

# Pricing and Information Disclosure in Markets with Loss-Averse Consumers\*

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## Abstract

We develop a theory of imperfect competition with loss-averse consumers. All consumers are fully informed about match value and price at the time they make their purchasing decision. However, a share of consumers is initially uncertain about their tastes and forms a reference point consisting of an expected match value and an expected price distribution, while the other consumers are perfectly informed all the time. Loss aversion in the match value dimension leads to less competitive outcome, while loss aversion in the price dimension leads to a more competitive outcome. Thus a market with uninformed consumers may be more or less competitive than a market with informed consumers. We explore the interplay between these two effects. We, also, derive implications for firm strategy and public policy concerning the firms' incentives to "educate" consumers about their own tastes. In particular, we show that private incentives to disclose information early on may be socially insufficient.

**Keywords:** Loss Aversion, Reference-Dependent Utility, Information Disclosure, Price Variation, Collusive Pricing, Advertising, Behavioral Industrial Organization, Imperfect Competition, Product Differentiation

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# 1 Introduction

Consumer information about price and match value of products is a key determinant of market outcomes. Previous work has emphasized the role of consumer information at the moment of purchase.<sup>1</sup> If consumers are loss-averse, information prior to the moment of purchase matters: Product information plays an important role at the stage at which loss-averse consumers form expectations about future transactions.

In this paper, we develop a theory of imperfect competition to investigate the competitive effects of consumer loss aversion. Our theory applies to inspection goods with the feature that consumers readily observe prices in the market but have to inspect products before knowing the match value between product characteristics and consumer tastes. As we show by examples, this applies to a large number of products and thus is important to understand market interaction and is relevant to formulate consumer protection policies, for instance, in the form of information disclosure rules.

Loss-aversion in consumer choice has been widely documented in a variety of laboratory and field settings starting with Kahneman and Tversky (1979). Loss-averse consumers have to form expectations about product performance. We postulate that, to make their consumption choices, loss-averse consumers form their probabilistic reference point based on expected future transactions which are confirmed in equilibrium. Here, a consumer's reference point is her probabilistic belief about the relevant consumption outcome held between the time she first focused on the decision determining the consumption plan—i.e., when she heard about the products, was informed about the prices for the products on offer, and formed her expectations—and the moment she actually makes the purchase.<sup>2</sup>

We distinguish between “informed” and “uninformed” consumers at the moment consumers form their reference point. Informed consumers know their taste *ex ante* and will perfectly foresee their equilibrium utility from product characteristics. Therefore, they will not face a loss or gain in product satisfaction beyond their intrinsic valuation.

Uninformed consumers, by contrast, are uncertain about their ideal product characteristic: they form expectations about the difference between ideal and actual product characteristic which will serve as a reference point when evaluating a product along its taste or

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<sup>1</sup> See e.g. Varian (1980), Janssen and Moraga-González (2004), and Armstrong and Chen (2008).

<sup>2</sup> For evidence that expectation-based counterfactuals can affect the individual's reaction to outcomes, see Blinder, Canetti, Lebow, and Rudd (1998), Medvec, Madey, and Gilovich (1995), and Mellers, Schwartz, and Ritov (1999). The general theory of expectation-based reference points and the notion of personal equilibrium have been developed by Koszegi and Rabin (2006) and Koszegi and Rabin (2007).

match value dimension. They will also face a gain or a loss relative to their expected distributions of price after learning the taste realization. Since all consumers become fully informed before they have to make their purchasing decision, we isolate the effect of consumer loss aversion on consumption choices and abstract from the effects of differential information at the moment of purchase.<sup>3</sup>

Consumers are loss-averse with respect to prices and match value and have rational expectations about equilibrium outcomes to form their reference point, as in Heidhues and Koszegi (2008). Firms are possibly asymmetric due to deterministic cost differences—this is common knowledge among the firms when the game starts.<sup>4</sup> They compete in prices for differentiated products. Consumers observe equilibrium prices before forming their reference point. Note that if prices are different between the firms, uninformed consumers will face either a loss or a gain in the price dimension depending on whether they buy the more or the less expensive product. Hence, an (ex ante) uninformed consumer's realized net utility depends not only on the price of the product she buys but also on the price of the product she does not buy.

A key modeling assumption is that a share of consumers are initially uninformed and form expectations before knowing the match value a particular product offers but after learning the prices of the offered products. This timing on information release and reference point formation appears appropriate for a number of industries. Let us provide some examples. First, prices of clothing and electronic devices are easily accessible (and are often advertised) in advance while, for inexperienced consumers, the match quality between product and personal tastes is impossible or difficult to evaluate before actually seeing or touching the product. A related example is high-end hifi-equipment and, in particular, loudspeakers. Price tags are immediately observed but it may take several visits to the retailers (on appointment) or even trials at home to figure out the match value of the different products under consideration—for example, because people differ with respect to the sound they like. In these markets potential cost differences may arise from size differences of producers and product-specific costs (or, as we allow in our extension, from different ex ante observable quality differences). Second, the housing market has the feature that the price is listed (and, in some countries, not negotiable) whereas the match value is only found out after visiting the flat. Third, price information on products sold over the internet—for example, CDs of a particular classical concert—is immediately available, while the match value is often determined only after listening to some of the material that is pro-

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<sup>3</sup>Our model can alternatively be interpreted as one in which consumers know their ideal taste ex ante but are exposed to uncertainty about product characteristics when they form their reference point.

<sup>4</sup>In the extension section we show that our analysis also applies to products of different qualities.

vided online. Fourth, competing services such as long-distance bus rides and flights are differentiated by departure times. Here consumers are perfectly aware of the product characteristics *ex ante*—i.e., price and departure time—but learn their preference concerning their ideal point of departure only at some later stage (after forming their probabilistic reference point but before purchase).

Our first main results is that, in markets with consumers described by a gain-loss utility and, more specifically, loss aversion, the competitive effect of such a behavioral bias depends on the importance of the price dimension relative to the match value dimension. In other words, whether the behavioral bias makes the market more or less competitive depends on the detail how gains and losses in the two dimensions enter the consumers' utility function. We show that a gain-loss utility with respect to prices leads to lower prices and, thus, is pro-competitive, whereas a gain-loss utility with respect to match value is anti-competitive. This even holds if gains and losses enter symmetrically into the utility function. Our model features a unique pure-strategy equilibrium and allows for clear-cut comparative statics results.

Our second main result is that, in asymmetric markets (and with equal weights assigned to the price and the match value dimension), price variation larger compared to a market without loss-averse consumers. This contrasts with the focal price result by Heidhues and Koszegi (2008).<sup>5</sup>

Our third main result is that loss aversion—or, more precisely, the presence of more *ex ante* uninformed, loss averse consumers—may lead to lower prices, in particular if the market is asymmetric (fixing the weights consumers put on gains and losses in the price relative to the match value dimension). Hence, the standard result that more informed consumers (or more consumers without a behavioral bias) lead to lower prices is challenged in our model when firms are strongly asymmetric, even if it holds in symmetric markets. The driving force also behind this result is that loss aversion in the price dimension has a pro-competitive effect while the effect of loss aversion in the taste dimension is anti-competitive. The pro-competitive effect dominates the anti-competitive effect if the size of loss aversion in the price dimension becomes sufficiently large. This occurs if the price difference is large, which is caused by strong cost asymmetries. In those markets

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<sup>5</sup>In a related setting to ours, Heidhues and Koszegi (2008) show that consumer loss aversion can explain the empirical observation that firms often charge the same price in differentiated product markets even if they have different costs. One of the distinguishing features of our model is that realized costs are public information and consumers observe prices before forming their reference point. For a detailed comparison, see below.

uninformed consumers are very reluctant to buy the expensive product and rather accept a large reduction in match value when buying the low-price product.

This paper contributes to the understanding of the effect of consumer loss aversion in market environments and is complementary to Heidhues and Koszegi (2008).<sup>6</sup> More broadly, it contributes to the analysis of behavioral biases in market settings, as in Eliaz and Spiegel (2006), Gabaix and Laibson (2006), and Grubb (forthcoming). An important issue in our paper, as also in Eliaz and Spiegel (2006), is the comparative statics effects in the composition of the population. However, whereas in their models this composition effect is behavioral in the sense that the share of consumers with a behavioral bias changes, we do not need to resort to this interpretation, although our analysis is compatible with it: We stress the composition effect to be informational in the sense that the arrival of information in the consumer population is changed (while the whole population is subject to the same behavioral bias).

Zhou (2008) provides another paper with consumer loss aversion that, in contrast to Heidhues and Koszegi (2008), predicts a pro-competitive effect of consumers being loss averse. In his model this is a clear-cut result. By contrast, we provide a taxonomy of different market environments and find that the impact of consumer loss aversion on competition depends on the particular specification of the gain-loss utility and on the degree of market asymmetry: If consumers experience a gain-loss utility in the price dimension only, the behavioral bias is pro-competitive; if they experience a gain-loss utility in the match value dimension only, the behavioral bias is anticompetitive. If both dimensions enter the utility function symmetrically, the result depends on the presence of consumer loss aversion: If gains and losses receive the same weights (i.e. no loss aversion), the bias is competitively neutral; otherwise, with consumer loss aversion the anti-competitive in the taste dimension dominates. In this last setting, we introduce asymmetries between firms and show that loss aversion increases competition compared to setting with fully rational consumers in strongly asymmetric markets and relaxes it in rather symmetric markets. Interestingly, in Zhou's models and in ours firms can manipulate consumers' reference point by choosing product prices. The main difference between the two models is that consumers in his model do not use an expectation-based reference point. Instead, consumers consider the product visited last as their reference point. Finally, our model has the additional feature that firms can influence the amount of loss-averse consumers by advertising, i.e. by disclosing information about product characteristics at an early stage.

The informational interpretation lends itself naturally to address questions about the ef-

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<sup>6</sup>See also Heidhues and Koszegi (2005) for a related monopoly model with a different timing.

fect of early information disclosure to additional consumers. We analyze information disclosure policies by firms and public authorities in the context of a behavioral industrial organization framework. We thus demonstrate the possible use of behavioral models to address policy questions in industrial organization. As stated above, our model has the feature that, absent behavioral bias, information disclosure policies are meaningless. Thus the behavioral bias is essential in our model to address these issues. In particular, we show that private and social incentives to disclose information at an early stage are not aligned. We also show that the more efficient (and, thus, larger) firm discloses information if firms have conflicting interests.

Our analysis also contributes to the literature on the economics of advertising (see Bagwell (2007) for an excellent survey). It uncovers the role of advertising as consumer expectation management. Note that at the point of purchase consumers are fully informed so that there is no role for informative advertising. However, since consumers are loss-averse, educating consumers about their preferences or, alternatively, about product characteristics, makes these consumers informed in our terminology. Advertising thus can remove the uncertainty consumers face when forming their reference point. This form of advertising can be seen as a hybrid form of informative and persuasive advertising because it changes preferences at the point of purchase—this corresponds to the persuasive view of advertising—, albeit due to information that is received *ex ante*—this corresponds to the informative view of advertising. It also points to the importance of the timing of advertising: for expectation management it is important to inform consumers early on.

Other marketing activities can also be understood as making consumers informed at the stage when consumers form their reference point. For instance, test drives for cars or lending out furniture, stereo equipment, and the like make consumers informed early on. Arguably, in reality uncertainty would otherwise not be fully resolved even at the purchasing stage. However, to focus our minds, we only consider the role of marketing activities on expectation formation before purchase. In short, in our model firms may use marketing to manage expectations of loss-averse consumers at an early stage.<sup>7</sup>

Our paper can be seen as complementary to the work on consumer search in product markets (see e.g. Varian (1980), Anderson and Renault (2000), Janssen and Moraga-González (2004), Armstrong and Chen (2008)). Whereas that literature focuses on the effect of differential information (and consumer search) at the purchasing stage, our paper abstracts from this issue and focuses on the effect of differential information at the expectation

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<sup>7</sup>For a complementary view see Bar-Isaac, Caruana, and Cunat (forthcoming).

formation stage which is relevant if consumers are loss aversion. Finally, our paper complements the large literature on spatially differentiated product markets by analyzing the competitive effects of consumer loss aversion.

We will discuss the connections to a number of the above cited contributions in more detail in the main text. The plan of the paper is as follows. In Section 2, we present the model. Here, we have to spend some effort to determine the demand of uninformed consumers. In Section 3, we characterize the duopoly equilibrium. In the accompanying Appendix B, we establish equilibrium uniqueness and equilibrium existence. Our existence proof requires to bound the parameters of our model, in particular, the two firms cannot be too asymmetric for equilibrium existence to hold. In Section 4, we analyze properties of the symmetric duopoly. Here, we also compare our findings to the duopoly model with a different timing of events that is inspired by Heidhues and Koszegi (2008) and provide an  $n$ -firm oligopoly analysis. In section 5, we analyze competition in asymmetric duopoly in more detail and derive our second and third main result. Here, we analyze the impact of the degree of asymmetry on equilibrium outcomes. We also establish comparative statics results with respect to the share of ex ante informed consumers. In Section 6 we provide extensions to models with different weights in the gain-loss utility and with different product qualities. Section 7 concludes. Some of the proofs are relegated to Appendix A. Results on equilibrium existence are provided in Appendix B. Some of the tables are contained in Appendix C.

## 2 The Model

### 2.1 Setup

Consider a market with two, possibly asymmetric firms,  $A$  and  $B$ , and a continuum of loss-averse consumers of mass 1. The firms' asymmetry consists of differences in marginal costs. Here, the more efficient firm is labeled to be firm  $A$ —i.e.,  $c_A \leq c_B$ . Firms are located on a circle of length 2 with maximum distance,  $y_A = 0$ ,  $y_B = 1$ . Firms announce prices  $p_A$  and  $p_B$  and product locations to all consumers.

Consumers of mass one are uniformly distributed on the circle of length 2. A consumer's location  $x$ ,  $x \in [0, 2)$ , represents her taste parameter. Her taste is initially—i.e., before she forms her reference point—known only to herself if she belongs to the set of informed consumers.



In our model, consumers' differential information applies to the date at which consumers determine their reference point and not to the date of purchase: a fraction  $(1 - \beta)$  of loss-averse consumers,  $0 \leq \beta \leq 1$ , is initially uninformed about their taste. As will be detailed below, they endogenously determine their reference point and then, before making their purchasing decision, observe their taste parameter (which is private information of each consumer). At the moment of purchase all consumers are perfectly informed about product characteristics, prices, and tastes.

All consumers have the same reservation value  $v$  for an ideal variety and have unit demand. Their utility from not buying is  $-\infty$  so that the market is fully covered.

Two remarks about our modeling choice are in order: First, we could alternatively work with the Hotelling line. Our model (in terms of market outcomes) is equivalent to the Hotelling model in which consumers are uniformly distributed on the  $[0, 1]$ -interval and firms are located at the extreme points of the interval. Second, the circle model allows for an alternative and equivalent interpretation about the type of information some consumers initially lack: at the point in time consumers form their reference point distribution, they all know their taste parameters but only a fraction  $(1 - \beta)$  does not know the location of the high- and the low-cost firm. These uninformed consumers only know that the two firms are located at maximal distance and that one is a high- whereas the other is a low-cost firm.

To determine the market demand faced by the two firms, let the informed consumer type in  $[0, 1]$ , who is indifferent between buying good  $A$  and good  $B$ , be denoted by  $\hat{x}_{in}(p_A, p_B)$ . Correspondingly, the indifferent uninformed consumer is denoted by  $\hat{x}_{un}(p_A, p_B)$ . Since market shares on  $[0, 1]$  and  $[1, 2]$  are symmetric, the firms' profits are:

$$\begin{aligned}\pi_A(p_A, p_B) &= (p_A - c_A)[\beta \cdot \hat{x}_{in}(p_A, p_B) + (1 - \beta) \cdot \hat{x}_{un}(p_A, p_B)] \\ \pi_B(p_A, p_B) &= (p_B - c_B)[\beta \cdot (1 - \hat{x}_{in}(p_A, p_B)) + (1 - \beta) \cdot (1 - \hat{x}_{un}(p_A, p_B))].\end{aligned}$$

The timing of events is as follows:

Stage 0.) Marginal costs  $(c_A, c_B)$  realize (and become common knowledge among firms)<sup>8</sup>

Stage 1.) Price setting stage: Firms simultaneously set prices  $(p_A, p_B)$

Stage 2.) Reference point formation stage: All consumers observe prices and

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<sup>8</sup>As mentioned above, without loss of generality, we consider realizations  $c_A \leq c_B$ .



- a) informed consumers observe their taste  $x$  (for them uncertainty is resolved)
- b) uninformed consumers form reference point distributions over purchase price and match value, as detailed below

Stage 3.) Inspection stage: Uninformed consumers observe their taste  $x$ —i.e., uncertainty is resolved for **all** consumers.

Stage 4.) Purchase stage: Consumers decide which product to buy:

- a) informed consumers make rational purchase decisions;
- b) (ex ante) uninformed consumers make rational purchase decisions, based on their utility that includes realized gains and losses relative to their reference point distribution.

At stage 1 we solve for subgame perfect Nash equilibrium, where firms foresee that uninformed consumers play a personal equilibrium at stage 2b. Personal equilibrium in our context simply means that consumers hold rational expectation about their final purchasing decision; for the general formalization see Koszegi and Rabin (2006).

## 2.2 Demand of informed consumers

Informed consumers ex ante observe prices and their taste parameter and therefore do not face any uncertainty when forming their reference point. Hence, their behavior is the same as in the standard Hotelling-Salop model. For prices  $p_A$  and  $p_B$  an informed consumer located at  $x$  obtains the following indirect utility from buying product  $i$

$$u_i(x, p_i) = v - t|y_i - x| - p_i,$$

where  $t$  scales the disutility from distance between ideal and actual taste on the circle. The expression  $v - t|y_i - x|$  then captures the match value of product  $i$  for consumer of type  $x$ . Denote the indifferent (informed) consumer between buying from firm  $A$  and  $B$  on the first half of the circle by  $\hat{x}_{in} \in [0, 1]$  and solve for her location given prices. The informed indifferent consumer is given by

$$\hat{x}_{in}(p_A, p_B) = \frac{(t + p_B - p_A)}{2t}. \quad (1)$$

Symmetrically, a second indifferent (informed) consumer type is located at  $2 - \hat{x}_{in}(p_A, p_B) \in [1, 2]$ . Without loss of generality we focus on demand of consumers between 0 and 1

and multiply by 2. Cost differences influence the location of indifferent consumers via prices: If asymmetric costs lead to asymmetric prices in equilibrium, then the indifferent informed consumer will also be located apart from  $1/2$  (resp.  $3/2$ ), the middle between  $A$  and  $B$ .<sup>9</sup>

### 2.3 Demand of uninformed consumers

Uninformed consumers do not know their ideal taste  $x$  ex ante. Since they cannot judge which product they will buy before they inspect products and learn their ideal taste  $x$ , they ex ante face uncertainty about their match value and purchase price (although they know firms' prices already). With regard to this uncertainty uninformed consumers form reference point distributions over match value and purchase price. Following Heidhues and Koszegi (2008) they will experience gains or losses in equilibrium depending on their realized taste and their purchase decision. These gains and losses occur in two dimensions, in a taste dimension (as determined by the fit between idiosyncratic taste and product characteristics) and in a price dimension. In both dimensions losses are evaluated at a rate  $\lambda$  and gains at a rate 1 with  $\lambda > 1$ . This reflects widespread experimental evidence that losses are evaluated more negatively than gains—we analyze the consequence of this assumption on market outcomes in Section 4. Three properties of this specification are worthwhile pointing out. First, consumers have gains or losses not about net utilities but about each product “characteristic”, where price is then treated as a product characteristic. This is in line with much of the experimental evidence on the endowment effect; for a discussion see e.g. Koszegi and Rabin (2006). Second, consumers evaluate gains and losses *across* products.<sup>10</sup> This appears to be a natural property for consumers facing a discrete choice problem: they have to compare the merits of the two products to each other. In other words, consumers view the purchasing decision with respect to these two problems as a single decision problem. Third, to reduce the number of parameters, we assume that the gain/loss parameters are the same across dimensions. This appears to be the natural benchmark—again we refer to Section 4 for an analysis under alternative assumptions.

While our setting is related to Heidhues and Koszegi (2008) our model has three distinguishing features. First, firms' deterministic costs are known by their competitor. This

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<sup>9</sup>E.g. if there are only informed consumers,  $\hat{x}_m = 1/2 + (c_B - c_A)/(6t)$  in equilibrium. This is closer to  $B$  for  $c_B > c_A$ . Thus, the low-cost firm serves a larger market share.

<sup>10</sup>Gains and losses also matter in the price dimension because, even though prices are deterministic, they are different across firms. Hence, a consumer who initially does not know her taste parameter is uncertain at this point in time about the price at which she will buy.

property is in line with a large part of the industrial organization literature on imperfect competition and is approximately satisfied in markets in which firms are well-informed not only about their own costs but also about their relative position in the market. Second, prices are already set before consumers form their reference point.<sup>11</sup> This property applies to markets in which consumers are from the start well-informed about the actual price distribution they face in the market. This holds in markets in which firms inform consumers about prices (but consumers are initially uncertain about the match value and thus their eventual purchasing decision) or in which prices are publicly posted.<sup>12</sup> Third, there is a fraction of  $(1 - \beta)$  of uninformed consumers who face uncertainty about their ideal taste  $x$  and a fraction of  $\beta$  informed consumers who know their ideal taste ex ante. As motivated in the introduction, various justifications for differential information at the ex ante stage can be given. Consumers differ by their experience concerning the relevant product feature. Alternatively, a share of consumers know that they will be subject to a taste shock between forming their reference point and making their purchasing decision. These consumers then do not condition their reference point on the ex ante taste parameter, whereas those belonging to the remaining share do.

Consider an uninformed consumer who will learn that she is located at  $x$  after her ideal taste is realized. Suppose firms set prices  $p_A$  and  $p_B$  in equilibrium. Then the uninformed consumer will buy from firm  $A$  if she is located close enough to firm  $A$ , i.e. if  $x \in [0, \hat{x}_{un}(p_A, p_B)] \cup [2 - \hat{x}_{un}(p_A, p_B), 2]$ , where  $\hat{x}_{un}(p_A, p_B)$  is the location of the indifferent (uninformed) consumer we want to characterize. Hence, the uninformed consumer at  $x$  will pay  $p_A$  in equilibrium with  $\text{Prob}[x < \hat{x}_{un}(p_A, p_B) \vee x > 2 - \hat{x}_{un}(p_A, p_B)]$  and  $p_B$  with  $\text{Prob}[\hat{x}_{un}(p_A, p_B) < x < 2 - \hat{x}_{un}(p_A, p_B)]$ . Since  $x$  is uniformly distributed on  $[0, 2]$  we obtain that  $\text{Prob}[x < \hat{x}_{un}(p_A, p_B) \vee x > 2 - \hat{x}_{un}(p_A, p_B)] = \hat{x}_{un}(p_A, p_B)$ , i.e., from an ex ante perspective  $p_A$  is the relevant price with probability  $\text{Prob}[p = p_A] = \hat{x}_{un}$ . Correspondingly, the purchase at price  $p_B$  occurs with probability  $\text{Prob}[p = p_B] = 1 - \hat{x}_{un}$ . Thus, the reference point distribution with respect to the purchase price  $p$  is discrete and can be expressed by

$$F(p) = \begin{cases} \hat{x}_{un} & \text{if } p \in [p_A, p_B) \\ 1 & \text{if } p \geq p_B. \end{cases}$$

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<sup>11</sup>This is particularly appropriate in market environments in which price information has been provided from the outset, while uninformed (or inexperienced) consumers observe the match value only when physically or virtually inspecting the product.

<sup>12</sup>Note that in an asymmetric market firms set different prices. Hence, although prices are deterministic, a consumer who does not know her taste parameter is uncertain about the price she will pay for her preferred product.

The reference point with respect to the match value is the reservation value  $v$  minus the expected distance between ideal and actual product taste times the taste parameter  $t$ . The distribution of the expected distance is denoted by  $G(s) = \text{Prob}(|x - y_\sigma| \leq s)$ , where the taste distance  $s \in [0, 1]$ , the location of the firm  $y_\sigma \in \{0, 1\}$ , and the consumer  $x$ 's purchase strategy in equilibrium for given prices  $\sigma \in \arg \max_{j \in \{A, B\}} u_j(x, p_j, p_{-j})$ .<sup>13</sup>

Since  $c_A \leq c_B$ , we restrict attention to the case  $\hat{x}_{un} \geq 1/2$ , i.e., firm  $A$  has a weakly larger market share than firm  $B$  also for uninformed consumers. Given that some uninformed consumers will not buy from their nearest firm,  $G(s)$  will be kinked. This kink is determined by the maximum distance  $|x - y_B|$  that consumers are willing to accept buying the more expensive product  $B$ ,  $s = 1 - \hat{x}_{un}$  because  $s \leq 1 - \hat{x}_{un}$  holds for consumers close to either  $A$  or  $B$ , while  $s > 1 - \hat{x}_{un}$  only holds for the more distant consumers of  $A$ . Hence, the distribution of  $s$  is

$$G(s) = \begin{cases} 2s & \text{if } s \in [0, 1 - \hat{x}_{un}] \\ s + (1 - \hat{x}_{un}) & \text{if } s \in (1 - \hat{x}_{un}, \hat{x}_{un}] \\ 1 & \text{otherwise.} \end{cases}$$

Note that if the indifferent uninformed consumer is located in the middle between  $A$  and  $B$ ,  $\hat{x}_{un} = 1/2$ , the expected distance between ideal and actual product taste,  $\mathbb{E}[s]$ , is minimized and equal to  $1/4$ .

Following Koszegi and Rabin (2006), after uncertainty is resolved consumers experience a gain-loss utility: the reference distribution is split up for each dimension at the value of realization in a loss part with weight  $\lambda > 1$  and a gain part with weight 1. In the loss part the realized value is compared to the lower tail of the reference distribution; in the gain part it is compared to the upper tail of the reference distribution.

Consider the gain-loss utility of an uninformed consumer located at  $x$ , at the moment she decides whether to purchase the product. Recall that at this point she knows her taste parameter  $x$ . The initially uninformed consumer now decides which product to buy taking into account her intrinsic utility from a product and her gain-loss utility when she

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<sup>13</sup>  $\sigma$  is a function of prices and consumer's location  $x$  conditional on consumer's expectation about equilibrium outcomes which are incorporated in their two-dimensional reference point distribution.  $\sigma$  states a consumer's personal equilibrium. This equilibrium concept was introduced by Koszegi and Rabin (2006) and requires that behavior-generating expectations must be optimal given expectations, i.e. must be self-fulfilling in equilibrium.

compares the price-taste combination of a product with her two-dimensional reference point distribution.

First, consider the indirect utility of an uninformed consumer from a purchase of product  $A$  when this consumer is located at  $x \in (1 - \hat{x}_{un}, 1]$ .<sup>14</sup>

$$u_A(x, p_A, p_B) = (v - tx - p_A) - \lambda \cdot \text{Prob}[p = p_A](p_A - p_A) + \text{Prob}[p = p_B](p_B - p_A) - \lambda \cdot t \int_0^x (x - s) dG(s) + t \int_x^1 (s - x) dG(s), \quad (2)$$

where the first term is the consumer's intrinsic utility from product  $A$ . The second term is the loss in the price dimension from not facing a lower price than  $p_A$ . This term is equal to zero because  $p_A$  is the lowest price offered in the market place. The third term is the gain from not facing higher price than  $p_A$ , which is positive. The last two terms correspond to the loss (gain) from not facing a smaller (larger) distance in the taste dimension than  $x$ . An uninformed consumer's indirect utility from a purchase of product  $B$  is derived analogously,

$$u_B(x, p_A, p_B) = \underbrace{v - t(1 - x) - p_B}_{\text{Intrinsic utility}} - \underbrace{\lambda \cdot \text{Prob}[p = p_A](p_B - p_A)}_{\text{Loss from facing a higher } p \text{ than } p_A} - \underbrace{\lambda \cdot t \int_0^{1-x} ((1-x) - s) dG(s)}_{\text{Loss from facing larger distance than 0}} + \underbrace{t \int_{1-x}^1 (s - (1-x)) dG(s)}_{\text{Gain from facing smaller distance than 1}} \quad (3)$$

This allows us to solve consumer's personal equilibrium by determining the location of the indifferent uninformed consumer  $\hat{x}_{un}$ .

**Lemma 1.** *Suppose that  $\hat{x}_{un} \in [1/2, 1)$ . Then  $\hat{x}_{un}$  is given by*

$$\hat{x}_{un}(\Delta p) = \frac{\lambda}{(\lambda - 1)} - \frac{\Delta p}{4t} - \underbrace{\sqrt{\frac{\Delta p^2}{16t^2} - \frac{(\lambda + 2)}{2t(\lambda - 1)} \Delta p + \frac{(\lambda + 1)^2}{4(\lambda - 1)^2}}}_{\equiv S(\Delta p)}. \quad (4)$$

where  $\Delta p \equiv p_B - p_A$ .<sup>15</sup>

<sup>14</sup>The indifferent uninformed consumer will be located at  $x = \hat{x}_{un}$ , therefore  $(1 - \hat{x}_{un}, 1]$  is the relevant interval for determining  $\hat{x}_{un}$ .

<sup>15</sup>For  $x \in [0, 1]$  consumer's personal equilibrium is described by

$$\sigma(x, \Delta p) = \begin{cases} A & \text{if } x \in [0, \hat{x}_{un}(\Delta p)], \\ B & \text{if } x \in (\hat{x}_{un}(\Delta p), 1]. \end{cases}$$

We relegate the proof of this lemma to Appendix A.1.

The square root,  $S(\Delta p)$ , is defined for  $\Delta p \in [0, \Delta \bar{p}]$  with

$$\Delta \bar{p} \equiv \frac{2t}{(\lambda - 1)} \left( 2(\lambda + 2) - \sqrt{(2(\lambda + 2))^2 - (\lambda + 1)^2} \right), \quad (5)$$

which is strictly positive for all  $\lambda > 1$ . It can be shown that for  $\lambda \geq 3 + 2\sqrt{5} \approx 7.47$ ,  $\hat{x}_{un}(\Delta p) \in [1/2, 1]$  for all  $\Delta p \in [0, \Delta \bar{p}]$ . If the degree of loss aversion is smaller,  $\lambda < 3 + 2\sqrt{5}$ ,  $\hat{x}_{un}(\Delta \bar{p})$  rises above one. Therefore, we have to define another upper bound on the price difference,  $\Delta \tilde{p}$ , with  $\Delta \tilde{p} < \Delta \bar{p}$  by the solution to  $\hat{x}_{un}(\Delta \tilde{p}) = 1$ . We can solve explicitly,

$$\Delta \tilde{p} = \frac{(\lambda + 3)t}{2(\lambda + 1)}. \quad (6)$$

The upper bound for the price difference (which depends on the parameters  $t$  and  $\lambda$ ) for which  $\hat{x}_{un}$  is defined as in equation ((4)) is given by:

$$\Delta p^{max} \equiv \begin{cases} \Delta \tilde{p}, & \text{if } 1 < \lambda \leq \lambda^c; \\ \Delta \bar{p}, & \text{if } \lambda > \lambda^c. \end{cases} \quad (7)$$

with  $\lambda^c \equiv 3 + 2\sqrt{5} \approx 7.47$ .<sup>16</sup>

It can be shown that the derivative of  $\hat{x}_{un}(\Delta p)$  with respect to  $\Delta p$ ,  $\hat{x}'_{un}(\Delta p)$ , is strictly positive for all  $\Delta p \in [0, \Delta p^{max}]$ :

$$\hat{x}'_{un}(\Delta p) = -\frac{1}{4t} - \frac{1}{2 \cdot S(\Delta p)} \cdot \left( \frac{\Delta p}{8t^2} - \frac{(\lambda + 2)}{2t(\lambda - 1)} \right)$$

Evaluated at  $\Delta p = 0$ , this becomes

$$\hat{x}'_{un}(0) = -\frac{1}{4t} + \frac{(\lambda + 2)}{2t(\lambda + 1)}.$$

$\hat{x}'_{un}(0)$  is approaching  $1/(2t)$  from below for  $\lambda \rightarrow 1$  and  $1/(4t)$  from above for  $\lambda \rightarrow \infty$ . This implies that, evaluated at  $\Delta p = 0$ , demand of ex ante uninformed consumers reacts less sensitive to price changes than demand of ex ante informed consumers—we return to this property in the following section. Moreover,  $\hat{x}_{un}(\Delta p)$  is strictly convex for all

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<sup>16</sup>Note that  $\Delta \tilde{p} \in [t \cdot (\sqrt{5} - 1)/2, t) \approx [0.618t, t)$  for  $1 < \lambda \leq \lambda^c$  and  $\Delta \bar{p} \in (t \cdot 2(\sqrt{3} - 2), t \cdot (\sqrt{5} - 1)/2) \approx (0.536t, 0.618t)$  for  $\lambda > \lambda^c$ .

$\Delta p \in [0, \Delta p^{max}]$  (see Figure 1).

$$\hat{x}_{un}''(\Delta p) = \frac{(3 + \lambda)(5 + 3\lambda)}{64t^2 \cdot (S(\Delta p))^3} > 0$$

We note that the degree of convexity of  $\hat{x}_{un}(\Delta p)$  is strictly increasing in  $\lambda$ .

## 2.4 Properties of the demand of uninformed consumers

In this subsection, we establish a number of properties of the demand of uninformed consumers and compare it to the demand of informed consumers—i.e., we compare  $\hat{x}_{un}(\Delta p)$  and  $\hat{x}_{in}(\Delta p)$  with one another.

The first property is a continuity property. For  $\lambda \rightarrow 1$ , the indirect utility function of uninformed consumers differs from the one of informed consumers only by a constant (this can be called a level effect).<sup>17</sup> Equation (24) collapses to a linear equation and we obtain  $\hat{x}_{un}(\Delta p) = \hat{x}_{in}(\Delta p)$  as a solution in this case. This means that if consumers put equal weights on gains and losses, the effect of comparing expectations with realized values exactly cancels out when a choice between two products is made.

The next properties refer to the sensitivity of demand with respect to price. The first derivative of  $\hat{x}_{in}(\Delta p)$  with respect to  $\Delta p$  is equal to  $1/(2t)$  for all  $\Delta p$ . Therefore,  $\hat{x}_{in}'(0)$  is strictly larger than  $\hat{x}_{un}'(0)$ . This implies that the demand of uninformed consumers, evaluated at equal prices, reacts less sensitive to price changes than the demand of informed consumers.

Evaluated at large price differences, this relationship is possibly reversed: for  $\Delta p \rightarrow \Delta \bar{p}$  the square root,  $S(\Delta p)$ , turns to zero and  $\hat{x}_{un}'(\Delta p)$  turns to infinity. Thus,  $\hat{x}_{un}'(\Delta \bar{p}) > \hat{x}_{in}'(\Delta \bar{p}) = 1/(2t)$ . Demand of uninformed consumers, evaluated at a large price difference, reacts more sensitive to an increase in the price difference than the demand of informed consumers. (This property is satisfied if the indifferent consumer at this price difference is strictly interior; otherwise some more care is needed, as is done in the following section.

Since  $\hat{x}_{in}'(\Delta p)$  is constant and  $\hat{x}_{un}'(\Delta p)$  continuous and monotone (with the required boundary properties), applying the mean value theorem, there exists a unique intermediate price

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<sup>17</sup>This continuity property holds in our specification where the gain-loss utility in the price and in the match value dimension enter with equal weights. This does not hold more generally, see the next section.



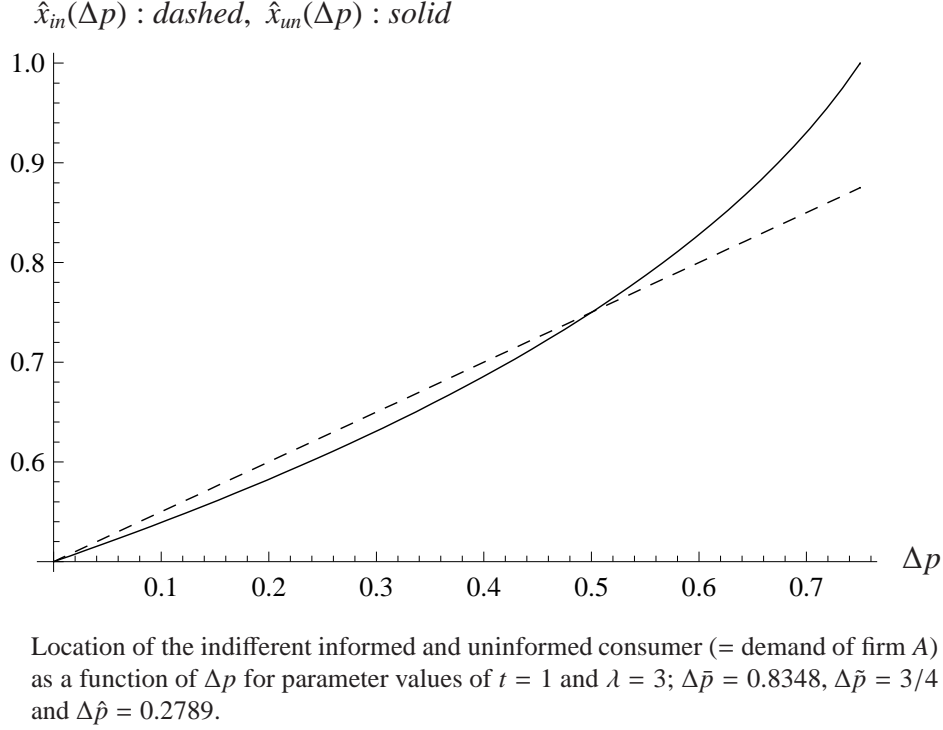


Figure 1: Demand of informed and uninformed consumers

difference  $\Delta \hat{p} \in [0, \Delta \bar{p}]$  such that  $\hat{x}'_{un}(\Delta \hat{p}) = \hat{x}'_{in}(\Delta \hat{p}) = 1/(2t)$ . This critical price difference can be explicitly calculated as

$$\Delta \hat{p} = \frac{t \left( 2\sqrt{2} \cdot (2(\lambda + 2)) - 3 \cdot \sqrt{(2(\lambda + 2))^2 - (\lambda + 1)^2} \right)}{\sqrt{2}(\lambda - 1)},$$

which is strictly positive for all  $\lambda > 1$  since  $\Delta \hat{p}(\lambda = 1) = 0$  and  $\Delta \hat{p}'(\lambda) > 0$ .

Hence, we find that *the demand of uninformed (or loss-averse) consumers is less price sensitive than the demand of informed consumers if the price difference is small,  $\Delta p < \Delta \hat{p}$* . The underlying intuition is that, for a small price difference, loss-averse consumers are harder to attract by price cuts because their gain from a lower price is outweighed by their loss in the taste dimension if they buy the other product. Thus, demand of loss-averse consumers reacts less sensitive to price in this range. For large price differences, however, their gain from lower prices starts to dominate their loss in the taste dimension if consumers switch to the cheaper producer. Therefore, *the demand of uninformed (or loss-averse) consumers is more price sensitive than the demand of informed consumers if the price difference is large,  $\Delta p > \Delta \hat{p}$* . More details on the competitive effects of the different dimensions of the gain-loss utility are provided in Section 4. In Section 5, we will see that this property is a driving force for our comparative static results in asymmetric markets.

### 3 Market Equilibrium

In this section, we characterize equilibrium candidates rearranging first-order conditions. In Appendix B, we provide conditions under which an interior equilibrium exists and under which it is unique. We start by establishing some properties of market demand which will be needed below.

#### 3.1 Properties of market demand

Using results from Section 2.4, we define the upper bound of firm  $A$ 's demand of uninformed consumers as<sup>18</sup>

$$\hat{x}_{un}(\Delta p^{max}) \equiv \begin{cases} \hat{x}_{un}(\Delta \bar{p}) = 1, & \text{if } 1 < \lambda \leq \lambda^c, \\ \hat{x}_{un}(\Delta \bar{p}) < 1, & \text{if } \lambda > \lambda^c. \end{cases} \quad (8)$$

Combining (1) and (4), we obtain the market demand of firm  $A$  as the weighted sum of the demand by informed and uninformed consumers,

$$\begin{aligned} q_A(\Delta p; \beta) &= \beta \cdot \hat{x}_{in}(\Delta p) + (1 - \beta) \cdot \begin{cases} \hat{x}_{un}(\Delta p), & \text{if } 0 \leq \Delta p < \Delta p^{max} \\ 1, & \text{if } t \geq \Delta p \geq \Delta p^{max} \end{cases} \\ &= \begin{cases} \frac{1}{2} - \frac{1}{4t}(1 - 3\beta)\Delta p + (1 - \beta)\frac{(\lambda+1)}{2(\lambda-1)} - (1 - \beta)S(\Delta p), & \text{if } 0 \leq \Delta p < \Delta p^{max} \\ \beta \cdot \frac{t+\Delta p}{2t} + (1 - \beta), & \text{if } t \geq \Delta p \geq \Delta p^{max} \end{cases} \\ &\equiv \begin{cases} \phi(\Delta p; \beta), & \text{if } 0 \leq \Delta p < \Delta p^{max} \\ \beta \cdot \frac{t+\Delta p}{2t} + (1 - \beta), & \text{if } t \geq \Delta p \geq \Delta p^{max}. \end{cases} \end{aligned} \quad (9)$$

The demand of firm  $A$  is a function in the price difference  $\Delta p$ , which is kinked at  $\Delta p^{max}$  and, for  $\Delta p^{max} = \Delta \bar{p}$ , discontinuous at  $\Delta p^{max}$ . It approaches one for  $\Delta p = t$ .<sup>19</sup> Firm  $B$ 's demand is determined analogously by  $q_B(\Delta p; \beta) = 1 - q_A(\Delta p; \beta)$ . In the following we are interested in interior equilibria in which both products are purchased by a strictly positive share of uninformed consumers—i.e.,  $\Delta p$  is lower than  $\Delta p^{max}$ .<sup>20</sup> We next state properties

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<sup>18</sup>  $\hat{x}_{un}(\Delta \bar{p}) = \frac{\lambda}{\lambda-1} - \frac{2(\lambda+2) - \sqrt{4(\lambda+2)^2 - (\lambda+1)^2}}{2(\lambda-1)} \in (\sqrt{3}/2, 1)$  for  $\lambda > \lambda^c$ ,

i.e.  $\hat{x}_{un}(\Delta \bar{p})$  is lower than one for  $\lambda > \lambda^c$ . This leads to a jump in demand of uninformed consumers at  $\Delta \bar{p}$  from  $\hat{x}_{un}(\Delta \bar{p})$  to one (see the definition of  $q_A(\Delta p; \beta)$ ), as  $\hat{x}'_{un}(\Delta \bar{p}) \rightarrow \infty$ .

<sup>19</sup> At  $\Delta p = t$  firm  $A$  serves also all distant informed consumers which are harder to attract than distant uninformed consumers because the former do not face a loss in the price dimension if buying from the more expensive firm  $B$ . For  $\Delta p > t$  demand of firm  $A$  shows a second kink. We ignore this region since we are interested in cases in which both firms face strictly positive demand.

<sup>20</sup> This corresponds to industries in which firms are not too asymmetric.

of  $\phi(\Delta p; \beta)$ , the demand of firm  $A$  in this case:<sup>21</sup>

**Lemma 2.** *For  $0 \leq \Delta p < \Delta p^{max}$ , the demand of firm  $A$ ,  $q_A(\Delta p; \beta) = \phi(\Delta p; \beta)$  is strictly increasing and convex in  $\Delta p$ .*

The proof is relegated to the appendix.

The derivative of the demand of  $A$  with respect to  $\beta$  is the difference of the demand of informed and uninformed consumers:

$$\frac{\partial \phi(\Delta p; \beta)}{\partial \beta} \equiv \phi_\beta = \hat{x}_{in}(\Delta p) - \hat{x}_{un}(\Delta p) = \frac{3}{4t}\Delta p - \frac{\lambda + 1}{2(\lambda - 1)} + S(\Delta p) \geq 0$$

with  $\phi_\beta = 0$  at  $\Delta p = 0$  and  $\Delta p = t/2$ . This derivative can be of positive or negative sign. We note that also the third derivative,  $\phi'''$ , is greater than zero. We also note that cross derivative of the demand of  $A$  w.r.t.  $\Delta p$  and  $\beta$ ,

$$\frac{\partial \phi'}{\partial \beta} \equiv \phi'_\beta = \hat{x}'_{in}(\Delta p) - \hat{x}'_{un}(\Delta p) = \frac{3}{4t} + \frac{1}{2S(\Delta p)} \cdot \left( \frac{\Delta p}{8t^2} - \frac{(\lambda + 2)}{2t(\lambda - 1)} \right),$$

is of ambiguous sign. This derivative has the boundary behavior that  $\phi'_\beta = 0$  at  $\Delta \hat{p}$ . and  $\phi'_\beta \rightarrow \infty$  for  $\Delta p = \Delta \bar{p}$ ; the latter holds because  $S(\Delta \bar{p}) = 0$ .

### 3.2 Equilibrium characterization

We next turn to the equilibrium characterization. At the first stage, firms foresee consumers' purchase decisions and set prices simultaneously to maximize profits. This yields the first-order conditions:

$$\frac{\partial \pi_i}{\partial p_i} = q_i + (p_i - c_i) \frac{\partial q_i}{\partial p_i} = 0, \quad i = A, B$$

If the solution has the feature that demand of each group of consumers, informed and uninformed, is positive, first-order conditions can be written as

$$\begin{aligned} \frac{\partial \pi_A}{\partial p_A} &= \phi - (p_A - c_A)\phi' = 0 & (FOC_A) \\ \frac{\partial \pi_B}{\partial p_B} &= (1 - \phi) - (p_B - c_B)\phi' = 0. & (FOC_B) \end{aligned}$$

We refer to a solution characterized by these first-order conditions as an interior solution.

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<sup>21</sup>We will use  $\phi$  as a short-hand notation for  $\phi(\Delta p; \beta)$ .

We will discuss the issue of sufficiency of first-order conditions as well as the issue of non-interior solutions and non-existence in Appendix B. The proof of equilibrium existence is non-standard since the profit function of the low-cost firm is not quasi-concave. We now turn to the characterization of interior solutions and denote an equilibrium with prices  $(p_A^*, p_B^*)$  that is determined by an interior solution as an interior equilibrium.

**Lemma 3.** *In an interior equilibrium with equilibrium prices  $(p_A^*, p_B^*)$ , the price difference  $\Delta p^* = p_B^* - p_A^*$  satisfies*

$$\Delta p^* = \Delta c + f(\Delta p^*; \beta) \quad \forall \beta \in [0, 1], \Delta p \text{ feasible}, \quad (10)$$

with  $\Delta c = c_B - c_A$  and  $f(\Delta p; \beta) = (1 - 2\phi)/\phi'$ .

*Proof.* Combining  $(FOC_A)$  and  $(FOC_B)$  yields the required equilibrium condition as a function of price differences.  $\square$

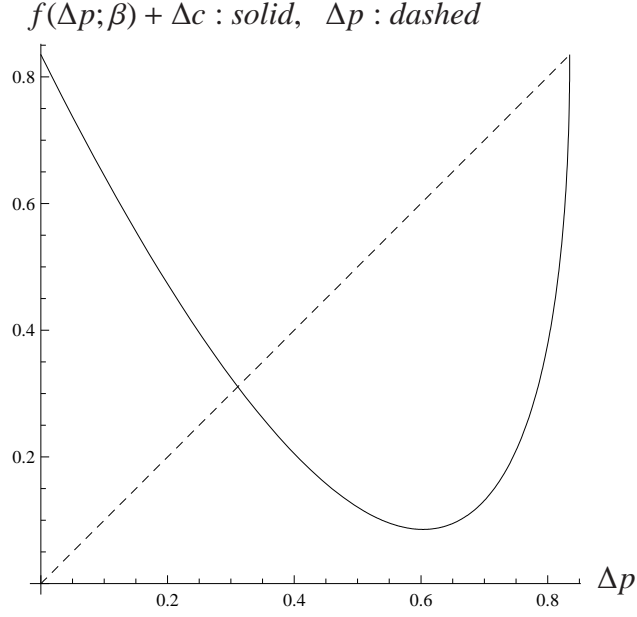
Thus, (10) implicitly defines the equilibrium price difference  $\Delta p^*$  as a function of the parameters  $\Delta c, \beta, \lambda$ , and  $t$ , where the latter two parameters affect the functional form of  $f$  via  $\phi$ .<sup>22</sup>

## 4 Competition in Symmetric Markets

In this section, we first characterize the equilibrium in symmetric duopoly and disentangle pro- and anti-competitive effects of the presence of ex ante uninformed loss-averse consumers. We relate our findings on the competitive effects of consumer loss aversion to a setting in which consumers form reference point before the firms have set their prices, as is also the case in Heidhues and Koszegi (2008). We also analyze an extension to an  $n$ -firm oligopoly.

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<sup>22</sup>Anderson and Renault (2009) face a similar fixed point problem as in (10). They consider a general differentiated product Bertrand duopoly with covered markets in which asymmetries arise due to quality differences between firms. The authors show uniqueness and existence of a pure-strategy price equilibrium under the assumption of strict log-concavity of firms' demand. Although strict log-concavity allows for some convexity of demand, in our setup this assumption is not met since for large price differences the convexity of the low-price firm's demand rises above any bound, i.e.  $\phi'' \rightarrow \infty$  for  $\Delta p \rightarrow \Delta \bar{p}$ .



Equilibrium condition (10) at  $\Delta c = \Delta \bar{p}$  for parameter values of  $\beta = 0$ ,  $t = 1$ , and  $\lambda = 3$ :  $\Delta \tilde{p} = 0.75$ ,  $\Delta \bar{p} = 0.8348$ .

Figure 2: Two potential interior equilibria

#### 4.1 Pro- and anti-competitive effects of consumer loss aversion

In contrast to Heidhues and Koszegi (2008), our framework allows us to explicitly solve for equilibrium markup in symmetric duopoly. The following result characterizes the symmetric equilibrium.

**Lemma 4.** *For  $\Delta c = 0$ , any equilibrium is unique and symmetric. Equilibrium prices are given by*

$$p_i^* = c_i + \frac{t}{1 - \frac{(1-\beta)(\lambda-1)}{2(\lambda+1)}}, i = A, B. \quad (11)$$

*Proof.* For  $\Delta c = 0$ , using equations (10) and (30), and the fact that  $f(0; \beta) = 0$ , we obtain that  $\Delta p^*(\beta) = 0$  is the unique equilibrium  $\forall \beta \in [0, 1]$  (provided it exists).<sup>23</sup> Rearranging the first-order conditions ( $FOC_i$ ) that are reported in Appendix B and using that  $\phi(0, \beta) = 1/2$  for all  $\beta$ , we obtain

$$p_i^* - c_i = \frac{\frac{1}{2}}{\phi'(0; \beta)} \quad \forall i \in \{A, B\}, \quad (12)$$

<sup>23</sup>We show in Appendix B that equilibrium existence implies uniqueness.

where

$$\begin{aligned}
\phi'(0; \beta) &= -\frac{1}{4t}(1 - 3\beta) - \frac{(1 - \beta)}{2(S(0))} \left( 0 - \frac{(\lambda + 2)}{2t(\lambda - 1)} \right) \\
&= -\frac{1}{4t}(1 - 3\beta) + \frac{(1 - \beta)}{2 \frac{\lambda+1}{2(\lambda-1)}} \left( \frac{(\lambda + 2)}{2t(\lambda - 1)} \right) \\
&= -\frac{1}{4t}(1 - 3\beta) + \frac{(1 - \beta)(\lambda + 2)}{2t(\lambda + 1)} \\
&= \frac{1}{4t(\lambda + 1)} \left( 2(\lambda + 1) - (1 - \beta)(\lambda - 1) \right).
\end{aligned}$$

Substituting into equation (12) yields (11).  $\square$

For  $\Delta p^*(\beta) = 0$ , on the equilibrium path, consumers do not experience gains and losses in the price dimension. In this situation uninformed consumers exclusively experience gains and losses in the taste dimension. Due to the potential loss in the taste dimension uninformed consumers are more willing to buy next door than informed consumers—i.e., they are harder to attract by low prices than their informed counterparts and, thus, the demand of uninformed consumers shows a lower price elasticity of demand. While this suggests consumer loss aversion has an anti-competitive effect, a correct understanding is more nuanced.

To this end, we have to disentangle various effects at play; we consider it useful to analyze the more general model in which consumers experience gain-loss utilities in the two dimensions, the price and the match value dimension, with different weights.<sup>24</sup> As one polar case we consider markets in which all consumers are uninformed ex ante and experience loss-aversion in the price dimension only. As the other polar case we consider markets in which all consumers are uninformed ex ante and experience loss-aversion in the match-value dimension only.

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<sup>24</sup>We introduce weights  $\alpha_p, \alpha_m \geq 0$  for the price and match value dimension of loss aversion. This means that the indirect gain/loss utility of buying product A in (2) adjusts to

$$\begin{aligned}
u_A(x, p_A, p_B; \alpha_p, \alpha_m) &= (v - tx - p_A) \\
&\quad + \alpha_p \cdot \left( -\lambda \cdot \text{Prob}[p = p_A](p_A - p_A) + \text{Prob}[p = p_B](p_B - p_A) \right) \\
&\quad + \alpha_m \cdot \left( -\lambda \cdot t \int_0^x (x - s) dG(s) + t \int_x^1 (s - x) dG(s) \right).
\end{aligned}$$

First consider the case that consumers experience a gain-loss utility in the price dimension only. Since gains relative to the expected price distribution enter positively and losses negatively the utility function, consumers find lower-priced products relatively more attractive than higher-priced products. Consequently, the price elasticity of demand is larger and the equilibrium is *more* competitive than in the standard Hotelling-Salop model. Formally, the unique symmetric equilibrium (if it exists) is characterized by  $p_p^* = c + 2t/(3 + \lambda)$ .<sup>25</sup> This proves that a gain-loss utility in the price dimension has an anti-competitive effect.<sup>26</sup> To obtain a better understanding, we take a closer look at ex ante uninformed consumers. Consider a small price decrease by firm A. Consumers observe the corresponding prices. They expect with some probability  $\hat{x}_p$  to end up buying the low-price firm. Hence, they have an expected gain of  $\hat{x}_p(p_B - p_A)$  when consuming product A and an expected loss of  $\lambda(1 - \hat{x}_p)(p_B - p_A)$  when consuming product B. This means that a price decrease yields a stronger utility difference in favor of the low-price product. This increases the price elasticity of demand and, everything else given, makes a price cut more attractive. The presence of ex ante uninformed consumers leads to a downward shift of best-response functions. Consequently (for best-response functions being upward sloping), the equilibrium is more competitive than in the standard Hotelling-Salop model.

Second consider the case that consumers experience a gain-loss utility in the match value dimension only. Comparing a market with ex ante uninformed to a market with ex ante informed consumers reveals that competition is less intense if consumers are ex ante uninformed. Straightforward computations show that  $p_m^* = c + t(1 + \lambda)$  which leads to a less competitive outcome than in the standard Hotelling-Salop model.<sup>27</sup> A price decrease for firm A implies that consumers are more likely to buy from firm A than firm B. This implies that the marginal consumer more often encounters a worse match from firm A. Since relatively bad matches enter negatively the gain-loss utility, the price elasticity of demand is lower and best-response function are shifted upward. Effectively, competition is less intense compared to the model with ex ante informed consumers.

The following remark summarizes the insights obtained above.

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<sup>25</sup>In general, as an extension of equation (11), the symmetric equilibrium prices are given by

$$p_i^* = c_i + \frac{t}{1 - \frac{(1-\beta)}{2} \frac{(2\alpha_m\lambda - \alpha_p(\lambda+1))}{(\alpha_m\lambda+1)}}, i = A, B,$$

provided an equilibrium exists.

<sup>26</sup>Note that this result does not rely on losses entering the utility function with a different weight than gains.

<sup>27</sup>Again we note that this result does not rely on losses entering the utility function with a different weight than gains.



**Remark 1.** *If consumers experience a gain-loss utility in the price dimension only, markets with ex ante uninformed consumer are more competitive than markets with ex ante informed consumers. By contrast, if consumers experience a gain-loss utility in the match-value dimension only, markets with ex ante uninformed consumers are less competitive than markets with ex ante informed consumers.*

This insight holds more generally; in particular, it does not rely on the assumption that taste parameters are uniformly distributed and that utility depends linearly on match value, defined as the distance between consumer and product. These assumptions are mainly made for computational reasons.

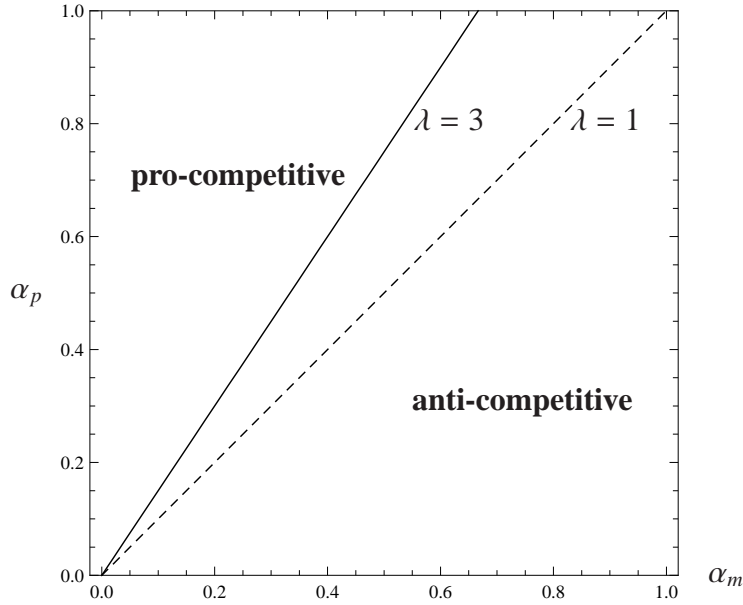


Figure 3: Pro- and anti-competitive effects of loss aversion

Let us now consider intermediate cases between the two polar cases. In our model, both dimensions entered with equal weights. For this case we obtain that the taste dimension dominates the price dimension (as follows from equation (11)) if consumers are loss-averse.

**Remark 2.** *Suppose that losses have a larger weight than gains and that the gain-loss utility enter with the same weights in the price and the match value dimension. Markets with ex ante uninformed consumers are less competitive than markets with ex ante informed consumers.*

Depending on the degree of loss aversion  $\lambda$ , there is a critical relationship of gains and losses in the price dimension relative to the match value dimension such that pro-competitive

and anti-competitive effects cancel out each other. This critical relationship is described by

$$\tilde{\alpha}_p(\alpha_m; \lambda) = \frac{2\lambda}{\lambda + 1} \cdot \alpha_m, \quad (13)$$

which turns out to be simply a ratio of weights on the price and match value dimension  $\alpha_p$  and  $\alpha_m$  for given  $\lambda$ .<sup>28</sup> This ratio is depicted in Figure 3. It shows the competitiveness of symmetric price equilibria (relative to the benchmark with ex ante informed consumers) for different weights in the two dimensions of loss aversion. It can be seen that for any positive degree of loss aversion ( $\lambda > 1$ ), markets are anti-competitive if weights are identical on the price and match value dimension. If the degree of loss aversion is increased, a relatively higher weight on the price dimension is required to balance the anti- and the pro-competitive effect. The figure reveals that even when gains and losses are weighted equally ( $\lambda = 1$ ; so that the utility function features reference dependence but not loss aversion), markets become anti- (resp. pro-) competitive if reference-dependent consumers, for a certain product category, put a relatively higher (resp. lower) weight on the match value dimension than on the price dimension.

## 4.2 Comparative statics in the symmetric duopoly model

In this subsection we focus on the case of two-dimensional consumer loss aversion. The following comparative static result states that, as the share of informed consumers increases, the firms' markup decreases. This result follows directly from differentiating (11) with respect to  $\beta$ .

**Proposition 1.** *For  $\Delta c = 0$  and  $\lambda > 1$ , equilibrium markup is decreasing in the share of informed consumers  $\beta$ .*

In other words, informed consumers exert a positive externality on uninformed consumers. This prediction is in line with alternative models from the search literature, where a larger share of consumers who do not know some products exert a negative externality on those who do. Nevertheless our framework is substantially different since all consumers are fully informed at the moment of purchase. Here, an externality arises due

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<sup>28</sup>This critical ratio can be derived by setting the symmetric equilibrium price with gain/loss utility and flexible weights in the both dimension of loss aversion equal to the symmetric equilibrium price with intrinsic utility only. The symmetric equilibrium price in the latter case is given by  $p_i^* = c_i + t$ , the standard Hotelling result.

to uncertainty at the moment consumers form their reference points. With respect to recent work with behavioral biases, our result is of interest in the light of claims that better informed consumers are cross-subsidized at the cost of less informed consumers. This, for instance, holds in Gabaix and Laibson (2006) where only a fraction of consumers are knowledgeable about their future demand of an “add-on service”, while other consumers are “naively” unaware of this. This shows that the particular type of behavioral bias is central to understand the competitive effect of changes in the composition of the consumer population.

The result implies that firms do not have an incentive to inform consumers at an early stage. However, there is a potential role of public authorities to inform consumers about their match value at an early point in time so that all uncertainty is resolved early on. This increases competitive pressure and thus lead to higher consumer surplus. As we already pointed out in the introduction, it is not required that public authorities aim at eliminating the behavioral bias directly (and thus to manipulate consumer preferences) but rather to disclose information at an early stage. This neutralizes the behavioral bias (but does not change the consumers’ utility function). This insight provides a novel rational for information disclosure by public authorities due to behavioral biases in the consumer population.

Two additional comparative static results follow immediately from Lemma 4. First, equilibrium markup is increasing in the degree of loss aversion,  $\lambda$ . For  $\lambda \rightarrow 1$  firms receive the standard Hotelling markup of  $t$ . Secondly, equilibrium markup is increasing in the inverse measure of industry competitiveness,  $t$ . For  $t \rightarrow 0$  firms face full Bertrand competition and markups converge zero for all levels of loss aversion. This shows that consumer loss aversion does not affect market outcomes in perfectly competitive environments and our results rely on the interaction of imperfect competition and behavioral bias. The second and third comparative statics results are rather obvious but still noteworthy.

Table 1 shows the variation of equilibrium markups in the share of informed consumers  $\beta$  and the degree of loss aversion  $\lambda$  for fully symmetric markets ( $\Delta c = 0$ ). We make the following observations: (1) The highest markup is reached when all consumers are uninformed and the degree of loss-aversion approaches its critical level for existence in symmetric markets  $\lambda = 1 + 2\sqrt{2} \approx 3.82843$ —compare Figure 10. (2) If the share of informed consumers is sufficiently large (above 57.7%) symmetric equilibria exist for all  $\lambda > 1$ . With such a large share of informed consumers the equilibrium markup is below its maximum level since the demand of informed consumers is more elastic and thus dampens the firms’ incentives to set higher prices.

Table 1: Symmetric Equilibrium: Equilibrium Markups

The table shows the variation of  $m_i^*(\Delta c = 0, \beta, \lambda) \equiv p_i^*(\Delta c = 0, \beta, \lambda) - c_i$  for all  $i \in \{A, B\}$  in  $\beta$  and  $\lambda$ .

$\beta$	$\lambda$	1	2	3	3.8284	5	7	9	$\infty$
1	1	1	1	1	1	1	1	1	1
0.8	1	1.03448	1.05263	1.06222	1.07143	1.08108	1.08696	1.11111	
0.6	1	1.07143	1.11111	1.1327	1.15385	1.17647	1.19048	1.25	
0.4	1	1.11111	1.17647	1.2132	1.25	1.29032	1.31579	-	
0.2	1	1.15385	1.25	1.30602	1.36364	-	-	-	
0	1	1.2	1.33333	1.41421	-	-	-	-	

### 4.3 Comparison to a model in which price information is not available ex ante

In this subsection, we discuss the outcome of the modified model in which consumers do not observe prices before forming their reference point. This is the limit case of the Heidhues-Koszegi model in which the cost uncertainty has vanished. Since consumers do not observe prices when forming their reference point, deviations from the equilibrium thus do not allow firms to manipulate consumer reference points. To simplify the analysis we set the share of ex ante informed consumer equal to zero,  $\beta = 0$ .

If consumers are loss-averse only in the price dimension, there is a continuum of equilibria: any price in the interval  $[c + t/(\lambda + 1), c + t/2]$  for all  $\lambda > 1$ . The unique equilibrium price in the setting in which prices are observed ex ante lies within this interval. We note that a market with ex ante uninformed, reference-dependent consumers features a more competitive price under both informational assumptions than a market in which consumers are informed ex ante. Also note that, for  $\lambda = 1$  the equilibrium under both informational assumption is the same and  $p^* = c + t/2$ .

If consumers are loss-averse only in the match-value dimension, there is a unique equilibrium  $p^* = c + t(\lambda + 1)$ . This price is the same that prevails if consumers learn prices before the reference point is formed and thus the timing of the price setting is immaterial to the outcome. The reason is that a local price deviation has only a second-order effect that is induced by consumer loss aversion; the price elasticity of demand remains locally unaffected so that we obtain the same solution to the system of first-order conditions of profit maximization.

If consumers are loss-averse in both dimensions, any price in the interval  $[c + t/(\lambda +$

1),  $c + (t/2)(\lambda + 1)$ ] constitutes an equilibrium. The unique equilibrium price in the setting in which prices are observed ex ante lies within this interval.

#### 4.4 Extension to $n$ -firm oligopoly and comparative statics in the number of firms

In this subsection, we provide an extension to an  $n$ -firm oligopoly. Suppose that the length of the circle is  $L = N$  (while the consumer mass is equal to 1); this implies that the equilibrium markup in the model with ex ante informed consumers (whose analysis coincides with the standard Salop model) are independent in the number of firms. As we will show in this subsection, this does not hold in the model with ex ante uninformed consumers. The intuition for this non-neutrality result is straightforward. In the duopoly model, consumers expect it to be likely to be affected by a price deviation and thus adjust their reference point distribution accordingly, while, given a large number of firms, the reference point distribution reacts very little to a deviation from the equilibrium strategy.

First, consider the case that consumers are loss-averse only in the price dimension—i.e.,  $\alpha_m = 0, \alpha_p = 1$ . The equilibrium markup is

$$m_p^*(n) = \frac{nt}{(\lambda - 1) + 2n}, \quad (14)$$

as illustrated in Figure 4. We find that the equilibrium markup is increasing in the number of firms. This confirms the general insight that, given a larger number of firms, the reference point distribution reacts less sensitive to individual price deviations. For  $n \rightarrow \infty$ , this markup converges to  $m_p^*(\infty) = t/2$ . Note that this is the upper bound on prices in the duopoly setting in which consumers form their reference point distribution before observing prices.<sup>29</sup>

Second, consider the case that consumers are loss-averse only in the match-value dimension—i.e.,  $\alpha_m = 1, \alpha_p = 0$ . The equilibrium markup is  $m_m^*(n) = (\lambda + 1)t$  and is independent of  $n$ .

Third, consider the case that consumers are loss-averse in both dimensions—i.e.,  $\alpha_m =$

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<sup>29</sup>Note that the set of equilibrium prices is not affected by the number of firms if consumers do not observe prices ex ante.

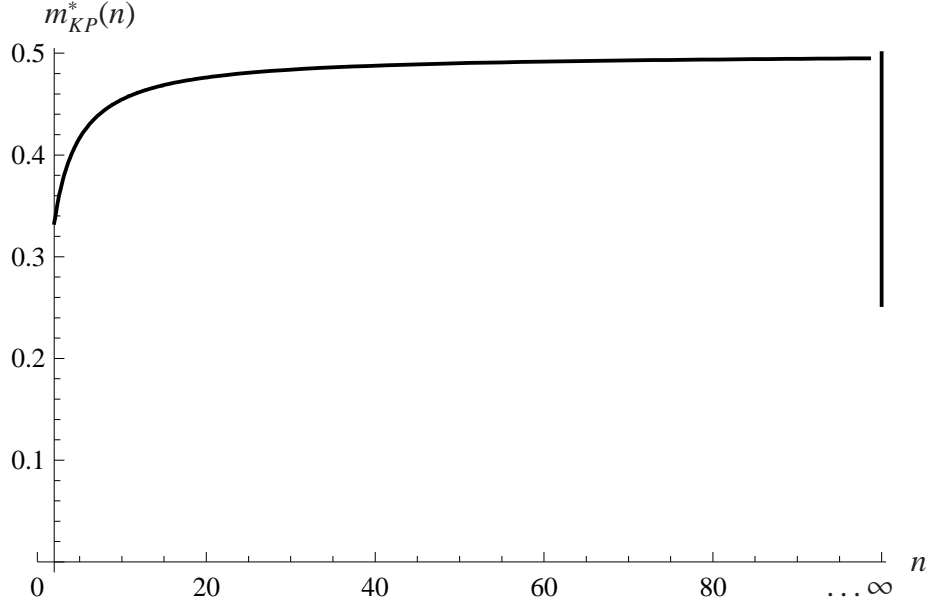


Figure 4: Markups with loss aversion in price dimension

1,  $\alpha_p = 1$ . The equilibrium markup is

$$m^*(n) = \frac{(\lambda + 1)nt}{(\lambda - 1) + 2n} \quad (15)$$

This is plotted in Figure 5. Again, we find that the equilibrium price is increasing in the number of firms because the reference price distribution reacts less sensitive to a price change after an increase of the number of firms. For  $n \rightarrow \infty$ , this markup converges to  $m^* = \lambda + 1t/2$ . Note that this is the upper bound on prices in the duopoly setting in which consumers form their reference point distribution before observing prices.

## 5 Cost Asymmetries

In this section, we take a first look at comparative statics properties of the asymmetric duopoly. Here we focus on the degree of cost asymmetry, i.e. the level of  $\Delta c = c_B - c_A$ .

**Proposition 2.** *The equilibrium price difference  $\Delta p^*(\Delta c, \beta)$  is an increasing function of the cost asymmetry between firms  $\Delta c$ . Moreover, the price difference reacts more sensitive to  $\Delta p$  than in a market in which all consumers are informed ex ante,  $d(\Delta p^*)/d(\Delta c) \geq 1/3$ .*

The proof of this proposition is presented in Appendix A.

This result says that the more pronounced the cost asymmetry the larger the price difference between high-cost and low-cost firm. This result shows that standard comparative

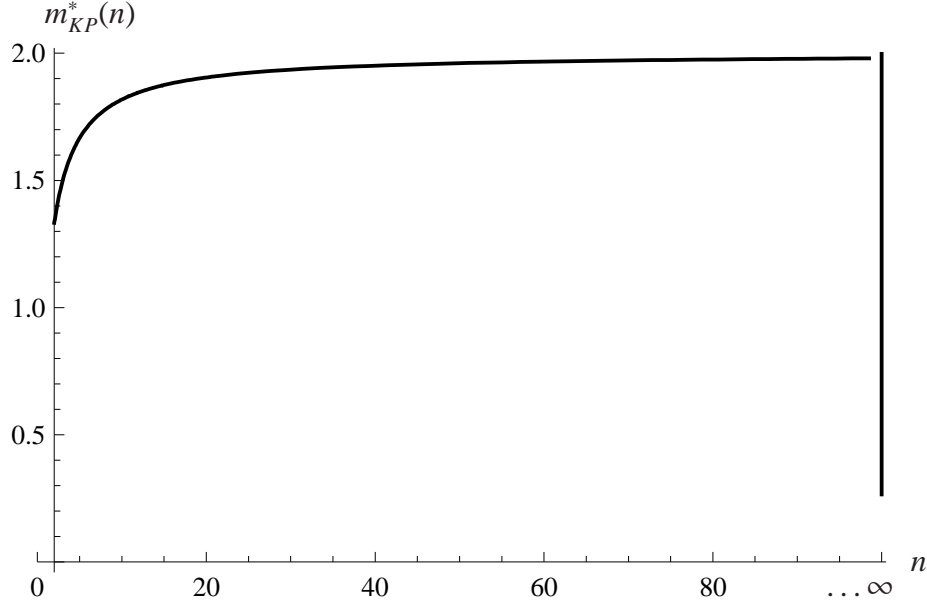


Figure 5: Markups with loss aversion in both dimensions

statics result with respect to cost difference are qualitatively robust to consumers being loss averse. However, in our model the marginal effect of an increase in cost differences on price variation is stronger if some consumers are loss averse—recall that a market in which all consumers are informed is observationally equivalent to the standard Hotelling case. Our model predicts exacerbated price variation in markets with uninformed loss-averse consumers.

This is different in spirit to Heidhues and Koszegi (2008) who found that price variation is reduced in markets with loss-averse consumers.

We next take a look at the individual prices set by the two firms. For comparative statics we use markups  $m_i^* \equiv p_i^* - c_i$ ,  $i \in \{A, B\}$  instead of prices because markups are net of individual costs and depend solely on cost differences.<sup>30</sup> Alternatively, we could use individual prices and consider a change of the rival's costs only.

First, we observe that the low-cost firm's markup is increasing or decreasing depending on the degree of market asymmetries (=cost differences) and the share of uninformed consumers in the market.

**Proposition 3.** *For  $\beta < 1$  and  $\lambda > 1$ , the equilibrium markup charged by the low-cost firm  $m_A^*(\Delta c) \equiv p_A^*(\Delta c, c_A) - c_A$  is either first increasing and then decreasing in the cost difference if the share of informed consumers  $\beta$  is high, or always monotonously*

<sup>30</sup>This follows directly from firms' first-order conditions.  $\Delta c$  affects  $p_i - c_i = \phi(\Delta p)/\phi'(\Delta p)$  via  $\Delta p$ .



decreasing if  $\beta$  is sufficiently low. For  $\beta = 1$  or  $\lambda \rightarrow 1$ ,  $m_A^*(\Delta c)$  is always monotonously increasing.

In the latter case—i.e., for  $\beta = 1$  or  $\lambda \rightarrow 1$ —when all consumers are informed or the behavioral bias vanishes we obtain the standard Hotelling result that the low-cost firm faces a larger markup in more asymmetric markets.

Note that, for  $\beta = 1$ ,  $dm_A^*/d(\Delta c)$  equals  $1/3$ . This, in particular, means that in the standard Hotelling world without behavioral biases ( $\beta = 1$ ) the markup of the more efficient firm is increasing in the cost difference. Thus, the proposition shows that a local increase of the cost difference may have the reverse effect under consumer loss aversion ( $\beta < 1$ ,  $\lambda > 1$ ).

This hold in strongly asymmetric markets since the price sensitivity of demand becomes larger than in the standard Hotelling world due to the dominating loss in the price dimension. Under very large cost differences firm A's markup might fall even below its level in the standard Hotelling world (compare Figure 6).

Second, we consider the markup of firm B.

**Proposition 4.** *The equilibrium markup charged by the high-cost firm  $m_B^*(\Delta c) \equiv p_B^*(\Delta c, c_B) - c_B$  is always decreasing in the cost difference.*

*Proof.*

$$\frac{dm_B^*(\Delta p^*(\Delta c))}{d(\Delta c)} = \frac{\partial m_B^*}{\partial(\Delta p^*)} \cdot \frac{\partial(\Delta p^*)}{\partial \Delta c},$$

where by  $(FOC_B)$

$$\frac{\partial m_B^*}{\partial(\Delta p^*)} = \frac{\partial p_B^*}{\partial(\Delta p^*)} = \frac{-(\phi')^2 - \phi'' \cdot (1 - \phi)}{(\phi')^2} < 0, \quad (16)$$

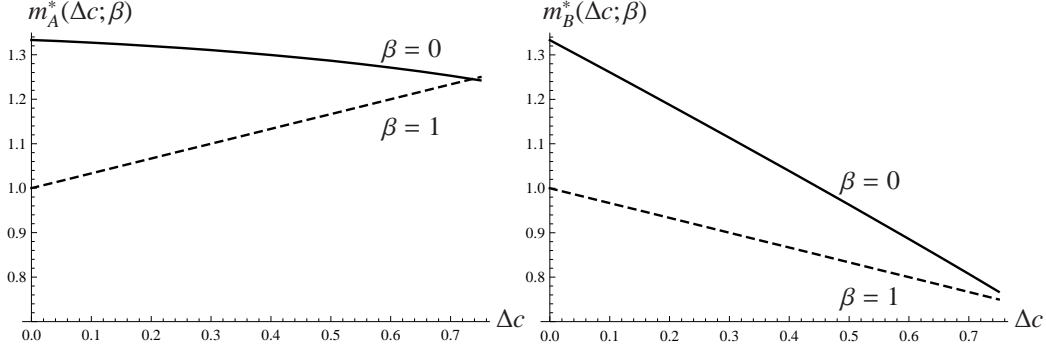
which is always negative for all  $\beta$ . Using (25) we obtain that

$$\frac{dm_B^*(\Delta p^*(\Delta c))}{d(\Delta c)} = -\frac{(\phi')^2 + \phi'' \cdot (1 - \phi)}{3(\phi')^2 + \phi''(1 - 2\phi)} < 0. \quad (17)$$

□

Note that for  $\beta = 1$ ,  $dm_B^*/d(\Delta c)$  is equal to  $-1/3$ . As it turns out, the qualitative finding that the equilibrium markup of the high-cost firm is decreasing in the cost difference is preserved under consumer loss aversion. Due to a level effect of high markups we find

that firm  $B$ 's markup is decreasing more strongly than in the standard Hotelling world without behavioral bias. However, the critical market asymmetry for which its markup drops below its Hotelling level has to be larger than for firm  $A$ . This is illustrated in Figure 6.



Equilibrium markups of firm  $A$  and  $B$  for markets in which either all consumers are uninformed ( $\beta = 0$ ) or informed (=benchmark case,  $\beta = 1$ ) as a function of cost differences  $\Delta c$  for parameter values of  $t = 1$  and  $\lambda = 3$ :  $\Delta c^{nd}(\beta = 0) = 0.75963$ .

Figure 6: Equilibrium markup of both firms

## 6 The Role of Information

In this section, we focus on comparative statics results with respect to  $\beta$ , the share of initially informed consumers. In other words, we investigate the effects of ex ante information disclosure on market outcomes in asymmetric markets. This allows us to provide a new perspective on information disclosure policies by public authorities and firms. In contrast to standard work on information disclosure policies, in our theory consumers are fully informed at the moment of purchase, independent of whether or not there is any information disclosure. Our theory hints at the role of timing of information disclosure. With respect to voluntary disclosure, we provide new insights into the firms' advertising and marketing activities.

### 6.1 The effect of ex ante information on prices and quantities

Our first result concerns the equilibrium price difference.

**Proposition 5.** *The equilibrium price difference  $\Delta p^*(\beta)$  is decreasing in  $\beta$ .*

The proofs of Section 6 are relegated to the Appendix.

The above proposition says that prices become more equal as the share of initially informed consumers increases, or, in other words, as the population average becomes less loss-averse

Put differently, more loss-averse consumers lead to larger price differences. This is in stark contrast to one of the main findings in Heidhues and Koszegi (2008) who show that, in their setting, consumers loss aversion is a rationale for focal prices compared to a setting without behavioral biases. In the latter firms would condition prices on their marginal costs. Using our terminology they compare a setting with mass 1 of uninformed consumers, i.e.,  $\beta = 0$ , to a setting with mass 0 of uninformed consumers, which corresponds to a world without behavioral bias. Their message is that consumer loss aversion tends to lead to (more) equal prices; our finding, by contrast, says that consumer loss aversion leads to larger price differences of asymmetric firms.

Let us now look at the individual prices set by the two firms. We first observe that the low-cost firm's price is monotone or inverse U-shaped in  $\beta$ , depending on the parameter constellation.

**Proposition 6.** *The equilibrium price charged by the low-cost firm  $p_A^*(\beta)$  may be increasing or decreasing in the share of informed consumers  $\beta$ :  $p_A^*(\beta)$  is monotonously increasing, monotonously decreasing or first increasing and then decreasing in  $\beta$ . It tends to be decreasing for small and increasing for large cost differences.*

The critical price difference (that implies the critical cost difference) at which price locally does not respond to  $\beta$  (c.p.  $\Delta p$ , i.e. partial effect) can be solved for analytically. The critical  $\Delta p$  is a function of  $\lambda$  and  $t$  and is independent of  $\beta$ :

$$\Delta p^{crit \partial p_A}(\lambda, t) = \frac{t}{4(3 + 5\lambda)} \left( (9 - (26 - 15\lambda)\lambda) + \sqrt{3} \cdot | -1 + 5\lambda | \sqrt{(2(\lambda + 2))^2 - (\lambda - 1)^2} \right)$$

For example, for parameters  $\lambda = 3$  and  $t = 1$  the critical price difference, at which the price of the low-cost firm reaches its maximum, satisfies  $\Delta p^{crit \partial p_A}(3, 1) = 0.2534$ . It is also insightful to evaluate the derivative in the limit as  $\beta$  turns to 1. In this case, we can also solve analytically for a critical  $\Delta p$  at which the total derivative of  $p_A$  is zero, i.e.  $\frac{dp_A^*(\Delta p^*(\beta); \beta)}{d\beta} = 0$ :

$$\Delta p^{crit dp_A}(\lambda, t) = t \frac{3(\lambda(31\lambda + 42) - 41) - \sqrt{21} \cdot |7 - 11\lambda| \sqrt{(\lambda + 3)(3\lambda + 5)}}{2(\lambda - 3)(9\lambda - 1)} \quad \text{at } \beta = 1.$$

For example,  $\Delta p^{crit dp_A}(3, 1) = 7/26 = 0.2692$  at  $\beta = 1$ . This means that, given parameters  $\lambda = 3$  and  $t = 1$ , if the equilibrium price difference satisfies  $\Delta p^*(1) < 0.2692$  a small

decrease in the share of informed consumers leads to a higher price of the more efficient firm,  $dp_A^*/d\beta < 0$ —the numerical results reported in Tables 3 and 4 illustrate this result. By contrast, for  $\Delta p^*(1) > 0.2692$ , the opposite holds—i.e.,  $dp_A^*/d\beta > 0$ —as illustrated in Table 5 in the Appendix.

The previous proposition implies that consumers who end up buying from the low-cost firm may actually be worse off when additional consumers become informed ex ante. Consider a change in policy from  $\beta$  to  $\beta'$  with  $\beta' > \beta$ —this parameterizes the market environment. Note that the majority of consumers buy from the low-price firm in both market environments. For a sufficiently large cost asymmetry, the equilibrium price of the low-cost firm is locally increasing for all environments between  $\beta$  and  $\beta'$ . Hence, all those consumers of the low-cost firm whose ex ante information is constant across the two market environments are worse off under information disclosure to a share of  $\beta' - \beta$  of initially uninformed consumers.

We now turn to the high-cost firm. Here, our result is qualitatively similar: The price tends to be decreasing in  $\beta$  for small cost differences and increasing for large cost differences.

**Proposition 7.** *The equilibrium price of the high-cost firm  $p_B^*(\beta)$  may be increasing or decreasing in the share of informed consumers  $\beta$ :  $p_B^*(\beta)$  is monotonously increasing, monotonously decreasing or first increasing and then decreasing in  $\beta$ .*

We solve for critical values at which the price change changes sign:

$$\Delta p^{crit \partial p_B}(\lambda, t) = \frac{t}{2(\lambda + 1)(\lambda + 7)} \left( (-23 + (\lambda - 10)\lambda) + |5 - \lambda| \sqrt{(2(\lambda + 2))^2 - (\lambda - 1)^2} \right)$$

For instance,  $\Delta p^{crit \partial p_B}(3, 1) = 0.3201$ . At  $\beta = 1$  we can solve analytically for a critical  $\Delta p$  at which the total derivative of  $p_B$  is zero, i.e.  $(dp_B^*(\Delta p^*(\beta); \beta))/d\beta = 0$ :

$$\Delta p^{crit dp_B}(\lambda, t) = \frac{t \left( 3(\lambda(17\lambda + 6) - 55) - \sqrt{15} \cdot |11 - 7\lambda| \sqrt{(\lambda + 3)(3\lambda + 5)} \right)}{4\lambda(3\lambda - 11)}$$

For instance,  $\Delta p^{crit dp_B}(3, 1) = 1/2 \cdot (5\sqrt{35} - 29) = 0.2902$  at  $\beta = 1$ . Thus, for  $\Delta p^*(1) < 0.2902$ , we obtain  $dp_B/d\beta < 0$  at  $\beta = 1 - \epsilon$  (compare Tables 3 and 4), while, for  $\Delta p^*(1) > 0.2902$ , we obtain  $dp_B/d\beta > 0$  at  $\beta = 1$  (compare Table 5). Thus, for these parameter values, the overall effect of a marginal increase in  $\beta$  can indeed become positive if cost asymmetries are sufficiently large.

Let us distinguish consumer groups by the product they consume. We observe that  $\Delta p^{crit dp_B}(\lambda, t) > \Delta p^{crit dp_A}(\lambda, t) \forall \lambda, t$ . Hence, for a larger range of cost parameters the

price of the high-cost firm is locally decreasing (compared to the low-cost firm). This implies that, focusing on the consumers whose ex ante information remains unchanged, there exists an intermediate range of values of  $\beta$  under which consumers of the low-cost product are worse off, whereas consumers of the high-cost product are better off after an increase in  $\beta$ . This means that in such cases additional information in the population benefits those consumers who purchase the high-cost product. Since the high-cost product only serves a niche market we call these consumers niche consumers. Then, informed niche consumers are more likely to benefit from an increase in  $\beta$  than the other informed consumers.<sup>31</sup>

The above observation helps us to shed some light on information acquisition by consumers. A particular application are consumer clubs that provide early information on match value to its members. Whether existing club members have an incentive to attract additional members depends on the market environment. Our above observation also indicates, that consumer clubs may be more likely to be formed by niche consumers. We also note that a forward-looking club may be willing to cope with increasing prices for a while with the understanding that, as the club further increases in size (reflected by an increase in  $\beta$ ) prices will eventually fall.

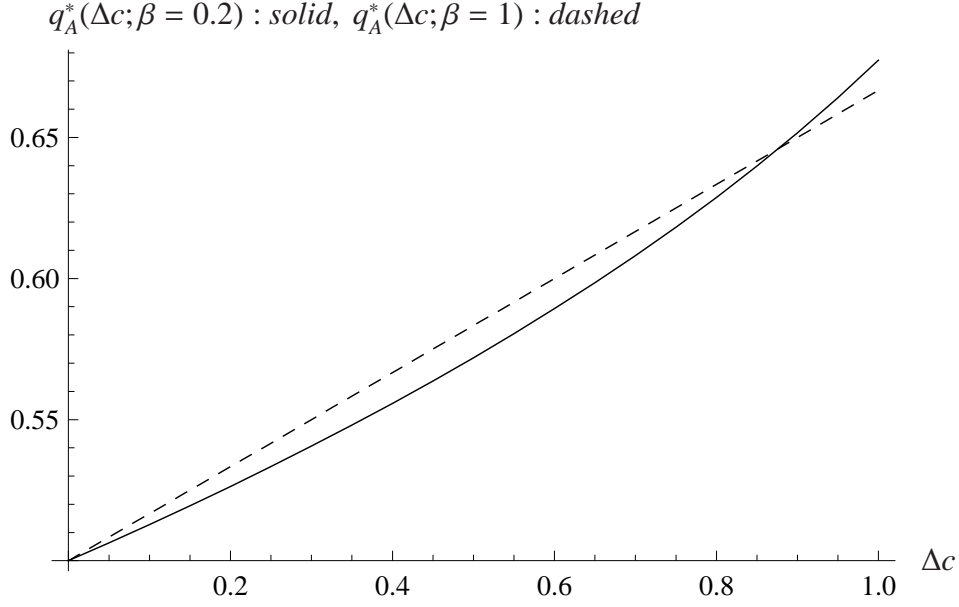
With respect to equilibrium demand our model generates the following predictions.

$$\begin{aligned} \frac{dq_A(\Delta p^*(\beta); \beta)}{d\beta} &= \beta \frac{d\hat{x}_{in}(\Delta p^*)}{d(\Delta p^*)} \cdot \frac{d(\Delta p^*)}{d\beta} + \hat{x}_{in}(\Delta p^*) + (1 - \beta) \frac{d\hat{x}_{un}(\Delta p^*)}{d(\Delta p^*)} \cdot \frac{d(\Delta p^*)}{d\beta} - \hat{x}_{un}(\Delta p^*) \\ &= \underbrace{\frac{\partial q_A(\Delta p^*)}{\partial(\Delta p^*)}}_{\oplus} \cdot \underbrace{\frac{d(\Delta p^*)}{d\beta}}_{\oplus} + (\hat{x}_{in}(\Delta p^*) - \hat{x}_{un}(\Delta p^*)) \geq 0, \end{aligned}$$

which is positive for small cost (resp. price) differences and negative for large cost (resp. price) differences (see also Figure 7). Hence, in rather symmetric markets the demand of the more efficient firm rises, as the share of informed consumers increases, as illustrated in Table 3 in the Appendix. This implies that in a market with consumer loss aversion (and a positive share of uninformed consumers) firm A's equilibrium demand is lower than in the standard Hotelling case.<sup>32</sup> Our result is reversed in strongly asymmetric markets in which the demand of the more efficient firm is decreasing in the share of informed consumers (compare Table 5 in the appendix).

<sup>31</sup>The effect on uninformed consumers is ambiguous from an ex ante perspective since they buy the low-cost and the high-cost product with positive probability.

<sup>32</sup>This is qualitatively in line with Heidhues and Koszegi (2008) who predict equal splits of demand between firms in asymmetric markets.



Equilibrium demand of firm A for markets with either many uninformed consumers ( $\beta = 0.2$ ) or only informed consumers (=benchmark case,  $\beta = 1$ ) as a function of cost differences  $\Delta c$  for parameter values of  $t = 1$  and  $\lambda = 3$ :  $\Delta c^{nd}(\beta = 0.2) = 1.00993$ .

Figure 7: Equilibrium demand of firm A

## 6.2 Incentives for information disclosure

In this subsection, we analyze private incentives to disclose information. To address this issue, we have to investigate the effect of  $\beta$  on profits. Here private information disclosure can be seen as the firms' management of consumer expectations (i.e. reference points). Note that in our simple setting information disclosure by one firm fully discloses the information of both firms since consumers make the correct inferences from observing the match value for one of the two products.<sup>33</sup>

$$\frac{d\pi_A(\Delta p^*(\beta), p_A^*(\beta); \beta)}{d\beta} = \frac{dp_A^*(\Delta p^*; \beta)}{d\beta} \cdot q_A(\Delta p^*; \beta) + (p_A^*(\Delta p^*; \beta) - c_A) \cdot \frac{dq_A(\Delta p^*; \beta)}{d\beta} \leq 0$$

$$\frac{d\pi_B(\Delta p^*(\beta), p_B^*(\beta); \beta)}{d\beta} = \frac{dp_B^*(\Delta p^*; \beta)}{d\beta} \cdot (1 - q_A(\Delta p^*; \beta))$$

<sup>33</sup>This is due to our assumption that firms necessarily locate at distance 1 from each other. It applies to either the setting in which uninformed consumers do not know their type before forming their reference point or they do not know the locations of firms in the product space.

$$-\left(p_B^*(\Delta p^*; \beta) - c_B\right) \cdot \frac{dq_A(\Delta p^*; \beta)}{d\beta} \leq 0$$

It is of interest to compare the size of the price effect to the size of the quantity effect for different degrees of market asymmetry. Numerical simulations suggest that the price effect dominates the quantity effect for all  $\lambda > 1$ . Thus, profits closely follow prices. Here, we confine attention to a single numerical example. The critical value of  $\Delta p$  such that  $d\pi_A(\cdot)/d\beta = 0$  at  $\beta = 1$  and  $\lambda = 3$  and  $t = 1$ ,  $c_A = 0.25$ , and  $c_B = 1$  is  $\Delta p = 0.2581$ . The critical values of  $\Delta p$  s.t.  $d\pi_B(\cdot)/d\beta = 0$  at the same values as above is  $\Delta p = 0.2870$ .<sup>34</sup> For comparison, we take a look at table 4 in the appendix: The critical value at  $\beta = 1$  is  $\Delta p^*(1) = 0.25$ . Hence, the critical values of  $\Delta p$  at  $\beta < 1$  are larger than  $\Delta p^*(1)$ . Moreover,  $\Delta p_B^{crit} > \Delta p_A^{crit}$ .

Our numerical example suggests that increasing the initial share of ex ante informed consumers first none, then one and then both firms gain from information disclosure. In case of conflicting interests it is the more efficient firm which locally gains from information disclosure as an expectation management tool.

Our numerical finding has direct implication for the observed advertising strategy of the firm. Our model predicts that it is rather more efficient firms that advertise product features and price and run promotions that allow consumers test-drives etc. This means that one should observe a positive correlation between efficiency level and advertising and marketing activities of the above mentioned form. We would like to stress that although all consumers will be fully informed at the moment of purchase, advertising content and price matters for firms if consumers are loss-averse. Without this behavioral bias it would be irrelevant whether or not a firm advertises price and characteristics.

### 6.3 The effect of ex ante information on consumer surplus

In this subsection, we analyze the effect of  $\beta$  on consumer surplus. More precisely, we answer the question: How is the surplus of the different consumer groups affected by an increase of the share of informed consumers?

First, consider the change in surplus of informed consumers. The aggregate consumer surplus for informed consumers is given by

$$CS_{in}(p_A(\beta), p_B(\beta)) = \int_0^{\hat{x}_{in}(\Delta p(\beta))} u_A(x, p_A(\beta)) dx + \int_{\hat{x}_{in}(\Delta p(\beta))}^1 u_B(x, p_B(\beta)) dx$$

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<sup>34</sup>Note that we have problems to obtain an analytical solution as a function of  $\lambda$  and  $t$  or  $c_B$  even for the special case  $\beta = 1$ .



The marginal effect of increasing the share of informed consumers on the surplus of the already informed consumers is

$$\begin{aligned} \frac{dCS_{in}}{d\beta} &= \int_0^{\hat{x}_{in}(\Delta p(\beta))} \underbrace{\frac{\partial u_A(x, p_A(\beta))}{\partial p_A(\beta)}}_{=-1} \cdot \frac{dp_A}{d\beta} \cdot dx + \int_{\hat{x}_{in}(\Delta p(\beta))}^1 \underbrace{\frac{\partial u_B(x, p_B(\beta))}{\partial p_B(\beta)}}_{=-1} \cdot \frac{dp_B}{d\beta} \cdot dx \\ &= -\hat{x}_{in}(\Delta p) \frac{dp_A}{d\beta} - (1 - \hat{x}_{in}(\Delta p)) \frac{dp_B}{d\beta} \geq 0. \end{aligned}$$

Consumer surplus of informed consumers may increase or decrease in the share of informed consumers. The sign of the derivative is determined by the weighted marginal price changes  $dp_i/d\beta$  of the two products. It is positive in markets with small cost differences because both prices decrease in the share of informed consumers ( $dp_i/d\beta < 0$ ). It is in markets with large cost differences in which the reverse is true. In markets with intermediate cost differences, the two prices move in different directions. Thus, some informed consumers are better off whereas others are worse off in response to an increase in the share of informed consumers.

Second, consider uninformed consumers. Evaluating the ex ante effect on uninformed consumers is more involved because gains and losses relative to their reference point have to be taken into account.

$$\begin{aligned} CS_{un}(p_A(\beta), p_B(\beta)) &= \left( \int_0^{1-\hat{x}_{un}(\Delta p(\beta))} \tilde{u}_A(x, p_A(\beta), p_B(\beta), \hat{x}_{un}(\Delta p(\beta))) dx \right. \\ &\quad \left. + \int_{1-\hat{x}_{un}(\Delta p(\beta))}^{\hat{x}_{un}(\Delta p(\beta))} u_A(x, p_A(\beta), p_B(\beta), \hat{x}_{un}(\Delta p(\beta))) dx \right) \\ &\quad + \int_{\hat{x}_{un}(\Delta p(\beta))}^1 u_B(x, p_A(\beta), p_B(\beta), \hat{x}_{un}(\Delta p(\beta))) dx, \end{aligned}$$

where  $u_A(x, \cdot)$  and  $u_B(x, \cdot)$  represent uninformed consumers' gain/loss utility for distant consumers of A and nearby consumers of B derived in (22) and (23), and

$$\begin{aligned} \tilde{u}_A(x, p_A(\beta), p_B(\beta), \hat{x}_{un}(\Delta p(\beta))) &= (v - tx - p_A) + (1 - \hat{x}_{un})(p_B - p_A) \\ &\quad - \lambda \cdot tx^2 + \frac{t}{2} \left( (1 - \hat{x}_{un})^2 - 2(1 - x)x + \hat{x}_{un}^2 \right), \end{aligned}$$

which demonstrates the gain/loss utility for nearby uninformed consumers of A.  $\tilde{u}_A(x, \cdot)$  differs from  $u_A(x, \cdot)$  only in the taste dimension of the gain/loss utility.

In contrast to intrinsic utility, the gain/loss utility also depends on reference point distributions which require knowledge of all prices and the location of the indifferent uninformed

consumer. Taking derivatives with respect to  $\beta$  we obtain

$$\begin{aligned} \frac{dCS_{un}}{d\beta} = & \int_0^{\hat{x}_{un}(\Delta p(\beta))} \left( \frac{\partial u_A(x, \cdot)}{\partial p_A} \cdot \frac{dp_A}{d\beta} + \frac{\partial u_A(x, \cdot)}{\partial p_B} \cdot \frac{dp_B}{d\beta} \right) \cdot dx \\ & + \left( \int_0^{1-\hat{x}_{un}(\Delta p(\beta))} \left( \frac{\partial \tilde{u}_A(x, \cdot)}{\partial \hat{x}_{un}} \cdot \frac{d\hat{x}_{un}(\Delta p)}{d\Delta p} \cdot \frac{d\Delta p}{d\beta} \right) \cdot dx \right. \\ & + \left. \int_{1-\hat{x}_{un}(\Delta p(\beta))}^{\hat{x}_{un}(\Delta p(\beta))} \left( \frac{\partial u_A(x, \cdot)}{\partial \hat{x}_{un}} \cdot \frac{d\hat{x}_{un}(\Delta p)}{d\Delta p} \cdot \frac{d\Delta p}{d\beta} \right) \cdot dx \right) \\ & + \int_{\hat{x}_{un}(\Delta p(\beta))}^1 \left( \frac{\partial u_B(x, \cdot)}{\partial p_A} \cdot \frac{dp_A}{d\beta} + \frac{\partial u_B(x, \cdot)}{\partial p_B} \cdot \frac{dp_B}{d\beta} + \frac{\partial u_B(x, \cdot)}{\partial \hat{x}_{un}} \cdot \frac{d\hat{x}_{un}(\Delta p)}{d\Delta p} \cdot \frac{d\Delta p}{d\beta} \right) \cdot dx. \end{aligned}$$

On top of consumers' intrinsic utility a price change also affects consumers' gains and losses with respect to the price dimension via the varying price difference. A change of the location of the indifferent uninformed consumer  $\hat{x}_{un}$  has an impact on consumers' gains/losses in both dimensions. The taste dimension is affected since an increase of  $\hat{x}_{un}$  shifts mass of the reference point distribution to the upper tail.<sup>35</sup> An impact on the price dimension occurs since the probability of buying at a specific price depends on the location at which consumers are indifferent between the two products. The equation of  $dCS_{un}/d\beta$  can be further simplified to

$$\begin{aligned} \frac{dCS_{un}}{d\beta} = & -\hat{x}_{un} \cdot \frac{dp_A}{d\beta} - (1 - \hat{x}_{un}) \cdot \frac{dp_B}{d\beta} \\ & + \left( (\lambda - 1)\hat{x}_{un}(1 - \hat{x}_{un}) + \Delta p (\hat{x}_{un} + \lambda(1 - \hat{x}_{un})) \cdot \frac{d\hat{x}_{un}}{d\Delta p} \right) \cdot \left( -\frac{d\Delta p}{d\beta} \right) \\ & - t \left( \frac{1}{2}(2\hat{x}_{un} - 1) \left( (\lambda - 1)(2\hat{x}_{un} - 1) + 2 \right) \right) \cdot \frac{d\hat{x}_{un}}{d\Delta p} \cdot \left( -\frac{d\Delta p}{d\beta} \right) \geq 0, \quad (18) \end{aligned}$$

where the first line shows marginal effect of  $\beta$  on intrinsic utility. This is analogous to the analysis of informed consumers above.

In the second line of equation (18) the marginal effect of  $\beta$  on the price dimension of consumers' gain/loss utility is depicted. An increase of the share of informed consumers has a positive overall impact on  $CS_{un}$ . This holds true for two reasons. Firstly, from Proposition 5 we obtain that the price difference is a decreasing function in the share of informed consumers. It turns out that a lower price difference (=seize of gains and losses in the price dimension) always reduces the losses for  $B$  consumers more in total terms than the gains for  $A$  consumers (consider the first term in second line). Secondly, a downward shift of the location of the indifferent uninformed consumer (caused by an reduction of

<sup>35</sup>It can be easily shown that  $G(s|\hat{x}'_{un})$  first-order stochastically dominates  $G(s|\hat{x}_{un})$  for all  $\hat{x}'_{un} > \hat{x}_{un}$  feasible.

the price difference) makes uninformed consumers of both firms better off with respect to gains/losses in the price dimension since the reference point distribution becomes skewed towards gains. This means that the probability of facing a loss in the price dimension decreases (for  $B$  consumers), while the probability of facing a gain in the price dimension increases (for  $A$  consumers).

The third line of equation (18) shows that the marginal effect of  $\beta$  on the match value dimension of consumers' gain/loss utility is always negative. A downward shift of the location of the indifferent uninformed consumer (caused by an increase in  $\beta$ ) decreases the probability of large taste differences ( $s \in (1 - \hat{x}_{un}, \hat{x}_{un}]$ ) keeping the probability of small taste differences ( $s \in [0, 1 - \hat{x}_{un}]$ ) constant.<sup>36</sup> Since the remaining uninformed consumers of firm  $B$  are located on the interval with small taste differences, they experience the same losses but lower gains. Thus, they are clearly worse off along the match value dimension of their gain/loss utility. The same holds true for nearby uninformed consumers of firm  $A$ . On top of lower gains, more distant consumers of  $A$  experience higher losses due to the downward shifted reference point distribution for the taste dimension. The overall effect of  $\beta$  on the taste dimension of consumers' gain/loss utility must, therefore, be negative.

The overall effect of  $\beta$  on  $CS_{un}$  is positive in rather symmetric markets ( $\Delta c$  small) since the effect of  $\beta$  on individual prices  $p_i$  is negative in these markets (compare  $CS_{in}$ ). By contrast, the effect is negative in more asymmetric markets. The surplus result that holds for informed consumers, thus, qualitatively carries over to uninformed consumers. The reason for this that the sign of the effect of  $\beta$  on both dimensions of consumers' gain/loss utility does not change with the asymmetry of the market.

It can be shown that for all  $\lambda > 1$  and  $\Delta c$  feasible the sum of the second and the third line of (18) is negative—i.e., the marginal effect of  $\beta$  on the taste dimension dominates its effect on the price dimension of consumers' gain/loss utility. However, this does not suffice to predict that the sign of  $dCS_{un}/d\beta$  is changing for a higher level of  $\beta$  in intermediately asymmetric markets since the price changes, which determine the sign change of consumer surplus, are weighted differently in case of uninformed instead of informed consumers. Table 4 in the Appendix illustrates that the effect of the weight difference dominates the negative effect of  $\beta$  on both dimensions of consumers' gain loss utility, i.e. the critical  $\beta$  at which the marginal consumer surplus of uninformed consumers switches sign is lower than the critical  $\beta$  for informed consumers.

**Remark 3.** *In symmetric and weakly asymmetric markets, all consumers whose information is unaffected are better off if more consumers become informed before forming their*

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<sup>36</sup>This argument also relies on the FOSD property of  $G(s|\hat{x}_{un})$ .

reference points. By contrast, in strongly asymmetric markets, all these consumers are worse off.

To determine the overall effect of  $\beta$  on aggregate consumer surplus of both consumer groups, an additional decomposition effect has to be taken into account. This effect stems from the change in consumer surplus of the group of formerly uninformed consumers who become informed. The overall effect of  $\beta$  on aggregate consumer surplus is determined by the first derivative of  $CS(\beta) = \beta \cdot CS_{in}(p_A(\beta), p_B(\beta)) + (1 - \beta) \cdot CS_{un}(p_A(\beta), p_B(\beta))$  with respect to  $\beta$ :

$$\begin{aligned} \frac{dCS}{d\beta} &= \beta \cdot \frac{dCS_{in}}{d\beta} + CS_{in} + (1 - \beta) \cdot \frac{dCS_{un}}{d\beta} - CS_{un} \\ &= \beta \cdot \frac{dCS_{in}}{d\beta} + (1 - \beta) \cdot \frac{dCS_{un}}{d\beta} + (CS_{in} - CS_{un}). \end{aligned}$$

It can be shown that the decomposition effect represented by  $(CS_{in} - CS_{un})$  is always strictly positive, which is intuitive since the group of uninformed consumers faces a lower average utility level due to the higher weight on losses than on gains. Although some uninformed consumers which receive high match value at low price are better off than their informed counterparts, the average utility of uninformed consumers is lower due to the losses in the taste dimension of consumers located apart from the product they purchase and the losses in the price dimension of  $B$  consumers (for illustration, see the tables in the Appendix).

It turns out that the decomposition effect always dominates the group-specific effects of  $\beta$  on consumer surplus. This means that the group of consumers who becomes informed is so much better off that its surplus increase always dominates the surplus change of the remaining uninformed consumers and the “old” informed consumers. This holds even in strongly asymmetric markets in which remaining uninformed and old informed consumers are worse off if the share of informed consumers increases. We summarize by the following remark.

**Remark 4.** *Consumer surplus is increasing the share of informed consumers  $\beta$ .*

The policy conclusions are straightforward: A public authority whose aim is consumer surplus (and who does not have distributional concerns about the effects of information disclosure) should always try to increase the share of informed consumers, possibly through the use of mandatory information disclosure rules. In symmetric and weakly asymmetric markets, all consumers will be better off. However, in strongly asymmetric markets most consumers suffer. Those consumers who become informed (due to the policy intervention) exert a negative externality on all other consumers.

## 7 Extensions

### 7.1 Relative weight on gain-loss utility

Consider next consumer preferences for which the intrinsic utility is weighted by one, while the gain-loss utility has a weight of  $\alpha > 0$ .<sup>37</sup> It could now be asked whether a change of the relative weight on the gain-loss utility has a different influence on the location of the indifferent uninformed consumer than a change in the degree of loss aversion  $\lambda$ . The next proposition shows that this is not the case.

**Proposition 8.** *Suppose the utility function of uninformed consumers shows an additional weight,  $\alpha > 0$ , on the gain-loss utility, i.e. all terms except for the intrinsic utility term in (22) (resp. (23)) are pre-multiplied by  $\alpha$ .*

*Then,  $\forall \lambda' > 1, \alpha' > 0 \exists \lambda > 1$  such that*

$$\hat{x}_{un}(\Delta p; \lambda, \alpha = 1) = \hat{x}_{un}(\Delta p; \lambda', \alpha'), \quad (19)$$

*where  $\hat{x}_{un}(\Delta p; \lambda, \alpha)$  is the location of the indifferent uninformed consumer given  $\alpha$ -extended preferences. Moreover,  $\lambda \geq \lambda'$  for  $\alpha' \geq 1$  and  $\lambda < \lambda'$  for  $\alpha' < 1$ .*

*Proof of Proposition 8.* The derivation of the indifferent uninformed consumer with  $\alpha$ -extended preferences is analogous to the derivation of the indifferent uninformed consumer for  $\alpha = 1$  provided in the proof of LEMMA 1. With  $\alpha$ -extended preferences the location equals

$$\hat{x}_{un}(\Delta p; \lambda, \alpha) = \frac{1 + \alpha(2\lambda - 1)}{2\alpha(\lambda - 1)} - \frac{\Delta p}{4t} - \sqrt{\frac{\Delta p^2}{16t^2} - \frac{(\alpha(2\lambda + 1) + 3)}{4\alpha t(\lambda - 1)}\Delta p + \frac{(\alpha\lambda + 1)^2}{4\alpha^2(\lambda - 1)^2}}. \quad (20)$$

By solving for  $\lambda$  in equation (19) we receive

$$\lambda(\lambda', \alpha') = \frac{1 + \alpha'(2\lambda' - 1)}{1 + \alpha'}. \quad (21)$$

Since  $\lambda(\lambda', \alpha' = 1) = \lambda'$  and  $\partial\lambda/\partial\alpha' = 2(\lambda' - 1)/(1 + \alpha')^2 > 0$ ,  $\lambda$  shows the required properties.  $\square$

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<sup>37</sup>For  $\alpha = 0$  we are obviously situated in a standard Salop world.

The previous proposition points out that for any change of the relative weight on gain-loss utility apart from one, there is an equivalent change of the degree of loss aversion,  $\lambda$ , which shows the same sign.

## 7.2 Asymmetric product quality

Our model is easily extended to allow for differences in product quality which are known to consumers at the beginning of the game. An informed consumer's utility function is  $u_i(x, p_i) = (v_i - p_i) - t|y_i - x|$ . We then distinguish between a quality-adjusted price dimension, which includes easily communicated product characteristics which are of unambiguous value to consumers and a taste dimension which includes those product characteristics whose value depends on the consumer type. We define quality-adjusted (or hedonic) prices  $\tilde{p}_i = p_i - v_i$ ,  $i \in \{A, B\}$  for all consumers and consider those to be relevant for consumers' purchase decision. The main difference arises for uninformed consumers when building their reference point distribution with respect to prices. Here, only the gain/loss in quality-adjusted prices  $\Delta\tilde{p} = \Delta p - \Delta v$  matters,  $\Delta v \equiv v_B - v_A$ . We label firms such that  $\Delta c - \Delta v > 0$  and call firm  $A$  the more efficient firm. In the following proposition we show that any market with asymmetric quality is equivalent to a market with symmetric quality and more asymmetric costs.

**Proposition 9.** *For any market with asymmetric quality represented by a vector  $(\Delta v, \Delta c)$  with  $\Delta c - \Delta v > 0$  there exists a market with symmetric quality represented by a vector  $(\Delta v', \Delta c')$  with  $\Delta v' = 0$ ,  $\Delta c' > 0$  such that market equilibria of both markets are the same, i.e.  $\Delta p^* - \Delta v = \Delta p'^*$ . Moreover,  $\Delta c' = \Delta c - \Delta v$ .*

As a special case, the asymmetry in the former market is generated by quality differences—i.e., firm  $A$  sells higher quality in a market with symmetric costs,  $\Delta v < 0$  and  $\Delta c = 0$ . Then, the costs asymmetry in the second market shows the same size in absolute terms as the quality difference in the first market,  $\Delta c' = -\Delta v$ .

In the proof we show that the optimization problems of the two consumer groups and the firms are the same in both markets.

*Proof of Proposition 9.* First consider informed consumers' utility: We find  $u_i(x, p_i) = (v_i - p_i) - t|y_i - x| = -\tilde{p}_i - t|y_i - x|$  for all  $i \in \{A, B\}$  in the first market and  $u_i(x, p'_i) = (v'_i - p'_i) - t|y_i - x|$  for all  $i \in \{A, B\}$  in the second market. Since in the second market quality levels are identical ( $\Delta v' = 0$ ), it holds true that  $\hat{x}_{in}(\Delta\tilde{p}) = \hat{x}_{in}(\Delta p')$  for  $\Delta p' = \Delta p - \Delta v$ . If

uninformed consumers use quality-adjusted prices for determining their reference point distribution in the price dimension we also receive  $\hat{x}_{un}(\Delta\tilde{p}) = \hat{x}_{un}(\Delta p')$  for  $\Delta p' = \Delta p - \Delta v$  by the same argument. Finally, compare firms' maximization problem for both markets. Firm A solves

$$\begin{aligned} \max_{\tilde{p}_A} \pi_A(\tilde{p}_A, \tilde{p}_B) &= (\tilde{p}_A + v_A - c_A)[\beta \cdot \hat{x}_{in}(\tilde{p}_B - \tilde{p}_A) + (1 - \beta) \cdot \hat{x}_{un}(\tilde{p}_B - \tilde{p}_A)] \quad \text{and} \\ \max_{p'_A} \pi_A(p'_A, p'_B) &= (p'_A - c'_A)[\beta \cdot \hat{x}_{in}(p'_B - p'_A) + (1 - \beta) \cdot \hat{x}_{un}(p'_B - p'_A)]. \end{aligned}$$

Firm A's equilibrium prices are identical iff markups in both markets are identical, i.e.  $\tilde{p}_A + v_A - c_A = p'_A - c'_A$ , and both demand functions are identical, i.e.  $\Delta p' = \Delta p - \Delta v$ . Analogously, for firm B this holds true iff  $\tilde{p}_B + v_B - c_B = p'_B - c'_B$  and  $\Delta p' = \Delta p - \Delta v$ . Finally, taking markup differences between firms we get  $\Delta\tilde{p} + \Delta v - \Delta c = \Delta p - \Delta c$  in first market and  $\Delta p' - \Delta c'$  in the second market. For  $\Delta p' = \Delta p - \Delta v$  both markup differences are the same iff  $\Delta c' = \Delta c - \Delta v$ .  $\square$

## 8 Conclusion

This paper has explored the impact of consumer loss aversion on market outcomes in imperfectly competitive markets. We did so in a Hotelling/Salop setting, which a standard work horse in the modern industrial organization literature. Consumer loss aversion only makes a difference compared to a market in which consumers lack this behavioral bias if they are uncertain about product characteristics or associated match value at an initial stage where they form their reference point distribution. Early information disclosure can thus be interpreted as expectation management. Such information disclosure can be achieved through advertising campaigns and promotional activities which do not generate additional information at the moment of purchase (at this point consumers would be informed in any case) but make consumers informed much in advance of their actual purchasing decision.

Our paper provides for a meaningful comparison to standard model of competition in the Hotelling-Salop tradition. It provides a nuanced view on the competitive effects of consumer loss aversion in differentiated product markets. Loss aversion, and more generally, a gain-loss utility, in the price dimension leads to more competitive outcomes, while the reverse holds in the match-value dimension. It is the interplay between this pro- and anti-competitive effect that determines whether the market is more or less competitive



compared to the standard Hotelling-Salop world. Empirical work may want to uncover the relative strength of those two effects.

We further explored how firm asymmetries and the share of uninformed loss-averse consumers in the population affect market outcomes. Here, we analyzed industries that are characterized by cost asymmetries. Alternatively, asymmetries with respect to observed product quality may be introduced (as explored in Section 7). Since there is a one-to-one relationship between these two models our insights are directly applicable to a model in which firms differ in observed product quality. Our model is able to generate reverse comparative statics results with respect to the effect of cost differences on the mark-up by the more efficient firm. Unfortunately, costs are typically not observable in empirical work, which makes it difficult to confront this implication of our theory with the data.

In our modeling effort we followed Heidhues and Koszegi (2008). Our framework, however, has notable differences to theirs: First, consumers and firms know the market environment; in particular, firms know the actual (asymmetric) cost realizations and consumers observe prices from the outset, whereas in Heidhues and Koszegi (2008) costs are private information. Second, consumers learn posted prices before they form their reference points, whereas in Heidhues and Koszegi (2008) consumers form their reference points before knowing posted prices. Our model is enriched by considering heterogeneous consumers who differ according to their knowledge of their preferences at the initial point in time when they form their (probabilistic) reference point. Our model delivers novel results. In particular, we show that the price difference between the two product *increases* in the share of uninformed loss-averse consumers, while Heidhues and Koszegi (2008) obtained focal pricing as a consequence of the presence of loss-aversion in the population. That is, in our setting the behavioral bias increases the observed price difference, whereas in Heidhues and Koszegi (2008) between asymmetric firms disappears. We also show that prices and profits *decrease* if the cost asymmetry is large.

Our theory provides a new perspective on information disclosure and advertising. Since all consumers are fully informed at the purchasing stage, standard theory would predict that it is irrelevant at which point in time prior to the purchasing stage information is revealed. Our theory predicts that consumer behavior and market outcomes depend on this timing.

Our results have implications for public policy and firms' advertising strategies. There are instances in which consumers would gain from more information whereas both firms would refrain from early information disclosure, namely when the market is symmetric



or weakly asymmetric. In these markets public information disclosure (which allows consumers to learn the products' match values) would enhance consumer surplus.

Moreover, our model predicts that advertising and other marketing instruments that allow for voluntary early information disclosure about match value are more prevalent in markets characterized by large asymmetries between firms. In these asymmetric markets, one or both firms gain from information disclosure because this leads to higher prices. Whenever firms have conflicting interests with respect to information disclosure, it is the more efficient firm that discloses information.

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## Appendix

### A Relegated Proofs

#### A.1 Relegated proof of Section 2

*Proof of Lemma 1.* Using the properties of the reference distributions, we rewrite the utility function further,

$$\begin{aligned}
 u_A(x, p_A, p_B) &= (v - tx - p_A) + (1 - \hat{x}_{un})(p_B - p_A) \\
 &\quad - \lambda \cdot t \left( \int_0^{1-\hat{x}_{un}} 2(x-s) ds + \int_{1-\hat{x}_{un}}^x (x-s) ds \right) + t \left( \int_x^{\hat{x}_{un}} (s-x) ds \right) \\
 &= (v - tx - p_A) + (1 - \hat{x}_{un})(p_B - p_A) \\
 &\quad - \lambda \cdot \frac{t}{2} \left( x^2 + 2x(1 - \hat{x}_{un}) - (1 - \hat{x}_{un})^2 \right) + \frac{t}{2} (\hat{x}_{un} - x)^2 \tag{22}
 \end{aligned}$$

$$\begin{aligned}
 u_B(x, p_A, p_B) &= (v - t(1-x) - p_B) - \lambda \cdot \hat{x}_{un}(p_B - p_A) - \lambda \cdot t \int_0^{1-x} 2((1-x)-s) ds \\
 &\quad + t \left( \int_{1-x}^{1-\hat{x}_{un}} 2(s - (1-x)) ds + \int_{1-\hat{x}_{un}}^{\hat{x}_{un}} (s - (1-x)) ds \right) \\
 &= (v - t(1-x) - p_B) - \lambda \cdot \hat{x}_{un}(p_B - p_A) - \lambda \cdot t(1-x)^2 \\
 &\quad + t \left( (x - \hat{x}_{un})^2 + \left( \frac{1}{2} - x - \hat{x}_{un} + 2x\hat{x}_{un} \right) \right). \tag{23}
 \end{aligned}$$

Next, we find the location of the indifferent uninformed consumer  $x = \hat{x}_{un}$  by setting  $u_A = u_B$ , where

$$\begin{aligned}
 u_A(\hat{x}_{un}, p_A, p_B) &= v - t\hat{x}_{un} - p_A + (1 - \hat{x}_{un})(p_B - p_A) - \lambda \cdot \frac{t}{2} \left( 1 - 2(1 - \hat{x}_{un})^2 \right) \\
 u_B(\hat{x}_{un}, p_A, p_B) &= v - t(1 - \hat{x}_{un}) - p_B - \lambda \cdot \hat{x}_{un}(p_B - p_A) - \lambda \cdot t(1 - \hat{x}_{un})^2 + 2t \left( \frac{1}{2} - \hat{x}_{un} \right)^2
 \end{aligned}$$

If she buys product  $A$  the indifferent uninformed consumer will experience no gain but the maximum loss in the taste dimension. If she buys product  $B$  she will experience a gain and a loss because distance could have been smaller or larger than  $1 - \hat{x}_{un}$ . With respect to the price dimension the indifferent uninformed consumer (like all other consumers) faces only a loss when paying price  $p_B$  and only a gain when paying price  $p_A$ .

$u_A(\hat{x}_{un}, p_A, p_B) = u_B(\hat{x}_{un}, p_A, p_B)$  can be transformed to the following quadratic equation in  $\hat{x}_{un}$ ,

$$0 = 2t(\lambda - 1) \cdot \hat{x}_{un}^2 - \left( (\lambda - 1)(p_B - p_A) - 4t\lambda \right) \cdot \hat{x}_{un} + \left( 2(p_B - p_A) + \frac{t}{2}(3\lambda + 1) \right) \quad (24)$$

Solving this quadratic equation w.r.t.  $\hat{x}_{un}$  leads to the expression given in the lemma.  $\square$

## A.2 Relegated proof of Section 3

*Proof of Lemma 2.*

$$\begin{aligned} \phi' &= \frac{\partial q_A(\Delta p; \beta)}{\partial \Delta p} = -\frac{\partial q_A(\Delta p; \beta)}{\partial p_A} = -\frac{\partial q_B(\Delta p; \beta)}{\partial \Delta p} = -\frac{\partial q_B(\Delta p; \beta)}{\partial p_B} \\ &= \beta \cdot \hat{x}'_{in}(\Delta p) + (1 - \beta) \cdot \hat{x}'_{un}(\Delta p) \\ &= -\frac{1}{4t}(1 - 3\beta) - \frac{(1 - \beta)}{2(S(\Delta p))} \underbrace{\left( \frac{\Delta p}{8t^2} - \frac{(\lambda + 2)}{2t(\lambda - 1)} \right)}_{\ominus} > 0 \end{aligned}$$

$\phi' > 0 \quad \forall \Delta p$  feasible and  $\forall \beta$ . At the boundaries we have

$$\begin{aligned} \phi'(0; \beta) &= -\frac{1}{4t}(1 - 3\beta) + (1 - \beta) \frac{(\lambda + 2)}{2t(\lambda - 1)} > 0 \\ \phi'(\Delta p \rightarrow \Delta \bar{p}; \beta < 1) &\rightarrow \infty \quad \text{since } S(\Delta \bar{p}) = 0. \end{aligned}$$

For  $0 \leq \Delta p < \Delta p^{max}$  the demand of  $A$  is convex in  $\Delta p$ . At the boundaries we have

$$\phi''(\Delta p; \beta) = (1 - \beta) \cdot \hat{x}''_{un}(\Delta p) = (1 - \beta) \cdot \frac{(3 + \lambda)(5 + 3\lambda)}{64t^2 \cdot (S(\Delta p))^3} \geq 0$$

$\phi'' > 0 \quad \forall \Delta p$  feasible and  $\forall \beta < 1$  since  $S(\Delta p) \geq 0$ :

$$\begin{aligned} \phi''(0; \beta) &= (1 - \beta) \cdot \frac{(3 + \lambda)(5 + 3\lambda)}{32t^2 \cdot \frac{(\lambda + 1)^3}{(\lambda - 1)^3}} > 0 \\ \phi''(\Delta p \rightarrow \Delta \bar{p}; \beta < 1) &\rightarrow \infty. \end{aligned}$$

$\square$

### A.3 Relegated proof of Section 5

*Proof of Proposition 2.*

$$\begin{aligned}\frac{d(\Delta p^*)}{d(\Delta c)} &= -\frac{(\phi')^2}{3(\phi')^2 + \phi''(1-2\phi)} \cdot (-1) \\ &= \frac{(\phi')^2}{3(\phi')^2 + \phi''(1-2\phi)}\end{aligned}\quad (25)$$

Since  $\phi'$  is strictly positive and denominator of  $d(\Delta p^*(\Delta c))/d(\Delta c)$  is equivalent to the tangent condition (34). We obtain that

$$\frac{d(\Delta p^*)}{d(\Delta c)} > 0 \quad (26)$$

if  $\Delta p < \Delta p^{ta}(\lambda, t)$  (the latter term is defined in Appendix B). Moreover, since  $\phi''(1-2\phi) = 0$  for  $\Delta c = 0$  (i.e.  $\Delta p = 0$ , compare symmetric equilibrium) and  $\phi''(1-2\phi) \leq 0$  for  $\Delta c > 0$  it holds true that  $d(\Delta p^*(\Delta c))/d(\Delta c) \geq 1/3$ .  $\square$

*Proof of Proposition 3.*

$$\frac{dm_A^*(\Delta p^*(\Delta c))}{d(\Delta c)} = \frac{\partial m_A^*}{\partial(\Delta p^*)} \cdot \frac{\partial(\Delta p^*)}{\partial \Delta c},$$

where by ( $FOC_A$ )

$$\frac{\partial m_A^*}{\partial(\Delta p^*)} = \frac{\partial p_A^*}{\partial(\Delta p^*)} = \frac{(\phi')^2 - \phi'' \cdot \phi}{(\phi')^2} \geq 0, \quad (27)$$

which may be positive or negative for  $\beta < 1$ . Firm A's markup is increasing in the price difference if the price difference is rather low and the share of uninformed consumers is not too high. It is decreasing for large price differences and/or if the share of uninformed consumers is high. Using (25) we obtain that

$$\frac{dm_A^*(\Delta p^*(\Delta c))}{d(\Delta c)} = \frac{(\phi')^2 - \phi'' \cdot \phi}{3(\phi')^2 + \phi''(1-2\phi)} \geq 0. \quad (28)$$

Hence  $m_A^*$  is not strictly increasing in  $\Delta p^*$ . Firm A's markup decreases in the price difference if the price difference, i.e. if the cost asymmetries in the industry, and/or the share of uninformed consumers become too large. (Compare markup of firm B.)  $\square$

## A.4 Relegated proofs of Section 6

*Proof of Proposition 5.* Recall that the equilibrium is implicitly characterized by

$$\Delta p - \Delta c - \frac{1 - 2\phi(\Delta p; \beta)}{\phi'(\Delta p; \beta)} = 0$$

The equilibrium price difference then satisfies

$$\begin{aligned} \frac{d\Delta p^*(\beta)}{d\beta} &= -\left(1 - \frac{-2(\phi')^2 - \phi''(1 - 2\phi)}{(\phi')^2}\right)^{-1} \left(-\frac{-2\phi' \frac{\partial \phi}{\partial \beta} - \frac{\partial \phi'}{\partial \beta}(1 - 2\phi)}{\phi'^2}\right) \\ &= -\frac{(\phi')^2}{3(\phi')^2 + \phi''(1 - 2\phi)} \cdot \left(\frac{2\phi' \phi_\beta + \phi'_\beta - 2\phi'_\beta \phi}{(\phi')^2}\right) \\ &= -\frac{2\phi' \phi_\beta + \phi'_\beta(1 - 2\phi)}{3(\phi')^2 + \phi''(1 - 2\phi)} \end{aligned}$$

We show that the numerator of  $\frac{d(\Delta p^*(\beta))}{d\beta}$ , denoted by  $N(\Delta p^*; \beta) = -(2\phi' \phi_\beta + \phi'_\beta(1 - 2\phi))$  is negative: For all  $\Delta p$  with  $0 \leq \Delta p \leq \Delta p^{max}$  and for all  $\beta \in [0, 1]$ , we can rewrite

$$\begin{aligned} N(\Delta p; \beta) &= -2\phi' \phi_\beta - \phi'_\beta(1 - 2\phi) = 2((1 - \beta)\hat{x}'_{un} + \beta\frac{1}{2t}) \cdot (\hat{x}_{un} - \hat{x}_{in}) \\ &\quad + (\hat{x}'_{un} - \frac{1}{2t})(1 - 2(1 - \beta)\hat{x}_{un} - 2\beta\hat{x}_{in}) \\ &= \frac{1}{t}(\hat{x}_{un} - \hat{x}_{in}) + (\hat{x}'_{un} - \frac{1}{2t})(1 - 2\hat{x}_{in}) \\ &= \frac{1}{t}\hat{x}_{un} + (\hat{x}'_{un})(1 - 2\hat{x}_{in}) - \frac{1}{2t} \\ &= \frac{1}{t}(\hat{x}_{un} + \frac{1}{2}) - \hat{x}'_{un}(2\hat{x}_{in} - 1) \\ &= -2t\hat{x}'_{un} \cdot (\hat{x}_{in} - \frac{1}{2}) + 1(\hat{x}_{un} - \frac{1}{2}) \\ &= -2t\hat{x}'_{un}(\Delta p)(\hat{x}_{in}(\Delta p) - \frac{1}{2}) + (\hat{x}_{un}(\Delta p) - \frac{1}{2}) \end{aligned}$$

Since  $N(0; \beta) = 0$  and

$$\begin{aligned} \frac{\partial N(\Delta p; \beta)}{\partial \Delta p} &= -\frac{1}{t} \left( 2t\hat{x}''_{un}(\Delta p)(\hat{x}_{in}(\Delta p) - \frac{1}{2}) + 2t(\hat{x}'_{un}(\Delta p))(\hat{x}'_{in}(\Delta p)) - \hat{x}'_{un}(\Delta p) \right) \\ &= -\frac{1}{t} \left( 2t\hat{x}_{un}(\Delta p)(\hat{x}_{in}(\Delta p) - \frac{1}{2}) + 0 - 0 \right) < 0 \end{aligned}$$

it holds that  $N(\Delta p^*; \beta) \leq 0$  for all admissible  $\Delta p, \beta$ .

Consider now the denominator of  $\frac{d(\Delta p^*(\beta))}{d\beta}$ , denoted by  $D(\Delta p^*; \beta) = 3(\phi')^2 + \phi''(1 - 2\phi)$ .

We show that, on the relevant domain of price differences,  $D(\Delta p^*; \beta)$  is strictly positive.

We have that

$$\begin{aligned} D(0; \beta) &= 3(\phi'(0; \beta))^2 + \phi''(0; \beta) \cdot 0 \\ &= 3(\phi'(0; \beta))^2 > 0 \end{aligned}$$

The sign of the derivative is of ambiguous sign:

$$\begin{aligned} \frac{\partial D(\Delta p; \beta)}{\partial \Delta p} &= 6\phi'\phi'' + \phi'''(1 - 2\phi) - 2\phi''\phi' \\ &= 4\phi'\phi'' + \phi'''(1 - 2\phi) \end{aligned}$$

Thus  $D(\Delta p^*; \beta)$  is not necessarily non-negative. However, since  $D(\Delta p^*; \beta)$  is equivalent to the tangent condition (34) which approaches zero at  $\Delta p = \Delta p^{ta}(\lambda, t)$  we conclude that

$$\frac{d\Delta p^*(\beta)}{d\beta} < 0 \quad (29)$$

for  $\Delta p < \Delta p^{ta}(\lambda, t)$ , which is the relevant domain for equilibrium existence.  $\square$

*Proof of Proposition 6.* We evaluate

$$\frac{dp_A^*(\Delta p^*(\beta); \beta)}{d\beta} = \frac{\partial p_A^*}{\partial(\Delta p^*)} \cdot \frac{\partial(\Delta p^*)}{\partial\beta} + \frac{\partial p_A^*}{\partial\beta},$$

where

$$\frac{\partial p_A^*}{\partial(\Delta p^*)} = \frac{(\phi')^2 - \phi'' \cdot \phi}{(\phi')^2} \geq 0,$$

which may be positive or negative. Hence  $p_A^*$  is not strictly increasing in  $\Delta p^*$ . Firm A's prices decreases in the price difference if the price difference becomes sufficiently large. In terms of the parameters of the model this means that the cost asymmetries in the industry (and the share of uninformed consumers) becomes sufficiently large.

$$\begin{aligned} \frac{\partial p_A^*}{\partial\beta} &= \frac{\phi'\phi_\beta - \phi'_\beta\phi}{(\phi')^2} \\ &= -\left[ ((1-\beta)\hat{x}'_{un} + \beta\hat{x}'_{in})(\hat{x}_{un} - \hat{x}_{in}) - (\hat{x}'_{un} - \hat{x}'_{in}) \cdot ((1-\beta)\hat{x}_{un} + \beta\hat{x}_{in}) \right] \cdot \frac{1}{\phi'^2} \\ &= -\left[ (1-\beta)(\hat{x}'_{un} - \frac{1}{2t})(\hat{x}_{un} - \hat{x}_{in}) - (1-\beta)(\hat{x}'_{un} - \frac{1}{2t})(\hat{x}_{un} - \hat{x}_{in})\frac{1}{2t}(\hat{x}_{un} - \hat{x}_{in}) - (\hat{x}_{un} - \frac{1}{2t})\hat{x}_{in} \right] \cdot \frac{1}{\phi'^2} \\ &= -\left[ \frac{1}{2t}\hat{x}_{un} - \hat{x}'_{un}\hat{x}_{in} \right] \cdot \frac{1}{\phi'^2} \end{aligned}$$

The numerator of  $\frac{\partial p_A^*}{\partial \beta}$  is independent of  $\beta$ .

$$\frac{\partial p_A^*}{\partial \beta}(\Delta p = 0) = -\frac{1}{2} \left( \frac{1}{2t} - \hat{x}'_{un}(0) \right) \cdot \frac{1}{\phi'(0)^2} < 0$$

$$\frac{\partial p_A^*}{\partial \beta}(\Delta p = \Delta \bar{p} - \epsilon) = -\frac{\left( \frac{1}{2t} \hat{x}_{un} - \hat{x}'_{un} \hat{x}_{in} \right)}{\phi'^2} > 0$$

for  $\epsilon$  small because the numerator is positive for  $\Delta p$  slightly less than  $\Delta \bar{p}$ . This implies that  $\frac{\partial p_A^*}{\partial \beta} = 0$  for a critical  $\Delta p \in (0, \Delta p^{max})$ ,  $\forall \beta$ .  $\square$

*Proof of Proposition 7.* We evaluate

$$\frac{dp_B^*(\Delta p^*(\beta); \beta)}{d\beta} = \frac{\partial p_B^*}{\partial(\Delta p^*)} \cdot \frac{\partial(\Delta p^*)}{\partial \beta} + \frac{\partial p_B^*}{\partial \beta},$$

$$\text{where } \frac{\partial p_B^*}{\partial(\Delta p^*)} = \frac{-(\phi')^2 - \phi''(1 - \phi)}{(\phi')^2} = -\left(1 + \frac{\phi''(1 - \phi)}{(\phi')^2}\right) < 0$$

In contrast to  $A$ , the price of  $B$  is always decreasing in  $\Delta p^*(\beta)$ .

$$\begin{aligned} \frac{\partial p_B^*}{\partial \beta} &= \frac{-\phi' \phi_\beta - \phi'_\beta (1 - \phi)}{(\phi')^2} \\ &= -\left[ -((1 - \beta) \hat{x}'_{un} + \beta \frac{1}{2t})(\hat{x}_{un} - \hat{x}_{in}) - (\hat{x}'_{un} - \frac{1}{2t})(1 - (1 - \beta) \hat{x}_{un} - \beta \hat{x}_{in}) \right] \cdot \frac{1}{(\phi')^2} \\ &= -\left[ -(1 - \beta)(\hat{x}'_{un} - \frac{1}{2t})(\hat{x}_{un} - \hat{x}_{in}) + (1 - \beta)(\hat{x}'_{un} - \frac{1}{2t})(\hat{x}_{un} - \hat{x}_{in}) \right. \\ &\quad \left. - \frac{1}{2t}(\hat{x}_{un} - \hat{x}_{in}) - (\hat{x}'_{un} - \frac{1}{2t})(1 - \hat{x}_{in}) \right] \cdot \frac{1}{(\phi')^2} \\ &= -\left[ -\frac{1}{2t}(\hat{x}_{un}) - (\hat{x}'_{un} - \frac{1}{2t}) + \hat{x}'_{un} \hat{x}_{in} \right] \cdot \frac{1}{(\phi')^2} \leq 0 \end{aligned}$$

$\square$

## B Equilibrium Existence and Uniqueness

### B.1 Equilibrium uniqueness

In Proposition 10 we state sufficient conditions under which an interior equilibrium is unique. Given parameters  $\lambda$  and  $t$ , the condition states that the cost asymmetry between



firms is not too large.<sup>38</sup>

**Proposition 10.** *An interior equilibrium is unique if*

$$\Delta c < \Delta c^u(\lambda) = \Delta \bar{p} \equiv \frac{2t}{(\lambda - 1)} \left( 2(\lambda + 2) - \sqrt{(2(\lambda + 2))^2 - (\lambda + 1)^2} \right), \quad (30)$$

where  $\Delta \bar{p}$  depicts the upper bound of  $\Delta p$  such that the  $S(\Delta p)$  in  $\hat{x}_{in}(\Delta p)$  is equal to zero.<sup>39</sup>

It is easy to check that  $\Delta c^u(\lambda)$  is strictly decreasing in  $\lambda$ . This means that in markets in which consumers show a higher degree of loss aversion cost asymmetries between firms should be less pronounced to meet the uniqueness condition.

*Proof.* We first consider the case of  $\lambda > \lambda^c \approx 7.47$ . We can derive a number of useful properties of  $f(\Delta p; \beta) = (1 - 2\phi)/\phi'$ :

$f(0; \beta) = 0/\phi'(0) = 0 \forall \beta$ ,  $f(\Delta \bar{p}; \beta) \rightarrow 0$  since  $\phi'(\Delta \bar{p}) \rightarrow \infty \forall \beta < 1$ , and  $f(\Delta \bar{p}; 1) = -2\Delta \bar{p} < 0$ .

$$f'(\Delta p; \beta) = \frac{-2(\phi')^2 - \phi''(1 - 2\phi)}{(\phi')^2} = -\left(2 + \frac{\phi''(1 - 2\phi)}{(\phi')^2}\right) \leq 0 \quad \forall \beta < 1,$$

since  $f'(0; \beta) = -2 < 0 \quad \forall \beta$  and  $f'(\Delta \bar{p}; \beta) \rightarrow \infty \quad \forall \beta < 1$ . Moreover,  $f'(\Delta p; 1) = -2 \quad \forall \Delta p$ .

It has to be shown that  $f(\Delta p; \beta)$  is strictly convex in  $\Delta p$  for  $\beta < 1$ . We find that

$$f''(\Delta p; \beta) = -\frac{(\phi' \phi''' - 2(\phi'')^2)(1 - 2\phi) - 2(\phi')^2}{(\phi')^3} > 0.$$

Figure 2 illustrates the equilibrium condition (10) at  $\Delta c = \Delta \bar{p}$ . Now, if  $\beta < 1$  by continuity of  $f(\Delta p)$  for  $\Delta p \in [0; \Delta \bar{p})$ ,  $f(0; \beta) = 0$ ,  $f(\Delta \bar{p}; \beta) \rightarrow 0$ ,  $f'(0; \beta) < 0$ ,  $f'(\Delta \bar{p}; \beta) \rightarrow \infty > 1$ , and strict convexity of  $f(\Delta p)$  for  $\beta < 1$ , we know that for  $\Delta c > \Delta \bar{p}$  there are two candidate interior equilibria since the  $(f(\Delta p) + \Delta c)$ -curve moves up and intersects the  $\Delta p$ -line twice. At  $\Delta c = \Delta \bar{p}$  a second solution to  $\Delta p = f(\Delta p; \beta < 1) + \Delta \bar{p}$  does not arise due to the discontinuity of  $\phi$  (resp.  $f(\Delta p; \beta < 1)$ ) at  $\Delta \bar{p}$ . Moreover, for values of  $\Delta c$  lower than  $\Delta \bar{p}$ ,  $(f(\Delta \bar{p}; \beta < 1) + \Delta c)$  is always smaller than  $\Delta \bar{p}$  and no second equilibrium can arise.

<sup>38</sup>Since  $t$  turns out to just be a scalar of equilibrium prices (cf. Section 4), we will neglect  $t$  as a parameter in Appendix B.

<sup>39</sup>Cf. equation (5).

If  $\beta = 1$ ,  $f(\Delta p; \beta)$  is strictly decreasing for all  $\Delta p$  and at most one intersection between  $f(\Delta p; 1) + \Delta c$  and  $\Delta p$  exists (standard Hotelling case).<sup>40</sup>

Secondly, in the case of  $1 \leq \lambda < \lambda^c$  all uninformed consumers buy from firm  $A$  at  $\Delta p = \Delta \tilde{p}$ , which is smaller than  $\Delta \bar{p}$ .<sup>41</sup> Since  $f$  is continuous here,  $f(\Delta \tilde{p}; \beta) < 0$ , and  $f(\Delta p; \beta) = (1 - 2(\beta \hat{x}_{in}(\Delta p) + (1 - \beta))) \cdot 2t/\beta$  is strictly decreasing for  $\Delta p > \Delta \tilde{p}$ ,  $\Delta c < \Delta \bar{p}$  is sufficient to rule out second equilibria in this case.  $\square$

## B.2 Equilibrium existence

For any interior solution, concavity of the profit functions would assure that the solution characterizes an equilibrium.

$$\begin{aligned} \frac{\partial^2 \pi_A}{\partial p_A^2} &= -2\phi' + (p_A - c_A)\phi'' < 0 & (SOC_A) \\ \frac{\partial^2 \pi_B}{\partial p_B^2} &= -2\phi' - (p_B - c_B)\phi'' < 0. & (SOC_B) \end{aligned}$$

Given the properties of  $\phi$  —particularly that  $\phi$  is strictly increasing and convex in  $\Delta p$  for  $\beta < 1$ —  $SOC_B$  holds globally, while  $SOC_A$  is not necessarily satisfied. Using that  $(p_A - c_A) = \phi/\phi'$  by  $FOC_A$ ,  $SOC_A$  can be expressed as follows

$$-2(\phi')^2 + \phi\phi'' < 0. \quad (31)$$

It can be easily shown that (31) is satisfied for small  $\Delta p$  while it is violated for  $\Delta p \rightarrow \Delta \bar{p}$  as  $\phi''$  goes faster to infinity in  $\Delta p$  than  $(\phi')^2$ .<sup>42</sup> This violation of  $SOC_A$  reflects that firm  $A$  has an increasing interest to non-locally undercut prices to gain the entire demand of uninformed consumers when  $\Delta p$  is large. The driving force behind this is that loss aversion in the price dimension increasingly dominates loss aversion in the taste dimension if price differences become large. Moreover, excessive losses in the price dimension if buying the expensive product  $B$  make also nearby consumers of  $B$  more willing to opt for product  $A$ .

The next proposition clarifies the issue of equilibrium existence. It deals with the non-concavity of firm  $A$ 's profit function by determining critical levels of market asymmetries

<sup>40</sup>An analytical solution for (10) can be determined in this case:  $\Delta p^* = \Delta c/3$ .

<sup>41</sup>Cf. Figure 2.

<sup>42</sup>This implies that  $\pi_A$  is not globally concave. It is easy to check that it is neither globally quasi-concave. This is illustrated in Figure 8. Moreover, the non-concavity of  $\pi_A$  is increasing in  $\Delta p$  (resp.  $-p_A$ ) for  $\Delta p \leq \Delta p^{max}$  (resp.  $p_A \geq p_B - \Delta p^{max}$ ).

and the degree of loss aversion such that firm  $A$  has no incentive to non-locally undercut prices. Here, we make use of the increasing convexity of firm  $A$ 's profit function which yields that stealing the entire demand of uninformed consumers is the unique optimal deviation of firm  $A$ . For notational convenience, we focus on the most critical setting for equilibrium existence. This is the one in which all consumers are uninformed.<sup>43</sup> In the proposition, moreover, it is shown that non-interior equilibria fail to exist.

**Proposition 11.** *Suppose all consumers are uninformed ( $\beta = 0$ ) and the degree of loss aversion,  $\lambda$ , lies within the interval  $(1, 1 + 2\sqrt{2}]$ . An interior equilibrium with prices  $(p_A^*, p_B^*)$  exists if and only if*

$$\Delta c \leq \Delta c^{nd}(\lambda) \equiv \Delta p^{nd}(\lambda) - f(\Delta p^{nd}(\lambda); 0), \quad (32)$$

with  $\Delta p^{nd}(\lambda)$  being implicitly determined by the following non-deviation condition

$$\Delta p^{nd}(\lambda) = \left\{ 0 \leq \Delta p < \Delta p^{max} \mid \Delta p = \Delta p^{max} - \frac{\phi(\Delta p) \cdot (1 - \phi(\Delta p))}{\phi'(\Delta p)} \right\}. \quad (33)$$

Moreover, any equilibrium is interior.

Before turning to the proof, let us comment on this proposition. The result shows that an equilibrium exists if firm  $A$  has no incentive to non-locally undercut prices. In fact, the incentive to undercut prices increases in more asymmetric industry or for more loss-averse consumers. For a low degree of loss aversion ( $1 < \lambda < 1 + 2\sqrt{2} \approx 3.828$ ) equilibrium exists if the cost difference between firms is not too large (see (32)).<sup>44</sup> In this case, an equilibrium exists for all values of  $\beta$ . However, if the degree of loss aversion rises further, equilibria only exist if there is a sufficiently large share of informed consumers which reduces the undercutting incentive of firm  $A$ . This tradeoff is illustrated in Table 2 below. The possible non-existence due to undercutting even holds for symmetric industries. Proposition 12 deals with this issue. It shows that if the share of informed consumers is sufficiently large, a symmetric equilibrium always exists; e.g. if 60% (which is greater than the derived upper bound of 57.7%) of the consumers are informed then an equilibrium exists in symmetric industries for *any* level of loss aversion  $\lambda > 1$ . See also Figure 10 below.

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<sup>43</sup>Adding more informed consumers always reduces the non-concavity of firm  $A$ 's profit function since the demand of informed consumers is linear. Thus, the derived upper bound on cost asymmetries with only uninformed consumers is sufficient for existence with a positive share of informed consumers.

<sup>44</sup>Note that according to experimental work on loss aversion  $\lambda$  takes the value of approximately 3, which is within this range.

Table 2: Non-deviation condition

Variation of  $\Delta p^{nd}$  and  $\Delta c^{nd}$  in  $\beta$  and  $\lambda$ . ( $t = 1$ )

	$\lambda = 3$		$\lambda = 6$		$\lambda = 9$	
$\beta$	$\Delta p^{nd}(\lambda, \beta)$	$\Delta c^{nd}(\lambda, \beta)$	$\Delta p^{nd}(\lambda, \beta)$	$\Delta c^{nd}(\lambda, \beta)$	$\Delta p^{nd}(\lambda, \beta)$	$\Delta c^{nd}(\lambda, \beta)$
1.0	-	-	-	-	-	-
0.8	0.648337	1.75869	0.372669	1.07069	0.294726	0.857815
0.6	0.543254	1.45317	0.23824	0.686206	0.150303	0.440498
0.4	0.459237	1.22329	0.107415	0.314749	0.000320	0.000959
0.2	0.377489	1.00993	-	-	-	-
0.0	0.278889	0.75963	-	-	-	-

In the proof we first provide the critical level of  $\Delta c$  for which the equilibrium condition in (10) is satisfied for candidate interior equilibria. We next identify the set of interior equilibria which are robust to non-local price deviations of firm A. Finally, the existence of non-interior equilibria is refuted.

*Proof of Proposition 11.*

1. To find an upper bound on  $\Delta c$  for which the equilibrium condition (10) is satisfied we determine the point at which  $f(\Delta p; \beta)$  is a tangent on the  $\Delta p$ -line. In Figure 2 this corresponds to an upward shift of the  $f(\Delta p; \beta)$ -curve.

*Tangent condition:*

$$f'(\Delta p; \beta) = 1 \quad \Leftrightarrow \quad 3(\phi')^2 + \phi''(1 - 2\phi) = 0 \quad (34)$$

An analytical solution to  $3(\phi')^2 + \phi''(1 - 2\phi) = 0$  can be found for  $\beta = 0$ .<sup>45</sup> Denote this critical price difference as  $\Delta p^{ta}(\lambda)$ .<sup>46</sup>

Then, the equilibrium condition in (10) is fulfilled if and only if  $\Delta c$  satisfies the following condition

$$\Delta c \leq \Delta c^{ta}(\lambda) \equiv \Delta p^{ta}(\lambda) - f(\Delta p^{ta}(\lambda); 0). \quad (35)$$

<sup>45</sup>This is sufficient since  $\beta = 0$  is the most critical case w.r.t. existence and uniqueness. The reason for this is that for  $\beta > 0$  there is a positive weight on the demand of informed consumers which is purely linear.

<sup>46</sup> $\Delta p^{ta}(\lambda)$  is decreasing in  $\lambda$ .

To derive  $\Delta c^{ta}(\lambda)$  from  $\Delta p^{ta}(\lambda)$ , the equilibrium condition (10) can directly be applied because at the tangent point there is a unique relationship between the two variables.<sup>47</sup>

2. We next rule out some candidate interior equilibria. First suppose that at  $\Delta p = \Delta p'$   $SOCA$  is not satisfied. Then  $\Delta p'$  depicts a profit minimum for firm A.  $\Delta p'$  cannot be an equilibrium. Now, define  $\Delta p^s(\lambda)$  as the critical price difference which satisfies the transformed second-order condition of firm A (31) with equality.  $\Delta p^s(\lambda)$  is unique by strictly increasing convexity of  $\pi_A$  in  $-p_A$ . Thus,  $SOCA$  holds for  $\Delta p \leq \Delta p^s(\lambda)$ . We next show that  $SOCA$  implies the tangent condition, i.e.  $\Delta p^s(\lambda) < \Delta p^{ta}(\lambda)$ . Rearranging (31) and (34) leads to

$$\frac{\phi}{2} \leq \frac{(\phi')^2}{\phi''}, \quad (31)'$$

$$\frac{(2\phi - 1)}{3} \leq \frac{(\phi')^2}{\phi''}. \quad (34)'$$

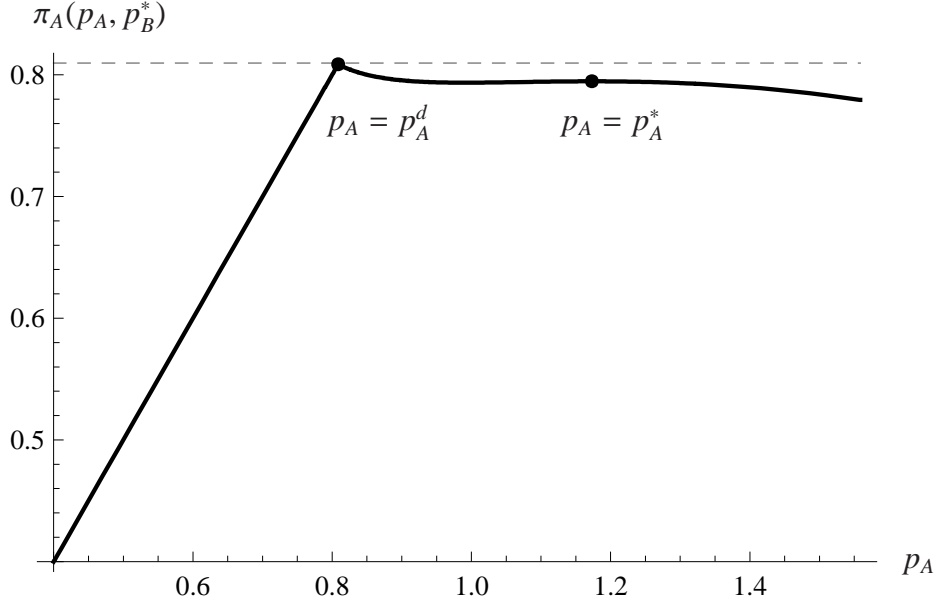
It is left to show that  $\phi/2 > (2\phi - 1)/3$  which is equivalent to  $\phi/6 - 1/3 < 0$ . This inequality is satisfied  $\forall \phi \in [1/2, 1]$ . Hence, it follows that a non-empty set of candidate interior equilibria is ruled out by local non-concavity.

Secondly, due to the increasing convexity of  $\pi_A$  in  $-p_A$  also some candidate interior equilibria which locally satisfy  $SOCA$  might be ruled out. This is the case when the convexity is sufficiently large and non-local price decreases become profitable for firm A. An example of this kind is presented in Figure 8. Given the increasing convexity of  $\pi_A$  the unique optimal deviation of firm A (if it exists) is characterized by firm A serving the entire market of uninformed consumers, i.e.  $p_A^d$  s.t.  $\Delta p^d = \Delta p^{max}$ . Decreasing  $p_A^d$  further is not profitable since firm A only attracts informed consumers while its profit margin goes down for informed and uninformed consumers.<sup>48</sup> Hence, in the following we will restrict our attention to price deviations of firm A that steal the entire demand of uninformed consumers. If deviating is profitable, firm A sets  $p_A^d = p_B^* - \Delta p^{max}$ . For  $\beta = 0$  the firm A's deviation profit,  $\pi_A^d$ , is equal to  $(p_A^d - c_A) \cdot 1$  since  $\phi(\Delta p^{max}, 0) = 1$ . Using that  $p_A^d = p_B^* - \Delta p^{max}$  we receive

$$\pi_A^d = \left( p_B^* - \Delta p^{max} - c_A \right) \cdot 1$$

<sup>47</sup>For  $\Delta c^u(\lambda) \leq \Delta c < \Delta c^{ta}(\lambda)$  there might arise two candidate interior equilibria. However as we see next, the second one does not survive the local  $SOCA$  criterion.

<sup>48</sup>For situations with  $\lambda \rightarrow 1$ , in which  $\Delta p^* > \Delta p^{max}$  can arise, it can be shown that non-concavity of  $\pi_A$  is not a problem.



Profit of firm A,  $\pi_A(p_A, p_B^*)$ , as a function of its own price  $p_A$  given  $p_B = p_B^*$  for  $\Delta c = 1$  ( $c_A = 0, c_B = 1$ ) and parameter values of  $\beta = 0$ ,  $t = 1$ , and  $\lambda = 3$ :  $p_A^* = 1.17309$ ,  $p_A^d = p_B^* - \Delta p^{max} = 0.80863$ ,  $p_B^* = 1.55863$ ,  $\Delta p^* = 0.385537$ , and  $\Delta p^{max} = \Delta \tilde{p} = 3/4$ .

Figure 8: Non-existence

$$\begin{aligned}
 &= \left( \frac{1 - \phi}{\phi'} + \Delta c - \Delta p^{max} \right) \cdot 1 && \text{by } FOC_B \\
 &= \left( \Delta p^* + \frac{\phi}{\phi'} - \Delta p^{max} \right) \cdot 1 && \text{by (10)} \quad (36)
 \end{aligned}$$

For non-deviation, firm A's profit is equal to  $\pi_A(\Delta p^*) = (p_A^* - c_A)\phi$ , which is equivalent to  $\phi^2/\phi'$  by  $FOC_A$ .

Thus, deviation of firm A is not profitable if and only if  $\pi_A(\Delta p^*) \geq \pi_A^d$ .<sup>49</sup> Rearranging yields

$$\Delta p^* \leq \Delta p^{max} - \frac{\phi \cdot (1 - \phi)}{\phi'}, \quad (37)$$

which describes the required non-deviation condition if we define  $\Delta p^{nd}(\lambda)$  as the price difference  $\Delta p \neq \Delta p^{max}$  that satisfies (37) with equality, i.e.

$$\Delta p^{nd}(\lambda) = \Delta p^{max} - \frac{\phi(\Delta p^{nd}(\lambda); 0) \cdot (1 - \phi(\Delta p^{nd}(\lambda); 0))}{\phi'(\Delta p^{nd}(\lambda); 0)}.$$

<sup>49</sup>We assume that firm A does not deviate from an interior strategy if it is indifferent between deviating and playing the interior best-response.

Lemma 5 below shows that  $\Delta p^{nd}(\lambda)$  is uniquely determined by this non-deviation condition if the trivial solution,  $\Delta p^{max}$ , is not considered and that the set of non-negative  $\Delta p^{nd}(\lambda)$  is non-empty for  $\lambda \in (1, 1 + 2\sqrt{2}]$ .

Again by utilizing the equilibrium condition (10) we receive existence of interior equilibria if and only if  $\Delta c \leq \Delta c^{nd}(\lambda) \equiv \Delta p^{nd}(\lambda) - f(\Delta p^{nd}(\lambda))$ .<sup>50</sup> Taken together due to increasing convexity of  $\pi_A$  the non-deviation condition implies local concavity of the firms' profit function and therefore, as shown above, the tangent condition. Thus, any price difference which satisfies the non-deviation condition lies in the set of candidate interior equilibria or equivalently,  $\Delta p^{nd}(\lambda) < \Delta p^s(\lambda) < \Delta p^{ta}(\lambda) < \Delta p^{max}(\lambda)$ .

3. Any equilibrium is interior because discontinuity of firm A's best response function due to non-concavity of its profit function rules out non-interior equilibria.

□

**Lemma 5.** For  $\beta = 0$  and  $\lambda \in (1, 1 + 2\sqrt{2}]$ ,  $\Delta p^{nd}(\lambda)$  is the unique non-trivial solution (i.e.  $\Delta p \neq \Delta p^{max}$ ) to the non-deviation condition in (33),

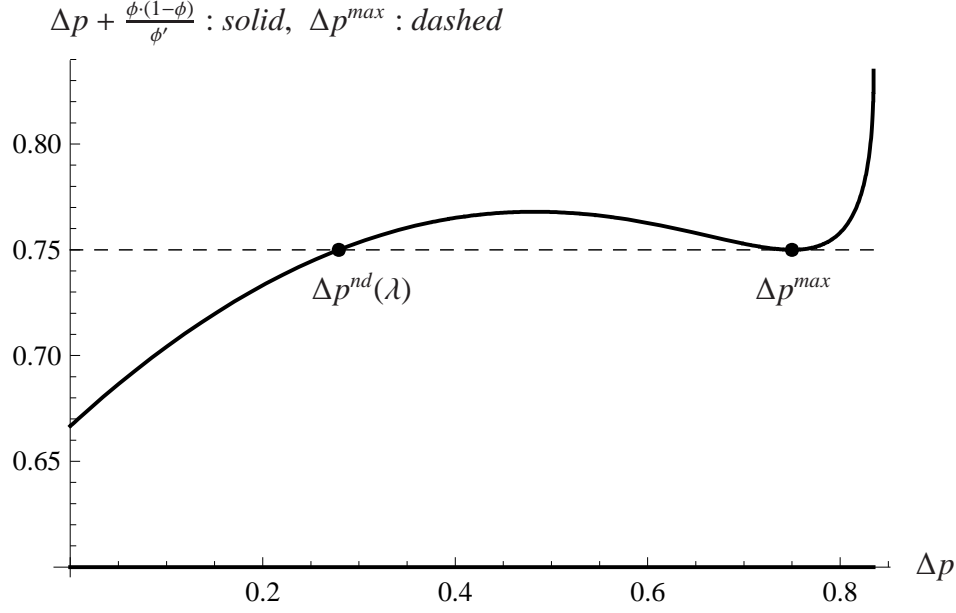
$$\Delta p = \Delta p^{max} - \frac{\phi(\Delta p) \cdot (1 - \phi(\Delta p))}{\phi'(\Delta p)}.$$

Moreover,  $\Delta p^{nd}(\lambda)$  is non-negative.

*Proof of Lemma 5.* First note that the non-deviation condition is trivially satisfied at  $\Delta p = \Delta p^{max}$  since  $\phi(\Delta p^{max}) = 1$  for  $\beta = 0$  (see Figure 9 below for a graphical illustration of the non-deviation condition). It can be shown that  $A(\Delta p) \equiv \Delta p + \phi(1 - \phi)/\phi'$  approaches  $\Delta p^{max}$  from above for  $\Delta p < \Delta p^{max}$ . For  $\Delta p \geq 0$  but  $\Delta p$  being small,  $A(\Delta p)$  is strictly increasing and strictly concave. Moreover,  $A(\Delta p)$  is continuous and exhibits at most one optimum for  $\Delta p \in [0, \Delta p^{max})$ . Taken together, there exists a unique  $\Delta p \in [0, \Delta p^{max})$  at which the non-deviation condition is satisfied if and only if at  $\Delta p = 0$ ,  $A(\Delta p)$  is smaller or equal than  $\Delta p^{max}$ . For  $\beta = 0$ ,  $A(0) = (\lambda + 3)/(4t(\lambda + 1))$  and  $\Delta p^{max} = \Delta \tilde{p} = (\lambda + 3)t/(2(\lambda + 1))$ . It is easy to check that the  $A(0) \leq \Delta p^{max}$  if and only if  $\lambda \in (1, 1 + 2\sqrt{2}]$ . Denoting the solution to the non-deviation condition by  $\Delta p^{nd}(\lambda)$  completes the proof.<sup>51</sup> □

<sup>50</sup>For  $\Delta c^u(\lambda) \leq \Delta c < \Delta c^{nd}(\lambda)$  the equilibrium condition (10) does not make a unique selection, i.e. there might arise a second solution to (10),  $\Delta p^{**}$ , which can be ruled out because by construction  $\Delta p^{**}$  is larger than  $\Delta p^{ta}(\lambda)$  and hence larger than  $\Delta p^{nd}(\lambda)$ . The unique interior equilibrium that survives the non-deviation condition would be selected by the following existence criterion  $\Delta c < \min\{\Delta c^{nd}(\lambda), \Delta c^u(\lambda)\}$ . This shows again that the uniqueness condition in Proposition 10 is only sufficient. Moreover, it can be seen here that any interior equilibrium (if it exists) must be unique.

<sup>51</sup>We receive  $\Delta p^{nd}(1 + 2\sqrt{2}) = 0$  and for  $\lambda \rightarrow 1$   $\Delta p^{nd}(\lambda) \rightarrow \Delta p^{max}$ .



Non-deviation condition of firm A, as a function of the price difference  $\Delta p$  for  $\Delta c = 0.25$  ( $c_A = 0.25, c_B = 0.5$ ) and parameter values of  $\beta = 0, t = 1$ , and  $\lambda = 3$ :  $\Delta p^{nd}(3) = 0.27889$ ,  $\Delta c^{nd}(3) = (\Delta p^{nd}(3) - f(\Delta p^{nd}(3); 0)) = 0.75963$ ,  $\Delta p^{max} = \Delta \tilde{p} = 3/4$ , and  $\Delta \tilde{p} = 0.83485$ . Non-deviation for  $\Delta p \leq \Delta p^{nd}(3) = 0.27889$ .

Figure 9: Non-deviation in asymmetric industries

If the degree of loss aversion becomes sufficiently high ( $\lambda > 1 + 2\sqrt{2} \approx 3.828$ ), the set of non-negative  $\Delta p^{nd}(\lambda)$  becomes empty. Here deviating is profitable even for symmetric industries ( $\Delta c = 0$ ). However, restricting the amount of uninformed consumers can reinforce existence of symmetric equilibria in this case. In Proposition 12 the critical level of  $\beta$  for symmetric equilibria to exist,  $\beta^{crit}(\lambda)$ , is derived as a function of  $\lambda$ .

**Proposition 12.** *An symmetric equilibrium with prices  $(p^*, p^*)$  exists if and only if  $\beta$  satisfies*

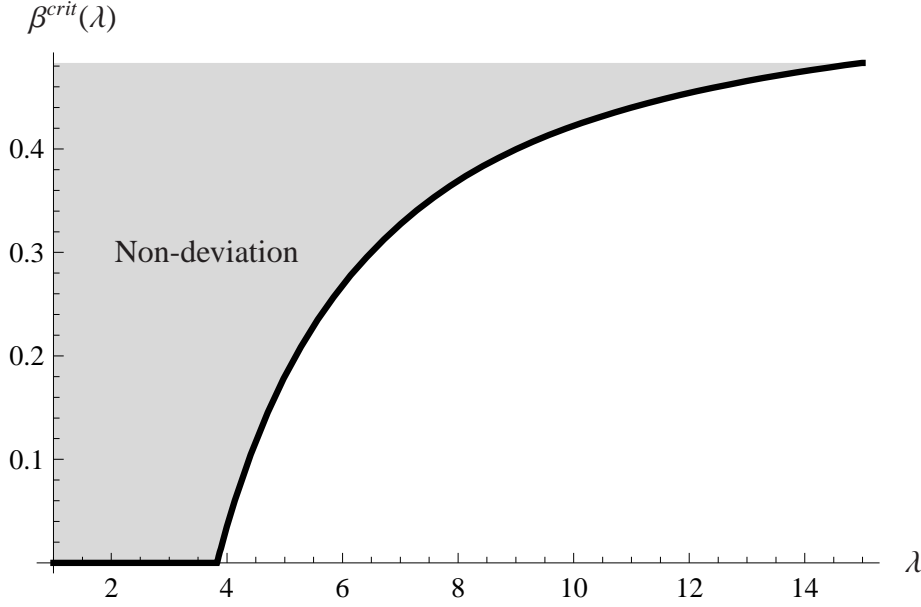
$$\beta \geq \beta^{crit}(\lambda), \quad (38)$$

with  $\beta^{crit}(\lambda)$  being an increasing function in  $\lambda$  which is expressed by

$$\beta^{crit}(\lambda) \equiv \begin{cases} 0, & \text{if } \lambda \in (1, 1 + 2\sqrt{2}]; \\ \beta_0^{crit}(\lambda) \in (0, 0.349], & \text{if } \lambda \in (1 + 2\sqrt{2}, \lambda^c]; \\ \beta_1^{crit}(\lambda) \in (0.349, 0.577), & \text{if } \lambda > \lambda^c \approx 7.47. \end{cases} \quad (39)$$

The critical level of  $\beta$  for existence of symmetric equilibria,  $\beta^{crit}(\lambda)$ , is depicted in Figure 10.





Critical share of informed consumers,  $\beta^{crit}(\lambda)$ , for which symmetric equilibria exist as a function of the degree of loss aversion  $\lambda > 1$ . Parameter values are  $\Delta c = 0$  and  $t = 1$ . Non-deviation for  $\beta \geq \beta^{crit}(\lambda)$ .

Figure 10: Non-deviation in symmetric industries

*Proof of Proposition 12.* It can be shown that at  $\Delta p = 0$  the non-deviation condition,  $A(\Delta p; \beta) - \Delta p^{max}$ , is continuous and monotonous in  $\beta$ .

For  $\lambda > 1 + 2\sqrt{2}$  the non-deviation condition can be reinforced at  $\Delta p = 0$  if  $\beta > 0$ . Solving for  $\beta^{crit}(\lambda)$  in  $A(0; \beta = \beta^{crit}(\lambda)) - \Delta p^{max} = 0$  yields

$$\beta_0^{crit}(\lambda) \equiv 1 - \frac{-\lambda(5\lambda + 14) + \sqrt{(3\lambda + 5)(\lambda(11\lambda(\lambda + 5) + 113) + 77)} - 13}{2(\lambda - 1)(\lambda + 3)}, \quad (40)$$

for  $\lambda \in (1 + 2\sqrt{2}, \lambda^c]$  (i.e.  $\Delta p^{max} = \Delta \tilde{p}$ ) and

$$\beta_1^{crit}(\lambda) \equiv 1 - \frac{37\lambda^3 - 21\Lambda\lambda^2 + 177\lambda^2 - 54\Lambda\lambda + 247\lambda - 21\Lambda - \Omega + 83}{2(12\lambda^3 - 7\Lambda\lambda^2 + 46\lambda^2 - 10\Lambda\lambda + 8\lambda + 17\Lambda - 66)} \quad (41)$$

with  $\Omega \equiv (4\lambda^6 - 2\Lambda\lambda^5 + 1596\lambda^5 - 918\Lambda\lambda^4 + 19848\lambda^4 - 9316\Lambda\lambda^3 + 91384\lambda^3 - 31228\Lambda\lambda^2 + 197268\lambda^2 - 42618\Lambda\lambda + 201868\lambda - 20366\Lambda + 78880)^{1/2}$

and  $\Lambda \equiv \sqrt{3\lambda^2 + 14\lambda + 15}$  for  $\lambda > \lambda^c$  (i.e.  $\Delta p^{max} = \Delta \bar{p}$ ). For  $\lambda \rightarrow \infty$  it holds that  $\beta_1^{crit}(\lambda) \rightarrow 1 - \frac{-37+21\sqrt{3}+\sqrt{4-2\sqrt{3}}}{-24+14\sqrt{3}} \approx 0.577$ . Compare Figure 10.  $\square$

We conclude this section by a numerical example. For  $\lambda = 3$ ,  $t = 1$  and  $\beta = 0$ , the following price differences arise  $\Delta p^{nd}(3) = 0.27889$ ,  $\Delta p^u(3) = 0.31072$ ,  $\Delta p^s(3) = 0.48259$ ,

$\Delta p^{ta}(3) = 0.69532$ ,  $\Delta p^{max} = \Delta \tilde{p} = 0.75$ , and  $\Delta \bar{p} = 0.83485$ . Moreover,  $\Delta c^{nd}(3)$  is equal to  $(\Delta p^{nd}(3) - f(\Delta p^{nd}(3); 0)) = 0.75963$ , i.e. an equilibrium exists for  $\Delta c < 0.75963$ . The remaining cost differences are equal to  $\Delta c^u(3) = 0.83485$ ,  $\Delta c^{ta}(3) = 1.40396$ . Since  $\Delta c < \Delta c^u(3)$  is sufficient for  $\Delta c < \Delta c^{nd}(3)$  any equilibrium is unique. Cf. Table 3 and 4 in Appendix C with  $\Delta c = 0.25$  and  $0.75$  at  $\beta = 0$ . An example for non-existence at  $\beta = 0$  is provided in Figure 8 with  $\Delta c = 1$ .

## C Tables

Table 3: Small Cost Differences:

The table shows the analytical solution of the market equilibria for parameter values of  $t = 1$ ,  $\lambda = 3$ ,  $c_A = 0.25$ ,  $c_B = 0.5$ :

$\beta$	$p_A^*(\beta)$	$p_B^*(\beta)$	$\Delta p^*(\beta)$	$q_A(\Delta p^*)$	$\hat{x}_{in}(\Delta p^*)$	$\hat{x}_{un}(\Delta p^*)$	$\pi_A^*$	$\pi_B^*$	$CS^*$	$CS_{in}^*$	$CS_{un}^*$
1.0	1.33333	1.41667	0.0833333	0.541667	0.541667	0.532453	0.586806	0.420139	1.37674	1.37674	1.16648
0.8	1.37274	1.45643	0.0836887	0.539995	0.541844	0.532597	0.606272	0.439961	1.29508	1.33717	1.12672
0.6	1.41524	1.49932	0.0840806	0.538326	0.54204	0.532755	0.627281	0.461361	1.21022	1.29448	1.08382
0.4	1.46121	1.54572	0.0845149	0.536662	0.542257	0.532931	0.650008	0.484522	1.12178	1.24832	1.03742
0.2	1.51103	1.59603	0.0849986	0.535002	0.542499	0.533127	0.674653	0.509652	1.02934	1.19828	0.987112
0.0	1.56518	1.65072	0.0855405	0.533347	0.54277	0.533347	0.701446	0.536986	0.932421	1.14388	0.932421

Table 4: Intermediate Cost Differences

The table shows the analytical solution of the market equilibria for parameter values of  $t = 1$ ,  $\lambda = 3$ ,  $c_A = 0.25$ ,  $c_B = 1$ :  
Prices of both firms are first increasing and then decreasing in  $\beta$ .

$\beta$	$p_A^*(\beta)$	$p_B^*(\beta)$	$\Delta p^*(\beta)$	$q_A(\Delta p^*)$	$\hat{x}_{in}(\Delta p^*)$	$\hat{x}_{un}(\Delta p^*)$	$\pi_A^*$	$\pi_B^*$	$CS^*$	$CS_{in}^*$	$CS_{un}^*$
1.0	1.5	1.75	0.25	0.625	0.625	0.605992	0.78125	0.28125	1.14063	1.14063	0.834921
0.8	1.5039	1.758	0.254109	0.62324	0.627054	0.60798	0.781477	0.285586	1.07357	1.13519	0.827071
0.6	1.50553	1.76414	0.25861	0.621651	0.629305	0.61017	0.780502	0.289112	1.00758	1.13188	0.821115
0.4	1.50448	1.76803	0.263546	0.62026	0.631773	0.612585	0.778104	0.29165	0.942908	1.13111	0.81744
0.2	1.50029	1.76925	0.26896	0.619097	0.63448	0.615251	0.774048	0.293008	0.879835	1.13332	0.816464
0.0	1.49248	1.76737	0.274896	0.618194	0.637448	0.618194	0.768092	0.292988	0.818625	1.13897	0.818625

The table shows the analytical solution of the market equilibria for parameter values of  $t = 1$ ,  $\lambda = 3$ ,  $c_A = 0.25$ ,  $c_B = 1.25$ : Non-existence for  $\beta = 0$  (see Figure 8).  $q_A(\Delta p^*)$  is decreasing in  $\beta$ , i.e. uninformed consumers are easier to attract than informed consumers. Reason: Due to large price differences loss aversion in price dimension dominates loss aversion in taste dimension. Uninformed consumers are more willing to buy the less expensive product.

[illegible]