Cash reserve policy, regulation and credibility in insurance

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ABSTRACT

The aim of this paper is to analyze the need for cash and default regulation in insurance. Proponents of deregulation argue that these requirements are useless as insurers would hold enough cash as soon as policyholders are fully informed about their default probability. Adding to the purpose the relationship between an insurer and her security holders (that is the issuance and dividend policy) we show that the second best (credible) cash reserve decided by the security holders is suboptimal whenever the return on cash inside the firm is smaller than outside. Because of limited commitment on recapitalization, disclosure of information may not be enough. Given these characteristics, State commitment to recapitalize could be an alternative regulation policy.

Subject headings: insurance, cash reserve, regulation, recapitalization

1. Introduction

Is cash regulation necessary, and, if so, which kind is the most accurate? In all countries that have insurance markets, regulation of insurance companies exists. The main motivation of such a regulation seems to be the protection of insurance buyers against the risk of insolvency of their insurers. This regulation generally takes the form of "technical" or "mathematical" reserves that insurers should at least carry in sufficiently liquid capital. These regulatory reserves are expressed as ratios of premium income and claims expenses. We want to focus in this paper on the necessity and the economic motivations of such regulation rules.

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The first rationale for cash regulation seems to be the potential asymmetry of information between insurers and policyholders. The buyer of insurance pays a premium against the promise that she will receive a payment if the specified random events occur. If the insurer does not hold enough reserves to fulfill this promise, the consumer is being cheated ex ante. This potential failure, itself, undermines the confidence on which the market is based. If the policyholders does not observe this default risk, the insurance market faces a typical "lemons problem": uncertainty about product quality – here solvency – may drive high-quality firms out of the market. Regulation is then intended to make sure that only "good" firms (with low risk of insolvency) are in the market.

Proponents of deregulation however argue that cash requirement is not an appropriate tool to mitigate this adverse selection problem. As the problem arises because policyholders are not conveniently informed about the risk of insolvency, "disclosure of information" policy, that is the public provision of information about insurers' risk of insolvency, is sufficient to solve the adverse selection issue. Private incentives are then sufficiently high to induce companies to hold enough liquid capital to optimally reduce the risk of insolvency.

Such a reasoning is however silent on other key actors of insolvency: shareholders or debt owners. Indeed, for a company to become insolvent, not only cash has to be insufficient to meet claims but it also has to be suboptimal to recapitalize (or impossible to issue debt). In the present paper we therefore want to focus on possible other reasons of cash regulation, beside the relation "insurer/policyholder" evoked above. Another bilateral relationship – between the insurer and her security holders – indeed seems to be of interest. It is now well established that agency problems may arise from the asymmetry of information between managers and shareholders. For example, managers can invest in inefficient projects that generate private benefits for them to the detriment of shareholders. Such an issue would therefore give security holders an incentive not to leave cash in the insurance company. The problem becomes clearer in presence of frictional capital market, if we assume that issuing new debt is costly. An interesting trade-off then arise between agency cost and recapitalization cost.

To study these mechanisms we build a dynamic model of insurance and analyze the dynamic of capital through the behavior of security holders. On the one hand, when the company is solvent, that is when assets are sufficient to meet claims, the security holders can either take dividend or issue new shares (or debt). On the other hand, if claims are too large for the current assets to cover it, the security holders choose whether to recapitalize the insurance company or to default.

When the capital market is frictionless, that is when issuing new shares is costless, the optimal strategy consists in taking dividend as long as it is possible – because of agency cost – and to recapitalize each time it is needed (provided the future value of the company is larger than the invested capital). However, if issuing new debt is costly, it can be optimal to leave some cash in the company. The optimal strategy can then be related to a well-known policy in inventory management: take dividend above a bottom limit, neither take dividend nor issue new debt if the ex-post (cash) reserve is positive but below the limit and issue new shares in order to meet claims when the current reserve is insufficient. Taking into account the effect of default on policyholders, we show that the first best policy implies – when recapitalization is costless – no default (shareholders always recapitalize) but no reserve. Although this dynamic is a good candidate for a complete information competitive equilibrium, it appears to be hardly implementable. Indeed, it implies an ex-ante commitment to recapitalize which is not credible. An efficient regulation would then consist in making this commitment credible or at least in guaranteeing that the company would always hold enough asset to continue operating. It therefore appears that State commitment to recapitalize can be a more efficient regulation than cash requirement.

Our work fits in the literature on cash reserve and solvency in insurance. Initiated by Borch (1981) in a model where shareholders can only invest in capital during the first period, this literature has then developed in analyzing the optimal dynamic choice of capital. Munch and Smallwood (1981) and Finsinger and Pauly (1984) for example analyze capital choices in a situation where the demand for insurance is elastic with respect to default risk. Both papers however assume that shareholders cannot recapitalize after claims are realized.

In a more recent paper, Rees, Gravelle and Wambach (1999) study a situation in which policyholders are fully informed of the default probability of their insurer. They show that, whereas an unconstrained insurer will optimally choose a corner solution (either zero or maximum), once the insured is informed about the probability of not being indemnified, the insurer's expected value is higher if it holds the maximum amount of capital. They however ignore the possibility of recapitalization when claims exceed assets. They indeed assume that contracts are not fully honored in these cases. Under this assumption insurers can commit on a default probability through their cash reserve. Being informed of the amount of cash their insurer holds, individuals can infer the probability of not being paid. Competition in insurance market then lead the companies to raise the maximum amount of cash. However, as we introduce recapitalization – that is the possibility to reinject cash when claims exceed assets – this mechanism no longer holds. Insurers then cannot commit on a default probability as they cannot commit on the behavior of their security holders. This creates a motive for an internal solution for cash reserve and therefore a room for cash regulation (minimal cash requirement or State guarantee) if this solution is suboptimal.

Blazenko, Parker and Pavlov (2007, 2008) analyze the concept of ”economic ruin” by modeling a situation where new shares can be issued in case of deficit. They however assume an exogenous dividend policy in the sense that a fixed return (the risk-free interest rate) is paid to shareholders whenever cash reserves are positive, and that insurer can continue operating with negative capital (debt). We however want to focus here on optimal (and therefore endogenous) issuance and dividend policy and we assume that shareholders has to recapitalize a company with negative capital if they want the company not to default. Finally, we want to focus on a different regulation scheme than Blazenko, Parker and Pavlov (2007, 2008). They indeed consider a regulation that requires an immediate cash contribution to offset a capital deficit when we model regulation as cash requirement or State guarantee.

The sequel of the paper is organized as follows. In the next section (Section 2) we present the model of dynamic insurance and its implication on the optimal issuance and dividend policy under different information and commitment settings. Such an approach allows us to capture the need for cash regulation and to analyze in Section 3 the efficiency of two forms of regulation: cash requirement and State guarantee. Conclusions and directions for future research are eventually provided in Section 4.

2. The model

We consider a discrete dynamic model where an insurance company, at each period, offers insurance contracts to n identical policyholders. Each policyholder incurs a per period loss $\widetilde{x}_i$. We denote by $\widetilde{x}_t$ the total claim at date t. We note f the distribution of $\widetilde{x}_t$, F its CDF and e its mean. Policyholders value wealth through a von Neumann, increasing and strictly concave, utility function u.

The timing inside a given period, from date t to $t+1$ is the following.

- At the beginning of each period the insurance company, if active, holds some assets m_t (cash reserves or liquid assets).
- He proposes a (full) insurance contract (premium π_t) to the n potential customers, who can either accept or refuse,
- so that (if the contract is accepted) total assets amounts to $A_t = n\pi_t + m_t$.
- For sake of simplicity, we will assume that claims occur at the beginning of the period. Two cases are then possible.

– Either, in a first case, the total claim x_t is lower than total assets $A_t = n\pi_t + m_t$. Shareholders of the company can then recapitalize or take dividends for the following period. Let $k_t(x_t)$ the amount of recapitalization (if negative, $-k_t(x_t)$ is the amount of dividends). In the following period, the insurance company then begins with a new cash reserve that amounts to: $m_{t+1} = \rho(A_t - x_t + k_t(x_t))$, where ρ is the return on the cash left in the company.

– Or, in a second case, the total claim x_t is larger than $A_t = n\pi_t + m_t$ and the company is potentially insolvent. The security holders can either refuse to keep on operating – in that case insured are not fully indemnified and A_t is simply equally shared among them – or subscribe to an issue of new securities (shares or debt[1]) to meet the claims. We suppose that there is a potential cost of issuance: 1 dollar of fresh cash in the company costs $\gamma \geq 1$ to the security holders. This can be, for example, explained by transaction costs (see Gomes 2001 for a justification and an evaluation of these costs). Notice that there is no possibility of negative balance sheet[2]. If $\widetilde{x}$ is larger than A_t the company must either stop or issue new shares.

We suppose that shareholders discount future with a discount rate δ. We make the assumption that $\delta\rho < 1$. Inside cash has a lower return ρ than outside opportunities δ^{-1}. There is no direct incentive to leave cash in the company since it has a better return outside. For instance, firms' managers may commit inside cash to inefficient projects that generate only private benefits for them to the detriment of outside security holders (see La Porta et al. 2000 for an overview of these problems).

In the paper we describe alternative information and commitment frameworks.

In a first part we suppose that insured cannot observe the cash reserve of the company (private information). In a second part we assume perfect observability (information disclosure). We then distinguish two cases: the case of full commitment on recapitalization and the case of the impossibility of full commitment.

[1] In the following model we focus on share issuance. Most of our results seems to remain with debt issuance but such a modeling would add interest paiement and debt maturity to the purpose.

[2] Note here that permanent debt would be suboptimal as it would induce commitment issues.

2.1. Private information on assets

Let us first assume that policyholders are unable to access information on their insurer cash reserve. In such cases, insurance choices are only governed by the premium offered by the insurer. This premium is accepted by policyholders if it is lower than some reservation level, π. Given this reservation level, the insurer define the optimal policy of cash and default. In a complete setting, this level would be endogenously determined and policyholders insured would perfectly anticipate the probability of default. Here, we don't describe completely this "lemons" effect and don't characterize the rational expectation equilibrium.

In the following we consider two cases regarding the cost of issuing new shares. We first assume costless cash (section 2.1.1) before adding opportunity costs of recapitalisation (section 2.1.2).

2.1.1. The frictionless case

In a frictionless world, fresh cash has no opportunity cost: $\gamma = 1$. We claim that, because of agency costs, the optimal policy is to maintain liquidity to zero and stop the activity when losses are too large. To show that, we are considering the optimal intertemporal decision of shareholders of a company starting with an initial cash m (we omit t indexes for sake of simplicity).

Let $V^*(m, \pi)$ the intertemporal optimal value of the company.

Using the Bellman principle, we have:

$$V^*(m, \pi) = \max_{I, k(x) \geq x - A} \int_I \left[\delta V^* \left(\rho \left(A - x + k(x) \right), \pi \right) - k(x) \right] f(x) dx, \tag{1}$$

where I is the range of the values of x for which security holders decide to keep on operating. Note here that, through I, our model endogenize bankruptcy. To that respect it can be related to the work of Leland and Toft (1996) that endogenously determine bankruptcy when studying optimal corporate debt with endogenous maturity.

A simple change of variables gives:

$$V^*(m, \pi) = \max_{I, H(x) \geq 0} \int_I \left[\delta V^* \left(H(x), \pi \right) - \frac{1}{\rho} H(x) + (m + n\pi) - x \right] f(x) dx \tag{2}$$

In the above equation $H(x)$ is the reserve chosen for the next period by the security holders. As we do not allow the company to operate with negative cash reserve (short term debt), $H(x)$ has to be positive.

Setting then $\arg\max_{H \geq 0}(\delta V^*(H,\pi) - \frac{1}{\rho}H) = H^*$ and $\max_{H \geq 0}(\delta V^*(H,\pi) - \frac{1}{\rho}H) = \delta W^*$, the equation (2) becomes:

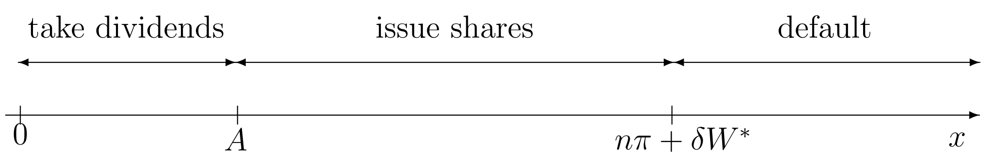
$$V^*(m,\pi) = \max_{I} \left(\int_I \left(\delta W^* + m + n\pi - x\right) f(x)dx \right) \tag{3}$$

By the envelop theorem we have, noticing I^* the optimal range of operating, $V_m^{*'}(m,\pi) = \mu(I^*)$ (where μ is the measure associated with F). This implies, as $\delta\rho < 1$, that $\delta V^*(H,\pi) - \frac{1}{\rho}H$ is a strictly decreasing function of H, which in turn implies that $H^* = 0$.

It is optimal to leave no cash reserve in the company. Even if $\delta\rho = 1$, that is even when cash has the same return inside and outside, $H^* = 0$ is an (now possibly non unique) optimal solution.

We can then deduce the optimal range I^*. It is optimal to keep on operating until the integrand of (3) is positive. We hence have $I^* = (0, b^*(m))$, where $b^*(m) = m + n\pi + \delta W^*$ is the default threshold.

The optimal policy can therefore be depicted as follows.



If we note $e_m = \mathbb{E}(x/x \leq b^*(m))$, we can write the complete solution:

$$\delta V^*(0,\pi) = \delta W^* = \delta \frac{F(b^*(0))(n\pi - e_0)}{1 - \delta F(b^*(0))}$$

The value of the company holding m and pricing π is then:

$$V^*(m,\pi) = F(b^*(m))\left(m + (n\pi - e_m) + \frac{\delta F(b^*(0))(n\pi - e_0)}{1 - \delta F(b^*(0))}\right) \tag{4}$$

with $e_m, b^*(m)$,solutions of:

$$\begin{cases} (1 - \delta F(b^*(0)))\, b^*(0) + \delta F(b^*(0)) e_0 = n\pi \\ F(b^*(m)) e_m = \int_0^{b^*(m)} x f(x) dx \\ b^*(m) = b^*(0) + m \end{cases} \tag{5}$$

On the following picture we represent in the plane (b, W) the functions $W = \int_0^b (b - x) f(x) dx$, $b = n\pi + \delta W$ and $W = b - e$. The intersection between the first two curves represents the point $(b^*(0), W^*)$, the intersection between the two lasts gives $W = \frac{n\pi - e}{1-\delta}$ which is the present value of the firm when it is systematically recapitalizes (never default). Obviously this value is smaller than W^*.

[Figure 1 about here.]

As $V^{*\prime}_m(m,\pi) = F(b^*(m))$ and $V^{*\prime\prime}_{mm}(m,\pi) = f(b^*(m)) \geq 0$, the value function is increasing and convex. The asymptote for large m is $m + (n\pi - e) + \delta V^*(0,\pi) = m + (n\pi - e) + \dfrac{\delta F(b^*(0))(n\pi - e_0)}{1 - \delta F(b^*(0))}$. The following picture gives the shape of V as a function of m.

[Figure 2 about here.]

The optimal strategy is the following: at the first period, shareholders take away all the money left if any, or reinject enough cash to meet claims. Then the liquidity is maintained to zero except when the total claim exceeds $\dfrac{\delta F(b^*(0))(n\pi - e_0)}{1 - \delta F(b^*(0))} + (m + n\pi)$. This value is exactly the present value of the firm computed with the "modified" discount factor $\delta F(b^*(0))$ that takes into account the probability of default. This simply means that security holders refuse to put more money than the present value of future returns.

It is worth noticing that the value of a firm holding m and pricing π, $V^*(m,\pi)$ is larger than the one obtained for a company that would adopt the non optimal strategy $I = [0, +\infty)$, for which the value is simply equal to the NPV: $m + \dfrac{n\pi - e}{1 - \delta}$.

With this optimal strategy from the point of view of security holders, the level of utility achieved by the consumer as long as he stays in the company is:

$$B(m,\pi) = \left(u(w-\pi) + \delta B(0,\pi)\right) F(b^*(m)) + \int_{b^*(m)}^{+\infty} u\left(w + \frac{m-x}{n}\right) f(x)dx$$

The above result must be contrasted with the ones obtained by Rees, Gravelle and Wambach (1999). In their model default is exogenous: if assets A are at least enough to meet claims, the insurer remains in business and receives a continuation value V that is the expected present value of being in the insurance business at the end of the first period. If claims costs turn out to be greater than assets, the insurer pays out her assets and defaults on the remaining claims, losing the right to the continuation value. In this framework, they find that it can be optimal, for the insurer, to put enough initial cash to avoid default, provided that insurance claims distributions belong to the class of "increasing failure rate" distributions on a bounded support. This result seems to be questionable since, in particular, there is no reason to assume that cash or assets must be put ex-ante and that ex-post recapitalization is impossible. This feature can, by the way, lead to accumulate ex ante a huge (and potentially infinite) level of cash up to the maximal value of total claims. It is as if there was an infinite cost of recapitalization and a zero cost of initial cash.

We obtain here a more nuanced result: default is endogenous and optimally decided when new cash needed is too large compared with the expected returns. This leads to a policy where permanent cash is useless. The only reason for permanent cash to be useful would be the case where recapitalization is costly.

In the following section we will introduce a (finite) cost of issuing new shares.

2.1.2. Opportunity cost of recapitalization

In this second part we assume that capital market imperfections make issues of new shares (or new debt) costly: $\gamma > 1$. In particular, when cash reserve becomes negative security holders have to choose between issuing new shares and to stop operating. The important consequence is that this cost creates an ex ante incentive to some precautionary policy which takes the form of cash reserves. Intuitively, when γ is low, reserves can be maintained to zero. But when γ becomes larger, it turns to be optimal to hold some permanent strictly positive reserves.

Suppose that shareholders can recapitalize when needed, and inject $k(x)$ in the firm at a cost $\gamma > 1$. When k is negative, that is when shareholders take dividends, there is no opportunity cost.

The optimal value of the firm holding m and pricing π becomes:

$$V(m,\pi) = \max_{k(x)\geq x-A,\ I} \min_{\alpha(x)\in\{1,\gamma\}} \int_I \left[\delta V(\rho(A-x+k(x)),\pi) - \alpha(x)k(x)\right] f(x)dx, \tag{6}$$

In the above equation, $\alpha(x)$ is optimally fixed to γ for x such that $k(x) \geq 0$ and to 1 elsewhere.

Setting, as previously, $H(x) = \rho(A-x+k(x))$ this equation becomes:

$$V(m,\pi) = \max_{H(x)\geq 0, I} \min_{\alpha(x)\in\{1,\gamma\}} \int_I \left[\delta V(H(x),\pi) - \alpha(x)\frac{1}{\rho}H(x) + \alpha(x)(m+n\pi-x)\right] f(x)dx \tag{7}$$

Therefore $\alpha^*(x) = 1$ if $H(x) \leq (A-x)$ and $\alpha^*(x) = \gamma$ if $H(x) \geq (A-x)$.

Let set $H^* = \arg\max_{H\geq 0}\left(\delta V(H,\pi) - \frac{1}{\rho}H\right)$ and $H^*_\gamma = \arg\max_{H\geq 0}\delta V(H,\pi) - \frac{\gamma}{\rho}H$.

The two thresholds H^* and H^*_γ have the following meanings: the optimal strategy consists in taking all cash reserves above H^* if any, and to recapitalize up to H^*_γ when needed.

As previously, set $\max\left(\delta V(H,\pi) - \frac{1}{\rho}H\right) = \delta W^*$ and $\max\left(\delta V(H,\pi) - \frac{\gamma}{\rho}H\right) = \delta W^*_\gamma$.

Lemma 1 $0 \leq H^*_\gamma \leq H^*$ *and* $W^*_\gamma \leq W^*$

Proof. see appendix ■

It is now easy to state the following proposition which gives the optimal strategy and the value of the firm.

Proposition 1 *The optimal cash policy is given by two optimal thresholds* $a^*(m) \leq A \leq b^*(m)$ *such that:*

if $x \leq a^*(m)$, $H(x) = H^*$*: take money above* H^*

if $a^*(m) \leq x \leq A, H(x) = \rho(A - x)$*: neither take money nor issue shares*

if $A \leq x \leq b^*(m)$, $H(x) = H^*_\gamma = 0$*: issue shares up to* $H^*_\gamma = 0$

if $x \geq b^*(m)$ *the company defaults.*

where $a^*(m)$*,* $b^*(m)$*, and* $V(m, \pi)$*, are solutions of:*

$$\begin{aligned} V(m,\pi) &= \max_{a \leq A \leq b} \int_{-\infty}^{a} \left[\delta W^* + (m + n\pi - x)\right] f(x)dx \\ &+ \int_{a}^{A} \delta V(\rho(m + n\pi - x), \pi) f(x)dx \\ &+ \int_{A}^{b} \left[\delta V(0,\pi) + \gamma(m + n\pi - x)\right] f(x)dx \end{aligned}$$

Proof. see appendix ■

Here, when issuing shares is necessary, the security holders put just enough cash to meet claims: $H^*_\gamma = 0$. When the profit is large, security holders take away dividends and leave some "precautionary reserve" $H^* \geq 0$. Intuitively, this cash reserve is larger as γ is large. Conversely, when γ is sufficiently low, this level can be maintained to 0. Indeed, the derivative of V w.r.t. m for $m = 0$ can be computed (thanks to the envelop theorem):

$$\begin{aligned} V'_m(0,\pi) &= F(a^*(0)) + \int_{a^*(0)}^{n\pi} \delta\rho V'(\rho(n\pi - x), \pi) f(x)dx \\ &+ \gamma\left(F(b^*(0)) - F(n\pi)\right) \end{aligned}$$

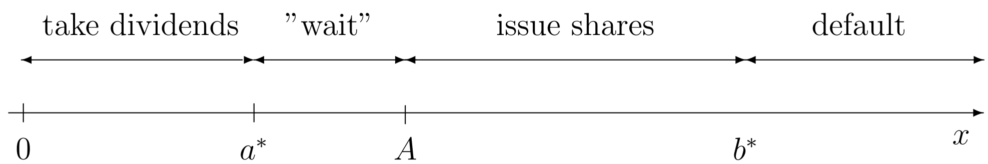
Suppose now that the optimal value is $H^* = 0$. This implies that $a^*(0) = n\pi$ and $b^*(0) = B$ is defined by:

$$\begin{cases} V(0,\pi)\left[1 - \delta F(B)\right] = \displaystyle\int_{-\infty}^{B} \left[(n\pi - x)\right] f(x)dx + (\gamma - 1)\int_{n\pi}^{B} \left[(n\pi - x)\right] f(x)dx \\ \gamma B = \delta V(0,\pi) + n\pi + (\gamma - 1)\, n\pi \end{cases}$$

The optimal reserve will be $H^* > 0$ if the previous expression of $V'_m(0, \pi)$ in which we take $a^*(0) = n\pi$ is greater than $\frac{1}{\delta\rho}$. That is:

$$F(n\pi) + \gamma (F(B) - F(n\pi)) > \frac{1}{\delta\rho}$$

In this case the optimal policy can be depicted as follows.



This barrier strategy can be related to the one defined in Décamps et al. (2008) on banking market and in Kulenko and Schmidli (2008) on insurance. The main difference is that, in their continuous framework, default never optimally occurs (they are therefore silent on commitment to recapitalize, central in our paper).

When γ is large enough the value function as a function of m has a concave part for small values of m.

Another interesting question concerns the variation of the default threshold $b^*(m)$ when γ increases. We know that $b^*(m) = \frac{\delta}{\gamma}V(0, \pi) + m + n\pi$. It is then easy to show that $V(0, \pi)$ is decreasing with γ, so that we can state the following proposition:

Proposition 2 *When the opportunity cost γ increases, both the precautionary reserve H^* and the probability of default increase.*

Proof. see appendix ■

A simple consequence of this proposition is that the level of permanent cash is not a signal of better solvency risk of the insurer. The precautionary reserve is not aimed at diminishing the risk of insolvency but at diminishing the cost of recapitalization.

2.2. Information disclosure on reserves

In the previous part, when deciding the reserve policy, shareholders did not take into account the impact of this policy on the welfare of policyholders. Now, we assume that customers can observe the level of reserve (for example through an information disclosure policy), anticipate the solvency of their insurer and therefore evaluate their expected utility. In order to be accepted, the contract must therefore induce a minimal level of expected utility. The optimal behavior of the company now corresponds the optimal policy: (premium, dividends, reserves) under the constraint that the insureds achieve a given per-period level of expected utility $\underline{u}$. This provides the company a new instrument and introduces a trade off between cash reserve and premiums as in Bourlès (2009). In this perfect information framework, this situation (optimal policy under some reservation level for the insureds), can be considered as a good candidate for a competitive equilibrium. We examine here the case $\gamma = 1$.

In a first paragraph we assume that the company can ex ante commit to recapitalize. In the second paragraph we suppose, on the contrary, that there is no way to commit on recapitalization.

2.2.1. Full commitment

The problem becomes the following:

$$\begin{aligned} V^{FC}(m,\underline{u}) &= \max_{I,k(.),\pi} \int_I -k(x)f(x)dx + \int_I \delta V^{FC}(\rho\,(A-x+k(x)))f(x)dx \\ \text{s.t.} & \quad \int_I u(w-\pi)f(x)dx + \int_{I^C} u\left(w+\frac{m-x}{n}\right)f(x)dx \geq \underline{u} \end{aligned} \tag{8}$$

That is:

$$\begin{aligned} V^{FC}(m,\underline{u}) &= \max_{I,H(.)\geq 0,\pi} \int_I (m+n\pi-x)\,f(x)dx + \int_I \left(\delta V^{FB}(H(x),u_0) - \frac{1}{\rho}H(x)\right) f(x)dx \\ \text{s.t.} & \quad \int_I u(w-\pi)f(x)dx + \int_{I^C} u\left(w+\frac{m-x}{n}\right)f(x)dx \geq \underline{u} \end{aligned} \tag{9}$$

As previously, set $H^{FC} = \arg\max \delta V^{FC}(H,\underline{u}) - \frac{1}{\rho}H$ and $\delta W^{FC} = \max \delta V^{FC}(H,u_0) - \frac{1}{\rho}H$.

Then:

$$\begin{aligned} V^{FC}(m,\underline{u}) &= \max_{I,\pi}\int_I (m+n\pi-x)\,f(x)dx + \int_I \delta W^{FC} f(x)dx \\ \text{s.t.} & \quad \int_I u(w-\pi)f(x)dx + \int_{I^C} u\left(w+\frac{m-x}{n}\right)f(x)dx \geq \underline{u} \end{aligned} \tag{10}$$

Let λ be the Lagrange multiplier associated with the constraint. Optimization with respect to π gives:

$$(n-\lambda u'(w-\pi))\,\mu(I)=0$$

Set $I=(0,\overline{b}]$. When total loss is larger than $\overline{b}$, the security holders provoke default. The derivative of the Lagrangian w.r.t.$\overline{b}$ is:

$$L(\overline{b}) \equiv \left(m+n\pi-\overline{b}\right)+\delta W^{FC}+\lambda\left(u(w-\pi)-u\left(w+\frac{m-\overline{b}}{n}\right)\right) \tag{11}$$

It is easy to see that $L(\overline{b})$ is a convex function with a minimum for $\overline{b}$ such that $\frac{\lambda}{n}u'\left(w+\frac{m-\overline{b}}{n}\right)=1$, that is precisely $\overline{b}=m+n\pi$. This minimum is exactly δW^{FC}. This value being essentially positive we can conclude that $L(\overline{b})$ is always positive, and hence that

$$\begin{aligned} V^{FC}(m,\underline{u}) &= m+n\underline{\pi}-e+\delta W^{FC} \\ \text{with} &: u(w-\underline{\pi})=\underline{u} \end{aligned} \tag{12}$$

As $\delta\rho<1$, $\arg\max\limits_{H\geq 0}\delta V^{FC}(H,\underline{u})-\frac{1}{\rho}H=0$: the optimal policy is again to maintain cash reserves to zero. Therefore

$$\begin{aligned} V^{FC}(m,u_0) &= m+\frac{(n\underline{\pi}-e)}{1-\delta} \\ \text{with} &: u(w-\underline{\pi})=\underline{u} \end{aligned}$$

As $\dfrac{F(b^*(0))(n\underline{\pi}-e_0)}{1-\delta F(b^*(0))} \geq \dfrac{(n\underline{\pi}-e)}{1-\delta}$, the next proposition holds.

Proposition 3 *The Full Commitment policy implies recapitalization in situations where it is not the (private) optimal behavior for security holders. Therefore, an insurer has a short term incentive to deviate from this First Best policy by provoking default. This makes the First Best allocation unstable even if policyholders perfectly observe cash reserves of their insurer.*

2.2.2. Credibility: second best policy

The optimal full commitment policy cannot be non cooperatively implemented since there is no way for the shareholders to commit to recapitalize when losses are larger than the expected present value of future income. An interesting question is then to find a second best credible policy. In this second best world, cash reserves allow to commit to some default probability.

Endogenously, shareholders provoke default when the present value of future income is lower than the claims, that is when x is larger than $m + n\pi + \delta W^{SB}$, where W^{SB} is the optimal second best value of the firm.

We hence have:

$$
\begin{aligned}
V^{SB}(m, \underline{u}) \quad &= \quad \int_0^{m+n\pi+\delta W^{SB}} \left(m + n\pi + \delta W^{SB} - x\right) f(x)dx \qquad (13)\\
\text{with } \pi \quad &\text{the} \quad \text{largest value of the premium } p \text{ such that,}\\
U(m, p) \quad &= \quad \int_0^{m+np+\delta W^{SB}} u(w-p)f(x)dx\\
&\quad + \int_{m+np+\delta W^{SB}}^{+\infty} u\left(w + \frac{m-x}{n}\right) f(x)dx \geq \underline{u} \qquad (14)\\
\text{where} \quad &: \quad \delta W^{SB} = \max \delta V^{SB}(H, \underline{u}) - \frac{1}{\rho} H \qquad (15)
\end{aligned}
$$

The difference with the full commitment situation considered in the previous section, is that I is now constrained to be the interval $(0, m + n\pi + \delta W^{SB}]$. This comes from the fact that the decision not to default has to be be ex ante credible.

Intuitively, increasing m allows to decrease the probability of default which in turn increases the value of the contract for the policyholders and hence allows an increase of $n\pi$. A natural question is to compare the return of this "investment" with its cost.

As π is implicitly defined by (14), we have:

$$n\frac{\partial\pi}{\partial m} = -n\frac{U_m}{U_\pi} = \frac{\displaystyle\int_b^{+\infty} u'\left(w + \frac{m-x}{n}\right) f(x)dx + n\left[u(w-\pi) - u\left(w - \pi - \frac{\delta W^{SB}}{n}\right)\right] f(b)}{u'(w-\pi)F(b) - n\left[u(w-\pi) - u\left(w-\pi-\frac{\delta W^{SB}}{n}\right)\right] f(b)},$$

with $b = m + n\pi + \delta W^{SB}$.

As π is the largest value providing the level $\underline{u}$, U_π is necessarily negative, and then $-n\frac{Um}{U_\pi} \geq 0$.

We hence have:

$$n\frac{\partial\pi}{\partial m} \geq \frac{(1-F(b))}{F(b)} \frac{\mathrm{E}\left[u'\left(w + \frac{m-x}{n}\right)/x \geq b\right]}{u'(w-\pi)}.$$

As u is strictly concave and $w + \frac{m-x}{n} \leq w - \pi$ when $x \geq b$, this turns to:

$$n\frac{\partial\pi}{\partial m} \geq \frac{(1-F(b))}{F(b)}.$$

Now differentiating (13) gives:

$$\frac{\partial V^{SB}}{\partial m}(m, \underline{u}) = F(b) + F(b)n\frac{\partial\pi}{\partial m} \geq 1.$$

This investment is profitable if this derivative is greater than $\frac{1}{\rho\delta}$.

Proposition 4 *Supposing W_0, b_0 and π_0 are solutions of*

$$\begin{cases} W = \displaystyle\int_0^b (b-x)f(x)dx \\ b = n\pi + \delta W \\ u(w-\pi)F(b) + \displaystyle\int_b^{+\infty} u\left(w - \frac{x}{n}\right) f(x)dx = \underline{u} \end{cases} \tag{16}$$

If

$$F(b_0) + (1-F(b_0))\frac{E\left[u'\left(w-\frac{x}{n}\right)/x \geq b_0\right]}{u'(w-\frac{b_0}{n})} > \frac{1}{\delta\rho} \tag{17}$$

it is optimal to hold positive permanent cash $H^{SB} > 0$

Proof. Easily proven by supposing that $H^{SB} = 0$ and minoring $\frac{\partial V^{SB}}{\partial m}(0, \underline{u})$by $F(b_0) + (1 - F(b_0))\frac{\mathrm{E}\left[u'\left(w - \frac{x}{n}\right)/x \geq b_0\right]}{u'(w - \frac{b_0}{n})}$. ■

On the following picture we represent in the plane (b, W), the curves $W = \displaystyle\int_0^b (b - x)f(x)dx$ and $u\left(w - \dfrac{b}{n} + \dfrac{\delta W}{n}\right)F(b) + \displaystyle\int_b^{+\infty} u\left(w - \dfrac{x}{n}\right)f(x)dx = \underline{u}$. The latter is an increasing curve with a slope smaller than $\dfrac{1}{\delta}$, $\left(= \dfrac{1}{\delta} \text{ for } W = 0\right)$ and an asymptote $b = n\underline{\pi} + \delta W$. The intersection of these two curves give the solution W_0, b_0 and π_0 if $H^{SB} = 0$. Obviously we have $W_0 < W^{FC} \leq W^*$ meaning that the credible proof value of the company is lower than under the full commitment policy.

[Figure 3 about here.]

When the condition of the previous proposition is fulfilled, $V^{SB}(m, \underline{u})$ is a function of m with a slope greater than $\frac{1}{\delta\rho}$ for $m = 0$, and a slope exactly equal to $\frac{1}{\delta\rho}$ for $m = H^{SB}$. For m going to infinity, the slope tends to 1as π tends to $\underline{\pi}$ (and hence $\frac{\partial \pi}{\partial m}$ tends to 0). The asymptote being $m + (n\underline{\pi} - e) + \delta W^{SB}$, the optimal credible cash reserve can be represented as follows.

[Figure 4 about here.]

3. Regulation

In this section, we intuitively analyze the possible impact of two possible regulation schemes: cash requirement (that is the minimal amount of cash insurers have to hold to continue operating) and State guarantee to recapitalize. As the lack of credibility on recapitalization has been shown to be the main issue in reserve policy, the second option may be preferable.

3.1. The impact of minimal cash requirement

Let us first examine the possible implications of cash requirement on optimal cash reserve. Such a regulation rule (chosen in most countries) constraints the insurance companies to hold a minimal amount of cash $\underline{m}$.

In our setting, this implies that recapitalization is needed as soon as claims exceed assets *minus this ceiling* $\underline{m}$ (and therefore, more often than without regulation, for a given amount of initial cash). Moreover – under this scheme – when new shares (or debt) have to be issued, security holders need to build up cash reserve up to the required minimum (and no longer up to zero). As, the amount of cash shareholders are willing to inject is still bounded by the present value of future returns these two mechanisms that (i) increases the need for recapitalization and (ii) potentially reduce the value of company, may lead to a perverse effect of cash requirement through an increase in the probability of default. Lastly, it appears that cash requirement may reduce the potential amount of "precautionary reserve" (by imposing early recapitalization) that could – as shown in Proposition 2 – reduce the cost of recapitalization.

The exact implication of reserve requirement (in particular on the value of company) however remains to be investigated and calls for further research. The analyze of the precise constrained program would for example allow us to discuss more precisely the impact of cash requirement on the value of the firm and to provide some interesting comparative statics on $\underline{m}$. Our analysis moreover seems to call for the study of an alternative policy that would consist in fixing a minimum amount of cash above which security holders are prevented to take dividend but do not constrain them to recapitalize above zero (when claims exceed assets). Such a policy would create precautionary reserves without increasing the need for recapitalization.

3.2. A solution to the credibility issue: State guarantee to recapitalize

The questionable efficiency of cash requirement and the fact that the first best policy cannot be implemented because of a credibility issue lead us to consider an alternative form of regulation: State guarantee to inject cash. We have shown in this paper that default occurs when security holders are reluctant to inject enough cash for the company to keep on operating. This issue will therefore be solved if the State commits – in these cases – to buy enough shares for the cash reserve not to be negative.

This however creates a typical moral hazard issue as it will then be easy for the shareholders to cheat on there capacity/willingness to add cash. They will then benefit from the cash injected by the government without bearing the costs. This issue however disappears if we assume that government can infer the value of the company and can be eased by assuming that the State conditions its intervention to a takeover of the company. Such a nationalization would lead to a null value of the company (from the point of view of shareholders).

Therefore the call for State capital would only be optimal if the amount of needed cash exceed the value of the company.

4. Conclusion

We highlight in the paper to role of the relationship between an insurer and its security holders in the need for cash and default regulation. We show that beside the informational issue between an insurer and its policyholders, regulation can be needed to solve an issue of credibility on recapitalization. It indeed appears in our work that the first best policy is not credible as it induces recapitalization in situation where shareholders have no incentive to inject cash.

Contrarily to existing literature we moreover show that an interior solution for cash reserve can be optimally chosen if recapitalization is costly. When the capital market is frictionless, that is when issuing new shares is costless, the optimal strategy consists in taking dividend as long as it is possible – because of agency cost – and to recapitalize each time it is needed (provided the future value of the company is larger than the invested cash). However, if issuing new shares is costly, it can be optimal to leave some cash in the company. The optimal strategy then consists in (a) taking dividend above a bottom limit, (b) neither take dividend nor issue new shares if ex-post cash reserves are positive but below the limit and (c) issue new shares in order to meet claims when current reserves are insufficient.

Taking into account the effect of default on policyholders, we show that the first best policy implies no default but no cash reserve. This policy however appears to be hardly implementable as it implies an ex-ante commitment to recapitalize which is not credible. An efficient regulation would therefore consist in making this commitment credible and therefore State commitment to recapitalize may be a more efficient regulation than cash requirement.

Is left for future research to analyze more precisely the optimal regulation. It would for example be interesting to evaluate the optimal cash reserve policy under cash requirement that is if cash reserve is constrained to be above a given level. We would then be able to define more exactly the efficient regulation. Another extension of interest would consist in studying the value of a share (and not of the company). Such a variant of our model may create an incentive for positive reserve (even with costless cash) as recapitalization – that is the issuance of new shares – would reduce the value of an existing share.

5. Appendix

5.1. Proof of Lemma 1

We have

$$\begin{cases} \delta W^* = \delta V(H^*,\pi) - \frac{1}{\rho}H^* \geq \delta V(H^*_\gamma,\pi) - \frac{1}{\rho}H^*_\gamma \\ \delta W^*_\gamma = \delta V(H^*_\gamma,\pi) - \frac{\gamma}{\rho}H^*_\gamma \geq .\delta V(H^*,\pi) - \frac{\gamma}{\rho}H^* \end{cases}$$

Summing up these two inequalities gives:

$$\frac{\gamma-1}{\rho}H^* \geq \frac{\gamma-1}{\rho}H^*_\gamma$$

Moreover $S(\gamma) = \max\limits_{H\geq 0}\delta V(H,\pi) - \frac{\gamma}{\rho}H$ is the supremum of decreasing affine (and hence convex) functions. It is hence a convex decreasing function, such that $S'(\gamma) = \frac{-H^*_\gamma}{\rho}$ (almost everywhere).

5.2. Proof of Proposition 1

We have:

$$V(m,\pi) = \max_{H(x)\geq 0, I} \int_I \left[\delta V(H(x),\pi) - \alpha(x)\frac{1}{\rho}H(x) + \alpha(x)\left(A-x\right)\right] f(x)dx \qquad (18)$$

With $\alpha(x) = 1$ if $H(x) < \rho\left(A-x\right)$ and $\alpha(x) = \gamma$ if $H(x) > \rho\left(A-x\right)$

take $H^*(x)$the optimal policy and define:

$I_1 = \left\{x, H^*(x) < \rho\left(A-x\right)\right\}, I_2 = \left\{x, H^*(x) = \rho\left(A-x\right)\right\}, I_3 = \left\{x, H^*(x) > \rho\left(A-x\right)\right\}$

We have:

$$\begin{aligned} V(m,\pi) &= \int_{I_1}\left[\delta V(H^*(x),\pi) - \frac{1}{\rho}H^*(x) + \left(A-x\right)\right] f(x)dx &(19)\\ &+ \int_{I_2}\delta V(H^*(x),\pi)f(x)dx &(20)\\ &+ \int_{I_1}\left[\delta V(H^*(x),\pi) - \frac{\gamma}{\rho}H^*(x) + \gamma\left(A-x\right)\right] f(x)dx &(21)\end{aligned}$$

Clearly in I_1, $H^*(x) = \min(\rho\,(A-x)\,, H^*)$, and in I_3, $H^*(x) = \max(\rho\,(A-x)\,, H^*_\gamma)$.

This in turn implies that:

$$V(m,\pi) = \max_{a \le b \le A \le \overline{x}} \int_{-\infty}^{a} [\delta W^* + (A-x)]\, f(x)dx \tag{22}$$

$$+ \int_{a}^{b} \delta V(\rho\,(A-x)\,, \pi) f(x)dx \tag{23}$$

$$+ \int_{b}^{\overline{x}} \left[\delta W^*_\gamma + \gamma\,(A-x)\right] f(x)dx \tag{24}$$

In order to find H^* and H^*_γ it is helpful to compute the first derivative of V Thanks to the envelop theorem we have:

$$V'_m(m,\pi) = \int_{I^*} \alpha^*(x) f(x)dx$$

Where we define $\alpha^*(x)$ by:

$$\begin{aligned}
\alpha^*(x) &= 1 \text{ if } x \le a^* \\
\alpha^*(x) &= \gamma \text{ if } x \ge b^* \\
\alpha^*(x) &= \delta\rho V'_m(\rho\,(A-x)\,, \pi) \text{ if } a \le x \le b
\end{aligned}$$

This implies that $V'_m(m,\pi)$ is smaller than γ, and then that $\delta V'_m(m,\pi)$ is smaller than $\delta\gamma$ (which is smaller than $\frac{\gamma}{\rho}$). This means that $H^*_\gamma = 0$ and hence that $b^* = A$ and $W^*_\gamma = V(0,\pi)$. Under the optimal cash policy the company issues new shares only in case of negative cash, that is only when claims are greater than reserves A.

Then the set of equations characterizing the solution of the problem are:

$$\left\{\begin{array}{rcl}
V(m,\pi) & = & \int_{-\infty}^{a^*} [\delta W^* + (m+n\pi-x)]\, f(x)dx \\
& & + \int_{a^*}^{A} \delta V(\rho\,(m+n\pi-x)\,, \pi) f(x)dx \\
& & + \int_{A}^{b^*} [\delta V(0,\pi) + \gamma\,(m+n\pi-x)]\, f(x)dx \\
\delta V(\rho\,(m+n\pi-a^*)\,, \pi) & = & \delta W^* + (m+n\pi-a^*) \\
\delta V(0,\pi) + \gamma\,(m+n\pi-b^*) & = & 0 \\
W^* & = & \max_{H \ge 0} \left(\delta V(H,\pi) - \frac{1}{\rho} H\right)
\end{array}\right.$$

5.3. Proof of Proposition 2

We know that:

$$\begin{aligned} V(m,\pi) &= \max_{a\leq A\leq b}\int_{-\infty}^{a}\left[\delta W^{*}+(m+n\pi-x)\right]f(x)dx \\ &\quad+\int_{a}^{A}\delta V(\rho\,(m+n\pi-x)\,,\pi)f(x)dx \\ &\quad+\int_{A}^{b}\left[\delta V(0,\pi)+\gamma\,(m+n\pi-x)\right]f(x)dx \end{aligned}$$

With the envelop theorem the derivative of V with respect to γ is hence:

$$V'_{\gamma}(m,\pi)=\int_{A}^{b^{*}(m)}\left[\delta V'_{\gamma}(0,\pi)+(m+n\pi-x)\right]f(x)dx.$$

In particular:

$$V'_{\gamma}(0,\pi)=\int_{A}^{b^{*}(0)}\left[\delta V'_{\gamma}(0,\pi)+(n\pi-x)\right]f(x)dx,$$

which give :

$$V'_{\gamma}(0,\pi)\left(1-\delta\left(F(b^{*}(0))-F(A)\right)\right)=\int_{A}^{b^{*}(0)}\left(n\pi-x\right)f(x)dx,$$

and implies $V'_{\gamma}(0,\pi)\leq 0$

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Fig. 1.— On the suboptimality of "automatical" recapitalization

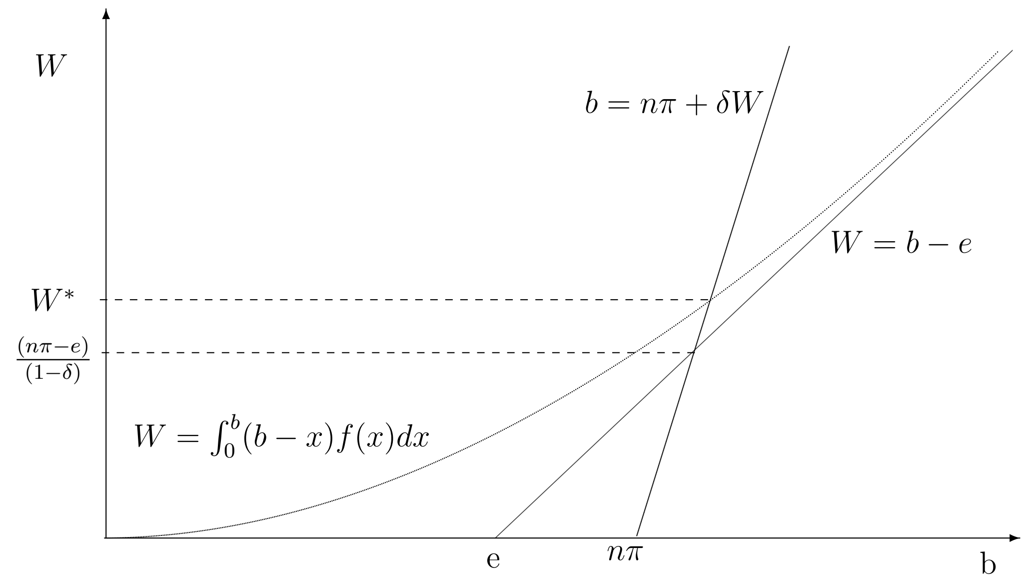

Fig. 2.— The value of a company holding cash m

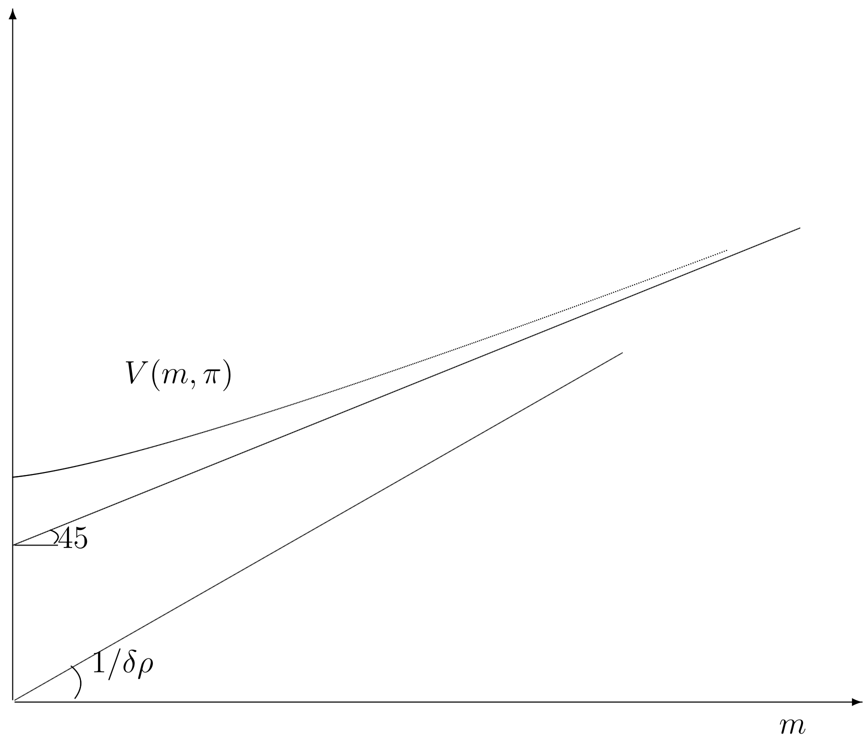

Fig. 3.— The credibility proof value of the firm

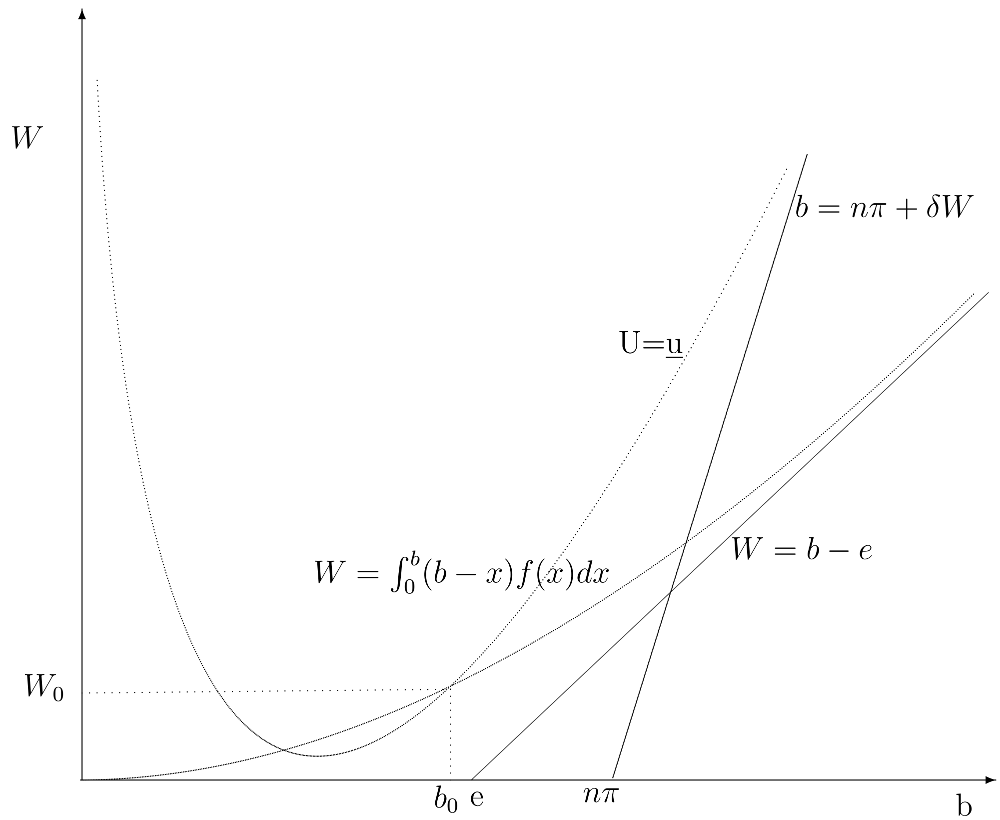

Fig. 4.— The second best cash reserve