Formal Education Versus Learning-by-Doing

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Abstract

This paper studies the efficiency of educational choices in a search-matching model where individuals face a tradeoff between acquiring formal education and learning-by-doing while on-the-job. The labor market is hierarchically segmented into two sectors. When their educational effort is successful, (educated) workers can directly obtain a high-skill / better-paying job; whereas when their effort is unsuccessful, uneducated workers must begin with a low-skill job, learn-by-doing and then search while on-the-job for a high-skill job. We state that low-skill firms suffer from a hold-up behavior by high-skill firms. As a consequence, the low-skill sector is insufficiently attractive and individuals devote too much effort to formal education. A self-financed tax and subsidy policy restores market efficiency.

Keywords: Formal education; Learning-by-doing; Market efficiency; On-the-job search; Search unemployment.

JEL Codes: H21, I20, J21, J64, J68.

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1 Introduction

Formal education is not the only way to acquire skills which provide workers with the opportunity of holding a good job. Learning-by-doing in a low-skill job and then searching (while on-the-job) for a high-skill job is another way of reaching the same goal. Do workers choose the right amount of formal education when faced with this tradeoff? If not, what type of public policy should be implemented?

Although human capital is generally measured by the amount of formal education, many skills are best learned on the job thanks to participation in the production process, *i.e.* learning-by-doing (see Arrow (1962)). In this sense, training also determines workers productivity. Following Arrow, we assume that a learning-by-doing process allows workers to acquire general human capital which is therefore transferable from one firm to another.

Nevertheless, during the past few decades, more and more individuals have chosen to reinforce the intensity of the effort for their formal education. This well-known phenomenon has been, among others, reported by Machin (1996) who states the existence of an increase in the relative share of graduates in the UK in the 1980s associated with a rise in the relative use of skilled labor. Acemoglu (2002) sums up the same empirical evidence for the US where a large increase occurred in the supply of more educated workers over the last sixty years, this phenomenon being particularly strong in the 1970s. Mincer (1994, 2003) also reports an increase in educated labor supply which was less fast than the demand in the 1980s, and faster than the demand in the 1990s. Moscarini and Vella (2008) depict this trend using the Current Population Survey from 1979 to 2004, outlining the increase in the high-school graduates until mid-1990's and the ongoing rise in the proportion of college graduates.

Did those private educational choices lead to an efficient outcome? The purpose of this paper is to shed some light on this issue. We argue that individuals tend to put too much emphasis on formal education relative to training in the workplace. The reason for this does not lie in the educational decisions by themselves. This distortion comes from the fact that firms with high-skill jobs underestimate the social cost of filling their vacancies with workers coming from low-skill jobs in which they have learned by doing. Firms create too many high-skill jobs. In response to this hold-up behavior, job creation is suboptimal in the low-skill sub-market. As a result high-skill jobs are too appealing and individuals' formal educational effort is too strong. This creates motivation for a government involvement.

To assess the consistency of our argument, we use a two sectors search matching model (in which workers have a finite life expectancy (Moen and Rosén (2004), Gavrel *et al.* (2010)) Contrary to these papers, our model assume that workers can become skilled via formal education. Before entering the labor market, new workers decide on their formal education effort. If they succeed in acquiring the required skills, they directly join the pool of applicants for good jobs. If they fail, they have to search for a low-skill job and they begin to learn while on-the-job. When the learning-by-doing process comes to its end, workers are endowed with the same skills as (formally) educated workers. They then can join the pool of applicants for good jobs.

First, we describe a (decentralized) stationary equilibrium of the labor market et its efficiency conditions. Assuming that firms internalize the well-known congestion effect (Hosios (1990), Pissarides (2000)). High-skill job creation appears to be too high; whereas low-skill job creation, as well as individuals' educational choices, are partially efficient. In other words, they are optimal for the equilibrium value of the tightness of the high-skill labor sector. This means that, in line with our intuition, the inefficiency comes entirely from an excessive creation of high-skill vacancies. Next, we compare the decentralized equilibrium with a social optimum. The results validate the consistency of our argument: low skill jobs are too few and individuals put too much emphasis on formal education.

Second, we show that a Tax and Subsidy Policy (TSP) can decentralize the social optimum. Taxes must be levied on (filled) good jobs. They make that the perceived hiring costs coincide with social ones. However, these taxes distort low-skill job creation as well as educational choices. In order to restore market efficiency, these taxes must be dedicated to the funding of two kinds of compensatory transfers. One is allocated to firms of the low-skill sub-market when they loose their workers who leave them for a better job. The other one is a reward that workers receive if their formal education is successful. The reason why rewarding the graduates is necessary is that taxes which have to be levied on high-skill jobs exaggeratedly lower the surplus for a match with such jobs, hence the returns to formal education for workers.

Economists have been interested in the efficiency of human capital investment for a long time. A controversial issue, going back to Pigou (1912), is that of governmental involvement aiming at enhancing skills. As firms would not have any interest in investing in workers' skills because of the risk that their experienced workers would quit for external opportunities, government subsidies seemed to be a necessary measure for improving training as well as schooling. By opposition, Becker (1964) pointed out that the solution for human capital inefficiency may be better loan markets rather than government regulation and training subsidies. A competitive labor market implies that workers are the only ones incentivized to invest in their general training, bearing the cost either directly out of pocket or by taking a wage cut. Therefore, unless workers are credit-constrained, the right amount of investment for the market to be efficient would be undertaken.

More recently, labor theory reexamined the issue of educational choice in the presence of market imperfections. Our paper is a contribution to this literature. In their survey about non-competitive theories of training, Acemoglu and Pischke (1999) argued that labor market imperfections, such as search frictions, allow to account for employer-provided on-the-job training, because firms are able to recoup their investment in human capital. Moen (1999) studies the efficiency of educational choices when workers compete for jobs. Indeed, firms search is not random. They rank their applicants and hier the best one. As a response to this recruitment behavior, workers use formal education to compete for jobs. Consequently the education effort can be too strong. The same result is exposed by Charlot and Decreuse (2007) who model a labor market consisting of two schooling levels and two sectors in which budget constrained workers, differing with respect to labor market ability, self-select their educational choice (see also Charlot and Decreuse (2005)). Low-ability workers race for degrees in order to obtain a job even if education can be costly. Such a behavior is not socially optimum leading the authors not to recommend educational subsidies.

In order to set up public policies which lead workers to get the efficient amount of training they need, economists investigated several forms of educational processes. In this manner Heckman, Lochner and Cossa (2002) investigated the impact of wage subsidies on skill formation by distinguishing two models of training: a learning-by-doing model where skills are acquired as a by-product of work, and an on-the-job training model where investing in training is rival with working, as in Becker (1964) or Ben Porath (1967). They state that contrary to on-the-job training models, learning-by-doing models predict that wage subsidies increase skill formation. The impact of some public policies on educational choices have recently been highlighted by Adda *et al.* (2006) who considered a model with formal education and on-the-job training. On the contrary, our contribution emphasizes the opposition between formal education and learning-by-doing.

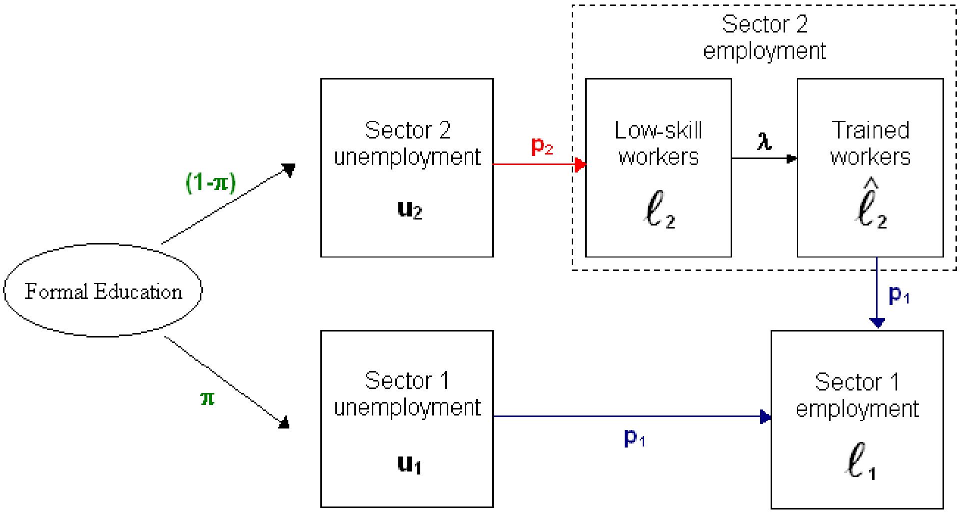
The paper is organized as follows: Section 2 outlines the analytical framework. We define a labor market decentralized (stationary) equilibrium in section 3. Section 4 studies market efficiency and states two main results: a decentralized equilibrium is partially efficient in terms of low-skill job creation and educational choices but inefficient in terms of high-skill job creation; the *laissez-faire* situation is inefficient. In section 5, we exhibit a self-financed fiscal policy which rewards educational success and leads to a social optimum. Finally, section 6 contains some concluding comments.

2 Analytical framework

The economy consists of two types of agents: workers and firms. Firms are infinity-lived whereas workers have a finite life expectancy of $1/m$. Time is continuous and parameter m measures the workers' labor market exit rate. Each worker who leaves the market is replaced with a newcomer. The measure of the total labor force is constant and normalized to one. All agents are risk-neutral and discount future payoffs at rate r ($r \geq 0$).

The labor market is segmented into two interacting sub-markets (sectors arranged into a hierarchy). Sector 2 offers low-skill jobs, while sector 1 offers high-skill jobs. Workers decide on their formal education effort e when entering the economy. If their effort is successful (which occurs with the probability π), workers will enter the pool of applicants for high-skill jobs (high-skill unemployment); whereas, workers with unsuccessful effort will enter the pool of applicants for low-skill jobs (see figure 1). Workers with low-skill jobs will therefore have to engage a learning-by-doing process in order to become skill enough to be employable in a high-skill firm. The expected duration of this process is denoted by $(1/\lambda)$. Workers thus acquire the required skills at Poisson rate λ. When the learning period comes to an end, workers engage in an on-the-job search process, hoping to get a high-skill job. The incentive to look for a high-skill job is the wage differentiation between sectors.

Figure 1: Workers' flows



When entering the labor market, firms choose the sub-market i ($i = 1, 2$) in which they will operate. They then create a single job in their chosen sub-market. Frictions exist that prevent the instantaneous matching of jobs with workers. Firms thus have to pay a cost, c, in order to keep their vacancy open. When matched with a worker, jobs yield output y_1 in sector 1, $\hat{y}_2$ in sector 2 when workers are trained and y_2 when workers are untrained (with $y_1 > \hat{y}_2 > y_2$). Wages are negotiated. Workers have a bargaining power of β and firms have a bargaining power of $(1 - \beta)$. Sector 1 offers the wage w_1; whereas, sector 2 offers the wage w_2 when workers are untrained and the wage $\hat{w}_2$ when workers had learned by doing.

Job creation results from the usual assumption of free entry in both sectors. Market frictions in sector-i are summarized in a constant-returns matching function that defines the arrival rate of workers to job vacancies $q_i(\theta_i)$ with $q_i'(\theta_i) < 0$. The arrival rate of job offers to searching workers $p_i = \theta_i q_i$ with $p_i'(\theta_i) > 0$ where θ_i is the sub-market tightness.

2.1 High-skill jobs

2.1.1 Asset values

In sub-market 1, the lifetime utility of an employed worker, called W_1, satisfies:

$$(r + m)(W_1 - U_1) = w_1 - (r + m)U_1 \tag{1}$$

where w_1 denotes workers' wage and U_1 is the lifetime utility of a high-skill worker when unemployed. We have:

$$(r + m)U_1 = d + p_1(W_1 - U_1) \tag{2}$$

with d being the value of leisure.
Regarding sector-1 firms, the value of a filled job, called J_1, verifies:

$$(r+m)(J_1 - V_1) = y_1 - w_1 - (r+m)V_1 \tag{3}$$

where V_1 is the asset value of a sector-1 firm whose job is vacant. We have:

$$rV_1 = -c + q_1(J_1 - V_1) \tag{4}$$

2.1.2 Private surplus and market tightness

When a worker and a firm meet and agree to form a match, the private surplus $S_1 = [W_1 - U_1] + [J_1 - V_1]$ of this match is shared between the worker and the firm according to their bargaining power. From equations (1) and (3), we deduce that the (private) surplus of a match in sub-market 1, satisfies:

$$(r+m)S_1 = y_1 - (r+m)(U_1 + V_1) \tag{5}$$

As the wage w_1 stems from static Nash bargaining, we have:

$$\beta S_1 = [W_1 - U_1] \tag{6}$$

We thus obtain:

$$(r + m + \beta p_1(\theta_1))S_1 = y_1 - d - (r+m)V_1$$

As already mentioned, in both sub-markets job creation results from the assumption of free-entry ($V_1 = 0$). We thus have:

$$(r + m + \beta p_1(\theta_1))S_1 = y_1 - d \tag{7}$$

Consequently, by using (4), the market tightness θ_1 is determined by the following equilibrium equation:

$$-c + q_1(\theta_1)(1-\beta)S_1 = 0 \tag{8}$$

This equilibrium equation is equivalent to the reduced form of the basic matching model (Pissarides (2000)). So, an increase in parameters c, β, d, r and m lowers the market tightness θ_1, whereas an increase in the output y_1 stimulates job creation in this sub-market.

2.2 Low-skill jobs

2.2.1 Asset values

When the training period comes to an end, the output of a worker in a low-skill job raises from y_2 to $\hat{y}_2$ and the worker begins to search (while on the job) for a high-skill vacancy. Her outside opportunities are defined by the lifetime utility of an unemployed worker in sub-market 1 (utility U_1). As Nash bargaining is static, the wage jumps from w_2 to $\hat{w}_2$.

It means that we first need to define the asset values associated with a match between a low-skill job and a trained worker) (hereafter referred to as an on-the-job seeker). So let $\hat{W}_2$ be the lifetime utility of such a worker. Using (6), one can show that this asset value satisfies:

$$(r + m + p_1)(\hat{W}_2 - U_1) = \hat{w}_2 + p_1\beta S_1 - (r + m)U_1 \tag{9}$$

Regarding sector 2 firms, the value of a low-skill job when matched with an on-the-job seeker, called $\hat{J}_2$, verifies:

$$(r + m + p_1)(\hat{J}_2 - V_2) = \hat{y}_2 - \hat{w}_2 - rV_2 \tag{10}$$

where V_2 is the value of a sector 2 vacancy.
From equations (9) and (10), we deduce that the (private) surplus of a match of a sector 2 firm with an on-the-job-seeker, $\hat{S}_2$. Knowing that $\hat{S}_2 = [\hat{W}_2 - U_1] + [\hat{J}_2 - V_2]$, the private surplus $\hat{S}_2$ satisfies:

$$(r + m + p_1)\hat{S}_2 = \hat{y}_2 + p_1\beta S_1 - (r + m)U_1 - rV_2 \tag{11}$$

Under the assumption of free-entry ($V_2 = 0$), the substitution of (2) into (11) yields:

$$(r + m + p_1(\theta_1))\hat{S}_2 = \hat{y}_2 - d \tag{12}$$

We can now define the asset values associated with a match between a sector 2 firm and a newcomer.
As Nash bargaining implies that:

$$\hat{W}_2 - U_1 = \beta\hat{S}_2,$$

we obtain that the lifetime utility of an unskilled worker when holding a sector 2 job, W_2, satisfies:

$$(r + m + \lambda)(W_2 - U_2) = w_2 + \lambda\beta\hat{S}_2 + \lambda U_1 - (r + m + \lambda)U_2 \tag{13}$$

where U_2 is the value of unemployment in this sub-market. We have:

$$(r + m)U_2 = d + p_2(W_2 - U_2) \tag{14}$$

On the firms' side, the value of a job when held by a newcomer verifies:

$$rJ_2 = y_2 - w_2 - m(J_2 - V_2) + \lambda(\hat{J}_2 - J_2) \tag{15}$$

Under the assumptions of free-entry ($V_2 = 0$) and Nash bargaining, the latter equation can be rewritten as:

$$(r + m + \lambda)J_2 = y_2 - w_2 + \lambda(1 - \beta)\hat{S}_2 \tag{16}$$

2.2.2 Private surplus and market tightness

The private surplus of an untrained worker matched with a sector 2 firm is such that $S_2 = [W_2 - U_2] + [J_2 - V_2]$. From equations (13) and (16), we deduce S_2 as a function of $\hat{S}_2$:

$$(r + m + \lambda)S_2 = y_2 + \lambda\hat{S}_2 + \lambda U_1 - (r + m + \lambda)U_2 \tag{17}$$

Finally, by using (2) and (14), one can see that equation (17) can be rewritten as follows:

$$\frac{(r + m + \lambda)(r + m + \beta p_2(\theta_2))}{r + m} S_2 = y_2 + \lambda\hat{S}_2 - d + \frac{\lambda}{r + m}\beta p_1(\theta_1)S_1 \tag{18}$$

According to equation (18), the tightness of sub-market 2 is a function of the tightness of sub-market 1 via the term $\beta p_1 S_1$. Equilibrium in sector 2 thus depends on the equilibrium in sector 1. This results from the fact that workers' asset values in sector 2 depends on workers' asset value in sector 1. This one way interdependence will play a crucial role in the efficiency study.
As a result, the assumption of free-entry determines the market tightness θ_2 by the following equilibrium equation:

$$-c + q_2(1 - \beta)S_2 = 0 \tag{19}$$

where the cost to keep a vacancy open, c, is assumed to be the same in both sub-markets.

2.3 Educational choices

When entering the labor market, a new worker decides on her formal education effort. Her effort, denoted by e, determines the probability π for becoming a high-skill worker. If she succeeds, she enters the pool of applicants for high-skill jobs; if she fails, she must search for a low-skill job and must begin a learning-by-doing process after finding one. The probability π is assumed to be an increasing and concave function $\pi(e)$ of the effort e ($\pi'(.) > 0$, $\pi''(.) < 0$).
The education effort is then obtained by maximizing the following objective:

$$ED \equiv -e + \pi(e)U_1 + (1 - \pi(e))U_2 \tag{20}$$

We obtain the following first order condition:

$$\pi'(e)(U_1 - U_2) - 1 = 0 \tag{21}$$

For obvious reasons, the effort e increases with the difference $(U_1 - U_2)$. From the concavity of function $\pi(.)$, we deduce that the second order condition is satisfied.
Using equations (2) and (14), we can rewrite the optimality condition as follows:

$$\pi'(e)\beta(p_1 S_1 - p_2 S_2) - (r + m) = 0 \tag{22}$$

The educational effort is an increasing function of the private surplus S_1, whereas it is a deceasing function of the private surplus S_2. In other words, workers would have an incentive to increase (reduce) their educational effort if the gain of holding a high-skill (low-skill) job rises.

3 Equilibrium

3.1 Definition

In sum, an equilibrium of the labor market can be defined as follows:

Definition 1. *An equilibrium of the labor market is a set of variables* $(S_1, \theta_1, \hat{S}_2, S_2, \theta_2, e)$ *which jointly satisfy equations (7), (8), (12), (18), (19) and (22).*

From market tightness and the probability π, one deduces the employment and unemployment levels in both sub-markets by using the conditions for flow-equilibrium.

3.2 Employment and unemployment levels

In steady state, employment and unemployment levels are deduced from the flow-equilibrium conditions. All employment (unemployment) variables are divided by the total labor force.
In sub-market 1, high-skill unemployment u_1 and high-skill employment l_1 are obtained from the following equations:

$$m\pi = (m + p_1)u_1 \tag{23}$$

$$ml_1 = p_1(u_1 + \hat{l}_2) \tag{24}$$

where $\hat{l}_2$ is the number of on-the-job seekers (*i.e.* the level of high-skill employment in sub-market 2).
In sub-market 2, low-skill unemployment u_2, low-skill employment l_2 and high-skill employment $\hat{l}_2$ are derived from the following conditions:

$$m(1 - \pi) = (m + p_2)u_2 \tag{25}$$

$$ml_2 + \lambda l_2 = p_2 u_2 \tag{26}$$

$$(m + p_1)\hat{l}_2 = \lambda l_2 \tag{27}$$

v_i denoting vacant jobs in the labor sub-market i, the sub-market tightness of sector 1 is given by $\theta_1 = v_1/(u_1 + \widehat{\ell_2})$ and the sub-market tightness of sector 2 is given by $\theta_2 = v_2/u_2$. From these flow-equilibrium conditions, we derive the impacts of variables θ_1, θ_2 and π on all employment and unemployment levels. Table 1 reports these partial derivatives. The variable η_i $(i = 1, 2)$ denotes the elasticity of rate q_i with respect to market tightness θ_i (in absolute value).
The tightness in sub-market 1 is independent of the one in sub-market 2. However owing to the interactions between the two sub-markets, high-skill employment depends on the

Table 1: Partial derivatives of employment and unemployment levels

	u_2	l_2
θ_1	0	0
θ_2	$-\frac{m(1-\pi)q_2(1-\eta_2)}{(m+p_2)^2}$	$\frac{m^2(1-\pi)q_2(1-\eta_2)}{(m+\lambda)(m+p_2)^2}$
π	$-\frac{m}{m+p_2}$	$-\frac{mp_2}{(m+\lambda)(m+p_2)}$

	$\hat{l}_2$	u_1	l_1
θ_1	$-\frac{\lambda l_2 q_1(1-\eta_1)}{(m+p_1)^2}$	$-\frac{m\pi q_1(1-\eta_1)}{(m+p_1)^2}$	$\frac{(u_1+\hat{l}_2)q_1(1-\eta_1)}{m+p_1}$
θ_2	$\frac{\lambda}{m+p_1}\frac{\partial l_2}{\partial \theta_2}$	0	$\frac{\lambda p_1}{m(m+p_1)}\frac{\partial l_2}{\partial \theta_2}$
π	$-\frac{\lambda m p_2}{(m+p_1)(m+\lambda)(m+p_2)}$	$\frac{m}{m+p_1}$	$\frac{p_1}{m+p_1} + \frac{p_1}{m}\frac{\partial \hat{l}_2}{\partial \pi}$

transition rates in sectors 1 and in sector 2. Therefore high-skill employment depends on job creation in the low-skill sub-market.

4 Efficiency

We now study the welfare properties of a decentralized equilibrium (Definition 1). As in Gavrel *et al.* (2010) firms do not internalize the social cost of hiring a high-skill worker coming from the low-skill sector, the creation of high-skill jobs appears to be too high. Due to this hold up phenomenon, job creation is suboptimal in the low-skill sub-market. As a consequence, educational choices are inefficient; workers devote too much effort to formal education.

Similar to Hosios (1990) and Pissarides (2000), let us consider a social planner who is only subject to search frictions and can redistribute income at no cost. In this case, the efficiency criterion is the social surplus. For the sake of expositional simplicity, the interest rate r is assumed to be equal to zero[1]. This assumption allows us to compare steady states according to the social surplus per period.
Denoted by Σ, the social surplus per head and per period is given by:

$$\Sigma = l_1 y_1 + l_2 y_2 + \hat{l}_2 \hat{y}_2 + (u_1 + u_2)d - \theta_1(u_1 + \hat{l}_2)c - \theta_2 u_2 c - me \tag{28}$$

Notice that in (28) the last term, me, measures the cost of formal education, per period.

[1]Main results extend to a positive interest rate. Proof is available from the authors upon request.

In what follows, for methodological reasons[2], we will also assume that the usual Hosios' condition holds true in both sub-markets[3], that is:

$$\eta_1 = \eta_2 = \beta$$

4.1 High-skill job creation

Let us first study the efficiency of the creation of high-skill jobs. Using Table 1, one can show that the derivative of the surplus Σ with respect to θ_1 has the same sign as:

$$HS \equiv (1-\eta_1)q_1 \left[y_1 - \left(\frac{\hat{l}_2}{u_1 + \hat{l}_2} \hat{y}_2 + \frac{u_1}{u_1 + \hat{l}_2} d \right) \right] - (m + \eta_1 p_1)c \tag{29}$$

As we know that:

$$\frac{\hat{l}_2}{u_1 + \hat{l}_2} \hat{y}_2 + \frac{u_1}{u_1 + \hat{l}_2} d > d$$

It results that firms create too many vacancies in a decentralized equilibrium (see equation (8)).

When the relevant outside option for sector 2 workers is unemployment, the on-the-job search with bargaining after the job-to-job transition yields an inefficiency that is not resolved by the Hosios rule. Moen and Rosén (2004) conjectured that in this case the wage level in poaching firms under the Hosios condition would be too low.

The intuition behind our inefficiency result is that with static Nash bargaining, firms underestimate the (social) opportunity cost of a match with a worker who comes from sub-market 2. This cost is given by the output $\hat{y}_2$ which is higher than the value of leisure. As a consequence, job creation in sub-market 1 is all the more inefficient as the share of on-the-job seekers in the pool of applicants for a high-skill job is large. In other words, firms of sub-market 2 suffer from a hold up behavior of firms of sub-market 1.

4.2 Low-skill job creation

One can show that the derivative of the social surplus Σ with respect to the market tightness θ_2 has the same sign as[4]:

$$\begin{aligned} LS \equiv \quad & (1-\eta_2)q_2 \frac{\lambda}{m(m+p_1)} [p_1(y_1 - d) - m\theta_1 c] \\ & + (1-\eta_2)q_2 \left[y_2 - d + \frac{\lambda(\hat{y}_2 - d)}{m + p_1} \right] - \frac{(m+\lambda)(m+\eta_2 p_2)}{m} c \end{aligned} \tag{30}$$

[2]We can show that the Hosios condition $\eta_2 = \beta$ is a necessary condition in order to restore efficiency in the low-skill sub-market.

[3]The matching functions are therefore Cobb-Douglas.

[4]Detailed calculus are available upon request from the authors.

We shall state that LS is equal to zero in a decentralized equilibrium (Definition 1). This means that the equilibrium value of the sub-market tightness θ_2 is partially efficient. In other words, it is socially optimal for the equilibrium value of sub-market tightness θ_1. Let us consider the expression:

$$X \equiv p_1(y_1 - d) - m\theta_1 c$$

For a nil interest rate, this expression can be rewritten as follows (see equation (7)):

$$X = p_1(m + \beta p_1)S_1 - m\theta_1 c$$

According to the equilibrium equation (8), we have:

$$m\theta_1 c = m(1 - \beta)p_1 S_1$$

Substitution into X then yields:

$$X = (m + p_1)\beta p_1 S_1$$

Under the Hosios' condition, this proves that the derivative of the social surplus with respect to the sub-market tightness θ_2 is zero in a decentralized equilibrium (see the equilibrium equation (19)).
At first glance, this (partial) efficiency result might look surprising as, via on-the-job search, employment in high-skill jobs depends positively on low-skill job creation (see Table 1); but, private surplus S_2 takes in account this externality through the term $\beta p_1 S_1$ (see equation (19)). In other words, holding a low-skill job gives workers the opportunity of getting a high-skill one. It raises the workers' surplus for a given wage. Assuming Nash bargaining, the firms' surplus rises as well, thus stimulating low-skill job creation.

4.3 Educational choices

One can check[5] that for a nil interest rate, the derivative of the social surplus with respect to the formal education effort has the same sign as:

$$\begin{aligned} E \equiv \quad & \left[1 - \frac{\lambda p_2}{(m+\lambda)(m+p_2)}\right]\left[\frac{p_1(y_1 - d)}{m + p_1} - \frac{m\theta_1 c}{m + p_1}\right] \\ & - \frac{m}{m+\lambda}\left[\frac{p_2}{m+p_2}(y_2 - d + \lambda\hat{S}_2) - \frac{m+\lambda}{m+p_2}\theta_2 c\right] - \frac{m}{\pi'(e)} \end{aligned} \tag{31}$$

Here also, we shall state that E is equal to zero in a decentralized equilibrium. In other words, the educational effort e appears to be partially efficient.
In order to verify the previous statement, let us first consider the quantity $\beta p_1 S_1$ (see Appendix for detailed calculus). From the definition of the private surplus S_1 (see equation (7)) and from the equilibrium equation (8), we obtain (for $r = 0$):

[5] idem

$$\frac{p_1}{m+p_1}(y_1-d)-\frac{m}{m+p_1}\theta_1 c=\beta p_1 S_1 \tag{32}$$

Substitution of equation (32) into equation (31) then yields:

$$E=\beta p_1 S_1-\frac{m}{m+\lambda}\left[\frac{p_2}{m+p_2}\left(y_2-d+\lambda\hat{S}_2+\frac{\lambda\beta p_1 S_1}{m}\right)-\frac{m+\lambda}{m+p_2}\theta_2 c\right]-\frac{m}{\pi'(e)} \tag{33}$$

Let us now consider the quantity $\beta p_2 S_2$ (see Appendix for detailed calculus). From the definition of the private surplus S_2 (see equation (18)) and by using equation (19), we have (for $r=0$):

$$\frac{p_2}{m+p_2}\left[y_2-d+\lambda\hat{S}_2+\frac{\lambda\beta p_1 S_1}{m}\right]-\frac{m+\lambda}{m+p_2}\theta_2 c=\frac{m+\lambda}{m}\beta p_2 S_2 \tag{34}$$

Substitution of (34) into (33) finally yields:

$$E=\beta p_1 S_1-\beta p_2 S_2-\frac{m}{\pi'(e)}$$

This shows that in a decentralized equilibrium, the derivative of the social surplus with respect to the educational effort e is nil (see equilibrium equation (22)). Notice that the Hosios' condition was not used in stating this point. Furthermore this result remains true whatever the value of workers' bargaining strength β is.
The following proposition summarizes these (partial) efficiency results:

Proposition 1. *A decentralized equilibrium of the labor market is partially efficient in terms of low-skill job creation (θ_2) and educational effort (e) but inefficient in terms of high-skill job creation (θ_1).*

It is worth noting that the efficiency of market tightness θ_2 is only partial as it only holds for the (decentralized) equilibrium value of market tightness θ_1. However, one can check that in the absence of an on-the-job search, a decentralized equilibrium would coincide with a social optimum (under the Hosios' condition).

4.4 Social optimum and decentralized equilibrium

A social optimum can be defined as follows:

Definition 2. *A social optimum is a set of variables (θ_1, θ_2, e) which jointly satisfy $HS = LS = E = 0$.*

The partial efficiency results we stated above are interesting in themselves as they enable us to understand why the decentralized equilibrium (Definition 1) is not a social optimum (Definition 2). We now use them to see how a decentralized equilibrium is located relative to a social optimum. We already know that job creation is beyond its optimal level in the high-skill sector. Market tightness θ_1 is too high. What can be said about job creation in the low-skill sub-market and the educational effort of entrant workers? Under the Hosios' condition ($\eta_1 = \eta_2 = \eta$), we state the following proposition (see proof in Appendix 2):

Proposition 2. *Relative to a social optimum, the low-skill job creation* (θ_2) *is too low in a decentralized equilibrium. As a consequence, individuals' education effort* (e) *is too high.*

determines θ_1 as a decreasing function of α.
In line with our intuition, relative to a social optimum, θ_2 is too low and e is too high in a decentralized equilibrium.

5 Optimal public policy

The *laissez-faire* situation is not an optimum. What then should a government do? We now present a self-financed Taxes and Subsidies Policy (TSP) leading to a social optimum. The same assumptions as above have been adopted. The interest rate is equal to zero and the Hosios condition holds on both sub-markets.

5.1 Taxing sector 1

As previously highlighted job creation is too high in sub-market 1. The government can decentralize the social optimum by implementing an appropriate fiscal policy. We now prove that in order to restore the efficiency, a tax τ could be levied in sub-market 1. Thus, the value of a filled job J_1 (see equation (3)) now depends on τ:

$$(r+m)(J_1 - V_1) = y_1 - \tau - w_1 - (r+m)V_1 \tag{35}$$

By comparison between (8) and the optimal condition (29), we obtain that the tax would restore the sub-market efficiency if it is equal to:

$$\tau = \frac{\hat{\ell}_2}{u_1 + \hat{\ell}_2}\hat{y}_2 + \frac{u_1}{u_1 + \hat{\ell}_2}d - d = \frac{\hat{l}_2}{\hat{l}_2 + u_1}(\hat{y}_2 - d) > 0 \tag{36}$$

Let α be the share of workers coming from sector 2 in the employment of sector 1:

$$\alpha = \frac{\hat{l}_2}{\hat{l}_2 + u_1}$$

The tax can therefore be written as:

$$\tau = \alpha(\hat{y}_2 - d)$$

and S_1 is now given by:

$$S_1 = \frac{y_1 - (\alpha \hat{y}_2 + (1-\alpha)d)}{m + \beta p_1}$$

Therefore, sector-1 equilibrium (8) becomes:

$$0 = -c + q_1(1-\beta)\frac{y_1 - (\alpha \hat{y}_2 + (1-\alpha)d)}{m + \beta p_1} \tag{37}$$

Equation (37) coincides with the optimality condition in sector 1 (29). With tax τ, the high-skill job creation becomes efficient. In short the pigovian tax τ makes sector 1 firms internalize the real cost of hiring a worker that comes from sector 2. However, implementing this tax does not only restore partial efficiency in sector 1, it also modifies efficiency results for sector 2 and for educational choices: job creation in sector 2 is no longer efficient and the same holds for the educational effort e.

These distortions lead to dedicate the tax τ to the funding of two compensatory transfers. The first one, denoted by σ_q is allocated to sector 2 firms when a worker quits her low-skill job. The transfer σ_q is given by:

$$\sigma_q = \frac{\tau}{m} \tag{38}$$

The second transfer, denoted by σ_e, is allocated to (entrant) workers whose educational effort e is successful. The transfer σ_e is given by:

$$\sigma_e = \frac{p_1}{m + p_1}\sigma_q \tag{39}$$

Before showing that these transfers offset the distortions created by the tax τ, we need to verify that the policy is self-financed. As there are $m\pi$ workers whose effort e is successful and $p_1\hat{l}_2$ quits, the government's expenditures are equal to:

$$m\pi\sigma_e + p_1\hat{l}_2\sigma_q = \left(\frac{m\pi}{m + p_1} + \hat{l}_2\right) p_1\sigma_q$$

From equations (23), (24), and (38), we deduce that:

$$\left(\frac{m\pi}{m + p_1} + \hat{l}_2\right) p_1\sigma_q = (u_1 + \hat{l}_2)p_1\sigma_q = ml_1\sigma_q = l_1\tau$$

As the government's receipts are given by $(l_1\tau)$ per period, this shows that the government's balanced budget constraint is satisfied for this self-financed TSP.

5.2 Subsidizing Sector 2

By restoring efficiency in sector 1, one has reduced job creation in sector 2 above the efficiency level. In order to restore the efficiency in the overall labor market, we propose to subsidize sector 2 firms whose workers leave for sector 1. With the compensatory transfer σ_q (see equation (38)), the private surplus S_2 now satisfies (for $r = 0$):

$$\frac{(m+\lambda)(m+\beta p_2)}{m}S_2 = y_2 - d + \lambda\frac{\hat{y}_2 - d}{m+p_1} + \lambda\frac{p_1}{m+p_1}\sigma_q + \frac{\lambda}{m}\beta p_1 S_1$$

On the other hand, using equation (37) one can see that, with the tax, the quantity $(\beta p_1 S_1)$ is now given by:

$$\beta p_1 S_1 = \frac{p_1(y_1-d) - m\theta_1 c}{m+p_1} - \frac{p_1}{m+p_1}m\sigma_q \tag{40}$$

Combining the two previous equations yields:

$$\frac{(m+\lambda)(m+\beta p_2)}{m}S_2 = y_2 - d + \lambda\frac{\hat{y}_2 - d}{m+p_1} + \frac{\lambda}{m}\frac{p_1(y_1-d) - m\theta_1 c}{m+p_1}$$

Substituting S_2 into equation (19) shows that transfer σ_q enables the restoration of the efficiency of the low-skill vacancy creation (see equation (30) for $\beta = \eta_2$).

5.3 Rewarding educational success

With the reward σ_e defined by equation (39), the private optimality condition (21) has to be rewritten as follows (for r=0):

$$\pi'(e)\left(U_1 + \frac{p_1}{m+p_1}\sigma_q - U_2\right) - 1$$

or

$$\pi'(e)\left(\beta p_1 S_1 + \frac{p_1}{m+p_1}\sigma_q - \beta p_2 S_2\right) = m$$

Using equation (40), the previous equation can be rewritten as:

$$\left(\frac{p_1}{m+p_1}(y_1-d) - \frac{m}{m+p_1}\theta_1 c - \frac{p_1}{m+p_1}\sigma_q + \frac{p_1}{m+p_1}\sigma_q - \beta p_2 S_2\right) - \frac{m}{\pi'(e)} = 0$$

or

$$\left(\frac{p_1}{m+p_1}(y_1-d) - \frac{m}{m+p_1}\theta_1 c - \beta p_2 S_2\right) - \frac{m}{\pi'(e)} = 0$$

As the efficiency of θ_2 is restored (despite the tax), $\beta p_2 S_2$ remains equal to:

$$\frac{m}{m+\lambda}\left[\frac{p_2}{m+p_2}\left(y_2 - d + \lambda\hat{S}_2 + \frac{\lambda\beta p_1 S_1}{m}\right) - \frac{m+\lambda}{m+p_2}\theta_2 c\right]$$

Therefore the social optimality condition (33) holds true.

At first glance, the idea of rewarding educational success might look counterintuitive as one could point out that educational effort is lower in a social optimum according to Proposition 2. The reason for this is that without subsidies private educational choices are no longer efficient for the optimum value of sub-market tightness θ_1 (computed with the tax τ). In the absence of a reward, the return to education (the opportunity to get a better-paying job) would be too weak thus leading to a reduction in formal education. The reward compensates for this effect.

The following proposition summarizes the above results:

Proposition 3. *With the TSP* $(\tau; \sigma_e; \sigma_q)$ *the decentralized equilibrium is a social optimum.*

6 Conclusion

In many countries, governments subsidize formal education and/or training through different channels. For example the first Clinton administration made skill upgrading a major priority. Are these subsidies justified?

We set up a model in which workers face a tradeoff between acquiring formal education (thus having the opportunity to obtain a good job directly) and learning-by-doing in a low-skill job, then searching (while on-the-job) for a high-skill job. We have stated that workers do not choose the right amount of formal education when faced with this tradeoff. Even if the decentralized equilibrium is partially efficient in terms of low-skill job creation and educational choices, it is inefficient in terms of high-skill job creation. Because high-skill job creation is too high and induces a hold-up behavior which penalizes low-skill jobs, a tax must be levied on high-skill firms. Therefore, the educational choices and low-skill job creation will not be partially efficient anymore. A self-financed Tax and Subsidy Policy restores the market efficiency. The tax should finance two compensatory transfers: a subsidy to low-skill jobs (the worker of which quits) and a reward aiming at encouraging educational effort.

In a decentralized equilibrium, workers tends to put too much stress on formal education. However according to our results, subsidies to education makes sense, even without credit constraint, as the tax that must be levied on high-skill firms makes formal education become deficient.

In a practical viewpoint, our results have to be taken with caution as one could object that this tax/transfer policy would be difficult to implement when the government is not perfectly able to know the output of each firm.

One could also argue that in presence of a reward allocated to graduates, the educational system can be tempted to award more diplomas in exchange of an increase in the tuition level. This phenomenon would affect the skill level of graduates.

To conclude we would like to emphasize an unexpected result of our study. Economists usually believe that education would not be high enough in the presence of search-frictions. Search-frictions create rents which implies that a part of the return to education goes to firms. Therefore, the incentive of workers to invest in their formal education would be too low. We have shown that this widespread view may be incorrect. Without on-the-job search, educational choices are perfectly efficient despite search-frictions. The reason for this is that firms have to pay for the cost of creating jobs that educated workers will hold.

As already mentioned, in our setting the inefficiency of the education effort in the *laissez-faire* situation doesn't come from private educational choices but from the presence of the hold-up phenomenon.

Appendix 1: optimality condition for educational choices

We show that the optimality condition for educational choices (31) is equal to zero in a decentralized equilibrium.

Let us consider the quantity $\beta p_1 S_1$. From equation (7), we deduce (for $r = 0$):

$$\beta p_1 S_1 = \frac{p_1}{m+p_1}(y_1 - d) + \frac{m}{m+p_1}(y_1 - d) - mS_1$$

By using the equilibrium equation (8), we obtain:

$$\frac{m}{m+p_1}(y_1 - d) - mS_1 = -\frac{m}{m+p_1}p_1(1-\beta)S_1 = -\frac{m}{m+p_1}\theta_1 c$$

The result is:

$$\frac{p_1}{m+p_1}(y_1 - d) - \frac{m}{m+p_1}\theta_1 c = \beta p_1 S_1$$

Substitution of the previous equation into equation (31) then yields:

$$E = \beta p_1 S_1 - \frac{m}{m+\lambda}\left[\frac{p_2}{m+p_2}\left(y_2 - d + \lambda\hat{S}_2 + \frac{\lambda\beta p_1 S_1}{m}\right) - \frac{m+\lambda}{m+p_2}\theta_2 c\right] - \frac{m}{\pi'(e)}$$

Let us consider the quantity $\beta p_2 S_2$. From equation (18), we deduce (for $r = 0$):

$$\frac{m+\lambda}{m}\beta p_2 S_2 = y_2 - d + \lambda\hat{S}_2 + \frac{\lambda\beta p_1 S_1}{m} - (m+\lambda)S_2$$

The latter equation can be rewritten as follows:

$$\begin{aligned}\frac{m+\lambda}{m}\beta p_2 S_2 = \quad & \frac{p_2}{m+p_2}\left[y_2 - d + \lambda\hat{S}_2 + \frac{\lambda\beta p_1 S_1}{m}\right] \\ & + \frac{m}{m+p_2}\left[y_2 - d + \lambda\hat{S}_2 + \frac{\lambda\beta p_1 S_1}{m}\right] - (m+\lambda)S_2\end{aligned}$$

By using the equilibrium equation (19), we obtain:

$$\begin{aligned}\frac{m}{m+p_2}\left[y_2 - d + \lambda\hat{S}_2 + \frac{\lambda\beta p_1 S_1}{m}\right] - (m+\lambda)S_2 \quad &= \quad \frac{m+\lambda}{m+p_2}(m+\beta p_2)S_2 - (m+\lambda)S_2 \\ &= \quad -\frac{m+\lambda}{m+p_2}p_2(1-\beta)S_2 = -\frac{m+\lambda}{m+p_2}\theta_2 c\end{aligned}$$

We thus have:

$$\frac{p_2}{m+p_2}\left[y_2 - d + \lambda\hat{S}_2 + \frac{\lambda\beta p_1 S_1}{m}\right] - \frac{m+\lambda}{m+p_2}\theta_2 c = \frac{m+\lambda}{m}\beta p_2 S_2$$

Substitution of the previous equation into (33) finally yields:

$$E = \beta p_1 S_1 - \beta p_2 S_2 - \frac{m}{\pi'(e)}$$

The optimality condition (31) coincides with the decentralized equilibrium (22).

Appendix 2: proof of proposition 2

First part of proposition 2

Proof. We first prove that tightness θ_2 is too low relative to the social optimum. Let us consider the system composed of the following two equations in (θ_1, θ_2):

$$(1-\eta)q_1[y_1 - d - a(\widehat{y}_2 - d)] - (m + \eta p_1)c = 0 \tag{41}$$

$$\begin{aligned} 0 = \quad & q_2(1-\eta)(y_2 - d) - \frac{m+\lambda}{m} c\eta p_2 - (m+\lambda)c \\ & + q_2(1-\eta)\frac{\lambda}{m}\left[\frac{p_1}{m+p_1}(y_1 - d) + \frac{m}{m+p_1}(\widehat{y}_2 - d) - \frac{m}{m+p_1}\theta_1 c\right] \end{aligned} \tag{42}$$

This system is parameterized with the scalar a which takes its values in the interval $[0, a^*]$. The limit a^* is the value of ratio $\frac{\hat{l}_2}{u_1+\hat{l}_2}$ when the social optimum is reached. Equation (42) is obtained by equalizing LS (defined by equation (30)) to zero, which is thus the (necessary) efficiency condition for tightness θ_2. This condition is identical to the decentralized equilibrium equation in sector 2 when $a = 0$ (see proposition 1).

For $a = 0$, the two previous equations describe the decentralized equilibrium. The social optimum is obtained for $a = a^*$.

Let us consider (41). This equation (implicitly) determines tightness θ_1 as a function in parameter a, denoted by $\theta_1(a)$. As $q_1'(\theta_1) < 0$ and $p_1'(\theta_1) > 0$, the parameter a has a negative impact on tightness θ_1 ($\theta_1'(a) < 0$). In accordance with proposition 1, we find that the decentralized equilibrium value of tightness θ_1 is greater than its optimal value: $\theta_1(a^*) < \theta_1(0)$.

The system composed of equations (41) and (42) also determines tightness θ_2 as an implicit function in a, $\theta_2(a)$. Let us consider equation (42). Its left hand side, LS, can be written as:

$$LS(.) = LS(\theta_1(a), \theta_2)$$

In order to deduce the impact of parameter a on tightness θ_2, we must sign the partial derivative of $LS(.)$ with respect to θ_1 and θ_2. Let us note $H(\theta_1)$ the term of equation (42) between brackets. The derivative of $H(\theta_1)$ with respect to θ_1 has the same sign that:

$$-q_1(1-\eta)(1-a)(\hat{y}_2 - d)$$

As the function $LS(.)$ is bounded by the social optimum ratio $\frac{\hat{l}_2}{u_1+\hat{l}_2}$, we know that $a < 1$. We therefore have $H'(\theta_1) < 0$. The partial derivative of $LS(.)$ with respect to θ_1 is thus negative.

Let us consider the impact of θ_2 on $LS(.)$. One must first determine the sign of $H(\theta_1)$ itself, which can be rewritten as:

$$\begin{aligned} H &= \frac{1}{m+p_1}\left[p_1(y_1 - d) + m(\hat{y}_2 - d) - m\theta_1 c\right] \\ &= \frac{1}{m+p_1}\left[\theta_1\left((m+\eta p_1)q_1\frac{y_1 - d - a(\hat{y}_2 - d)}{m+\eta p_1} - mc\right) + (m + ap_1)(\hat{y}_2 - d)\right] \end{aligned}$$

Knowing that (41) is equivalent to:

$$mc = m(1-\eta)q_1 \frac{y_1 - d - a(\hat{y}_2 - d)}{m + \eta p_1}$$

We obtain by substitution:

$$H = \frac{1}{m+p_1}\left[\eta(m+p_1)p_1 \frac{y_1 - d - a(\hat{y}_2 - d)}{m+\eta p_1} + (m + ap_1)(\hat{y}_2 - d)\right] > 0$$

$H(\theta_1) > 0$ and $q_2'(\theta_2) < 0$, therefore the partial derivative of $LS(.)$ with respect to θ_2 is (strictly) negative (see equation (42)).

The impact of parameter a on θ_2 is obtained by equalizing to zero the differential of $LS(.)$:

$$dLS(.) = \frac{\partial LS}{\partial \theta_1}\theta_1'(a)da + \frac{\partial LS}{\partial \theta_2}d\theta_2 = 0$$

where

$$\frac{d\theta_2}{da} = -\frac{\frac{\partial LS}{\partial \theta_1}\theta_1'(a)}{\frac{\partial LS}{\partial \theta_2}} > 0$$

The derivative of the implicit function $\theta_2(a)$ is thus (strictly) positive. As a consequence the optimal value of θ_2 is higher than its decentralized equilibrium value ($\theta_2(a^*) > \theta_2(0)$). This proves the first part of proposition 2. □

Second part of proposition 2

Proof. We now show that the educational effort e is too high relative to the social optimum. The necessary optimality condition of effort e (equation (31)) can be written as:

$$E = \frac{p_1(y_1 - d) - m\theta_1 c}{m+p_1} - \frac{m}{m+\lambda}\left[\frac{p_2}{m+p_2}\left(y_2 - d + \frac{\lambda}{m}H\right) - \frac{m+\lambda}{m+p_2}\theta_2 c\right] - \frac{m}{\pi'(e)} = 0 \tag{43}$$

where p_i and θ_i (for $i = 1, 2$), as well as $H(\theta_1)$, are deduced from equations (41) and (42).

Taking (41) and (42) into account, equation (43) determines e as an implicit function $e(a)$ of parameter a. The decentralized equilibrium value of e is given by $e(0)$ and its optimal value by $e(a^*)$. We must sign the derivative of $e(a)$. The left side term, E, of (43) can be written as:

$$E(.) = E(\theta_1(a), \theta_2(a), e)$$

For a given H, the derivative of $E(.)$ with respect to θ_1 has the same sign that:

$$(1-\eta)q_1(y_1 - d) - (m + \eta p_1)c$$

We deduce from equation (41) that the previous equation is (strictly) positive for $a > 0$. As the term H is a decreasing function of θ_1, we thus deduce that the derivative of $E(.)$ with respect to θ_1 is (strictly) positive.

The derivative of $E(.)$ with respect to θ_2 has the same sign that:

$$-q_2(1-\eta)\left(y_2 - d + \frac{\lambda}{m}H\right) + \frac{m+\lambda}{m}(m+\eta p_2)c$$

We deduce from equation (42) that the previous equation is nil. The derivative of $E(.)$ with respect to θ_2 is therefore nil.

The impact of parameter a on the educational effort $e(a)$ is obtained by equalizing the differential $E(.)$ to zero:

$$dE(.) = \frac{\partial E}{\partial \theta_1}\theta_1'(a)da + \frac{\partial E}{\partial \theta_2}\theta_2'(a)da + \frac{m}{\pi'(e)^2}\pi"(e)de = 0$$

where

$$\frac{de}{da} = -\frac{\frac{\partial E}{\partial \theta_1}\theta_1'(a)}{\frac{m}{\pi'(e)^2}\pi"(e)}$$

The function $\pi(.)$ being concave, the derivative of the implicit function $e(a)$ is thus (strictly) negative. As a consequence, the optimal value of educational effort e est lower than its decentralized equilibrium value ($e(a^*) < e(0)$). This proves the second part of proposition 2. $\square$

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