Trade Liberalization, Offshoring and Firm Heterogeneity *

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September 12, 2009

JOB MARKET PAPER

Abstract

This paper analyses the impact of trade liberalization in a model where heterogeneous firms can freely offshore their production. Firms choose whether to produce, and if so whether to sell on the domestic market only or on the export market as well. Simultaneously, they also choose where to locate their production. The paper shows that the interaction between heterogeneity in firm productivity and the possibility of offshoring production dramatically alters the impact of trade liberalization. Three main results emerge from this interaction:

i) Intra-industry factor reallocation towards the most productive firms, which is induced by trade liberalization, operates at the world level, but not necessarily at the country level and thus trade liberalization can lead to average productivity losses in some countries; ii) Trade liberalization may reverse country specialization independently of any country size effect; iii) The relation between trade liberalization and trade growth is non-linear, even in the absence of trade in intermediate goods.

J.E.L: F12, F15, R30

Keywords : Trade Liberalization, Offshoring, Firm Heterogeneity, Trade Patterns, Average Productivity

* **Acknowledgments:** I am grateful to Philippe Martin for his encouragment and advice. I also thank Matthieu Crozet, Fabrice Defever, Didier Laussel, Thierry Mayer, Marc Melitz, Toshihiro Okubo and Julien Vauday, as well as seminar and conference participants at University Paris 1, European University Institute, University of Granada, SMYE Conference, RIEF Meetings, EEA Conference, AFSE Congress. All remaining errors are mine.

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1 Introduction

Newly available firm-level data offer a strong evidence of an important dispersion in size and productivity among firms even in narrowly defined industries. The data show that only a small fraction of firms export and that exporters are larger and more productive than non-exporters (see Eaton *et al* 2004, Bernard and Jensen 2004, Bernard *et al* 2007). This new evidence led to the introduction into new trade theories of firm efficiency heterogeneity. Probably the most influential model, achieved by Melitz (2003), allows the analysis of the coexistence of firms with different productivity levels within general equilibrium analysis.[1] This recent literature highlights that trade is carried out by the most efficient firms, and that trade liberalization induces average productivity gains through the reallocation of factors towards those firms. Importantly, this literature assumes that firms stay in their country of origin and therefore do not choose the location of their production.

The ongoing trend of world globalization, however, is leading to a large reorganization of production location across countries. According to UNCTAD, world FDI inflows rose by 600% from 1990 to 2000 to reach 1.4 trillion dollars in 2000. Moreover, the growing share of intra-firm trade, which accounts for roughly one third of world trade (see Zeile, 1997, Antras, 2003), also provides evidence that firms relocate at least part of their production abroad. This important transformation of the world economy naturally pushes forward the question of countries' attractiveness as a major issue in the public debate. But the efficiency of firms which offshore their production is a marginal concern in this debate.

The literature has already widely explored these two important stylized facts, both theoretically and empirically, but has not focused on the interaction between offshoring and firm heterogeneity and its consequences on countries' average productivity and trade patterns. This paper aims to fill this gap on the theoretical side.

In order to do so, we introduce the possibility for firms to freely offshore their production in a Melitz-type framework. In this context, we show how the impact of trade liberalization on

[1] See also Bernard, Eaton, Jensen and Kortum (2003).

a country's average productivity and trade patterns depends on the productivity of firms that decide to offshore their production. The interaction between offshoring and firm heterogeneity yields three results that contrast with the existing literature and are worth highlighting:

First, the intra-industry factor reallocation towards the most productive firms induced by trade liberalization operates at the world level, but not necessarily at the country level. With firm heterogeneity, the impact of trade integration is to eliminate those firms with the lowest productivity as well as to enable the largest and most productive firms to expand (Melitz, 2003). With the possibility of offshoring, this story of how trade liberalization increases aggregate productivity in all countries through a self-selection process becomes more complex. Indeed, the self-selection mechanism is also affected by the possibility of offshoring when firms are heterogeneous. If trade integration leads the most productive firms to relocate, aggregate productivity in the country that experiences this outflow of firms may actually fall. This paper shows that such a scenario is possible for high-wage countries.

Second, trade liberalization may reverse country specialization independently of any country size effect. Since trade is carried out by the most efficient firms, the net exporter of the manufactured good is the country that attracts the most productive firms, and not necessarily the one that is the most attractive overall.[2] As trade gets freer, we show that the location choices of the most productive firms change. This result is a novel theoretical explanation of how trade liberalization may reverse "ex ante" comparative advantage. Indeed, the trade literature only provides explanations for this possibility through the Home Market Effect (see Laussel and Paul, 2007, for a recent contribution).

Third, trade liberalization may increase world trade growth in a non-linear way without any international fragmentation of the production process. This last result provides an alternative explanation of an empirical fact analyzed by Yi (2003): The response of world trade growth to a change in tariffs is highly non-linear. As shown by Yi (2003), this empirical fact is difficult

[2]Note that it also means that the two standard definitions of the Home Market Effect, i.e. the larger country i) attracts a disproportionate share of world production ii) is a net exporter of the manufactured good, are not equivalent.

to rationalize using standard trade models. His explanation is based on international trade in intermediate goods. Using a Ricardian model of trade, he shows that trade liberalization induces a fragmentation of the production process over a larger range of countries, which increases the number of trade flows per final good sold. Similarly to Yi (2003), we find a non-linear response of world trade growth to a decrease in trade costs even if we do not model intermediate inputs.

These results are the consequences of the interaction between offshoring and firm heterogeneity. They would not hold in a model with heterogeneous firms but without offshoring, nor in a model with symmetric firms and offshoring.

In this paper, we define offshoring in the following way: A firm producing a good and choosing to offshore its production abroad stops production at home and transfers all the production process to another country. An important aspect of our analysis concerns the productivity of the firm and how it is affected by the decision to locate production abroad. In the existing literature, the presence of wage differentials between countries is considered as one of the main motives for firms to offshore their production. In this case, the direct consequence of offshoring is that the observed productivity of a firm is necessarily affected by its new location. It is however less clear why this offshoring decision should affect the idiosyncratic productivity of the firm. We will thus assume that the firm is able to freely "travel" with its idiosyncratic productivity.[3]

We first develop a two-country variant of the model by Chaney (2008), which serves as a benchmark. [4]Specifically, we assume that the two countries are symmetric in all ways except in their labor input coefficient in the numeraire sector. This generates a comparative advantage, as well as an incentive for some firms to relocate their production to the other country.

We first display the model's conclusions concerning average productivity and trade patterns when firms are immobile. We then relax the assumption commonly made in the literature that

[3]This assumption is also motivated by the fact that the firm is solely defined by its productivity in these models. If the firm was forced to draw another productivity level when offshoring its production abroad, it would be less clear how and why we should consider this firm as the same.

[4]His model reproduces the main mechanisms of Melitz's model, but is static rather than dynamic. This simplification, together with the introduction of a numeraire sector that can be freely traded, leads to a more tractable model. The main cost of this simplification is that it is no longer possible to consider a free entry condition *per se* (i.e. the number of potential entrants is fixed), but rather an endogenous number of firms in equilibrium.

firms stay in their country of origin and allow them to freely offshore their production.

Since firms differ in productivity levels, we are able to determine their incentives to choose to exit the market, to produce domestically or to export as well, in each market, with respect to their productivity. The number of firms, their status (domestic or exporter), and the location of their production are thus endogenously determined in this version of the model. By comparing the results obtained with and without firm mobility, we stress the impact of offshoring and infer the three results presented above.

Several implications of firm heterogeneity have been analyzed in the trade literature. Importantly for this paper, Helpman, Melitz and Yeaple (2004) have studied firms' decisions to make foreign direct investments. The possibility of making foreign direct investments in order to produce domestically in both countries is not taken into account in our model, since it would lead to very similar results to those provided by the above authors and would not affect the main qualitative results of the paper.

Firm heterogeneity has also been introduced in economic geography models. A first attempt to tackle this issue was provided by Baldwin and Okubo (2006). Based on a simple economic geography model, they show that the most efficient firms should relocate first towards larger countries, implying that standard empirical measures of agglomeration economies are biased and likely to be overestimated. Their results depend, however, on the initial distribution of firms between countries. A more recent paper (Okubo et al 2009) introduces two kinds of firms (low and high-cost firms) in an economic geography model with a quadratic utility function. They show that high and low productivity firms do not have the same location incentives, leading to a spatial sorting of firms. This feature is also present in this paper, but is enriched by a selection between domestic firms and exporters. The papers mentioned consider all firms as exporters and therefore cannot tackle the issue of factor reallocations from domestic firms to exporters. Neither do they provide clear conclusions on trade patterns for the same reason. In this paper, we simultaneously endogenize the location of production and firms' decisions to export with respect to trade costs, which allows an assessment of the impact of offshoring on average productivity and trade patterns.

Another recent body of research has studied the consequences of the possibility of offshoring parts of the production process on firm boundaries (see Antras, 2003 & 2005, Antras and Helpman, 2004, Grossman and Helpman, 2003 & 2005). A central question in this literature is how trade liberalization changes the incentives to offshore parts of the production process through foreign direct investments or through outsourcing inputs from independent foreign suppliers. Another stream in this literature studies how the possibility of offshoring some "tasks" in the production process amends the conclusions of the Hecksher-Olhin model (Grossman and Rossi-Hansberg, 2006, Baldwin and Robert-Nicoud, 2007). Here, we do not consider different modes of organizations. This simplifying assumption, however, allows us to easily deal with factor reallocations among firms due to trade liberalization, and gives the possibility of studying trade patterns in a general equilibrium analysis. This in turn enables us to single out the consequences of the interaction between offshoring and firm heterogeneity.

The remainder of the paper is organized as follows. In section 2, we present the basic assumptions of the model in a closed economy framework. In section 3, we study the open economy equilibrium, when firms are forced to stay in their country of origin. This case serves as a benchmark. In section 4, we study the open equilibrium with offshoring and derive the main results of this paper. Concluding remarks are provided in section 5.

2 Model Framework: Closed Economy

2.1 Demand

Preferences of a representative consumer in country j are depicted by a quasi-linear utility function U with preference parameter μ, and a CES sub-utility function over the continuum of varieties produced in sector M:

$$U = \mu \ln C_M + C_N \quad C_M = \left(\int_{i \in \Omega} c_i^{1-1/\sigma} \right)^{1/(1-1/\sigma)} \text{ where } \sigma > 1 \text{ and } \mu > 0 \tag{1}$$

C_M and C_N denote consumption of the M composite manufactured good and the numeraire good, respectively. Ω is the set of available varieties that has to be determined in equilibrium. The constant elasticity of substitution between any two varieties is given by σ. The parameter μ measures the demand for the composite good. The use of a quasi-linear utility function ensures that expenditures devoted to the composite manufacturing good are constant (and equal to μ), whatever the level of income.

The CES nature of consumer preferences generates the following demand function for any variety i:

$$c_i = \mu L_j P_j^{\sigma-1} p_i^{-\sigma} \qquad\qquad P_j = \left(\int_{i\in\Omega} p_i^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}, \tag{2}$$

where p_i is the price of variety i, L_j the population in country j, and P_j is the perfect price index in this country.

Importantly, we interpret the perfect price index as an inverse index of market competition. In this paper, we will therefore say that market competition strengthens when the perfect price index in market j decreases. Indeed, in this case the market share of firm i will decrease independently of market size, preferences and firms' idiosyncratic marginal costs. The perfect price index in country j can decrease if average marginal costs are lower and/or if the number of firms operating in this market increases. Note that with our definition of market competition we do not refer to a change in mark-ups, which in our model are constant, due to the CES nature of preferences.[5]

2.2 Production

The numeraire good is produced under perfect competition with α_j units of labour per unit of output. Hence, normalizing the price of good N to one, the wage in country j is $\frac{1}{\alpha_j}$.

Any operating firm i in the M sector bears a fixed overhead labor cost C_D,[6] and a constant marginal production cost a_i. The cost of producing q units of good i with marginal cost a_i in

[5]Note that Melitz and Ottaviano (2008) have a different definition and refer to market competition as a change in mark-ups.

[6]This fixed cost reflects the cost of building a plant, and also all costs entailed by the legal system and the standards applied in the country.

country j is thus:

$$C_i(q) = (a_i q + C_D)\, w_j \tag{3}$$

Given the demand function (2), the optimal price charged by firm i is a constant mark-up over its marginal cost. Hence, a firm with marginal cost a_i will charge a price $p_i = \frac{\sigma}{\sigma-1} a_i w_j$. It follows that profits of a firm with marginal cost a_i are:

$$\pi_{ij} = \underbrace{\frac{\mu}{\sigma} L_j P_j^{\sigma-1} \left(\tfrac{\sigma}{\sigma-1} w_j\right)^{1-\sigma}}_{\text{market access}} a_i^{1-\sigma} - w_j C_D \tag{4}$$

We call market access the component of demand addressed to a firm which is independent of its productivity and thus common to all firms. The other component of demand is specific to the firm; its marginal cost a_i.

We follow Chaney (2008) in assuming that firm marginal costs are drawn from a Pareto distribution. This assumption is becoming increasingly common (see for instance Do and Levchenko 2008, Arkolakis 2008). Not only motivated by empirical evidence (see Axtell, 2001, Luttmer, 2007), this assumption also allows us to make simple analytic statements in the model without offshoring and to make results easily comparable with Chaney (2008) and Helpman *et al* (2004). However, the main results derived in the model with offshoring do not depend on this assumption.

Precisely, we assume that marginal costs are comprised between 0 and 1 and that these marginal costs are drawn from a Pareto distribution $F(a)$ with a shape parameter ρ:

$$F(a) = (a)^{\rho} \;\; \text{, with } 0 < a < 1 \tag{5}$$

Finally, we assume that there is a group of entrepreneurs proportional to the size of the country. Hence, the total mass of potential entrants in country j is fixed and proportional to L_j.

2.3 Equilibrium

In equilibrium, each active firm earns non-negative profits. Equation (4) defines the profit expression for any firm and thus defines the least efficient active firm in the economy. We label as a_D the marginal cost drawn by the least efficient firm, which earns zero profits:

$$a_D = \tfrac{\sigma-1}{\sigma}\left(\frac{\mu L_j w_j^{\sigma}}{\sigma C_D}\right)^{\frac{1}{\sigma-1}} P_j \tag{6}$$

Firms which have drawn a marginal cost above a_D choose not to produce, whereas firms which have drawn a marginal cost below a_D realize pure profits and constitute the mass of active firms in equilibrium.[7]

The definition of a_D in (6) together with (2) allows us to solve the model for a_D and P:

$$P_j^{1-\sigma} = \left(L_j \int_0^{a_D} \left(\tfrac{\sigma}{\sigma-1}\right)^{1-\sigma} a^{1-\sigma} w_j^{1-\sigma} dF(a)da\right) = L_j \left(\tfrac{\sigma}{\sigma-1}\right)^{1-\sigma} w_j^{1-\sigma} \tfrac{\rho}{1-\sigma+\rho} a_D^{1-\sigma+\rho} \tag{7}$$

$$a_D = \left(\frac{\mu\left(1-\sigma+\rho\right)}{\sigma\rho C_D w_j}\right)^{\frac{1}{\rho}} \tag{8}$$

The marginal cost drawn by the least efficient firm is lower, the higher the fixed cost C_D and the wage w_j, since an increase in any of those two increases the cost function for a given quantity produced. This threshold, together with $F(a)$, is sufficient to determine the mass of active firms in equilibrium.

This allows to compute the inverse of the average productivity, what we can call the average marginal cost in this economy, which we denote by $\widetilde{a}_j$. We define the average marginal cost as the mean quantity of labor that is used to produce one good M in this economy.[8] Accordingly, the average marginal cost is computed as the arithmetic mean of the marginal costs drawn, weighted by the distribution of sales in market j, namely $s_j(a) = \left(\frac{\mu L_j w_j^{1-\sigma}}{P_j}\right) a^{1-\sigma}$ and the distribution $dF(a)$:

[7]Parameters are chosen such that $a_D < 1$.

[8]This definition is slightly different from Melitz (2003), who computes the average marginal cost (actually the average productivity) as the mean quantity of labor that is used to "produce" one unit of the representative consumer's welfare. While very similar, we will clarify below why we prefer the measure used here.

$$\begin{aligned} \widetilde{a}_j &= \frac{L_j \int_0^{a_D} a * s_j(a) dF(a) da}{L_j \int_0^{a_D} s_j(a) dF(a) da} \\ \widetilde{a}_j &= \tfrac{1-\sigma+\rho}{2-\sigma+\rho} a_D \end{aligned} \tag{9}$$

3 Open economy

We now consider two open economies A and B, that engage in international trade.

Trade in good N is assumed to be costless, while trade in the composite good M is costly, forcing firms that engage in international trade to incur both variable and fixed costs. Variable trade costs take the standard form of iceberg trade costs, so that for one unit of a good to be delivered, $\tau > 1$ units must be shipped. This variable cost increases the marginal selling cost and thus the consumer price for imported varieties.

The fixed cost of participating in the export market is C_X, which, like the domestic fixed cost, is an overhead labor cost. The fixed cost of exporting is consistent with the need to obtain information about the foreign market, the need to alter product characteristics in order to meet market-specific standards, and the need to create a distribution network in the foreign country.[9] The literature provides much evidence for the presence of such costs in export markets (see Das et al, 2006, for an estimation of these sunk costs for Columbian manufacturing plants).

The two countries are perfectly symmetric except for their labor input coefficients in the production of good N (α_j). Specifically, we assume that $\alpha_B = 1$ and $\alpha_A = \frac{1}{\gamma}$ with $\gamma > 1$, introducing a comparative advantage in good N for country A.

Free trade in good N ensures price equalization across countries. Besides, perfect competition forces marginal cost pricing in both countries, which induces the world price of good N to be such that: $p_N^W = w_B = \frac{w_a}{\gamma}$.[10] Using good N as a numeraire, we normalize p_N^W to one which pins

[9]Many papers highlight the importance of sunk costs associated with engaging in international trade. We consider here fixed rather than sunk costs, as in Yeaple (2005) and Chaney (2008), since the framework considered is static.

[10]We assume that μ is sufficiently low to ensure that both countries produce the numeraire good.

down wages in the two countries: $w_B = 1 < w_a = \gamma$.

3.1 Firm selection

Most related papers impose a condition on the size of export fixed costs and variable trade costs relative to domestic fixed costs in order to avoid any incentive for firms to serve the foreign market without serving their domestic market, and thus act as export-platforms. Here, we assume instead that firms cannot export if they do not also serve their domestic market. Accordingly, firms choose to be exporters only if their pure profits from serving both local and foreign markets are larger than those from their local market only.

Profits of domestic firms (D firms) and export firms (X firms) in each market are:

$$\begin{aligned} \pi_{DA}(a_i) &= \beta P_A^{\sigma-1} (\gamma a_i)^{1-\sigma} - \gamma C_D && (10) \\ \pi_{DB}(a_i) &= \beta P_B^{\sigma-1} a_i^{1-\sigma} - C_D && (11) \\ \pi_{XA}(a_i) &= \beta \left(P_A^{\sigma-1} + \phi P_B^{\sigma-1} \right) (\gamma a_i)^{1-\sigma} - \gamma (C_D + C_X) && (12) \\ \pi_{XB}(a_i) &= \beta \left(P_B^{\sigma-1} + \phi P_A^{\sigma-1} \right) a_i^{1-\sigma} - (C_D + C_X), && (13) \end{aligned}$$

where $\beta = \frac{\mu}{\sigma} \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma}$ and $\phi = \tau^{1-\sigma}$, which is an index of trade freeness, ranging from 0 to 1.

Each firm chooses its status (D or X) so as to maximize its profits, taking the decision of all other firms as given. The partitioning of firms among status thus depends exclusively on their marginal costs:

$$\begin{aligned} &DA \text{ firm if } \pi_{DA}(a_i) \geq \pi_{XA}(a_i) \text{ and } \pi_{DA}(a_i) \geq 0 \\ &XA \text{ firm if } \pi_{XA}(a_i) \geq \pi_{DA}(a_i) \text{ and } \pi_{XA}(a_i) \geq 0 \\ &DB \text{ firm if } \pi_{DB}(a_i) \geq \pi_{XB}(a_i) \text{ and } \pi_{DB}(a_i) \geq 0 \\ &XB \text{ firm if } \pi_{XB}(a_i) \geq \pi_{DB}(a_i) \text{ and } \pi_{XB}(a_i) \geq 0 \end{aligned} \qquad (14)$$

An active domestic firm therefore decides to become an exporter if the additional profits from export sales cover at least the extra fixed costs implied by the decision to export. The more profitable a firm is (lower a_i), the larger the additional profits from exporting. Hence, the most efficient firms decide to export while less efficient firms serve their domestic market only.

3.2 Equilibrium

Each firm chooses its status taking the decision of the other firms as given. We label as a_{Dj} the marginal cost drawn by the least efficient domestic firm in country j, which is indifferent between exiting the market or being a domestic firm. Similarly, a_{Xj} is the marginal cost drawn by the least efficient export firm in country j, which is indifferent between serving both markets or only the local one. From (10), (11), (12), (13) and (14), we obtain:

$$a_{DA} = \left(\frac{\beta}{\gamma^{\sigma} C_D}\right)^{\frac{1}{\sigma-1}} P_A \quad a_{DB} = \left(\frac{\beta}{C_D}\right)^{\frac{1}{\sigma-1}} P_B \tag{15}$$

$$a_{XA} = \left(\frac{\phi\beta}{\gamma^{\sigma} C_X}\right)^{\frac{1}{\sigma-1}} P_B \quad a_{XB} = \left(\frac{\phi\beta}{C_X}\right)^{\frac{1}{\sigma-1}} P_A \tag{16}$$

Firms in market j that have drawn a $a > a_{Dj}$ do not produce, while if $a_{Xj} < a \leq a_{Dj}$, they serve the domestic market only and if $a \leq a_{Xj}$, they serve both their domestic market and the foreign market (with $j = A, B$).[11]

These threshold definitions determine the status of each firm with respect to its marginal cost and allow further to define the price indices (P_A and P_B) in each country:[12]

$$\begin{aligned} P_A^{1-\sigma} &= \lambda\left(\gamma^{1-\sigma}\int_0^{a_{DA}} a_i^{1-\sigma} f(a)da + \phi\int_0^{a_{XB}} a_i^{1-\sigma} f(a)da\right) \\ P_B^{1-\sigma} &= \lambda\left(\phi\gamma^{1-\sigma}\int_0^{a_{XA}} a_i^{1-\sigma} f(a)da + \int_0^{a_{DB}} a_i^{1-\sigma} f(a)da\right) \end{aligned} \tag{17}$$

The definition of the marginal cost thresholds for domestic firms in (15) may be interpreted as the free entry conditions in each country, since they define the marginal cost for which domestic pure profits are equal to zero. Given $F(a)$, these cutoffs determine the mass of active firms in

[11] If $a_{Xj} \geq 1$, there is no partitioning of firms between the domestic and export markets since all firms export. We assume C_X to be sufficiently large to get such a partitioning.

[12] with $\lambda = \frac{\rho}{1-\sigma+\rho}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}$

equilibrium.

(15), (16) and (17) allow to solve for the cutoffs:

$$a_{DB}^{\rho} = \frac{\beta}{C_D\lambda}\left(\phi\gamma^{1-\sigma}\left(\frac{\phi C_D}{\gamma^{\sigma}C_X}\right)^{\frac{\rho}{\sigma-1}-1}+1\right)^{-1} \quad (18)$$

$$a_{DA}^{\rho} = \frac{\beta}{\gamma^{\sigma}C_D\lambda}\left(\gamma^{1-\sigma}+\phi\left(\frac{\phi\gamma^{\sigma}C_D}{C_X}\right)^{\frac{\rho}{\sigma-1}-1}\right)^{-1} \quad (19)$$

$$a_{XA}^{\rho} = \frac{\beta}{C_D\lambda}\left(\gamma\frac{C_X}{C_D}+\left(\frac{\gamma^{\sigma}C_X}{\phi C_D}\right)^{\frac{\rho}{\sigma-1}}\right)^{-1} \quad (20)$$

$$a_{XB}^{\rho} = \frac{\beta}{C_D\lambda}\left(\gamma\left(\frac{C_X}{\phi\gamma^{\sigma}C_D}\right)^{\frac{\rho}{\sigma-1}}+\frac{C_X}{C_D}\right)^{-1} \quad (21)$$

The model differs from that of Chaney (2008) in assuming a different labor input coefficient in the production of good N, inducing a wage gap between the two countries. Production of good M is cheaper in country B, which allows more firms to enter this market (for both domestic and exporting firms: $a_{DB} > a_{DA}$ and $a_{XB} > a_{XA}$, see figure 1). This leads to an important departure from the standard symmetric model: market competition does not equalize between countries. Production of good M being cheaper in market B, this market hosts more firms in equilibrium.

This feature can be measured by the ratio of the two price indices: $\widehat{P} = \frac{P_A}{P_B}$. Intuitively, this ratio is higher than 1, because country B hosts more firms. Put differently, firms in country A should benefit from a greater demand in order to make enough profits to cover the higher fixed cost induced by the higher unit labor cost. It follows that market access has to be larger in country A for both domestic and exporting firms. The calculation of the relative perfect price index $\widehat{P}$ allows to capture how this measure of relative competition between the two countries evolves with trade liberalization:

$$\widehat{P} = \frac{P_A}{P_B} = \gamma^{\frac{\sigma(1-\sigma+\rho)}{(\sigma-1)\rho}} \left(\frac{\gamma^{1-\sigma} + \phi \left(\frac{\phi\gamma^{\sigma} C_D}{C_X} \right)^{\frac{\rho}{\sigma-1}-1}}{\phi\gamma^{1-\sigma} \left(\frac{\phi C_D}{\gamma^{\sigma} C_X} \right)^{\frac{\rho}{\sigma-1}-1} + 1} \right)^{\frac{1}{\rho}} > 1 \tag{22}$$

$$\frac{\partial \widehat{P}}{\partial \phi} > 0 \tag{23}$$

The relative price index $\widehat{P}$ is greater than 1 as soon as $\gamma > 1$. Note that $\frac{\partial \widehat{P}}{\partial \phi} > 0$, which means that a decrease in trade costs magnifies the market competition differential between the two countries. Both countries benefit from trade liberalization ($\frac{\partial P_j}{\partial \phi} < 0$ for $j = A, B$) but country B benefits more from a greater openness to trade in good M because of its comparative advantage ($\frac{\partial P_B}{\partial \phi} < \frac{\partial P_A}{\partial \phi}$).

Lemma 1:

When firms cannot offshore their production, country B is a net exporter of good M, for any level of trade costs. Trade liberalization always increases its trade surplus in good M .

To foster intuition, figure 1 shows how each marginal cost threshold reacts to a decrease in trade costs.

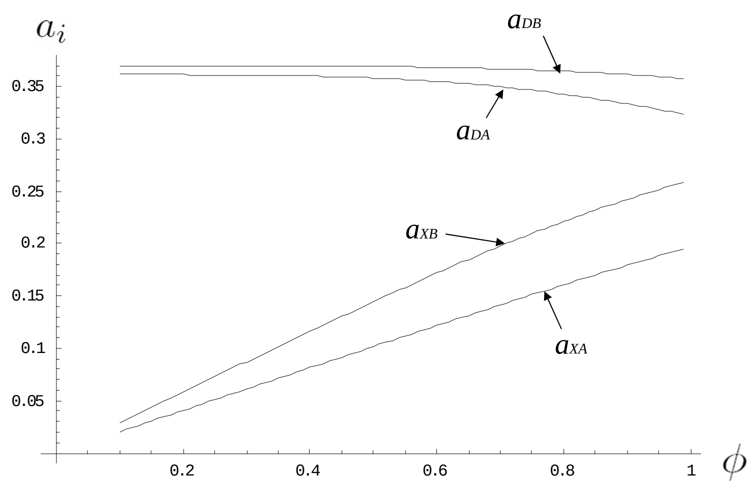
3.3 Average productivity

In Melitz (2003), the measure of average productivity is made possible by the symmetry of the countries, which ensures that exports are the mirror image of imports. Melitz's measure of average productivity cannot however be readily used in our setup with asymmetric countries. We therefore use a measure of average productivity based on production, not on utility as in Melitz.[13]

The inverse of the average productivity in each country, i.e. what we call the average marginal cost, is computed following the methodology presented in the closed economy section. We must

[13] A measure of average productivity based on utility should take into account the average productivity of imports rather than exports, but this in turn could no longer be understood as the average productivity in a country if imports and exports are not perfectly symmetric.

Figure 1: Marginal cost thresholds and trade liberalization



however take into account the asymmetry of market access between countries for each firm status. This implies that two firms having drawn the same marginal cost in each country do not have the same size, because they do not benefit from the same market access. Consequently, we must weight the marginal cost of each firm by its specific market access in order to obtain a consistent measure of the average marginal cost in each country:

$$\widetilde{a_A} = \frac{\int_0^{a_{XA}} a * \phi s_B(a)dF(a)da + \int_0^{a_{DA}} a * s_A(a)dF(a)da}{\int_0^{a_{XA}} \phi s_B(a)dF(a)da + \int_0^{a_{DA}} s_A(a)dF(a)da}$$
$$\widetilde{a_B} = \frac{\int_0^{a_{XB}} a * \phi s_A(a)dF(a)da + \int_0^{a_{DB}} a * s_B(a)dF(a)da}{\int_0^{a_{XB}} \phi s_A(a)dF(a)da + \int_0^{a_{DB}} s_B(a)dF(a)da},$$

where $s_j(a)$ is the distribution of sales in location j, net of possible trade costs.

The average marginal cost decreases in both countries under falling trade costs, due to the reallocation effect put forward in Melitz (2003): the least efficient firms in each market are forced to exit, new firms become exporters, and the intensive margin of the exporters grows. Each of these effects leads to lower the average marginal cost in both countries.

Lemma 2:

When firms cannot offshore their production, trade liberalization induces intra-industry factor reallocations towards the most productive firms and leads to efficiency gains in all countries, for any unit labor cost differential.

The above analysis has presented a static two-country model of trade with heterogeneous firms, introducing a comparative advantage in the numeraire good for country A. We have, however, assumed so far that firms are forced to produce in their country of origin, i.e. the country where they have drawn their marginal cost. We now investigate the consequences of relaxing this assumption in order to emphasize how offshoring, in interaction with firm heterogeneity, alters the standard results presented above.

4 Offshoring

This section develops a variant of the previously described model, in which firms are allowed to *freely* offshore their production to the other country. While the assumption of costless offshoring may seem extreme, it allows to emphasize how offshoring affects trade patterns and average productivity in each country. We assume that firms simultaneously choose their status (non-producer, domestic producer or exporter) and the location of their production (A or B). This gives four possibilities for active firms, defining four firm types: to be a domestic firm in country A or country B; to be an exporter in country A or country B. Since we have assumed no costs associated with offshoring, firm's country of origin has no impact on its decision.[14]

[14] An extension of this model with fixed costs associated with any offshoring decision yields the same qualitative results as those presented in this section if offshoring costs are not too high. It would however imply that the firm's country of origin would play a role in its decision, i.e. two firms with the same marginal costs but not located in the same country may take different decisions.

In this version of the model, the number of active firms, their status (D or X) as well as their location (A or B) are thus endogenous.

4.1 Firms selection

Formally, the profit expressions are the same as in the previous section (see equations 10 to 13). However, firms now face two additional possibilities and thus compare the profits that they would make in each of the four possible situations. The optimal strategy of a firm depends only on its marginal cost. Indeed, as is standard in Dixit-Stiglitz monopolistic competition models, firms are small enough such that they interact neither in their price decisions nor in their status and location decisions. Each firm thus chooses the type that maximizes its profits in equilibrium, taking the decision of all other firms as given. We can summarize the firms' decisions as follows:

$$\begin{array}{ll} DA \text{ Firm if } \pi_{DA}(a_i) \geq \left\{ \begin{array}{l} \pi_{DB}(a_i) \\ \pi_{XB}(a_i) \\ \pi_{XA}(a_i) \end{array} \right. \geq 0 & DB \text{ Firm if } \pi_{DB}(a_i) \geq \left\{ \begin{array}{l} \pi_{DA}(a_i) \\ \pi_{XB}(a_i) \\ \pi_{XA}(a_i) \end{array} \right. \geq 0 \\ XA \text{ Firm if } \pi_{XA}(a_i) \geq \left\{ \begin{array}{l} \pi_{DB}(a_i) \\ \pi_{DA}(a_i) \\ \pi_{XB}(a_i) \end{array} \right. \geq 0 & XB \text{ Firm if } \pi_{XB}(a_i) \geq \left\{ \begin{array}{l} \pi_{DB}(a_i) \\ \pi_{DA}(a_i) \\ \pi_{XA}(a_i) \end{array} \right. \geq 0 \end{array} \quad (24)$$

It is worth noting that the presence of four possible firm types does not mean that all these types necessarily coexist in equilibrium for all levels of trade costs. As an example, exporting firms in A do not operate when trade is perfectly free, since they would have access to the same world market as exporting firms from B, but they would incur a higher unit labor cost.[15]

We must therefore determine the existing firm types in equilibrium with respect to variable trade costs ϕ. This allows us to compute both price indexes as a function of existing firm types

[15]Wages could adjust in this model, for instance by introducing diminishing returns to scale in the numeraire sector. This would however make the model much more difficult to solve. Also, the results we present come from the different location incentives of large and small firms, based on the wage differential between the two countries. As long as country A is still the high wage country, the results hold.

and solve for the equilibrium with respect to trade costs.

First, we present a formal method for determining the existing firm types in equilibrium.[16]

4.1.1 Formal conditions for the existence of a firm type

Suppose that firms can choose their type among m possible firm types, where each of these firm types is characterized by an exogenous fixed cost. We rank these m possible firm types with respect to their exogenous fixed cost, from the cheapest to the costliest.[17] Thus, type 1 has the lowest fixed cost while type m has the highest. Type l ($l \in (2; m)$) must provide a larger market access than any "cheaper" type (types 1 to $l-1$), in order to generate larger sales (and profits) to cover the higher fixed cost. Otherwise, no firm would choose type l since profits would be less than those obtained under another type. This result is independent of the firms' marginal cost since market access and fixed costs are common to all firms. Importantly, this condition is necessary but not sufficient to ensure the presence of firm type l. It follows that:

Lemma 3:

A firm type characterized by a lower market access and a higher fixed cost than any other firm type cannot exist in equilibrium.

Corollary: In equilibrium, the ranking of existing firm types with respect to their fixed cost must be the same as the one with respect to their market access.

Proof. see appendix A ■

This first step eliminates a number of possible firm types. This leaves us with n different types (with $n \leq m$) ranked similarly with respect to their fixed cost and their market access. However, we must ensure that each of these possible firm types maximizes the profits of a subset of firms in order to be present in equilibrium. We now provide the condition for this.

[16]The interest in going through this methodology is twofold. In this model, it is possible to know which firm types will coexist in equilibrium. In addition, the same methodology can be applied to any other heterogeneous firm trade model with any number of different types considered, including different organizational modes for firms (FDI, outsourcing, export plateforms), and could be extended to a model with n countries. This methodology however only applies if there is no strategic interaction among firms and if fixed costs are independent of their decisions.

[17]In our model, $m = 4$, but this method could be applied for any $m > 1$.

Define as a_{l+1} the marginal cost of the firm that is indifferent between types $l+1$ and l (given by $\pi_{l+1}(a_{l+1}) = \pi_l(a_{l+1})$). Considering only these two types, all firms with a marginal cost below a_{l+1} will prefer type $l+1$ instead of type l, while all firms with a marginal cost above a_{l+1} will prefer type l rather than $l+1$. We can thus define $n-1$ marginal cost thresholds, each of which splits the mass of active firms into two groups, considering two "neighboring" types according to the above-defined ranking. Note that lemma 3 ensures that $a_l > 0$, for $l \in \{2; n\}$. The n*th* marginal cost threshold is given by the zero profit condition. Indeed, type 1 cannot be compared with a "cheaper" type. Firms that are not productive enough to make positive profits under type 1 exit the world market. This last threshold thus defines the marginal cost of the firm that is indifferent between entering the (world) market and not operating (and is given by $\pi_1(a_1) = 0$). Thanks to the distribution of marginal costs among firms, this marginal cost also drives the mass of active firms in equilibrium (given by $2F(a_1)$).

We obtain the sufficient condition for the existence of each possible firm type from:

Lemma 4:

A firm type l exists in equilibrium if and only if $a_{l+1} < a_l$, and $a_l > 0$.

Proof. see appendix B ■

The above lemma ensures that marginal costs thresholds are ranked inversely with respect to their indexes (l) in equilibrium. Firm types that do not respect this condition cannot be chosen by any firm in equilibrium. We thus have $k \leq n$ types that respect this condition.

It follows that a firm with a marginal cost a_i such that $a_{l+1} < a_i < a_l$ will choose type l. Firms with marginal costs above a_1 will exit the (world), market while firms with a marginal cost below a_k will choose type k.

4.1.2 Existence of firm types with respect to trade costs in the model

We can now determine which firm types exist in our model, for any level of trade costs. A decrease in trade costs indeed affects the market access of each firm type. As a consequence, the presence of a given firm type depends on the level of trade costs.

Quite naturally, we assume that the fixed cost of being an export firm is always higher than that of being a domestic firm worldwide.[18] Given that we have assumed that A is more expensive than B ($\gamma > 1$), we can rank fixed costs from the cheapest to the costliest as follows: $DB < DA < XB < XA$.

We further restrict the parameters in order to concentrate on the case where some domestic firms operate in the two countries for any level of trade costs.[19] Not only is this case the only realistic and interesting one according to micro-level data, it also allows us to focus on the decision whether to export or not and from where, which drives trade patterns.

We turn to the existence of exporting firms in each country with respect to trade costs. As long as trade costs are not infinite ($\phi > 0$), some firms always choose to export. Their location choice however depends on the level of trade costs.

According to lemma 3, since XA is the "costliest" type considered, the presence of some exporting firms in country A only requires that they benefit from a larger market access than any other type. When this is verified, all firms with a marginal cost below a_{XA} choose to export from country A. Formally, we obtain that the marginal cost of the firm indifferent between exporting from A and from B (a_{XA}) is characterized by:

$$\pi_{XA}(a_{XA}) = \pi_{XB}(a_{XA}) \Longleftrightarrow a_{XA} = \left(\frac{\beta \left(\left(\gamma^{1-\sigma} - \phi \right) P_A^{\sigma-1} - (1 - \phi \gamma^{1-\sigma}) P_B^{\sigma-1} \right)}{(\gamma - 1)\left(C_D + C_X \right)} \right)^{\frac{1}{\sigma-1}}$$

$$a_{XA} \geq 0 \Longleftrightarrow \phi < \frac{\widehat{P}^{\sigma-1}\gamma^{1-\sigma} - 1}{\widehat{P}^{\sigma-1} - \gamma^{1-\sigma}} = \overline{\phi}$$

Hence, some firms export from country A in equilibrium only if trade costs are high enough.[20]

[18]We therefore assume that the fixed cost of being an exporter from country B ($C_X + C_D$) is larger than that paid by a domestic firm in country A (γC_D), leading to the assumtion that $C_X - (\gamma - 1) C_D > 0$.

[19]Intuitively, these parameters restrictions must verify that the elasticity of substitution σ and the labor cost differential γ are not too large for some domestic firms to choose to locate in A rather than B, while the fixed cost of exporting C_X must be high enough compared to C_D to ensure that some firms always prefer to be domestic in A rather than being exporters in B.

[20]Formally, we should also check the existence of export firms in A if no firm chooses to export from B (i.e. if firm type XB does not exist in equilibrium). In this case, we must verify that the marginal cost of the firm indifferent between being an export firm in A or just being a domestic firm in A is positive, which is always true:

In this case, the main determinant of the location choice is the profitability of the domestic market. Locating production in country A is costlier but, as a result, this also provides greater local sales since competition is weaker than in market B. Therefore, the most productive firms choose to export from A even if this is not the best location to export from. Indeed, any firm exporting from country B would make greater profits from exporting than if it were located in country A. However, greater domestic sales in market A overcome the lower sales in the export market.

According to lemma 4, export firms in country B only operate if $a_{XA} < a_{XB}$ and $a_{XB} > 0$.[21] We obtain that the marginal cost of the firm indifferent between exporting from B and being a domestic firm in A (a_{XB}) is given by:[22]

$$\pi_{XB}(a_{XB}) = \pi_{DA}(a_{XB}) \Longleftrightarrow a_{XB} = \left(\frac{\beta \left(P_B^{\sigma-1} - \left(\gamma^{1-\sigma} - \phi \right) P_A^{\sigma-1} \right)}{C_X - (\gamma - 1) C_D} \right)^{\frac{1}{\sigma-1}}$$

$$a_{XA} < a_{XB} \Longleftrightarrow \phi > \frac{\widehat{P}^{\sigma-1}\gamma^{1-\sigma} - 1}{\widehat{P}^{\sigma-1} - \gamma^{-\sigma}\left(1 - (\gamma - 1)\frac{C_D}{C_X}\right)} = \underline{\phi}$$

A subset of firms exports from country B only if their profits are greater than if they were domestic or exporters in country A. It follows from the above condition that some exporters operate from B only when trade is sufficiently free, which allows them to have an easy enough access to market A. It follows that trade costs must be low enough to provide sufficient sales from exporting from B to compensate for the lower level of domestic sales compared to A.

These two trade cost thresholds determine the existence or the absence of exporting firms in A and B, independently of the marginal cost distribution. For high trade costs (e.g. $\phi < \underline{\phi}$) no firm exports from country B, while for low trade costs (e.g. $\phi > \overline{\phi}$) no firm exports from

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$\pi_{XA}(a_{XA}) = \pi_{DA}(a_{XA}) \Longleftrightarrow a_{XA} = P_B \left(\frac{\beta\phi}{\gamma^{\sigma} C_X} \right)^{\frac{1}{\sigma-1}} > 0, \forall \phi > 0.$

[21] These conditions only apply if some firms choose to be exporters from A. If firm type XA does not exist, for the existence of some exporters in country B, only $a_{XB} > 0$ is required.

[22] The condition $a_{XB} > 0$ also leads to a constraint on trade costs: $a_{XB} > 0 \Leftrightarrow \phi > \frac{\gamma^{1-\sigma}\widehat{P}^{\sigma-1} - 1}{\widehat{P}^{\sigma-1}}$, but the range of ϕ for which $a_{XB} > 0$ is larger than that ensuring $a_{XB} > a_{XA}$.

country A. For intermediate trade costs (e.g. $\underline{\phi} < \phi < \overline{\phi}$) firms export from both countries, and international trade is characterized by intra-industry trade in good M between the two countries.[23]

4.2 Equilibrium and intra-industry factor reallocations

The above determination of the existing firm types in equilibrium with respect to trade costs allows us to compute price indexes for any level of trade costs and therefore solve for the equilibrium. For expositional ease, we only present a graphical analysis of the equilibrium with respect to trade costs, based on the results of the previous section, which are independent of the marginal cost distribution. We relegate to appendix D the methodology to solve for $\widehat{P}$ using the Pareto distribution, which allows to obtain the equilibrium values of the marginal cost thresholds.

Based on the results obtained in the previous section, figure 3 shows how the choices of firms in both countries to exit (EXIT), to remain domestic (DB, DA) and to export (XB, XA) depend on their marginal cost (vertical axis) and on trade costs (horizontal axis):

Note that this figure is only a stylized representation of the equilibrium. Marginal cost thresholds may evolve in a non-linear way with ϕ; this linear representation is only motivated by simplicity.

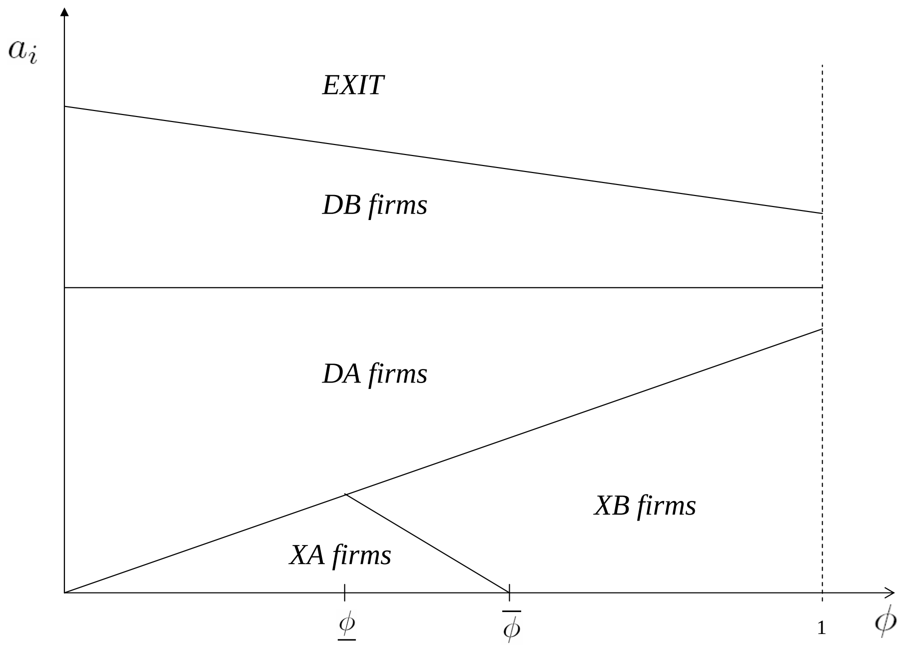
4.2.1 Trade liberalization and self-selection of firms into firm types

Trade liberalization (an increase in ϕ) affects the market access of each firm type, and as a consequence alters the incentive to remain domestic, to export or to relocate. This effect of trade liberalization is shown in figure 3 as moving to the right from $\phi = 0$ to $\phi = 1$.

The ranking of firm types necessarily leads the most efficient firms to choose to export, while less efficient firms choose to be domestic producers. The assumption of costless offshoring allows firms to freely choose their type among the four possible ones. Their country of origin has thus no impact on their decisions.

[23]Intuitively, we must have $\underline{\phi} < \overline{\phi}$. Otherwise, no firms would export for intermediate trade costs while they would export for large trade costs. This result can however be easily demonstrated with the Pareto distribution.

Figure 2: Firm types with respect to ϕ



At the world level, the self-selection mechanisms present in Melitz (2003) and Chaney (2008) are also present in our framework. Figure 3 shows that as trade costs decrease, the total mass of exporters increases (the subset $XA \cup XB$ expands). Access to the foreign market is indeed facilitated, providing new incentives to export. This increases competition everywhere since both markets are better integrated. As a consequence, the least efficient domestic firms worldwide are forced to exit (the subset $DA \cup DB$ falls).

At the country level, the story is richer. As is standard in the literature, the market competition differential, induced by the wage differential, is magnified as trade costs decrease ($\frac{\partial \widehat{P}}{\partial \phi} > 0$ the formal proof of this result is provided with the Pareto distribution in appendix D). Trade liberalization thus increases the market access of exporting firms in country B more than in country A, and decreases the market access of domestic firms in country B more than in country A. These effects alter the location decisions of domestic and export firms.

Consider first exporting firms. Using the results of the previous section, figure 3 shows that for high trade costs (i.e. $\phi < \underline{\phi}$) those firms only locate in country A. However, when $\phi > \underline{\phi}$, some firms decide to relocate and export from country B (XB expands). Interestingly, in this model, the increase in exports from the low-wage country results from the relocation of firms from the high-wage country. Trade liberalization increases the profitability of exporting from B faster than from A. The reason is that, as trade costs decrease, differences in production costs become more important than market access in the choice of location. Note that exporting firms that choose to relocate from A to B are the least efficient, while domestic firms that choose to relocate from A to B are the most efficient.

The impact of trade liberalization on the location choice of domestic firms is more complex and ambiguous. Whether domestic firms relocate from A to B or from B to A depends on two opposite forces: on the one hand trade liberalization reduces the profitability of domestic firms everywhere, which gives incentives for domestic firms to relocate from A to B since producing in B is cheaper. However, trade liberalization increases local competition faster in B than in A ($\frac{\partial \widehat{P}}{\partial \phi} > 0$), which gives incentives for domestic firms to relocate from B to A. The relative strength of these two opposite effects depends on the parameters of the model. Because it is not the main focus here, we merely show an example in figure 3 where the two effects cancel out.

4.2.2 Intra-industry factor reallocations

The intra-industry reallocations from less productive to more productive firms put forward by Melitz (2003) still operates in our model, but only at the world level. As trade costs decrease (ϕ rises), the most productive domestic firms decide to engage in international trade, therefore increasing the worldwide number of exporters, while the least efficient firms are forced to exit the (world) market, due to the increased competition. These effects improve the world average productivity of firms. However, they do not necessarily operate in both countries, because of the possibility of firms freely offshoring their production.

When trade costs are high (i.e. $\phi < \underline{\phi}$), trade liberalization generates intra-industry factor reallocations towards the most productive firms in country A, because the most efficient domestic firms become exporters and exporters increase in size. In country B, the exit of the least efficient firms also reallocates factors towards the most productive firms. Hence, when trade costs are high, trade liberalization produces average productivity gains in both countries.

When trade costs are intermediate (i.e. $\underline{\phi} < \phi < \overline{\phi}$), trade liberalization forces the least efficient firms to exit in country B, but also produces incentives for some firms in A to offshore their production in country B and export from there. These firms become the most efficient firms in this country. This tends to reallocate factors towards the most productive firms and therefore improves average productivity in country B. In contrast, the least efficient exporting firms and the most efficient domestic firms in country A offshore their production to country B. The impact of trade liberalization on average productivity in country A thus depends on the productivity of these firms compared to the average productivity of firms remaining in the country.

Finally, when trade costs are low enough (i.e. $\phi > \overline{\phi}$), trade liberalization induces intra-industry factor reallocations towards the least efficient firms in country A, since the most efficient domestic firms offshore their production to country B. As a consequence, average productivity necessarily decreases in country A. In country B, the exit of the least productive firms helps to improve average productivity. Moreover, new firms become exporters from B. However, the offshoring of firms from country A to country B could lead to factor reallocation towards less productive firms in country B, because these new firms may be less productive than the average productivity in B. This composition effect could lead to an average efficiency loss in both countries (but not at the world level).

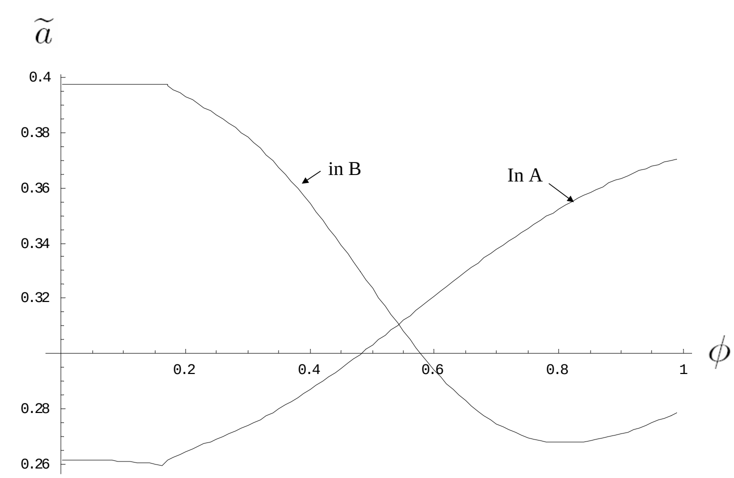
This analysis of the impact of trade liberalization on the offshoring decisions of firms with different productivity levels underlines the possibility of factor reallocations from more to less productive firms, in contrast to the situation where firms cannot choose where to locate their production. From this follows:

Proposition 1:

While trade liberalization necessarily induces intra-industry factor reallocation towards the most productive firms at the world level, it can induce intra-industry factor reallocation towards the least productive firms in the high-wage country.

According to proposition 1, trade liberalization may concentrate all efficiency gains in the low wage country and induce efficiency losses in the other. To foster intuition, figure 4 presents a simulation of the average marginal cost in each country. As in the previous section, average marginal costs are computed by weighting firms' marginal cost by their market access. The formal expressions for the average marginal costs in the two countries are provided in appendix C:[24]

Figure 3: Average marginal costs and trade liberalization



This figure illustrates that a country may experience a fall in efficiency induced by trade liberalization. Note however that our measure of average productivity being based on production and not on utility, this fall in efficiency does not mean a fall in welfare.

[24]The parameters values used are: $\rho = 6$, $\sigma = 2$, $\mu = 0.2$, $C_D = 4$, $C_X = 4.5$ and $\gamma = 1.01$.

We turn now to a more precise discussion of the further consequences of the presence of both heterogeneous firms and offshoring on the direction of net trade flows, and the trade response to trade liberalization.

4.3 Trade flows

The direction of trade flows depends on the level of trade costs in the model. For high trade costs, exporting firms are located in country A only, which is thus necessarily the net exporter of good M, whereas for low trade costs, exporting firms are only located in country B, which thus becomes the net exporter of the manufactured good after trade liberalization. Intra-industry trade only occurs for intermediate trade costs.

This specific feature of our model means that an "ex ante" comparative advantage[25] cannot explain the direction of net trade flows. This results from the interaction between firm heterogeneity and offshoring. Indeed, section 3.2 shows that without offshoring, country B is the net exporter of the manufactured good, whatever the extent of market integration. Furthermore, it can be shown that the same applies when firms are identical and can offshore production. In this latter case, all firms would be exporters, and the net exporter of the manufactured good would be country B, since country B always hosts a larger share of the world manufacturing production.

The source of this new result is that the most efficient firms do not have the same location incentives as the least efficient ones. The net exporter of the manufactured good is thus not the country that attracts the largest share of world production, but the one that attracts the most productive firms, i.e the exporters.

To our knowledge, the existing literature in trade only provides explanations for a reversal of net trade flows induced by trade liberalization based on the Home Market Effect (HME), introduced by Krugman (1980).[26] The present model proposes an alternative explanation for the

[25]The productivity in the numeraire sector is lower in B than in A so that B has an "ex ante" comparative advantage in the manufactured good.

[26]As an example, in the same model with symmetric firms, suppose that trade costs are large and that country A has a larger demand for good M than country B. According to the HME, this country should host a dispro-

reversal of net trade flows even if countries have identical demand for the manufactured good. The HME is thus not at work here. In contrast to explanations based on the HME, country A is able to be a net exporter of the manufactured good for large trade costs *even if* it hosts less than 50% of the world manufacturing production, because it attracts the most efficient firms: those that export.

Proposition 2:

The interaction between firm heterogeneity and offshoring is able to reverse the direction of net trade flows following trade liberalization, independently of any country size effect.

This result is also important because it means that the two standard definitions of the HME, i.e. the larger country i) attracts a disproportionate share of world production ii) is a net exporter of the manufactured good, are not equivalent.[27]

4.4 Trade response to trade liberalization

Another feature of our model is its ability to produce a non-linear response of world manufactured trade growth to variable trade costs. Indeed, Yi (2001) points out that: "*the response of exports to tariffs has increased sharply since the mid-1980s [...]. For example, between 1962 and 1985 the elasticity of trade with respect to tariffs was 7, while between 1986 and 1999 it was 50. This non-linear effect is a qualitative puzzle from the perspective of the standard models*". The theoretical mechanism that Yi proposes to explain this non-linearity is based on the fragmentation of the production process induced by trade liberalization. The source of non-linearity in our model can be interpreted as a complement to the explanation by Yi. It is the non-linear response of the two margins of trade to trade liberalization. The intuition behind this result is as follows: In the first phase (when trade costs are large), trade liberalization leads to an increase in exports (both on

portionate share of world production with respect to its market size and thus should be a net exporter of the manufactured good. However, as trade costs decrease, country B becomes a more attractive location for firms because of the cheaper unit labor cost. Beyond a threshold value of trade costs, country B becomes a net exporter of the manufactured good.

[27]Indeed, according to Head et al (2002): "(the HME) means that the large country hosts a disproportionate share of firms. Moreover, this condition implies that the large country will run a trade surplus in the increasing returns sector". The model shows however that a country (here country B) can attract a disproportionate share of firms while running a trade deficit in the increasing returns sector.

the intensive and the extensive margins) from the high wage country to the low wage country. The latter being more competitive, this increase cannot be very large. In a later phase, when trade liberalization has led exporters to relocate to the low wage country, further trade liberalization generates a large increase in exports to the high wage country. Both trade margins indeed react faster, since $\frac{\partial(\beta\phi P_A^{\sigma-1})}{\partial\phi} > \frac{\partial(\beta\gamma^{1-\sigma}\phi P_B^{\sigma-1})}{\partial\phi}$, because $\frac{\partial\widehat{P}}{\partial\phi} > 0$. As soon as the low wage country attracts some exporting firms, both trade margins are more sensitive to trade liberalization, inducing a convex non-linear response of world manufactured trade to trade liberalization. Hence:

Proposition 3:

Trade liberalization induces a convex non-linear response of world manufactured trade to trade costs when firms can offshore their production.

It is important to note that the source of this non-linearity differs from that proposed by Yi (2003), and could thus be considered as an alternative explanation for this non-linearity. In our model, we do not introduce intermediate inputs, so this mechanism is absent.

5 Conclusion

In this paper we have presented the theoretical consequences of the interaction of two stylized facts recently highlighted by the literature on trade. First, there is much evidence that exporting firms are very different in productivity and size compared to non-exporters. Second, the globalization process has strongly reshaped the geography of world production by creating incentives for firms to offshore their production to foreign countries. We have shown that the interaction between these two facts has important consequences for countries' average productivity and trade patterns. The main idea behind this result is that it is not possible to understand the impact of offshoring without taking into account the productivity of firms that offshore their production. This question is of high political relevance. We can no longer only focus on which countries are the most attractive for economic activities; a crucial question is which countries attract the most efficient firms. The paper further shows that trade liberalization plays an important role because it affects location incentives of large and small firms differently.

We therefore consider that this paper sheds a new light on the importance of offshoring in shaping the impact of trade liberalization. Three important results that contrast with the existing literature emerge from this interaction: i) Trade liberalization may concentrate all average efficiency gains in one country, the other country experiencing an average productivity loss. ii) Trade liberalization can reverse the direction of net trade flows, independently of any country size effect. iii) Trade liberalization may increase world trade growth in a non-linear way, even in the absence of trade in intermediate goods. Importantly, these results would not hold if firms were forced to stay in their country of origin, and neither would they if we had considered a trade model with symmetric firms that have the possibility of offshoring their production.

The assumption of costless offshoring has been made to stress the impact of offshoring in the model, but may seem unrealistic. It would however be possible to introduce such costs, which would allow a link to be made between the two versions of the model. If offshoring costs were prohibitive, the output of the model would be similar to the one described in section 3. A gradual decrease in offshoring costs would bring the output of the model closer to that of section 4, until offshoring costs were nil.

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APPENDICES

A proof of lemma 3

Proof. Suppose a firm of type l has rank $l+p$ with respect to its market access. There exist $m-l$ types that have higher fixed costs than type l, while there exist $m-l-p$ types that have a larger market access. It follows that among the $m-l$ types that have higher fixed costs, at least p of them are characterized by a lower market access than type l. These types cannot exist in equilibrium. We can thus eliminate these firm types until $p=0$. (A similar reasoning applies if type l has rank $l-p$ with respect to its market access). ■

B proof of lemma 4

Proof. Suppose that $a_{l+1} > a_l$. According the definition of a_{l+1}, all firms with a marginal cost below a_{l+1} prefer type $l+1$ rather than l. Similarly, all firms with a marginal cost above a_l prefer type $l-1$ rather than l. There is thus no marginal cost for which a firm would prefer type l considering the two alternative types $l+1$ and $l-1$. Type l is strictly dominated either by type $l+1$ or $l-1$ and thus cannot exist in equilibrium. ■

C Expressions for average marginal costs in each country with respect to trade freeness:

$\phi < \underline{\phi}$

$$\widetilde{a_A} = \frac{1-\sigma+\rho}{2-\sigma+\rho} \frac{\phi a_{XA}^{2-\sigma+\rho} + \widehat{P}^{\sigma-1} a_{DA}^{2-\sigma+\rho}}{\phi a_{XA}^{1-\sigma+\rho} + \widehat{P}^{\sigma-1} a_{DA}^{1-\sigma+\rho}}$$

$$\widetilde{a_B} = \frac{1-\sigma+\rho}{2-\sigma+\rho} \frac{a_{DB}^{2-\sigma+\rho} - a_{DA}^{2-\sigma+\rho}}{a_{DB}^{1-\sigma+\rho} - a_{DA}^{1-\sigma+\rho}}$$

$$\underline{\phi} < \phi < \overline{\phi} \qquad \begin{array}{l} \widetilde{a_A} = \frac{1-\sigma+\rho}{2-\sigma+\rho} \dfrac{\left(\widehat{P}^{\sigma-1}+\phi\right) a_{XA}^{2-\sigma+\rho} + \widehat{P}^{\sigma-1}\left(a_{DA}^{2-\sigma+\rho} - a_{XB}^{2-\sigma+\rho}\right)}{\left(\widehat{P}^{\sigma-1}+\phi\right) a_{XA}^{1-\sigma+\rho} + \widehat{P}^{\sigma-1}\left(a_{DA}^{1-\sigma+\rho} - a_{XB}^{1-\sigma+\rho}\right)} \\ \widetilde{a_B} = \frac{1-\sigma+\rho}{2-\sigma+\rho} \dfrac{\left(1+\phi\widehat{P}^{\sigma-1}\right)\left(a_{XB}^{2-\sigma+\rho} - a_{XA}^{2-\sigma+\rho}\right) + a_{DB}^{2-\sigma+\rho} - a_{DA}^{2-\sigma+\rho}}{\left(1+\phi\widehat{P}^{\sigma-1}\right)\left(a_{XB}^{1-\sigma+\rho} - a_{XA}^{1-\sigma+\rho}\right) + a_{DB}^{1-\sigma+\rho} - a_{DA}^{1-\sigma+\rho}} \end{array}$$

$$\phi > \overline{\phi} \qquad \begin{array}{l} \widetilde{a_A} = \frac{1-\sigma+\rho}{2-\sigma+\rho} \dfrac{a_{DA}^{2-\sigma+\rho} - a_{XB}^{2-\sigma+\rho}}{a_{DA}^{1-\sigma+\rho} - a_{XB}^{1-\sigma+\rho}} \\ \widetilde{a_B} = \frac{1-\sigma+\rho}{2-\sigma+\rho} \dfrac{\left(1+\phi\widehat{P}^{\sigma-1}\right) a_{XB}^{2-\sigma+\rho} + a_{DB}^{2-\sigma+\rho} - a_{DA}^{2-\sigma+\rho}}{\left(1+\phi\widehat{P}^{\sigma-1}\right) a_{XB}^{1-\sigma+\rho} + a_{DB}^{1-\sigma+\rho} - a_{DA}^{1-\sigma+\rho}} \end{array}$$

D Formal solution for the model with offshoring:

In this appendix, we provide the solution of the model with offshoring when firms' productivity levels are Pareto distributed. This gives a formal proof that $\frac{\partial \widehat{P}}{\partial \phi} > 0$.

When trade costs are high (i.e. $\phi < \underline{\phi}$), there are only three existing firm types: DB, DA, and XA, which allows to define the relevant marginal cost thresholds that pin down the firm types chosen by each active firm:

$a_{DB} = P_B \left(\frac{\beta}{C_D}\right)^{\frac{1}{\sigma-1}}$, $a_{DA} = \left(\frac{\beta\left(P_A^{\sigma-1}\gamma^{1-\sigma} - P_B^{\sigma-1}\right)}{(\gamma-1)C_D}\right)^{\frac{1}{\sigma-1}}$, and $a_{XA} = P_B \left(\frac{\beta\phi}{\gamma^{\sigma} C_X}\right)^{\frac{1}{\sigma-1}}$.

This in turn allows to define the price indexes expressions in each country:

$$\begin{aligned} P_A^{1-\sigma} &= \lambda \gamma^{1-\sigma} a_{DA}^{1-\sigma+\rho} \\ P_B^{1-\sigma} &= \lambda \left(a_{DB}^{1-\sigma+\rho} - a_{DA}^{1-\sigma+\rho} + \phi\gamma^{1-\sigma} a_{XA}^{1-\sigma+\rho}\right) = \frac{\beta}{C_D a_{DB}^{\sigma-1}} \end{aligned}$$

We first define all marginal cost thresholds relative to the marginal cost threshold of the least efficient firm worldwide a_{DB}:

$\widehat{a_{DA}} = \frac{a_{DA}}{a_{DB}} = \left(\frac{\left(\widehat{P}^{\sigma-1}\gamma^{1-\sigma} - 1\right)}{(\gamma-1)}\right)^{\frac{1}{\sigma-1}}$ and $\widehat{a_{XA}} = \frac{a_{XA}}{a_{DB}} = \left(\frac{\phi C_D}{\gamma^{\sigma} C_X}\right)^{\frac{1}{\sigma-1}}$

This gives an implicit solution for the relative price index:

$$\widehat{P}^{\sigma-1}=\frac{1-\widehat{a_{DA}}^{1-\sigma+\rho}+\phi\gamma^{1-\sigma}\widehat{a_{XA}}^{1-\sigma+\rho}}{\gamma^{1-\sigma}\widehat{a_{DA}}^{1-\sigma+\rho}}=\frac{1-\left(\frac{\left(\widehat{P}^{\sigma-1}\gamma^{1-\sigma}-1\right)}{(\gamma-1)}\right)^{\frac{\rho}{\sigma-1}-1}+\phi\gamma^{1-\sigma}\left(\frac{\phi C_D}{\gamma^\sigma C_X}\right)^{\frac{\rho}{\sigma-1}-1}}{\gamma^{1-\sigma}\left(\frac{(\widehat{P}\gamma^{1-\sigma}-1)}{(\gamma-1)}\right)^{\frac{\rho}{\sigma-1}-1}}.$$

Rearranging terms, we obtain the following equation, which pins down $\widehat{P}$ and ensures that $\frac{\partial\widehat{P}}{\partial\phi}>0,\ \forall\ \phi$:

$$h(\widehat{P})-\left(1+\phi^{\frac{\rho}{\sigma-1}}\left(\frac{C_D}{C_X}\right)^{\frac{\rho}{\sigma-1}-1}\gamma^{1-\frac{\sigma\rho}{\sigma-1}}\right)=0 \tag{25}$$

$$\text{with } h(\widehat{P})=\left(\gamma^{1-\sigma}\widehat{P}^{\sigma-1}+1\right)\left(\gamma^{1-\sigma}\widehat{P}^{\sigma-1}-1\right)^{\frac{\rho}{\sigma-1}-1}\left(\frac{1}{(\gamma-1)}\right)^{\frac{\rho}{\sigma-1}-1}$$

The definition of the price index in country B provides the relation between the threshold a_{DB} and $\widehat{P}$, which allows to solve for all other endogenous thresholds:

$$a_{DB}=\left(\lambda\frac{C_D}{\beta}\left(1-\left(\frac{\left(\widehat{P}^{\sigma-1}\gamma^{1-\sigma}-1\right)}{(\gamma-1)}\right)^{\frac{\rho}{\sigma-1}-1}+\phi\gamma^{1-\sigma}\left(\frac{\phi C_D}{\gamma^\sigma C_X}\right)^{\frac{\rho}{\sigma-1}-1}\right)\right)^{\frac{-1}{\rho}}$$

When trade costs are intermediate (i.e. $\underline{\phi}<\phi<\overline{\phi}$), the four firm types coexist. We can then define the relevant marginal costs thresholds that pin down the firm types chosen by each active firm:

$a_{DB}=\left(\frac{\beta}{C_D}\right)^{\frac{1}{\sigma-1}}P_B$, $a_{DA}=\left(\frac{\beta\left(P_A^{\sigma-1}\gamma^{1-\sigma}-P_B^{\sigma-1}\right)}{(\gamma-1)C_D}\right)^{\frac{1}{\sigma-1}}$, $a_{XB}=\left(\frac{\beta\left(P_B^{\sigma-1}-\left(\gamma^{1-\sigma}-\phi\right)P_A^{\sigma-1}\right)}{C_X-(\gamma-1)C_D}\right)^{\frac{1}{\sigma-1}}$, and $a_{XA}=\left(\frac{\beta\left(\left(\gamma^{1-\sigma}-\phi\right)P_A^{\sigma-1}-(1-\phi\gamma^{1-\sigma})P_B^{\sigma-1}\right)}{(\gamma-1)(C_D+C_X)}\right)^{\frac{1}{\sigma-1}}$.

We obtain the following definitions for price indexes:

$$\begin{aligned}P_A^{1-\sigma} &= \lambda\left(\left(\gamma^{1-\sigma}-\phi\right)\left(a_{XA}^{1-\sigma+\rho}-a_{XB}^{1-\sigma+\rho}\right)+\gamma^{1-\sigma}a_{DA}^{1-\sigma+\rho}\right)\\ P_B^{1-\sigma} &= \lambda\left(a_{XB}^{1-\sigma+\rho}+a_{DB}^{1-\sigma+\rho}-a_{DA}^{1-\sigma+\rho}-\left(1-\phi\gamma^{1-\sigma}\right)a_{XA}^{1-\sigma+\rho}\right)=\frac{\beta}{C_D a_{DB}^{\sigma-1}}\end{aligned}$$

Using the same notations as above, we obtain the definitions for the relative marginal cost thresholds:

$\widehat{a_{DA}} = \left(\frac{\left(\widehat{P}^{\sigma-1}\gamma^{1-\sigma}-1\right)}{(\gamma-1)}\right)^{\frac{1}{\sigma-1}}$, $\widehat{a_{XB}} = \widehat{P}\left(\frac{C_D\left(1-\left(\gamma^{1-\sigma}-\phi\right)\right)}{C_X-(\gamma-1)C_D}\right)^{\frac{1}{\sigma-1}}$, and $\widehat{a_{XA}} = \left(\frac{C_D\left(\left(\gamma^{1-\sigma}-\phi\right)\widehat{P}^{\sigma-1}-1+\phi\gamma^{1-\sigma}\right)\right)}{(\gamma-1)(C_D+C_X)}\right)^{\frac{1}{\sigma-1}}$.

Following the methodology presented above, we find that the relative price index is given by:

$$g(\widehat{P})\left(\frac{C_D}{(\gamma-1)(C_D+C_X)}\right)^{\frac{\rho}{\sigma-1}-1} - f(\widehat{P})\left(\frac{C_D}{C_X-(\gamma-1)C_D}\right)^{\frac{\rho}{\sigma-1}-1} + h(\widehat{P}) - 1 = 0 \qquad (26)$$

$$\text{with}: \; g(\widehat{P}) = \left(\left(\gamma^{1-\sigma}-\phi\right)\widehat{P}^{\sigma-1} + \left(1-\phi\gamma^{1-\sigma}\right)\right)\left(\left(\gamma^{1-\sigma}-\phi\right)\widehat{P}^{\sigma-1} - \left(1-\phi\gamma^{1-\sigma}\right)\right)\right)^{\frac{\rho}{\sigma-1}-1}$$

$$\text{and}: \; f(\widehat{P}) = \left(1+\widehat{P}^{\sigma-1}\left(\gamma^{1-\sigma}-\phi\right)\right)\left(1-\left(\gamma^{1-\sigma}-\phi\right)\widehat{P}^{\sigma-1}\right)^{\frac{\rho}{\sigma-1}-1}$$

which leads again to $\frac{\partial\widehat{P}}{\partial\phi} > 0, \forall\, \phi$.

Finally, when trade costs are low (i.e. $\phi > \overline{\phi}$), we again only have three active firm types: DB, DA and XB. The marginal cost thresholds are given by:

$a_{DB} = P_B\left(\frac{\beta}{C_D}\right)^{\frac{1}{\sigma-1}}$, $a_{DA} = \left(\frac{\beta\left(P_A^{\sigma-1}\gamma^{1-\sigma}-P_B^{\sigma-1}\right)}{(\gamma-1)C_D}\right)^{\frac{1}{\sigma-1}}$, and $a_{XB} = \left(\frac{\beta\left(P_B^{\sigma-1}-\left(\gamma^{1-\sigma}-\phi\right)P_A^{\sigma-1}\right)}{C_X-(\gamma-1)C_D}\right)^{\frac{1}{\sigma-1}}$

while the relevant price indexes now take the following form:

$$\begin{aligned} P_A^{1-\sigma} &= \lambda\left(\left(\phi-\gamma^{1-\sigma}\right)a_{XB}^{1-\sigma+\rho} + \gamma^{1-\sigma}a_{DA}^{1-\sigma+\rho}\right) \\ P_B^{1-\sigma} &= \lambda\left(a_{XB}^{1-\sigma+\rho} + a_{DB}^{1-\sigma+\rho} - a_{DA}^{1-\sigma+\rho}\right) = \frac{\beta}{C_D a_{DB}^{\sigma-1}} \end{aligned}$$

The relative price index is now given by:

$$h(\widehat{P}) - f(\widehat{P})\left(\frac{C_D(\gamma-1)}{C_X-(\gamma-1)C_D}\right)^{\frac{\rho}{\sigma-1}-1}\left(\frac{1}{(\gamma-1)}\right)^{\frac{\rho}{\sigma-1}-1} - 1 = 0 \qquad (27)$$

we get $\frac{\partial\widehat{P}}{\partial\phi} > 0, \forall\, \phi$.