyncratic risks, result in a significant deterioration of the US net foreign asset position, and a small fall in the world interest rates.

Future research should generalise this analysis in at least two directions: first, one should also take account of the change in aggregate macroeconomic risk, which declined over the period of analysis. And second, an adequate equilibrium of the world economy should not only take into account advanced countries with deficits and emerging surplus economies, but also countries like Germany or Japan, that experienced surpluses yet have relatively developed domestic financial markets. In this context, the model's prediction of an inverse U-shape relationship between net foreign asset positions and individual income risk is especially interesting.

6 References

1. Alvarez, Fernando and Urban J. Jermann (2000). “Efficiency, Equilibrium, and Asset Pricing with Risk of Default”, Econometrica Vol. 68 No. 4, 775-798.

2. Atkeson, Andrew and Robert E. Lucas Jr. (1995). “Efficiency and Equality in a Simple Model of Unemployment Insurance”, Journal of Economic Theory, Vol. 66 No. 1, pp. 64-88.

3. Ayagari, S. Rao (1993). “Uninsured idiosyncratic Risk and Aggregate Saving”, Federal Reserve Bank of Minneapolis Working Paper 502.

4. Brandolini, Andrea and Timothy M. Smeeding (2007). “Inequality Patterns in Western-Type Democracies: Cross-Country Differences and Time Changes”, Luxembourg Income Study Working Paper No. 458.

5. Broer, Tobias (2009b). “Stationary equilibrium distributions under limited commitment”, European University Institute, mimeo.

6. Broner, Fernando A., Alberto Martin and Jaume Ventura (2006). “Sovereign Risk and Secondary Markets”, mimeo.

7. Floden, Martin (2008). “A note on the accuracy of Markov-chain approximations to highly persistent AR(1) processes”, Economics Letters Vol. 99 No. 3, 516-520.

8. Fogli, Alessandra and Fabrizio Perri (2006). “The “Great Moderation” and the US External Imbalance”, Monetary and Economic Studies Vol 26 (December), 209-225.

9. Heathcote, Jonathan, Kjetil Storesletten and Giovanni L. Violante (2008a). “The Macroeconomic Implications of Rising Wage Inequality in the United States”, NBER Working Paper 14052.

10. Heathcote, Jonathan, Fabrizio Perri and Giovanni L. Violante (2008b). “Cross-Sectional Facts for Macroeconomists: United States (1967-2006)”, mimeo.

11. Kehoe, Timothy J. and David K. Levine (2001). “Liquidity constrained markets versus debt constrained markets”, Econometrica Vol. 69 No. 3, 575-598.

12. Krueger, Dirk and Fabrizio Perri (2006). “Does Income Inequality Lead to Consumption Inequality? Evidence and Theory”, Review of Economic Studies Vol 73, 169-99.

13. Krueger, Dirk and Fabrizio Perri (2005). “Public or Private Risk Sharing”, mimeo.

14. Lane, Philip R. and Gian Maria Millesi-Ferreti (2007). “The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 19702004”, Journal of International Economics Vol. 73 No. 2, 223-250.

15. Marcet, Albert and Ramon Marimon (2009). “Recursive Contracts”, mimeo.

16. Mendoza Enrique G., Vincenzo Quadrini and Jose-Victor Rios-Rull (2007). “Financial Integration, Financial Deepness and Global Imbalances”, NBER Working Paper 12909.

17. Perloff, Jeffrey M. and Ximing Wu (2005). “China’s Income Distribution 19852001”, The Review of Economics and Statistics Vol 87 No. 4, 763775.

18. Roubini, Nouriel and Brad Setser (2004). “The US as a Net Debtor: The Sustainability of the US External Imbalances”, mimeo 2004.

19. Roubini, Nouriel (2005). Discussion of “Current Account Fact and Fiction” by David Backus and Frederic Lambert, mimeo.
At https://nber15.nber.org/c/2005/si2005/ifm/roubini.ppt.

20. Schmitt-Grohé, Stephanie and Martin Uribe (2003). “Closing small open economy models”, Journal of International Economics Vol. 61, 163-185

21. Storesletten, Kjetil, C. Telmer, and A. Yaron (2004). “Consumption and Risk Sharing over the Life-Cycle”, Journal of Monetary Economics, Vol. 51 No. 3, 609633.

22. Tauchen, George and Robert Hussey (1991), “Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models”, Econometrica Vol. 59, 371-396.

23. Uhlig, Harald (1996). “A Law of Large Numbers for Large Economies”, Economic Theory Vol. 8 no. 1, 41-50.

24. Zeldes, Stephen P. (1989). “Consumption and liquidity constraints: an empiricial investigation”, Journal of political economy Vol. 97, 305-346.

7 Appendix

7.1 Proof of existence and uniqueness

Result: *For every given world interest rate $R_{min} < R < \frac{1}{\beta}$, there exists a unique equilibrium allocation in the small country that is equal to the solution of the planner's problem for an appropriate weighting function μ in the social welfare function.*

Proof

I prove the existence of a unique solution to the planner's problem by checking that the conditions for a simplified version of Proposition 3 in Marcet and Marimon (2009) hold in this economy. Given the finite space of individual endowments Z we can apply a version of Tychonoff's theorem to see that the Euclidian product Z^T is compact for countable T. So the exogenous vector of individual states lies in a compact (Borel) subset of the Euclidian Space R^T. And of course, the discrete transition function satisfies the Feller property (Assumption A1 in Marcet and Marimon (2009)). Second, given the No-Ponzi condition, for any given B_t, R, Y the set of feasible consumption allocations $c_{i,t} : \int_{\mathbb{I}} c_i, t \leq \frac{Y}{R-1} + B_t, \forall t$ is just a simplex, so the choice vector lies in a compact and convex set (Assumption A2 in Marcet and Marimon (2009)). Third, note that our objective function is continuous, but unbounded. However, since aggregate resources are bounded each period, so is $\int_{\mathbb{I}} U(c)$ (Assumption B1 in Marcet and Marimon (2009)). Finally, individual autarky is incentive compatible and resource feasible. So the constraint set is convex, compact, and non-empty.[14]

Given the continuous objective function, the original sequential problem (6) therefore has a solution. Also, Marcet and Marimon (2009) show that, given any initial weighting function μ, these conditions suffice to show that an allocation $\{c_{i,t}\}, i \in \mathbb{I}, t \geq 0$ solves the original problem if and only if there is a sequence of multipliers $\gamma_{i,t},\ i \in \mathbb{I}, t \geq 0$ such that $\{c_{i,t}, \gamma_{i,t}\},\ i \in \mathbb{I}, t \geq 0$ solves the saddle-point functional equation (7).

Uniqueness of the equilibrium is assured by the strict concavity of the utility function u.■

[14]Strictly, we have to show that the constraint set has a non-empty interior, or that there is a real number $\varepsilon > 0$, such that $\int_{\mathbb{I}} c_i, t - Y \geq \varepsilon$ and $\int_{\mathbb{I}}[E[\sum_{t=0}^{\infty}(\mu_{i,t} + \gamma_{i,t})U(c_{i,t}) - W(z_i)] > \varepsilon$.

In fact, without knowing the solution of the problem, the existence of $\varepsilon > 0$ is not trivial to prove. However, once we have the solution, the condition is easy to check. For now, I show the existence of ε for the i.i.d. version of the special case, with $p = q = 1/2$ and $B_{t+1} = B_t = 0$. For this case it is easy to see that as long as the income uncertainty is large enough, or $\epsilon > \nu : \frac{U'(y_0+\nu)}{U'(y_0-\nu)} = \frac{2-\beta}{\beta}$, there are numbers $\xi, \widehat{\varepsilon} > 0$ such that a programme of the form $c(y_h) = y_h - \xi, c(y_l) = y_h + \xi - \widehat{\varepsilon}$ fulfills the conditions above. Intuitively, the expected discounted gain from higher consumption in future low-income states is large enough to allow a ressource-feasible reallocation of current consumption from high to low income agents. Thus the interior of the constraint set is strictly non-empty (Assumption B2 in Marcet and Marimon (2009)). But, as we will see, this history independent sharing rule is not optimal.

8 Tables and figures

Table 1:
Stationary assets and consumption inequality - different calibrations

I	**Baseline**						
	year	R	β	σ	Assets/GDP	Var(log(c))	Save in default?
	1980	1.034	0.96	1	0	0.034	No
	2003	1.034	0.96	1	-0.56	0.04	No
II	**Save in default**						
	year	R	β	σ	Assetss/GDP	Var(log(c))	Save in default?
	1980	1.025	0.96	1	-0.04	0.07	At 2.5%, not in t=0
	2003	1.025	0.96	1	-0.15	0.09	At 2.5%, not in t=0
III	**Save in default,** $\sigma = 2$						
	year	R	β	σ	Assetss/GDP	Var(log(c))	Save in default?
	1980	1.025	0.96	2	-0.79	0.04	At 2.5%, not in t=0
	2003	1.025	0.96	2	-1.23	0.05	At 2.5%, not in t=0

Table 2:
Income risk and savings in a simple world economy - variances of income components for China

	permanent	persistent	transitory	Gini income	Gini consumption
1985	0.08	0.0038	0.03	0.19	0.17
2001	0.08	0.057	0.10	0.27	0.21

The table reports the variances of components of an income process for Chinese urban regions that has the same structure as that reported in the text for the US: in the absence of information on group-specific attributes, (between-group) permanent income differences are modelled as a log-uniform distribution with 5 support points, while within-group income risk is the sum of a an AR(1) process with persistence parameter 0.9989 (discretized as a 5-state Markov process), plus a purely transitory binary shock (see the text for details). The parameters are chosen to target the Gini coefficients for consumption and income from Perlach and Wu (2005) for urban regions, and a zero net foreign asset position in 1980.

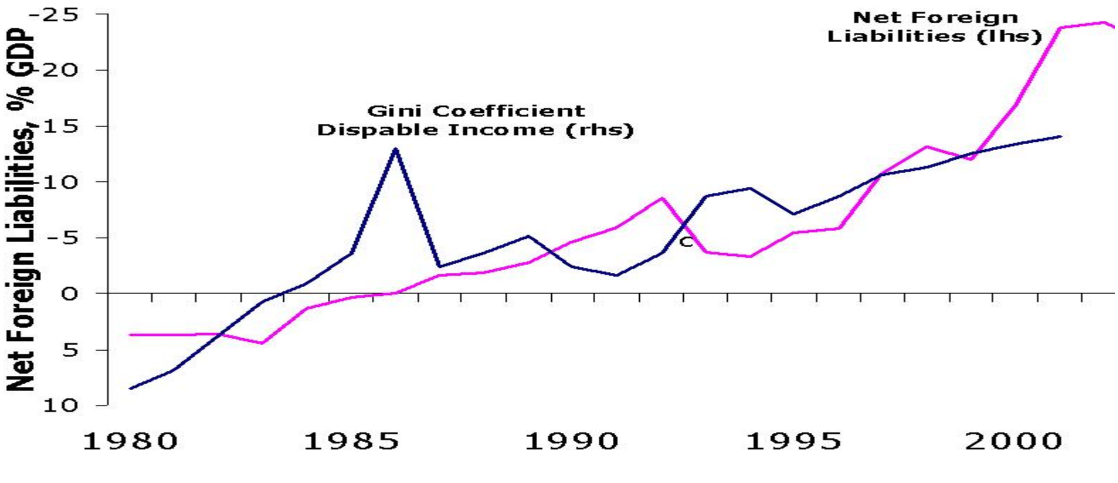

Figure 1: US current account and Gini coefficients. Source: IMF and Brandolini et al (2007)

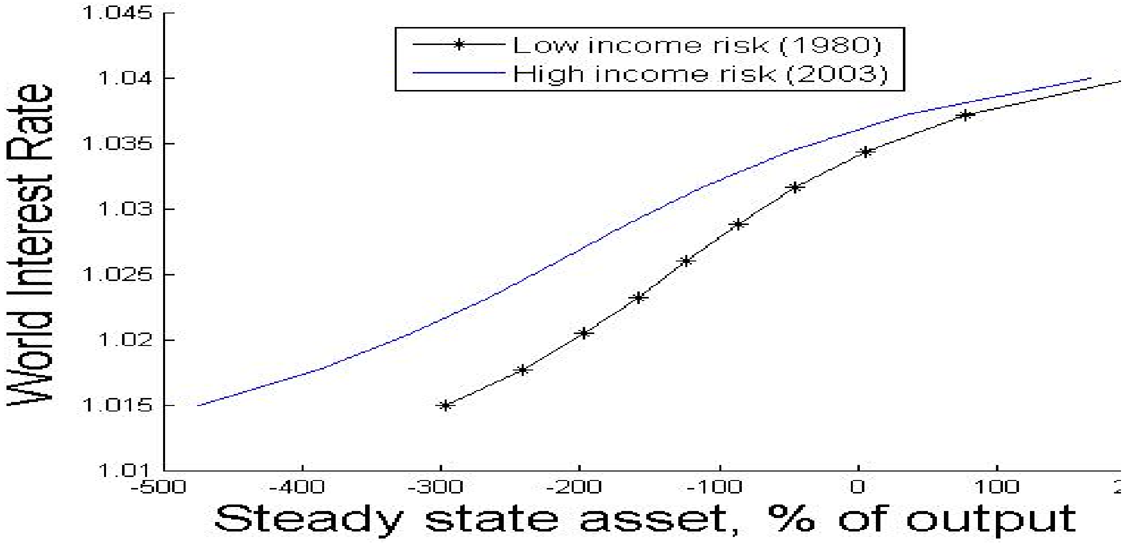

Figure 2: Asset demand function, baseline calibration.

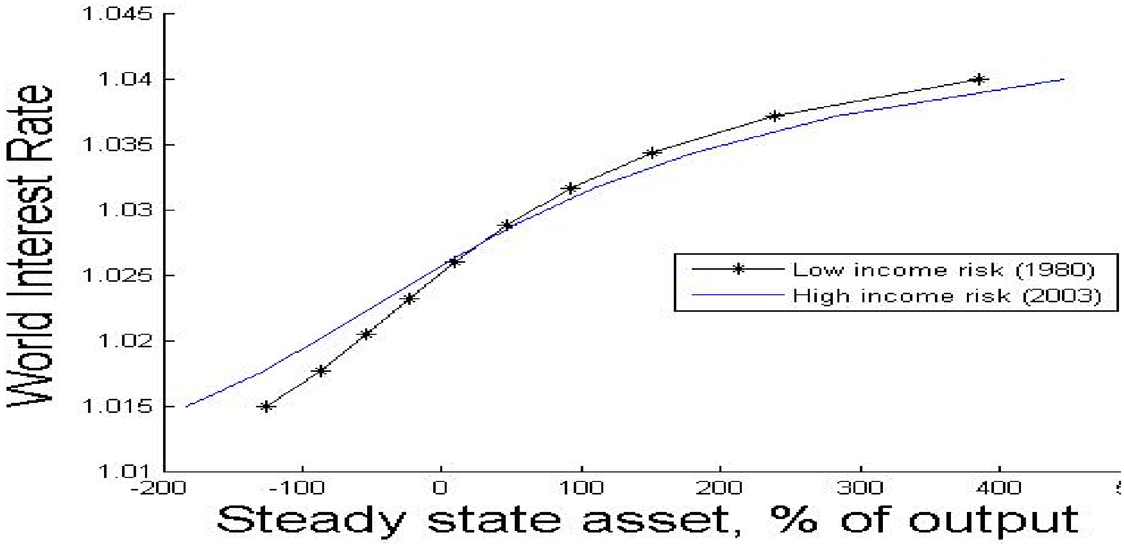

Figure 3: Asset demand function, log-preferences, saving at world interest rate but not in t=0.

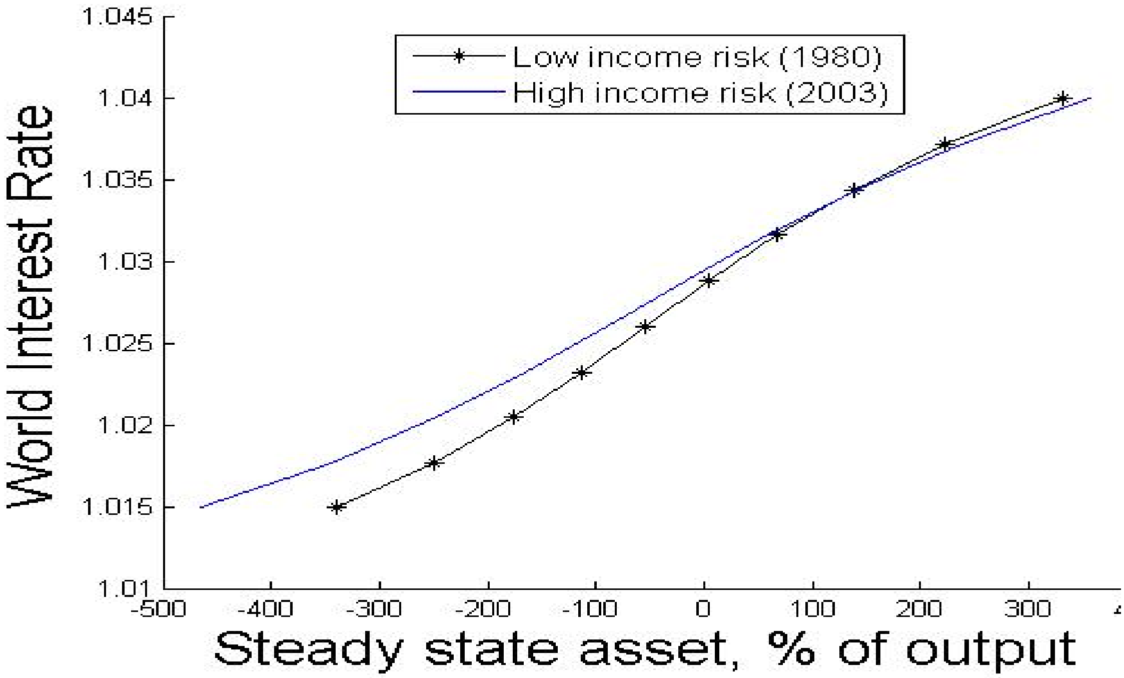

Figure 4: Asset demand function, higher risk aversion ($\sigma = 2$), saving at world interest rate but not in t=0.

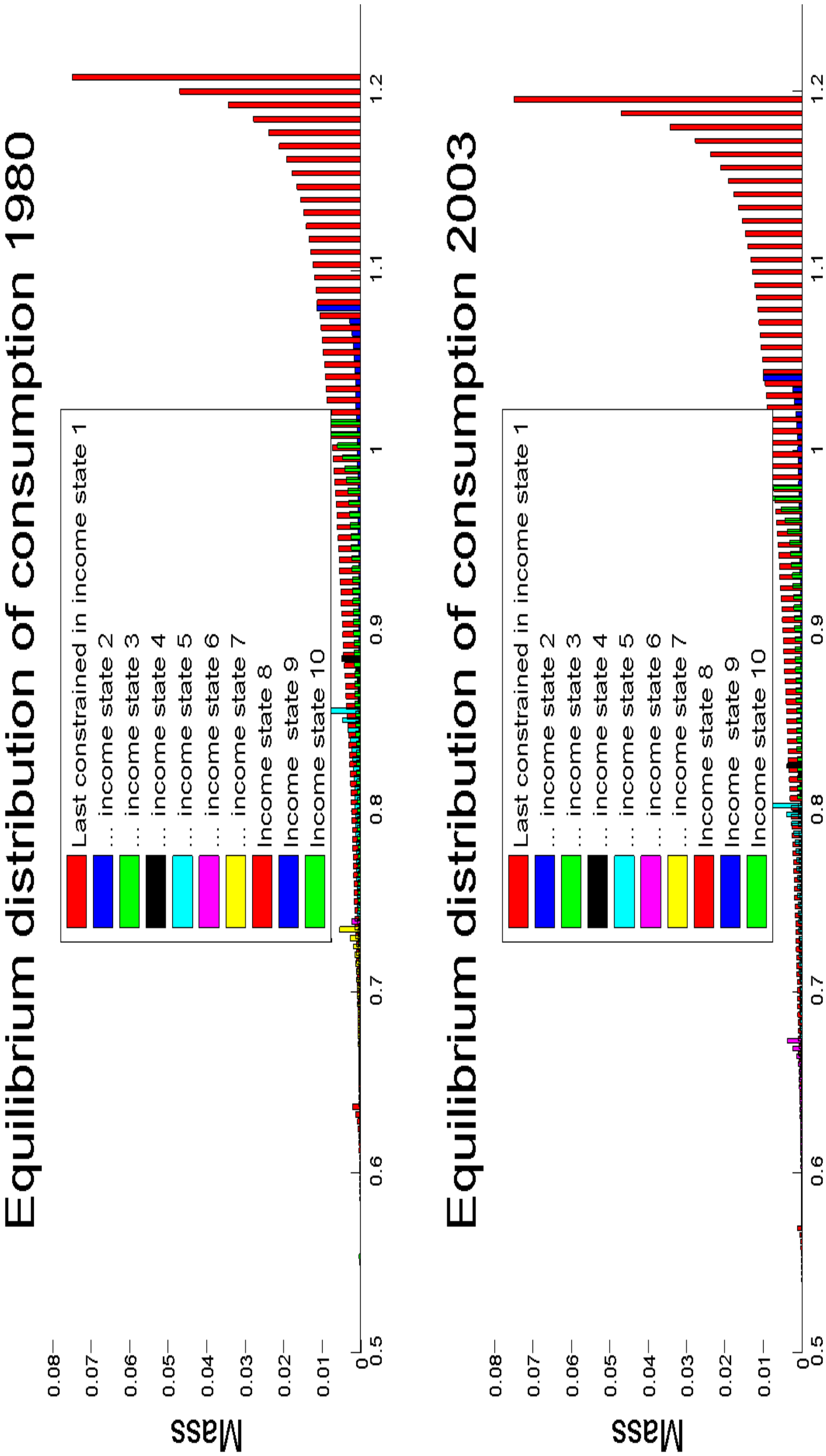

Figure 5: The consumption distribution in 1980 and 2003, baseline calibration (log-preferences, no savings in autarky).

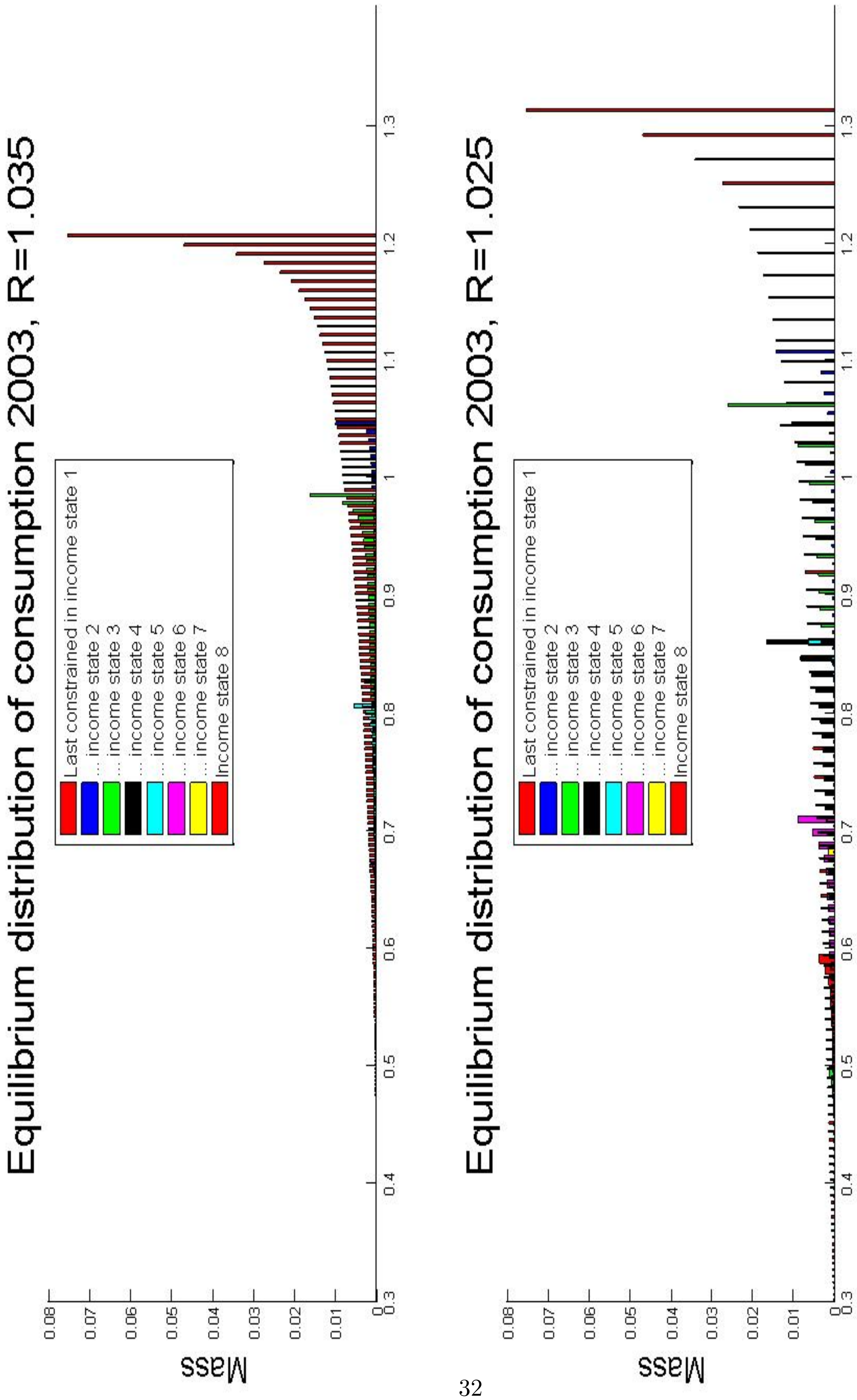

Figure 6: The consumption distribution in 2003 (high income risk), with high and lower interest rates.

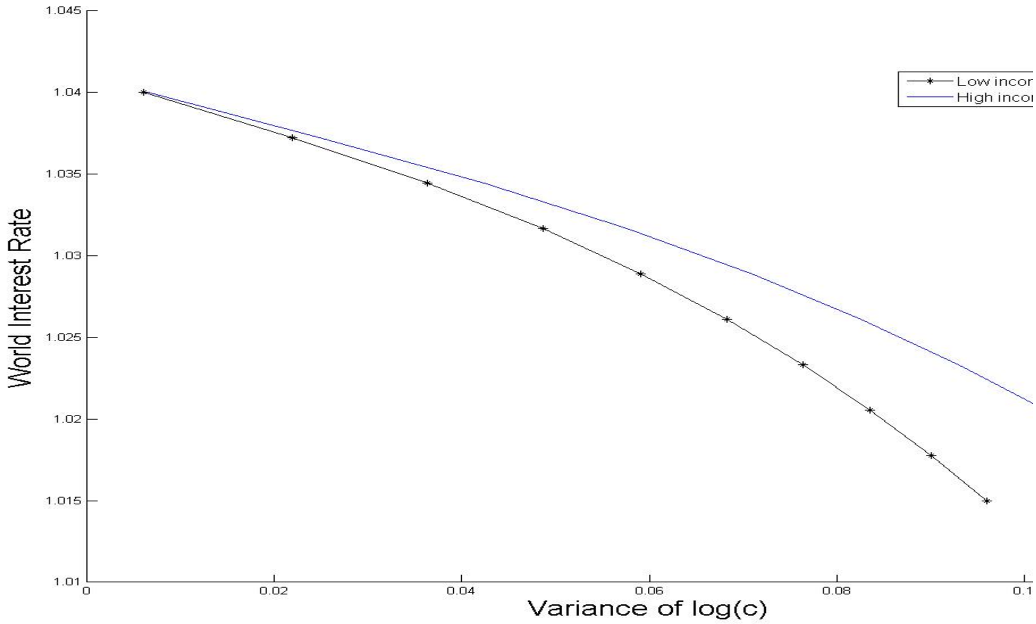

Figure 7: Variance of log(c), baseline calibration.

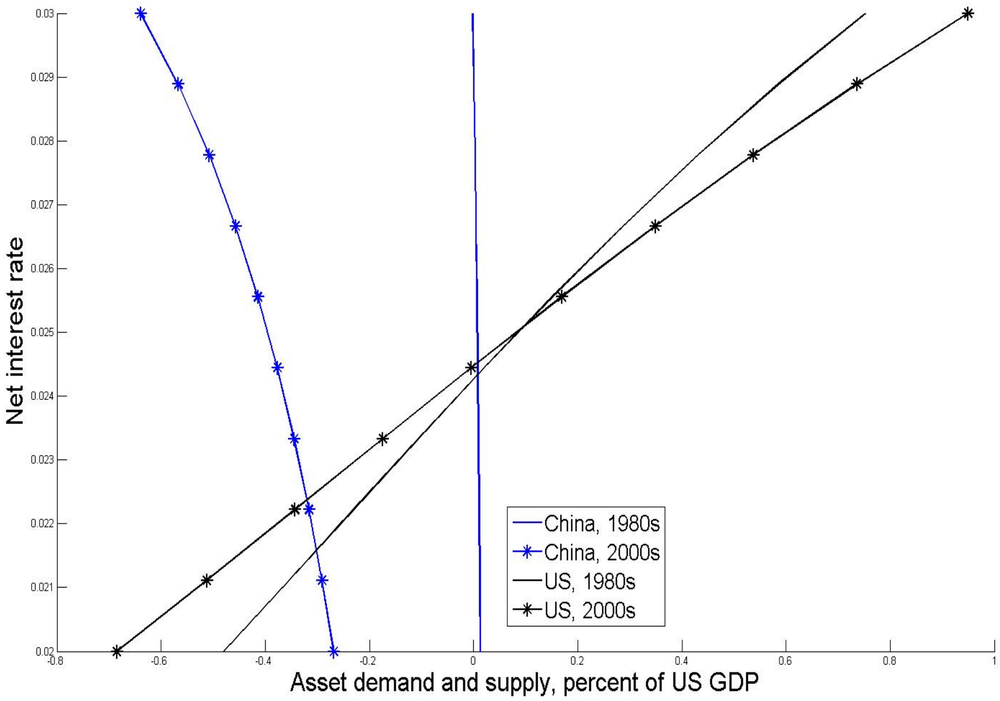

Figure 8: Asset demand and supply in a two country world economy.
The picture depicts US asset supply together with asset demand by China, which has a negative sign.

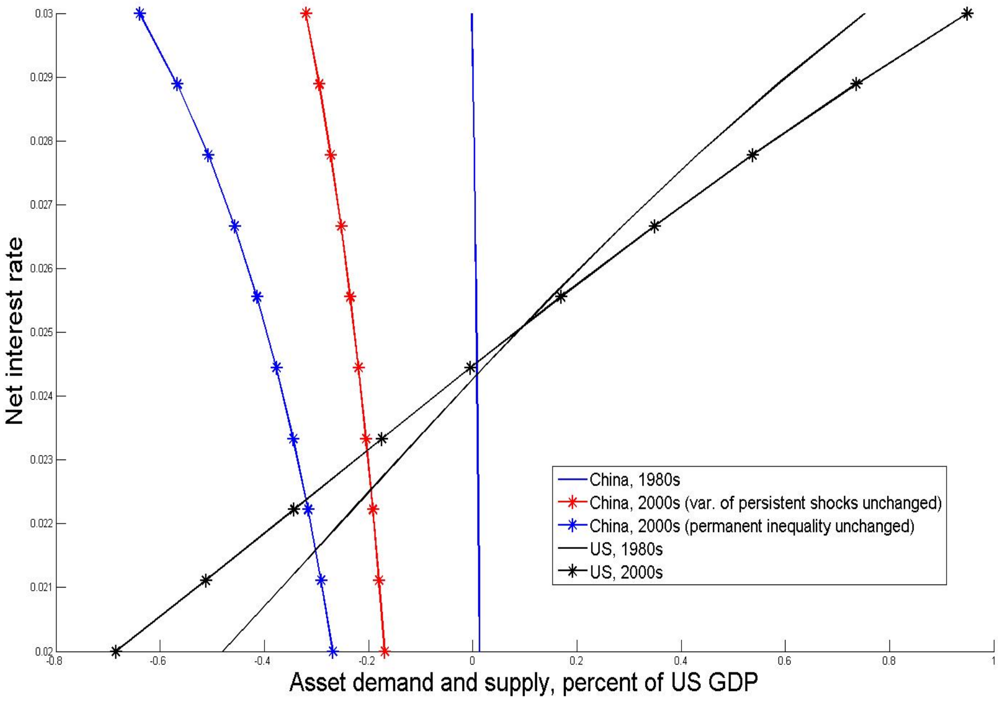

Figure 8: Asset demand and supply in a two country world economy.
The picture depicts US asset supply together with asset demand by China, which has a negative sign.