Pride and Diversity in Social Economics*

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Abstract

We study a two-period economy in which agents' preferences take into account relative economic position. The study builds on a decision theoretic analysis of the social emotions that underly these concerns, i.e., envy and pride, which respond to social losses and gains, respectively. The analysis allows individual differences in their relative importance and, in the tradition of Prospect Theory, summarizes these differences in the geometric properties of the externality function that represents relative outcome concerns.

Our main result is that envy leads to conformism in consumption behavior and pride to diversity. We thus establish a link between emotions that are object of study in psychology and neuroscience, and important features of economic variables, in the first place the equilibrium distribution of consumption and income. This research provides a tool to relate experimental and empirical studies of individual preferences for relative position and important features of macro data.

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1 Introduction

Empirical and experimental literature has established the importance in individual choices of relative outcome concerns, especially in consumption and income. External habits, keeping up with the Joneses and other regarding preferences are common names used for this phenomenon. Early classic contributions are Veblen (1899) and Dusenberry (1949); the latter in particular is an early attempt to provide explanations of aggregate economic behavior on the basis of individual preferences over relative position. Recent empirical works show direct effects on well being of individuals, as measured by happiness indicators, of relative income (see, for example, Luttmer, 2005, and Dynan and Ravina, 2007). Other works explain economic behavior as motivated in part by concerns for relative position. For example, Charles, Hurst and Roussanov (2009) shows that visible consumption in luxury goods is well explained by status seeking (and that the level of consumption is declining in the income of the reference group). These studies use data sets and evidence of different nature, some experimental and some empirical. Though at the moment there is no wide agreement on the properties and strength of these features, these studies indicate that the effects are significant.

Our paper studies at a theoretical level how different shapes of individual preferences of agents with these social concerns affect the nature of the equilibrium and in particular the degree of inequality in the economy. Our analysis is based on the decision theoretic analysis of social preferences we pursued in Maccheroni, Marinacci, and Rustichini (2008). There we provide behavioral conditions that deliver objective functions in which, to a standard utility term that depends on agents' own outcomes, is added a positional index that quantifies agents' relative outcome concerns that arise by comparing their own outcomes with those of their peers. The positional index introduces a social dimension in agents' objective functions and establishes a direct link (in addition to prices) among their choices as agents' well being now also depends on their peers' choices and outcomes. Since the foundation of the representation is behavioral, the hypothesis underlying the representation are testable. Even more important for the purpose of the current paper, these features of individual preferences are reduced to few simple factors, which admit simple parametric representations that can be used in the theoretical analysis of equilibrium behavior and thus provide a foundation for calibration analysis.

Specifically, in Maccheroni et al (2008) we identify two basic social emotions that, through agents' attitudes toward them, determine the shape of the positional index, that is, envy and pride. By envy we mean the negative emotion that agents experience when their outcomes fall below those of their peers, and by pride the positive emotion that they experience when they have better outcomes than their peers. Envy and pride can be viewed as the emotions that arise when agents experience, respectively, a social loss and a social gain. Like attitudes toward standard private losses and gains, also these social attitudes may well vary across individuals. As detailed in the next section, the shape of the positional index reflects these different attitudes, thus making it possible to carry out comparative statics analysis in agents' attitudes toward envy and pride.

Two-Period Economy Our investigation of the economic consequences of these social preferences is based on an economy of Robinson Crusoe-type agents who live two periods in their own "islands," where they work/produce, consume, and save/store for their own future consumption a single consumption good. With traditional "asocial" preferences these agents would be on their own and the economy would be in equilibrium once they solve their individual intertemporal problems. With social preferences, however, this is no longer the case: even though their choices are still independent – that is, their outcomes are not affected by peers' choices – now agents' well being also depends on their peers' outcomes. In this economy of Robinson Crusoes, agents are thus linked only via their relative outcome concerns. This makes this setup especially well suited for our purposes since it allows to study in "purity" the equilibrium consequences of these relative concerns, with no room left for other possible interdependencies among agents' actions that may affect the analysis.

Our main finding is that in these social economies envy leads to conformism in equilibrium, pride to diversity. Specifically, suppose that our Robinson Crusoes are identical and that their labor supply is inelastic. In this case the choice problem they face is to select consumption in each of the two periods they live. For, they can save in the first period and store what saved for consumption in the next period. When deciding how much to consume in the first period, agents face a trade-off: if they increase consumption today they will increase their relative ranking today, but, ceteris paribus, also decrease their standing in the next period. They thus compare a positive effect today with a possible negative effect in the next period. This intertemporal trade-off (also noted, for example, in Binder and Pesaran, 2001, and Arrow and Dasgupta, 2007) points to a crucial feature of the preferences: the attitudes they exhibit toward social gains and losses, that is, the relative strength of the effect on individuals' welfare of being either in a dominant or in a dominated position in their reference group.

To better see how these social attitudes affect choices and the solution of the intertemporal trade-offs, we consider as illustration the two polar cases of pure envy and pure pride. The equilibrium set will be completely different in the two cases: it will be conformist in the case of pure envy (all agents consume the same) and diversified in the pure pride case (identical agents choose a different consumption). Agents with pure envy preferences only care about the situation in which their consumption is below the average value. The positional index has in this case a concave kink at the origin – defined by the simple inequality (7) below – that turns out to force equilibria to be symmetric: all agents choose the same consumption. A kink at zero, that is, at the reference point in the space of social gains and losses, is consistent with the view originating in Prospect Theory that a change in sign induces a change in marginal evaluations.

In contrast, agents with pure pride preferences have a positional index with a convex kink at the origin, and this feature changes completely the structure of the equilibrium set. This local convexity turns out to be enough to make all equilibria non-symmetric: although agents are identical, they will choose different consumptions. Some will choose to have a dominant position in the current period, at the expense of a dominated one in the future, and others will choose the opposite.

Since all agents are identical, the asymmetry in behavior caused by pride is noteworthy. This asymmetry only arises out of social concerns, not because of any need of the agents to equilibrate their actions in terms of overall available resources (in fact, in this economy of Robinson Crusoes there is no trade). The transmission channel that our analysis examines is saving behavior. Empirically, our analysis thus suggests that, ceteris paribus, in economies where envy prevails agents' saving behavior should be more homogeneous, with a lower degree of inequality in outcomes in the economy. The opposite should be true if, instead, pride prevails.[1]

Summing up, envy and pride – modeled here as the correspondents of social gains and losses, with a similar psychological nature – turn out to have very different implications for the underlying equilibria.[2] This is a novel insight of our analysis, which is made behaviorally well founded by the analysis of Maccheroni et al (2008), with its behavioral characterization of agents' objective functions, in particular of the shape of their positional indexes. Besides its intrinsic interest, from the methodological standpoint this behavioral foundation is important because it opens the possibility of an estimation of suitable parametric specifications of these objective functions via micro and experimental data. Once these estimates are provided in separate studies, the equilibrium effects could be determined and perhaps calibrated, just as it is done typically with other features of preferences like risk aversion.

Related Literature and Outline There is a large literature that investigates the economic consequences of relative outcome concerns. In Maccheroni et al (2008) we provide a detailed bibliography. Recent relevant surveys include Sobel (2005), Clark, Frijters, and Shields (2008), Fershtman (2008), and Heffetz and Frank (2008).

Our analysis is especially related to two strands of literatures. The first one considers the link between the shape of positional indexes and some important features of the equilibrium (see in particular Clark and Oswald, 1994 and 1998). The shape of these indexes (in particular their concavity/convexity properties) is taken as a given: these results suggest that a behavioral foundation would enlighten the conclusions. In this tradition, Dupor and Liu (2003) focus on overconsumption.

The second strand models relative concerns via comparisons of ordinal ranks in outcomes. For example, Frank (1985a) studies how these comparisons affect the demand of positional goods, that is, goods on which consumers exhibit relative concerns, and that of nonpositional ones.[3] More recently, Hopkins and Kornienko (2004) use these comparisons in a game

[1] It would be interesting to know whether these different saving behaviors would affect growth, at least at a theoretical level. In our two-period analysis we cannot, however, address this question, which is left for future research.

[2] As in Maccheroni et al (2008), we consider emotions as a first approximation explanation of behavior that is sufficient for choice theoretic purposes, in an utilitarian tradition that traces its origin back at least to Bentham's classic pleasure and pain calculus. Evolutionary perspectives on relative concerns has been recently considered by Rayo and Becker, 2007, and Samuelson, 2004.

[3] The distinction between positional and nonpositional goods is due to Hirsch (1976) (see also Frank, 1985b).

theoretic approach to status and to its consequences on income distribution. Our paper provides, *inter alia*, a link between these strands of literature and bases its analysis on a behaviorally specified foundation, which, as mentioned before, reduces the concern for relative positions to few simple factors.

The paper is organized as follows. Section 2 presents the basic elements of the social decision theory we developed in Maccheroni et al (2008). Section 3 introduces the two-period economy that we study, while Section 4 shows how overconsumption and workaholism may arise in it. Section 5 contains the paper's main result that shows how envy leads to conformism in consumption behavior and pride to diversity. Section 6 concludes, while the Appendix collects all proofs.

2 Social Decision Theory

In this preliminary section we summarize the essential features of the social decision theory we introduced and axiomatized in Maccheroni et al (2008).

Following Savage (1954)'s tradition, acts are measurable functions $f : S \to \mathbb{R}$ from a state space S to a consequence space C.[4] In social decision theory, an agent o has preferences over acts' profiles $\left(f_o, (f_i)_{i\in I}\right)$ that represent the situation in which agent o takes act f_o, while each member i of the agent's reference group I takes act f_i. Agent o then evaluates this situation according to:

$$V\left(f_o, (f_i)_{i\in I}\right) = \int_S u\left(f_o(s)\right) dP(s) + \int_S \varrho\left(v\left(f_o(s)\right), \sum_{i\in I} \delta_{v(f_i(s))}\right) dP(s). \tag{1}$$

The first term of this representation is familiar. The index $u\left(f_o(s)\right)$ represents the agent's intrinsic utility of the realized outcome $f_o(s)$, and P represents his subjective probability over the state space S. The first term thus represents his direct subjective expected utility from act f_o.

The effect on o's welfare of the outcome of the other individuals is reported in the second term. The index $v\left(f_o(s)\right)$ represents the social value that o attaches to outcome $f_o(s)$. Given a profile of acts, agent o's peers will get outcomes $\left(f_i(s)\right)_{i\in I}$ once state s obtains. If o does not care about the identity of who gets the value $v\left(f_i(s)\right)$, then he will only be interested in the distribution of these values. This distribution is represented by the term $\sum_{i\in I} \delta_{v(f_i(s))}$ in (1) above, where δ_x is the measure giving mass one to x.

The positional index ϱ is an externality that models agent o's relative outcome concerns. It is increasing in the first component and stochastically decreasing in the second. These monotonicity properties of ϱ reflect o's different attitudes towards his own outcomes and those of his peers. In particular, ϱ is increasing in the first component because o positively values his own outcome $f_o(s)$, while ϱ is stochastically decreasing in the second one since

[4] In Maccheroni et al (2009) we use the Anscombe and Aumann (1963) version of Savage's setting, where consequences are lotteries.

o negatively values his peers' outcomes, and so benefits from a stochastically dominated distribution of their outcomes.

As mentioned in the Introduction, the social emotions that underlie these negative attitudes toward peers' outcome are envy and pride. This is discussed at length by Maccheroni et al (2008), who provide the behavioral axioms that deliver the choice criterion (1), with these monotonicity properties of the positional index ϱ.

The index $v : C \to \mathbb{R}$ may or may not be equal to $u : C \to \mathbb{R}$. In particular, the two indices are equal if agent o evaluates his peers' outcomes only through his own utility function, that is, according to the user value that their outcomes have for him. In contrast, if peers' outcomes are valued beyond their user value, say because of status concerns, then the indices u and v may differ as the latter keeps track of this further social concerns about peers' outcomes. For example, we can envy our neighbor's Ferrari both because we would like to drive it (user value) and because of its symbolic/status value. The index ν reflects the overall, cumulative, "outcome externality" that the agent perceives, that is, his overall relative outcome concerns. Thus, u and v agree if user value considerations prevail, but they may well differ if symbolic/status considerations matter.

Specifications In Maccheroni et al (2008) we study several possible specifications of the general choice criterion (1). Among them, the following average specification is especially important:

$$V\left(f_o, (f_i)_{i\in I}\right) = \int_S u\left(f_o\left(s\right)\right) dP\left(s\right) + \int_S \varrho\left(v\left(f_o\left(s\right)\right), \frac{1}{|I|}\sum_{i\in I} v\left(f_i\left(s\right)\right)\right) dP\left(s\right). \quad (2)$$

Here, agent o only cares about the average social value $|I|^{-1}\sum_{i\in I} v\left(f_i\left(s\right)\right)$. For example, if $v\left(x\right) = x$, then (2) becomes

$$V\left(f_o, (f_i)_{i\in I}\right) = \int_S u\left(f_o\left(s\right)\right) dP\left(s\right) + \int_S \varrho\left(f_o\left(s\right), \frac{1}{|I|}\sum_{i\in I} f_i\left(s\right)\right) dP\left(s\right),$$

where only the average outcome $|I|^{-1}\sum_{i\in I} f_i\left(s\right)$ appears, as it is the case in many specifications used in applications.

It is also possible to give behavioral conditions such that in (2) we actually have $\varrho\left(z,t\right) = \gamma\left(z-t\right)$ for some increasing $\gamma : \mathbb{R} \to \mathbb{R}$ with $\gamma\left(0\right) = 0$. Here relative concerns are modelled through the difference between the value of o's outcome and that of the peers' average outcome. This is the simple and convenient specification that we will use in this paper. That is, we will study an economy where agents rank acts' profiles according to

$$V\left(f_o, (f_i)_{i\in I}\right) = \int_S u\left(f_o\left(s\right)\right) dP\left(s\right) + \int_S \gamma\left(v\left(f_o\left(s\right)\right) - \frac{1}{|I|}\sum_{i\in I} v\left(f_i\left(s\right)\right)\right) dP\left(s\right). \quad (3)$$

2.1 Kinks and Comparative Envy

As observed in the Introduction, an outcome profile where your peers get a socially better outcome than yours can be viewed as social loss; conversely, a profile where you get more than them can be viewed as a social gain. In particular, individuals might well have different attitudes toward such social gains and losses, similarly to what happens for standard private gains and losses. The distinction between gains and losses is inspired by Prospect Theory, with its focus on *private* gains and losses. While there is no presumption that preferences in social and private domains are the same (there is indeed some evidence that they are not), we keep the basic intuition that the change across domains may change the evaluation of one additional unit, that is, may introduce a discontinuity in marginal utility. Kinks in the objective function add to the mathematical complexity of the analysis, but they are necessary to study appropriately this view.

These different social attitudes will play a key role in the paper and it is therefore important to understand how to model them via the positional index ϱ. Given a "fair" event E with $P(E) = 1/2$, say that an agent o that features the choice criterion (1) is *more envious than proud* (or *averse to social losses*), relative to a given $x_o \in C$, if

$$V(x_o, x_o) \geq V(x_o, x_i E y_i) \tag{4}$$

for all $x_i, y_i \in C$ such that $(1/2)\, v(x_i) + (1/2)\, v(y_i) = v(x_o)$.[5] The intuition is that agent o tends to be more frustrated by envy than satisfied by pride. That is, assuming wlog that $v(x_i) \geq v(y_i)$, he is more scared by the social loss (x_o, x_i) than lured by the social gain (x_o, y_i).

Maccheroni et al (2008) show that agent o is more envious than proud, relative to an $x_o \in C$, if and only if

$$\varrho(v(x_o), v(x_o) + h) \leq -\varrho(v(x_o), v(x_o) - h), \qquad \forall h > 0. \tag{5}$$

In particular,

$$D_+ \varrho(v(x_o), v(x_o)) \leq D_- \varrho(v(x_o), v(x_o)). \tag{6}$$

In other words, a concave kink at $v(x_o)$ reveals a more envious than proud attitude at x_o. In the special case $\varrho(z, t) = \gamma(z - t)$ – that is, in (3) – condition (6) becomes

$$\gamma'_+(0) \leq \gamma'_-(0). \tag{7}$$

Here, a kink at 0 thus reveals a global (i.e., at all points x_0) more envious than proud attitude. This is a remarkable feature of the specification (3), which makes it especially tractable.

[5] Here, $x_i E y_i$ denotes the act that gives x_i if event E obtains and y_i otherwise. It can be proved that the choice of E is immaterial in the definition (events E with $P(E) = 1/2$ are often called ethically neutral in decision theory).

3 Social Equilibria in a Storage Economy

Our analysis is based on a two-period storage economy where agents have the specification (3) of the general choice criterion (1). We later specialize this economy in order to focus on two distinct important economic phenomena that arise with our preferences, that is, overconsumption/workaholism and conformism/anticonformism.

We consider economies with a continuum of individually negligible agents. There are two main reasons for this modeling choice: it simplifies an already complicated derivation and it allows to abstract from strategic interactions among agents that might otherwise arise, so that we can better focus on the interdependencies due to the social dimension of preferences, as will be shown momentarily.

Formally, the set I of agents is a complete nonatomic probability space (I, Λ, λ). In particular, we denote by M^n the collection of all Λ-measurable functions $\phi : I \to \mathbb{R}^n$ and by L^n the subset of M^n consisting of bounded functions.

There is a single consumption good, which can be either consumed or saved. We consider a *storage economy*, in which a storage technology is available that allows agents to store for their own future consumption any amount of the consumption good that they do not consume in the first period.

As we will see momentarily, in the storage economy there is no room for trade: each agent produces, consumes, and saves/stores for his own future consumption. There are no markets and prices, and, with conventional asocial objective functions, this economy is in equilibrium (Definition 1) when agents just solve their individual intertemporal problems (8).

As a result, it is an equilibrium notion limited in scope, with no need of considering any form of mutual compatibility of agents' choices. If, however, agents have our social objective functions, this is no longer the case. In fact, when agents' own consumption choices are affected by their peers' choices, a link among all such choices naturally emerges. Even without any trading, in this case there is a sensible notion of mutual compatibility of the agents' choices and, therefore, a more interesting equilibrium notion becomes appropriate (Definition 2).

In storage economies, therefore, interaction among agents is only due to the social dimension of consumption. This allows us to study the equilibrium effects of this social dimension in "purity," without other factors intruding into the analysis. This is why we consider these economies. Later, in Section 6, we will briefly discuss a market economy.

We turn now to the formal model. We assume that the storage technology gives a real (gross) return $R > 0$. Agents live two periods and in each of them they work and consume; in period one they can also store. In the first period each agent i selects a consumption/effort pair $(c_{i,0}, e_{i,0}) \in \mathbb{R}_+^2$, evaluated by a utility function $u_i : \mathbb{R}_+^2 \to \mathbb{R}$. Effort is transformed in consumption good according to an individual production function $F_{i,0} : \mathbb{R}_+ \to \mathbb{R}_+$.

In the second period there is technological uncertainty, described by a stochastic production function $F_{i,s} : \mathbb{R}_+ \to \mathbb{R}_+$ that depends on a finite space S of states of Nature, endowed with a probability P. With the usual abuse of notation we set $S = \{1, 2, ..., S\}$ and $S_0 = \{0, 1, 2, ..., S\}$, and we write p_s instead of $P(s)$. The production functions $\{F_{i,s}\}_{s \in S_0}$ use a physical capital, whose amount is exogenously fixed in each period and state (capital accumulation is thus not studied here).

In the second period too, each agent i works and consumes. He thus selects in each state s a consumption/effort pair $(c_{i,s}, e_{i,s}) \in \mathbb{R}^2_+$, again evaluated by the same utility function u_i of the first period.

Finally, effort is a limited resource: for each i there is a vector $h_i \in \mathbb{R}^{S+1}_+$ such that $e_{i,s}$ cannot exceed $h_{i,s}$ for all $s \in S_0$.

Summing up, the intertemporal problem of agent i in the storage economy is:

$$\max_{(c_i, e_i) \in B_i} U_i(c_i, e_i), \tag{8}$$

where

$$U_i(c_i, e_i) = u_i(c_{i,0}, e_{i,0}) + \beta \sum_{s \in S} p_s u_i(c_{i,s}, e_{i,s}), \qquad \forall (c_i, e_i) \in \mathbb{R}^{S+1}_+ \times \mathbb{R}^{S+1}_+,$$

and B_i is the subset of $\mathbb{R}^{S+1}_+ \times \mathbb{R}^{S+1}_+$ consisting of all (c_i, e_i) such that:

(i) $(c_i, e_i) \in \mathbb{R}^{S+1}_+ \times \prod_{s=0}^{S} [0, h_{i,s}]$;

(ii) $c_{i,0} \le F_{i,0}(e_{i,0})$;

(iii) $c_{i,s} = F_{i,s}(e_{i,s}) + R(F_{i,0}(e_{i,0}) - c_{i,0})$ for all $s \in S$.

The set B_i is never empty since in every period and state each agent can consume all he produces. Next we make a first assumption on the storage economy.

H.1 For each agent $i \in I$:

- (i) $u_i : \mathbb{R}^2_+ \to \mathbb{R}$ is continuous.
- (ii) $F_{i,s} : \mathbb{R}_+ \to \mathbb{R}_+$ is increasing and continuous for all $s \in S_0$.

This assumption guarantees that the (nonempty) set B_i is compact, and that the objective function U_i is continuous. By the Weierstrass Theorem, problem (8) thus admits a solution.

Say that a consumption/effort profile $(c, e) \in M^{S+1} \times M^{S+1}$ is *feasible* if $(c_i, e_i) \in B_i$ for all $i \in I$.

Definition 1 *A feasible consumption/effort profile* (c^*, e^*) *is an* asocial equilibrium *for the storage economy if*

$$U_i(c_i^*, e_i^*) \ge U_i(c_i, e_i), \qquad \forall (c_i, e_i) \in B_i,$$

for λ-almost all $i \in I$.

As mentioned before, this equilibrium notion just requires that agents individually solve their problems (8), with no interaction whatsoever among themselves.

We turn now to our social preferences, adapted to our continuum setup. Assume that the preferences of our agents are represented by the preference functional (3). Given a common social value function $v : \mathbb{R}_+ \rightarrow \mathbb{R}$, the social objective function V_i of each agent now depends on the entire profile of consumption and effort, as follows:

$$V_i(c,e) = U_i(c_i,e_i) + \gamma_i\left(v(c_{i,0}) - \int_I v(c_0)\, d\lambda\right) + \beta \sum_{s\in S} p_s \left[\gamma_i\left(v(c_{i,s}) - \int_I v(c_s)\, d\lambda\right)\right],$$

for all $(c,e) \in L_+^{S+1} \times L_+^{S+1}$. Here we are assuming that only consumption has a social dimension, while effort (and so leisure) is only valued privately. That is, consumption is more positional than effort/leisure, in the terminology of Hirsch (1976) and Frank (1985). This was a classic assumption in Veblen's analysis, and is justified by the lower degree of observability of effort relative to consumption. For this reason, effort is not an argument of the function γ.

Remark. In the social objective function V_i there are both outcomes that are socially valued (i.e., consumption) and outcomes that are only privately valued (i.e., effort). Moreover, there are two periods, 0 and 1, and so V_i is an intertemporal criterion. Though, strictly speaking, these features are not covered by the basic representation (1), and so by (3), simple extensions of these basic representations cover them (see Maccheroni et al, 2008, for details).

The equilibrium notion relevant for our social preferences is a Nash equilibrium for a continuum of agents.

Definition 2 *A feasible consumption/effort profile* $(c^*, e^*) \in L^{S+1} \times L^{S+1}$ *is a* social equilibrium *for the storage economy if*

$$V_i(c^*,e^*) \geq V_i\left(c_i, c_{-i}^*, e_i, e_{-i}^*\right), \qquad \forall (c_i,e_i) \in B_i, \tag{9}$$

for λ*-almost all* $i \in I$.

This equilibrium notion requires a mutual compatibility of agents' choices and is thus qualitatively very different from that of Definition 1, a difference entirely due to the social dimension of our preferences.

A key theoretical issue is the existence of social equilibria. To prove this result, which ensures that there are no inconsistencies in our analysis, we need the following mild assumption. Point (i) says that the effort and production capacities are limited, while the other points are standard assumptions.

H.2 The following conditions are satisfied:

(i) $\sup_{i \in I} (F_{i,s}(h_{i,s}) + h_{i,s}) < \infty$ for all $s \in S_0$.

(ii) $\gamma_i : \mathbb{R} \to \mathbb{R}$ is increasing and continuous, with $\gamma_i(0) = 0$, for all $i \in I$.

(iii) the real valued functions $u_{(\cdot)}(x, y)$, $h_{(\cdot),s}$, $F_{(\cdot),s}(z)$, and $\gamma_{(\cdot)}(t)$ are Λ-measurable on I for each fixed $(x, y, z, t) \in \mathbb{R}^3_+ \times \mathbb{R}$ and $s \in S_0$.

(iv) $v : \mathbb{R}_+ \to \mathbb{R}$ is increasing and continuous.

We can now prove a general existence result for storage economies. The proof relies on existence results of Schmeidler (1973) and Balder (1995).

Theorem 1 *In a storage economy satisfying assumptions H.1 and H.2 there exists a social equilibrium.*

4 Consumerism: Overconsumption and Workaholism

The first phenomenon we consider is how overconsumption and workaholism can arise in a social equilibrium. This is an often mentioned behavioral consequence of concerns for relative consumption and here Proposition 1 shows how it emerges in our general analysis.[6]

We focus on a single period version of the storage economy. In fact, as pointed out in the Introduction, trade-offs arise in more general intertemporal settings (i.e., consuming more today leads to lower saving and, possibly, to lower future consumption). The tendency to overconsumption and workaholism that here we identify in the single period setting might be then offset by other forces.

To ease notation, we drop the subscripts 0; that is, c_i and e_i stand for $c_{i,0}$ and $e_{i,0}$, respectively. The asocial problem of each agent $i \in I$ is then given by

$$\max_{(c_i, e_i) \in B_i} u_i(c_i, e_i) \tag{10}$$

where $B_i = \left\{(c_i, e_i) \in \mathbb{R}^2_+ : 0 \le e_i \le h_i, \ c_i = F_i(e_i)\right\}$.

Here a feasible consumption/effort profile $(c^*, e^*) \in L \times L$ is an asocial equilibrium if (c_i^*, e_i^*) is a solution of problem (10) for λ-almost all $i \in I$.

H.3 For each agent $i \in I$:

(i) u_i is twice continuously differentiable on $\mathbb{R}^2_{++}$, $\partial u_i / \partial x > 0$, and the Hessian matrix $\nabla^2 u_i$ is negative definite.

(ii) F_i is twice differentiable on $\mathbb{R}_{++}$, $F_i' > 0$ and $F_i'' < 0$.

Lemma 1 *If H.1, H.2, and H.3 hold, then there exists a (λ-a.e.) unique asocial equilibrium.*

[6] Empirical evidence on this phenomenon can be found, for example, in Bowles and Park (2005) and the references therein. Recent anecdotal evidence is reported in Rivlin (2007), who describes Silicon Valley workaholic executives as "working class millionaires."

The social objective functions V_i take the form

$$V_i(c,e) = u_i(c_i,e_i) + \gamma_i\left(v(c_i) - \int_I v(c)\,d\lambda\right),$$

and a feasible pair $(c^*,e^*) \in L \times L$ is a social equilibrium if (9) holds.

To state the result we need a condition and some notation. The special form that B_i has in this case guarantees that a consumption/effort profile $(c,e) \in M \times M$ is feasible if and only if $e_i \in [0,h_i]$ and $c_i = F_i(e_i)$ for all $i \in I$. Under H.2-(i), feasible profiles are thus determined by effort profiles that belong to the supnorm closed and convex set $E = \{e \in L : 0 \le e \le h\}$. The value of the social objective function can be thus written as

$$W_i(e) = u_i(F_i(e_i), e_i) + \gamma_i\left(v(F_i(e_i)) - \int_I v(F_\iota(e_\iota))\,d\lambda(\iota)\right) \quad \forall e \in E, i \in I. \tag{11}$$

An equilibrium (c^*,e^*) is *internal* if $e^* \in \text{int}E$ and *strongly Pareto inefficient* if it is strongly Pareto dominated, that is, there is $\varepsilon > 0$ and a feasible $(c,e) \in M \times M$ such that

$$V_i(c,e) \ge V_i(c^*,e^*) + \varepsilon$$

for λ-almost all $i \in I$.

We can now state the needed assumption.

H.4 The following conditions are satisfied:

(i) v is differentiable on $\mathbb{R}_{++}$ and $v' > 0$.

(ii) γ_i is differentiable on $\mathbb{R}$ for all $i \in I$, and $\inf_{(i,t)\in I\times\mathbb{R}} \gamma_i'(t) > 0$.

(iii) $\sup_{|x|,|y|,|t|\le n,\, i\in I} |u_i(x,y) + \gamma_i(t)| < \infty$ and $\sup_{|x|\le n,\, i\in I} |v'(x) + F_i'(x)| < \infty$ for all $n \in \mathbb{N}$.[7]

(iv) $W : E \to L$ is strictly differentiable on $\text{int}E$.[8]

Proposition 1 *If H.1-H.4 hold, then internal social equilibria are strongly Pareto inefficient and exhibit overconsumption and workaholism.*[9]

Overconsumption and workaholism thus characterize equilibria in the single period version of the storage economy. We studied here in detail the Pareto inefficiency of the equilibria to stress the negative features of these equilibria.

This result confirms a well known intuition about social preferences (see, e.g., Dupor and Liu, 2003, for a related point). The next section will show a genuine novel economic insight of our analysis.

[7] This implies $W(E)$ consists of bounded functions.

[8] See pages 30-32 of Clarke (1983) for the properties of strict differentiability.

[9] That is, λ-a.e., $c_i^* > \hat{c}_i$ and $e_i^* > \hat{e}_i$, where (c^*,e^*) and $(\hat{c},\hat{e})$ are, respectively, social and asocial consumption and effort equilibrium pairs.

5 Equilibrium Conformism and Anticonformism

We now study how conformism and anticonformism can characterize the consumption choices of agents in social equilibria, depending, as anticipated in the Introduction, on whether either envy or pride prevails among agents.

Since the rise of anticonformism is our main interest, in order to better focus on this issue we consider a version of the storage economy in which agents are *identical* (so that the social dimension of their preferences is the only possible cause of heterogeneous consumption choices). We also assume that labor is supplied inelastically, say $e_{i,s} = \bar{e}_s > 0$ for all $i \in I$ and $s \in S_0$. To ease notation, we set $F_0(\bar{e}_0) = \bar{x}_0 > 0$ and $F_s(\bar{e}_s) = \bar{x}_s > 0$, and we drop effort as argument of the utility function u. This is the version of the two-period economy that we discussed in the Introduction.

In this case, the asocial intertemporal problem of each identical agent i is

$$\max_{c_i \in [0, \bar{x}_0]} U(c_i), \tag{12}$$

where $U(c_i) = u(c_i) + \beta \sum_{s \in S} p_s u(\bar{x}_s + R(\bar{x}_0 - c_i))$ for all $c_i \in [0, \bar{x}_0]$. A (first period) consumption profile $c \in M$ is feasible if it belongs to the set $C = \{c \in M : 0 \leq c \leq \bar{x}_0\}$, and is an asocial equilibrium if c_i solves problem (12) for λ-almost all $i \in I$. Clearly, all asocial equilibria are symmetric (i.e., constant λ-almost everywhere) provided U is unimodal.

H.5 The following conditions are satisfied:

(i) $u : \mathbb{R}_+ \to \mathbb{R}$ is continuous on $\mathbb{R}_+$, strictly concave, strictly increasing, and differentiable on $(0, +\infty)$.

(ii) $U'_+(0) > 0 > U'_-(\bar{x}_0)$.

Lemma 2 *If H.5 holds, then there exists a (λ-a.e.) unique asocial internal equilibrium.*

Given $\gamma : \mathbb{R} \to \mathbb{R}$ and $v : \mathbb{R}_+ \to \mathbb{R}$ and $c \in C$, agent i's social objective function becomes

$$\begin{aligned} V_i(c) = {} & u(c_i) + \beta \sum_{s \in S} p_s u(\bar{x}_s + R(\bar{x}_0 - c_i)) + \gamma \left(v(c_i) - \int_I v(c_\iota)\, d\lambda(\iota) \right) \\ & + \beta \sum_{s \in S} p_s \left[\gamma \left(v(\bar{x}_s + R(\bar{x}_0 - c_i)) - \int_I v(\bar{x}_s + R(\bar{x}_0 - c_\iota))\, d\lambda(\iota) \right) \right]. \end{aligned}$$

Here a $c^* \in C$ is a social equilibrium if $V_i(c^*) \geq V_i(c_i, c^*_{-i})$ for all $c_i \in [0, \bar{x}_0]$ and for λ-almost all $i \in I$.

We will use the following assumption on γ and v.

H.6 The following conditions are satisfied:

(i) $v : \mathbb{R}_+ \to \mathbb{R}$ is continuous, concave, and strictly increasing on $\mathbb{R}_+$.

(ii) $\gamma : \mathbb{R} \rightarrow \mathbb{R}$ is increasing and $\gamma(0) = 0$.

We can now state the main result of the paper. We consider two polar cases, one in which agents exhibit pure envy, that is, $\gamma(t) = 0$ for all $t \geq 0$, and one in which they exhibit pure pride, that is, $\gamma(t) = 0$ for all $t \leq 0$.[10] We show that envy leads to conformism, that is, all social equilibria are symmetric, while pride leads to diversity, that is, all social equilibria are asymmetric. The intuition behind the theorem is easier to see if one considers a simple case of a two-player game where only relative concerns matter. As the function γ changes from the pure envy to the pure pride preference, the set of Nash equilibria is easily seen to change from symmetric to the asymmetric one. In our continuum case, with both direct and indirect effects, things are more complicated. In this regard, observe how the presence of a kink $D_{+}\gamma(0) > 0$ is key for the result.

Theorem 2 *Suppose assumptions H.5 and H.6 hold. Then:*

(i) All social equilibria are asymmetric provided $\gamma(t) = 0$ for all $t \leq 0$ and $D_{+}\gamma(0) > 0$.

(ii) All social equilibria are symmetric provided $\gamma(t) = 0$ for all $t \geq 0$, and $D_{+}\gamma(t) > 0$ for all $t < 0$.

Moreover, in (ii) condition $D^{-}\gamma(0) = 0$ implies that the asocial symmetric equilibrium is the unique social equilibrium.

6 Conclusions

We have derived simple and general conclusions on the relationship between the degree of inequality in an economy and the relative weight of envy and pride in the objective function of the typical agent in the economy. As we remarked in the Introduction, the diversity in consumption behavior caused by pride is the most remarkable part of the result because agents are identical. By point (i) of Theorem 2, in all equilibria necessarily some agents will choose to consume more today, that is, to have a dominant position today, while other agents will choose the opposite, that is, they will save more today in order to consume more tomorrow and have then a dominant position. This diversity in equilibrium behavior is entirely due to the social dimension of preferences. In fact, in this storage economy there is no trade and agents' actions do not need to equilibrate in terms of resources, as remarked in the Introduction.

In the storage economy there was no room for trade. A simple modification of the storage economy that allows trade is to change the "saving technology" by assuming that agents no longer can store the consumption good for future consumption. They can, however, borrow and lend amounts of the consumption good, which is now also a real asset. Agents can save by lending any amount of the consumption good that they do not consume in the first

[10] Though for simplicity we consider this case, the result holds more generally under a weaker condition that only requires agents to be more proud than envious.

period. As a result, in the (real) asset economy agents interact by trading in the real asset market.

Though for brevity we do not study in detail this economy, it is worth observing that here conformism/anticonformism correspond to no trade/trade. In fact, conformism means that all social equilibria are symmetric, and, by the market clearing condition, it is easy to see that in such equilibria there is no trade in the real asset market. In contrast, this market operates in the asymmetric equilibria of the anticonformism case. As a result, in this market economy, envy leads to autarky, pride to trade.

7 Appendix: Proofs and Related Material

7.1 Chain Rules for Dini Derivatives

Let $a, b \in \mathbb{R}$ with $a < b$. If $f, g : (a, b) \to \mathbb{R}$, set

$$\begin{aligned}
\limsup_{x \to a^+} f(x) &\equiv \lim_{\delta \to 0^+} \sup_{h \in (0,\delta)} f(a+h) = \inf_{\delta > 0} \sup_{h \in (0,\delta)} f(a+h), \\
\liminf_{x \to a^+} f(x) &\equiv \lim_{\delta \to 0^+} \inf_{h \in (0,\delta)} f(a+h) = \sup_{\delta > 0} \inf_{h \in (0,\delta)} f(a+h), \\
\limsup_{x \to b^-} f(x) &\equiv \lim_{\delta \to 0^+} \sup_{h \in (0,\delta)} f(b-h) = \inf_{\delta > 0} \sup_{h \in (0,\delta)} f(b-h), \\
\liminf_{x \to b^-} f(x) &\equiv \lim_{\delta \to 0^+} \inf_{h \in (0,\delta)} f(b-h) = \sup_{\delta > 0} \inf_{h \in (0,\delta)} f(b-h).
\end{aligned}$$

These limits always exist in $[-\infty, +\infty]$,[11] with the conventions:

$$\begin{aligned}
(-\infty) \dot{+} (+\infty) = (+\infty) \dot{+} (-\infty) = +\infty = (-\infty) \dot{-} (-\infty) = (+\infty) \dot{-} (+\infty), \\
(-\infty) \underset{\cdot}{+} (+\infty) = (+\infty) \underset{\cdot}{+} (-\infty) = -\infty = (-\infty) \underset{\cdot}{-} (-\infty) = (+\infty) \underset{\cdot}{-} (+\infty).
\end{aligned}$$

It is easy to see that $\limsup_{x \to a^+} f(x) = \sup \{ \limsup_{n \to \infty} f(x_n) : (a, b) \ni x_n \to a^+ \}$, and that there exists $(a, b) \ni \bar{x}_n \to a^+$ such that $\lim_{n \to \infty} f(\bar{x}_n) = \limsup_{x \to a^+} f(x)$.[12]

Let $f : [a, b] \to \mathbb{R}$, and set

$$D^+ f(c) \equiv \limsup_{x \to c^+} \frac{f(x) - f(c)}{x - c} \quad \text{and} \quad D_+ f(c) \equiv \liminf_{x \to c^+} \frac{f(x) - f(c)}{x - c}$$

for all $c \in [a, b)$, and, analogously,

$$D^- f(c) \equiv \limsup_{x \to c^-} \frac{f(x) - f(c)}{x - c} \quad \text{and} \quad D_- f(c) \equiv \liminf_{x \to c^-} \frac{f(x) - f(c)}{x - c}$$

for all $c \in (a, b]$. It is easy to see that, if $c \in [a, b)$ is a local maximum, then

$$D_+ f(c) \le D^+ f(c) \le 0,$$

[11] To be precise, we should write $d > \delta > 0$, where $d \in (0, b-a)$, rather than $\delta > 0$. But, no confusion should arise.

[12] With analogous results for the other limits.

and, analogously, if $c \in (a, b]$ is a local maximum, then

$$0 \leq D_{-}f(c) \leq D^{-}f(c).$$

Next we provide a chain rule that will be useful in the sequel:

Proposition 2 *Let $v : [\alpha, \beta] \to [a, b]$ be strictly increasing, onto, and concave. Then, given any $f : [a, b] \to \mathbb{R}$, we have:*

(i) $D^{+}(f \circ v)(\gamma) = v'_{+}(\gamma) D^{+}f(v(\gamma))$ *provided either* $\gamma \in (\alpha, \beta)$ *or* $\gamma = \alpha$ *and either* $v'_{+}(\alpha) \neq +\infty$ *or* $D^{+}(f)(v(\alpha)) > 0$.

(ii) $D_{+}(f \circ v)(\gamma) = v'_{+}(\gamma) D_{+}f(v(\gamma))$ *provided either* $\gamma \in (\alpha, \beta)$ *or* $\gamma = \alpha$ *and either* $v'_{+}(\alpha) \neq +\infty$ *or* $D_{+}f(v(\alpha)) > 0$.

(iii) $D^{-}(f \circ v)(\gamma) = v'_{-}(\gamma) D^{-}f(v(\gamma))$ *provided either* $\gamma \in (\alpha, \beta)$ *or* $\gamma = \beta$ *and* $v'_{-}(\beta) \neq 0$.

(iv) $D_{-}(f \circ v)(\gamma) = v'_{-}(\gamma) D_{-}f(v(\gamma))$ *provided either* $\gamma \in (\alpha, \beta)$ *or* $\gamma = \beta$ *and* $v'_{-}(\beta) \neq 0$.

7.2 Proofs and Related Analysis

For each agent $i \in I$, set

$$W_i(x, y, z) \equiv \sum_{s \in S_0} \pi_s u_i(x_s, y_s) + \sum_{s \in S_0} \pi_s \left[\gamma_i \left(v(x_s) - z_s\right)\right] \tag{13}$$

for all $(x, y, z) \in \mathbb{R}_{+}^{S+1} \times \mathbb{R}_{+}^{S+1} \times \mathbb{R}^{S+1}$, where $\pi_0 = 1$ and $\pi_s = \beta p_s$ for all $s \in S$. Notice that $V_i(c, e) =$

$$\sum_{s \in S_0} \pi_s u_i(c_{i,s}, e_{i,s}) + \sum_{s \in S_0} \pi_s \gamma_i \left(v(c_{i,s}) - \int_I (v \circ c_s)\, d\lambda\right) = W_i\left(c_i, e_i, \left[\int_I (v \circ c_s)\, d\lambda\right]_{s \in S_0}\right)$$

for all $(c, e) \in L_{+}^{S+1} \times L_{+}^{S+1}$.

Lemma 3 *If (c^*, e^*) is a social equilibrium, then $(c^*, e^*) \in L^{S+1} \times L^{S+1}$ and (c_i^*, e_i^*) is a solution of problem $\max_{(x,y) \in B_i} W_i(x, y, z)$ where $z_s \equiv \int_I (v \circ c_s^*)\, d\lambda$ for all $s \in S_0$, for λ-almost all $i \in I$. The converse is true up to a λ-negligible variation of (c^*, e^*).*[13]

The simple proof is omitted.

Lemma 4 *If H.1 holds, then B_i is compact and nonempty for all $i \in I$.*

[13] A λ-negligible variation of a function on a measure space (I, Λ, λ) is a function that coincides λ-almost everywhere with the original one.

Proof. Since $F_{i,s}$ is increasing, then

$$B_i = \left\{ (x,y) \in \mathbb{R}_+^{S+1} \times \prod_{s=0}^{S} [0, h_{i,s}] : F_{i,0}(y_0) \geq x_0, \ x_s = F_{i,s}(y_s) + R(F_{i,0}(y_0) - x_0) \ \forall s \in S \right\}$$

that is

$$\begin{aligned} B_i = \ & \left([0, F_{i,0}(h_{i,0})] \times \prod_{s=1}^{S} [0, F_{i,s}(h_{i,s}) + RF_{i,0}(h_{i,0})] \times \prod_{s=0}^{S} [0, h_{i,s}] \right) \\ & \cap \left\{ (x,y) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} : F_{i,0}(y_0) - x_0 \geq 0 \right\} \\ & \cap \left\{ (x,y) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} : F_{i,1}(y_1) + R(F_{i,0}(y_0) - x_0) - x_1 = 0 \right\} \\ & \ldots \\ & \cap \left\{ (x,y) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} : F_{i,S}(y_S) + R(F_{i,0}(y_0) - x_0) - x_S = 0 \right\} \end{aligned} \tag{14}$$

which is compact since the functions $(x,y) \mapsto F_{i,0}(y_0) - x_0$ and $(x,y) \mapsto F_{i,s}(y_s) + R(F_{i,0}(y_0) - x_0) - x_s$ are continuous for all $s \in S$. Moreover, for all $y \in \prod_{s=0}^{S} [0, h_{i,s}]$, $\left((F_{i,s}(y_s))_{s=0}^{S}, (y_s)_{s=0}^{S} \right) \in B_i$, which implies $B_i \neq \emptyset$. ■

Proof of Theorem 1. For each agent i, consider the strategy set $B_i \subseteq \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1}$ and the payoff function $W_i(x,y,z) = \sum_{s \in S_0} \pi_s u_i(x_s, y_s) + \sum_{s \in S_0} \pi_s [\gamma_i (v(x_s) - z_s)]$ for all $(x,y,z) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \times \mathbb{R}^{S+1}$, where $\pi_0 = 1$ and $\pi_s = \beta p_s$ for all $s \in S$.

Since $\mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1}$ and $\mathbb{R}^{S+1}$ are Polish spaces, assumptions 2.1 and 2.2 of Balder (1995) hold. Since B_i is nonempty and compact for all $i \in I$, assumption 2.3 of Balder (1995) holds. For every $i \in I$, $W_i : \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \times \mathbb{R}^{S+1} \to \mathbb{R}$ is continuous, and so assumptions 2.4 and 2.6 of Balder (1995) hold.

Assumptions H.1.ii and H.2.iii guarantee that the real valued functions on $I \times \left(\mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \right)$ defined by $(i,(x,y)) \mapsto F_{i,0}(y_0) - x_0$ and by $(i,(x,y)) \mapsto F_{i,s}(y_s) + R(F_{i,0}(y_0) - x_0) - x_s$ are Caratheodory functions for all $s \in S$, hence they are $\Lambda \times \mathcal{B}\left(\mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \right)$-measurable and the graphs of the correspondences $i \mapsto \left\{ (x,y) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} : F_{i,0}(y_0) - x_0 \geq 0 \right\}$ and $i \mapsto \left\{ (x,y) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} : F_{i,s}(y_s) + R(F_{i,0}(y_0) - x_0) - x_s = 0 \right\}$ are $\Lambda \times \mathcal{B}\left(\mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \right)$-measurable.

Moreover, the functions $i \mapsto F_s(i, h_s(i))$ are Λ-measurable for all $s \in S_0$, since $i \mapsto (i, h_s(i))$ is Λ-measurable on I and $(i,t) \mapsto F_{i,s}(t)$ is a Caratheodory function on $I \times \mathbb{R}_+$. Therefore the graph of the correspondence $i \mapsto [0, F_{i,0}(h_{i,0})] \times \prod_{s=1}^{S} [0, F_{i,s}(h_{i,s}) + RF_{i,0}(h_{i,0})] \times \prod_{s=0}^{S} [0, h_{i,s}]$ is $\Lambda \times \mathcal{B}\left(\mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \right)$-measurable.

By (14), for all $i \in I$, the graph of the correspondence $B : i \mapsto B_i$ is $\Lambda \times \mathcal{B}\left(\mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \right)$-measurable and assumption 2.5 of Balder (1995) holds.

For every $z \in \mathbb{R}^{S+1}$, $W_{(\cdot)}(\cdot, z) : (i,(x,y)) \mapsto W_i((x,y), z)$ is a Caratheodory function on $I \times \left(\mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \right)$. Hence, it is $\Lambda \times \mathcal{B}(\mathbb{R}^n)$-measurable, and so assumption 2.7 of Balder (1995) holds.

Now define $g_s : I \times \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \to \mathbb{R}$ by $g_s(i,(x,y)) = v(x_s)$ for all $s \in S_0$. Let $s \in S_0$, clearly $g_s(i,(\cdot,\cdot))$ is continuous on $\mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1}$ for all $i \in I$ and $g_s(\cdot,(x,y))$ is constant –

hence Λ-measurable – on I for all $(x,y) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1}$. Therefore g_s is a Caratheodory function for all $s \in S_0$, in particular, it is $\Lambda \times \mathcal{B}(\mathbb{R}^n)$-measurable.

For all $i \in I$, $\inf_{(x,y)\in B_i} g_s(i,(x,y)) = \inf_{(x,y)\in B_i} v(x_s) \geq v(0)$, $\sup_{(x,y)\in B_i} g_s(i,(x,y)) = \sup_{(x,y)\in B_i} v(x_s) \leq \sup_{x_s\in[0,F_{i,s}(h_{i,s})+RF_{i,0}(h_{i,0})]} v(x_s) = v(F_{i,s}(h_{i,s}) + RF_{i,0}(h_{i,0}))$, and $i \mapsto v(F_{i,s}(h_{i,s}) + RF_{i,0}(h_{i,0}))$ is Λ-measurable and bounded (by H.2.i). Therefore, assumption 3.4.2 of Balder (1995) holds.

Finally, nonatomicity of λ guarantees that assumption 3.4.1 of Balder (1995) holds too.

Therefore, by Theorem 3.4.1 of Balder (1995) there exists a λ-measurable almost everywhere selection (c^*,e^*) of the correspondence $B : i \mapsto B_i$ such that for λ-almost all i, $(c_i^*,e_i^*) \in \arg\max_{(x,y)\in B_i} \left(\sum_{s\in S_0} \pi_s u_i(x_s,y_s) + \sum_{s\in S_0} \pi_s \left[\gamma_i\left(v(x_s) - m_s(c^*,e^*)\right)\right]\right)$ where $m_s(c^*,e^*) =$ $\int_I g_s(\iota,(c_\iota^*,e_\iota^*))\,d\lambda(\iota) = \int_I v\left(c_{\iota,s}^*\right) d\lambda(\iota)$. Since B_i is never empty, wlog, we can assume that $(c_i^*,e_i^*) \in B_i$ for all $i \in I$. Then, by Corollary, we only have to check that (c^*,e^*) is bounded. This is easily obtained, setting $\Xi = \max_{s\in S_0}(\sup_{i\in I}(F_{i,s}(h_{i,s}) + h_{i,s}))$ which is finite by H.2.i, and observing that, for all $i \in I$, $B_i \subseteq [0,F_{i,0}(h_{i,0})] \times \prod_{s=1}^S [0,F_{i,s}(h_{i,s}) + RF_{i,0}(h_{i,0})] \times \prod_{s=0}^S [0,h_{i,s}] \subseteq [0,\Xi] \times \prod_{s=1}^S [0,(1+R)\Xi] \times \prod_{s=0}^S [0,\Xi]$. ■

Proof of Lemma 1. For all $i \in I$, problem (10) is equivalent to $\max_{0\leq y\leq h_i} u_i(F_i(y), y)$. Setting $U_i(y) = u_i(F_i(y), y)$ for all $y \in \mathbb{R}_+$ it is easily checked that

$$U_i'(y) = \nabla u_i(F_i(y), y) \begin{bmatrix} F_i'(y) \\ 1 \end{bmatrix} = F_i'(y) \frac{\partial u_i}{\partial x}(F_i(y), y) + \frac{\partial u_i}{\partial y}(F_i(y), y)$$

and

$$U_i''(y) = \begin{bmatrix} F_i'(y) \\ 1 \end{bmatrix}^\intercal \nabla^2 u_i(F_i(y), y) \begin{bmatrix} F_i'(y) \\ 1 \end{bmatrix} + F_i''(y) \frac{\partial u_i}{\partial x}(F_i(y), y).$$

H.3 guarantees that $U_i'' < 0$ on $(0,h_i)$, in particular, U_i is concave on $[0,h_i]$ and U_i' is strictly decreasing on $(0,h_i)$.

If U_i' never vanishes, since derivatives have the Darboux property, then U_i is either strictly increasing or decreasing on $[0,h_i]$ and the maximum is achieved at $y_i^* = h_i$ or $y_i^* = 0$ (and nowhere else).

If U_i' vanishes at some y_i^* in $(0,h_i)$, then y_i^* is the unique maximum (U_i' is strictly decreasing on $(0,h_i)$).

We can conclude that if an equilibrium profile (c^*,e^*) exists, then it is, λ-a.e. unique since it must satisfy $c_i^* = F_i(y_i^*)$ and $e_i^* = y_i^*$ for λ-almost all $i \in I$. ■

For any function f on $\mathbb{R}$, set

$$D^{\pm}f(t) = \lim\sup_{h\to 0\pm} \frac{f(t+h)-f(t)}{h} \text{ and } D_{\pm}f(t) = \lim\inf_{h\to 0\pm} \frac{f(t+h)-f(t)}{h}. \tag{15}$$

Next proposition shows that H.1, H.2, and H.4-(iii) imply that $W : E \to L$ is well defined.

Proposition 3 *If H1-H2 hold, then* $W_{(\cdot)}(e) : i \mapsto W_i(e)$ *is* Λ*-measurable for all* $e \in E$*. Moreover,* $W(e) \in L$ *for all* $e \in E$ *provided*

$$\sup_{i\in I}\left(\sup_{(x,y,z)\in[0,n]^2\times[-n,n]} |u_i(x,y)+\gamma_i(z)|\right) < \infty \quad \forall n \in \mathbb{N}. \tag{16}$$

Proof. Fix $e \in E$. The function $i \mapsto F_i(e(i))$ is Λ-measurable, since $i \mapsto (i, e(i))$ is Λ-measurable from I to $I \times \mathbb{R}_+$ and $(i,t) \mapsto F_i(t)$ is a Caratheodory function from $I \times \mathbb{R}_+$ to $\mathbb{R}_+$. H.2-(i) implies that the function $i \mapsto F_i(e(i))$ is also bounded.

Set $m^* = \int v(F_\iota(e_\iota))\, d\lambda(\iota)$.

For every $i \in I$, the real valued function on $\mathbb{R}^2_+$ defined by $(x,y) \mapsto u_i(x,y)+\gamma_i(v(x)-m^*)$ is continuous, and for every $(x,y) \in \mathbb{R}^2_+$, the real valued function on I defined by $i \mapsto u_i(x,y)+\gamma_i(v(x)-m^*)$ is Λ-measurable. Hence, the real valued function on $I \times \mathbb{R}^2_+$ defined by

$$(i,(x,y)) \mapsto u_i(x,y)+\gamma_i(v(x)-m^*) \tag{17}$$

is $\Lambda \times \mathcal{B}(\mathbb{R}^2_+)$-measurable (being a Caratheodory function).

Conclude that $i \mapsto (i, F_i(e_i), e_i)$ is Λ-measurable from I to $I \times \mathbb{R}^2_+$ (since $i \mapsto F_i(e(i))$ is Λ-measurable), and its composition with (17) delivers measurability of $W_{(\cdot)}(e)$.

Finally $\sup_{i\in I}|W_i(e)| = \sup_{i\in I}|u_i(F_i(e_i), e_i)+\gamma_i(v(F_i(e_i))-m^*)|$. By H.2.i, it follows that $\Xi = \sup_{i\in I}(F_i(h_i)+h_i) < \infty$ hence $(0,0) \le (F_i(e_i), e_i) \le (F_i(h_i), h_i) \le (\Xi,\Xi)$ for all $i \in I$, moreover $v(F_i(e_i)) \in [v(0), v(\Xi)]$ and $v(F_i(e_i)) - m^* \in [v(0)-m^*, v(\Xi)-m^*]$ for all $i \in I$, thus $\sup_{i\in I}|W_i(e)| \le \sup_{i\in I}\left(\sup_{(x,y,z)\in[0,\Xi]^2\times[v(0)-m^*,v(\Xi)-m^*]}|u_i(x,y)+\gamma_i(z)|\right)$ which is finite if (16) holds. ■

Lemma 5 *If H.1-H.3 and H.4.i hold, then all social equilibria* (c^*, e^*) *are such that* $c^*_i \ge \hat{c}_i$ *and* $e^*_i \ge \hat{e}_i$ λ*-a.e. If* (c^*, e^*) *is internal and* $D^+\gamma_i > 0$*, then,* $e^*_i > \hat{e}_i$ *and* $c^*_i > \hat{c}_i$ λ*-a.e..*

Proof. Notice that, by Lemma 3, if a pair $(c^*, e^*) \in L \times L$ is a social equilibrium, then, setting $m^* = \int (v \circ c^*)\, d\lambda$, (c^*_i, e^*_i) is a solution of problem

$$\max_{(x,y)\in B_i} u_i(x,y)+\gamma_i(v(x)-m^*). \tag{18}$$

For all $i \in I$, problem (18) is equivalent to $\max_{0\le y\le h_i} u_i(F_i(y), y)+\gamma_i(v(F_i(y))-m^*)$.

If (c^*, e^*) is a social equilibrium and $(\hat{c}, \hat{e})$ is the asocial equilibrium, then for λ-almost all $i \in I$,

$$e^*_i \in \arg\max_{0\le y\le h_i} u_i(F_i(y), y)+\gamma_i(v(F_i(y))-m^*),$$

$$c^*_i = F(e^*_i), \quad m^* = \int v(F_\iota(e^*_\iota))\, d\lambda(\iota),$$

$$\hat{e}_i \in \arg\max_{0\le y\le h_i} u_i(F_i(y), y), \quad \hat{c}_i = F(\hat{e}_i).$$

If $e^*_i = h_i$ or $\hat{e}_i = 0$, then $e^*_i \ge \hat{e}_i$ and $c^*_i = F_i(e^*_i) \ge F_i(\hat{e}_i) = \hat{c}_i$.

If $e^*_i = 0$ and $\hat{e}_i > 0$, then:

- either U_i' never vanishes in $(0, h_i)$,[14] then U_i is strictly increasing (it cannot be strictly decreasing, otherwise $\hat{e}_i = 0$), but also $\gamma_i\left(v(F_i(\cdot)) - m^*\right)$ is increasing and the inclusion $0 \in \arg\max_{0 \le y \le h_i} u_i\left(F_i(y), y\right) + \gamma_i\left(v(F_i(y)) - m^*\right) = \arg\max_{0 \le y \le h_i} U_i(y) + \gamma_i\left(v(F_i(y)) - m^*\right)$ is absurd,

- or U_i' vanishes at $\hat{e}_i \in (0, h_i)$, then U_i' – being strictly decreasing – must be positive in a right neighborhood of 0, again $u_i\left(F_i(\cdot), \cdot\right) + \gamma_i\left(v(F_i(\cdot)) - m^*\right)$ is strictly increasing in a right neighborhood of 0, which is absurd.

It follows that, if $e_i^* = 0$, then $\hat{e}_i = 0$, thus $e_i^* \ge \hat{e}_i$ and $c_i^* = F_i\left(e_i^*\right) \ge F_i\left(\hat{e}_i\right) = \hat{c}_i$.

Finally, if $e_i^* \in (0, h_i)$, then

$$D^+\left[u_i\left(F_i(\cdot), \cdot\right) + \gamma_i\left(v(F_i(\cdot)) - m^*\right)\right]\left(e_i^*\right) \le 0 \tag{19}$$

that is $U_i'\left(e_i^*\right) + F_i'\left(e_i^*\right) v'\left(F_i\left(e_i^*\right)\right) D^+\gamma_i(v(F_i\left(e_i^*\right)) - m^*) \le 0$ and

$$U_i'\left(e_i^*\right) \le -F_i'\left(e_i^*\right) v'\left(F_i\left(e_i^*\right)\right) D^+\gamma_i(v(F_i\left(e_i^*\right)) - m^*). \tag{20}$$

By monotonicity, $D^+\gamma_i \ge 0$, therefore $U_i'\left(e_i^*\right) \le 0$, which implies $e_i^* \ge \hat{e}_i$ because from the proof of Lemma 1 we know that U_i is concave on $[0, h_i]$ with a unique maximum. Again $c_i^* = F_i\left(e_i^*\right) \ge F_i\left(\hat{e}_i\right) = \hat{c}_i$.

Suppose that (c^*, e^*) is an internal social equilibrium and $D^+\gamma_i > 0$. Then (20) delivers $U_i'\left(e_i^*\right) < 0$, which implies $e_i^* > \hat{e}_i$ because from the proof of Lemma 1 we know that U_i is concave on $[0, h_i]$ with a unique maximum. It follows that $c_i^* = F_i\left(e_i^*\right) > F_i\left(\hat{e}_i\right) = \hat{c}_i$. ■

Proposition 4 *If assumptions H.1-H.4 hold, then all internal social equilibrium profiles (e^*, c^*) are strongly inefficient.*

Proof. Suppose, *per contra*, that $(c^*, e^*) \in L \times L$ is a social equilibrium with $e^* \in \operatorname{int}(E)$ and (c^*, e^*) is not strongly inefficient. Let $f : L \to \mathbb{R}$ be defined by $f(\xi) = \operatorname{essinf}_\lambda\left[\xi - W\left(e^*\right)\right]$ for all $\xi \in L$. Then, $f\left(W\left(e^*\right)\right) = 0$ and $f\left(W(e)\right) \le 0$ for all $e \in E$. Moreover, f is a concave niveloid.[15]

Next we show that there is no concave niveloid $f : L \to \mathbb{R}$ such that e^* solves the problem $\max_{e \in E}\left(f \circ W\right)(e)$, which is absurd.

First observe that Gateaux differentiability of W guarantees that for all $e \in \operatorname{int}E$ there exists a linear and continuous operator $\nabla W(e) : L \to L$ such that

$$\lim_{t \to 0} \frac{W(e + tk) - W(e)}{t} = \nabla W(e)(k) \in L \tag{21}$$

for all $k \in L$. Arbitrarily choose $e \in \operatorname{int}E$ and $k \in L$, (21) means that

$$\lim_{t \to 0} \left\| \frac{W(e + tk) - W(e)}{t} - \nabla W(e)(k) \right\|_{\sup} = 0.$$

[14] U_i is defined in the proof of Lemma 1.

[15] A functional $f : M \to \mathbb{R}$ is a niveloid (see Maccheroni, Marinacci, and Rustichini, 2006) if, for all ψ and φ in M and $c \in \mathbb{R}$, we have: (i) $\varphi \ge \psi$ implies $f(\varphi) \ge f(\psi)$; (ii) $f(\varphi + c) = f(\varphi) + c$.

A fortiori, for all $i \in I$, $\frac{W_i(e+tk)-W_i(e)}{t} \to \nabla W(e)(k)_i$ as $t \to 0$, but for all $i \in I$

$$\frac{W_i(e+tk)-W_i(e)}{t} = \frac{u_i(F_i(e_i+tk_i), e_i+tk_i) + \gamma_i\left(v(F_i(e_i+tk_i)) - \int v(F_\iota(e_\iota+tk_\iota))\,d\lambda(\iota)\right)}{t} + \\ - \frac{u_i(F_i(e_i), e_i) + \gamma_i\left(v(F_i(e_i)) - \int v(F_\iota(e_\iota))\,d\lambda(\iota)\right)}{t}.$$

It is relatively easy to show that, this implies

$$\begin{aligned}\nabla W(e)(k)_i = \; & k_i U_i'(e_i) + \gamma_i'\left(v(F_i(e_i)) - \int v(F_\iota(e_\iota))\,d\lambda(\iota)\right) \times \\ & \times \left(v'(F_i(e_i))F_i'(e_i)\,k_i - \int v'(F_\iota(e_\iota))\,F_\iota'(e_\iota)\,k_\iota d\lambda(\iota)\right)\end{aligned} \tag{22}$$

for λ-almost all $i \in I$.

If $f : L \to \mathbb{R}$ is concave niveloid, then it is Lipschitz and its superdifferential at each point consists of probability charges that are absolutely continuous with respect to λ.

By a chain rule for the Clarke differential (see Theorem 2.3.10 in Clarke, 1983), we have that $f \circ W$ is Lipschitz near e and $\partial(f \circ W)(e) \subseteq \partial f(W(e)) \circ \nabla W(e)$. That is, for all $\mu \in \partial(f \circ W)(e)$ there is $\nu \in \partial f(W(e))$ such that $\mu = \nu \circ \nabla W(e)$. Therefore, for all $k \in L$

$$\mu(k) = \nu(\nabla W(e)(k)) = \int_I \nabla W(e)(k)\,d\nu$$
$$= \int_I \left[k_i U_i'(e_i) + \gamma_i'\left(v(F_i(e_i)) - \int v(F_\iota(e_\iota))\,d\lambda(\iota)\right)\left(v'(F_i(e_i))F_i'(e_i)\,k_i - \int v'(F_\iota(e_\iota))\,F_\iota'(e_\iota)\,k_\iota d\lambda(\iota)\right)\right] d\nu(i).$$

If, as assumed *per contra*, e^* is a local maximum of $f \circ W$ on E, then $\partial(f \circ W)(e^*) \ni 0$, and there exists a probability charge $\nu \in \partial f(W(e^*))$ such that $\nu \circ \nabla W(e^*) = 0$, that is, for all $k \in L$

$$\int_I \left[k_i U_i'(e_i^*) + \gamma_i'\left(v(F_i(e_i^*)) - \int v(F_\iota(e_\iota^*))\,d\lambda(\iota)\right)\left(v'(F_i(e_i^*))F_i'(e_i^*)\,k_i - \int v'(F_\iota(e_\iota^*))\,F_\iota'(e_\iota^*)\,k_\iota d\lambda(\iota)\right)\right] d\nu(i) = 0 \tag{23}$$

But, for all $i \in I$, problem (18) is equivalent to

$$\max_{0 \le y \le h_i} u_i(F_i(y), y) + \gamma_i(v(F_i(y)) - m^*). \tag{24}$$

Therefore, if (c^*, e^*) is an internal social equilibrium,

- $c_i^* = F_i(e_i^*)$ for all $i \in I$.
- e_i^* is a solution of problem (24) for λ-almost all $i \in I$.
- $m^* = \int v(F_\iota(e_\iota^*))\,d\lambda(\iota)$.

In particular, first order conditions implied by the second point, see (19) and recall that now γ_i is differentiable, amount to $U_i'(e_i^*) + \gamma_i'(v(F_i(e_i^*)) - m^*)\,v'(F_i(e_i^*))F_i'(e_i^*) = 0$ for λ-almost all $i \in I$. Which plugged into (23) delivers,

$$\int v'(F_\iota(e_\iota^*))\,F_\iota'(e_\iota^*)\,k_\iota d\lambda(\iota) \int_I -\gamma_i'(v(F_i(e_i^*)) - m^*)\,d\nu(i)$$

$= 0$ for all $k \in L$.

Since v' and F'_ι are positive and λ is σ-additive, then $\int v'\left(F_\iota\left(e^*_\iota\right)\right) F'_\iota\left(e^*_\iota\right) k_\iota d\lambda\left(\iota\right) > 0$ for some $k \in L$ (e.g. $k_\iota = 1$ for all $\iota \in I$) and it must be the case that $\int_I -\gamma'_i\left(v(F_i\left(e^*_i\right)) - m^*\right) d\nu\left(i\right) = 0$, which is absurd since γ'_i is bounded away from 0. ■

Proof of Proposition 1. It follows from Lemma 5 and Proposition 4. ■

Proof of Lemma 2. Clearly U is continuous and strictly concave on $[0, \bar{x}_0]$. Therefore $\arg\max_{x\in[0,\bar{x}_0]} U\left(x\right)$ is a singleton. The conditions on the directional derivatives provide internality. ■

Proof of Theorem 2. Set

$$W\left(x\right) = U\left(x\right) + \gamma\left(v\left(x\right) - m^*_0\right) + \beta \sum_{s\in S} p_s \left[\gamma\left(v(\bar{x}_s + R(\bar{x}_0 - x)) - m^*_s\right)\right]$$

for all $x \in [0, \bar{x}_0]$, and $y_s = \bar{x}_s + R(\bar{x}_0 - x)$ for all $s \in S$. For all $x^* \in [0, \bar{x}_0)$,

$$\begin{aligned} D^+W\left(x^*\right) \leq{}& \limsup_{h\to 0^+} \frac{U\left(x^* + h\right) - U\left(x^*\right)}{h} \\ &\dot{+} \limsup_{h\to 0^+} \frac{\gamma\left(v\left(x^* + h\right) - m^*_0\right) - \gamma\left(v\left(x^*\right) - m^*_0\right)}{h} \\ &\dot{+}\beta \sum_{s\in S} p_s \limsup_{h\to 0^+} \frac{\gamma\left(v(y^*_s - Rh) - m^*_s\right) - \gamma\left(v\left(y^*_s\right) - m^*_s\right)}{h} \end{aligned}$$

and

$$\begin{aligned} D_+W\left(x^*\right) \geq{}& \liminf_{h\to 0^+} \frac{U\left(x^* + h\right) - U\left(x^*\right)}{h} \\ &+ \liminf_{h\to 0^+} \frac{\gamma\left(v\left(x^* + h\right) - m^*_0\right) - \gamma\left(v\left(x^*\right) - m^*_0\right)}{h} \\ &+ \beta \sum_{s\in S} p_s \liminf_{h\to 0^+} \frac{\gamma\left(v(y^*_s - Rh) - m^*_s\right) - \gamma\left(v\left(y^*_s\right) - m^*_s\right)}{h}. \end{aligned}$$

Analogously, for all $x^* \in (0, \bar{x}_0]$,

$$\begin{aligned} D^-W\left(x^*\right) \leq{}& \limsup_{h\to 0^-} \frac{U\left(x^* + h\right) - U\left(x^*\right)}{h} \\ &\dot{+} \limsup_{h\to 0^-} \frac{\gamma\left(v\left(x^* + h\right) - m^*_0\right) - \gamma\left(v\left(x^*\right) - m^*_0\right)}{h} \\ &\dot{+}\beta \sum_{s\in S} p_s \limsup_{h\to 0^-} \frac{\gamma\left(v(y^*_s - Rh) - m^*_s\right) - \gamma\left(v\left(y^*_s\right) - m^*_s\right)}{h} \end{aligned}$$

and

$$
\begin{aligned}
D_{-}W\left(x^{*}\right) \geq\ & \liminf{}_{h\to 0^{-}} \frac{U\left(x^{*}+h\right)-U\left(x^{*}\right)}{h} \\
& + \liminf{}_{h\to 0^{-}} \frac{\gamma\left(v\left(x^{*}+h\right)-m_{0}^{*}\right)-\gamma\left(v\left(x^{*}\right)-m_{0}^{*}\right)}{h} \\
& + \beta\sum_{s\in S} p_{s} \liminf{}_{h\to 0^{-}} \frac{\gamma\left(v(y_{s}^{*}-Rh)-m_{s}^{*}\right)-\gamma\left(v\left(y_{s}^{*}\right)-m_{s}^{*}\right)}{h}.
\end{aligned}
$$

(i) Consider any symmetric consumption profile, where all agents consume the same amount $x^{*}\in[0,\bar{x}_{0}]$ in the first period (i.e. $c_{i}^{*}=x^{*}$ for all $i\in I$), and $y_{s}^{*}=\bar{x}_{s}+R(\bar{x}_{0}-x^{*})$ in each state in the second period. Then

$$
m_{0}^{*}\left(c^{*}\right)=\int_{I} v\left(x^{*}\right) d\lambda\left(\iota\right)=v\left(x^{*}\right) \text{ and } m_{s}^{*}\left(c^{*}\right)=\int_{I} v\left(\bar{x}_{s}+R\left(\bar{x}_{0}-x^{*}\right)\right) d\lambda\left(\iota\right)=v(y_{s}^{*}) \quad \forall s\in S.
$$

For $x^{*}\in[0,\bar{x}_{0})$

$$
\begin{aligned}
D_{+}W\left(x^{*}\right) &\geq U_{+}'\left(x^{*}\right)+\liminf{}_{h\to 0^{+}} \frac{\gamma\left(v\left(x^{*}+h\right)-v\left(x^{*}\right)\right)-\gamma\left(0\right)}{h} \\
&\quad + \beta\sum_{s\in S} p_{s} \sup_{\delta>0} \inf_{h\in(0,\delta)} \frac{\gamma\left(v(y_{s}^{*}-Rh)\right)-v\left(y_{s}^{*}\right))-\gamma\left(v\left(y_{s}^{*}\right)-v\left(y_{s}^{*}\right)\right)}{h} \\
&= U_{+}'\left(x^{*}\right)+\liminf{}_{h\to 0^{+}} \frac{\gamma\left(v\left(x^{*}+h\right)-v\left(x^{*}\right)\right)-\gamma\left(0\right)}{h}
\end{aligned}
$$

since $\gamma_{|(-\infty,0]}\equiv 0$. Moreover $v_{0}:\mathbb{R}_{+}\to\mathbb{R}$ defined by $v_{0}\left(h\right)=v\left(x^{*}+h\right)-v\left(x^{*}\right)$ for all $h\in[0,+\infty)$ is concave, strictly increasing and continuous with $v_{0}\left(0\right)=0$ and $\left(v_{0}\right)_{+}'\left(0\right)=v_{+}'\left(x^{*}\right)$, thus $D_{+}W\left(x^{*}\right)\geq U_{+}'\left(x^{*}\right)+D_{+}\left(\gamma\circ v_{0}\right)\left(0\right)$. The assumption $D_{+}\gamma\left(0\right)>0$ allows to apply a chain rule for Dini derivatives so that $D_{+}W\left(x^{*}\right)\geq U_{+}'\left(x^{*}\right)+\left(v_{0}\right)_{+}'\left(0\right)D_{+}\gamma\left(v_{0}\left(0\right)\right)=U_{+}'\left(x^{*}\right)+v_{+}'\left(x^{*}\right)D_{+}\gamma\left(0\right)$. Analogously, for $x^{*}\in(0,\bar{x}_{0}]$

$$
D^{-}W\left(x^{*}\right)\leq U_{-}'\left(x^{*}\right)\mathbin{\dot{-}}\beta\sum_{s\in S} p_{s}\liminf{}_{h\to 0^{+}} \frac{\gamma\left(v(y_{s}^{*}+Rh)-v\left(y_{s}^{*}\right)\right)-\gamma\left(0\right)}{h}.
$$

since $\gamma_{|(-\infty,0]}\equiv 0$. Moreover, for all $s\in S$, $v_{s}:\mathbb{R}_{+}\to\mathbb{R}$ defined by $v_{s}\left(h\right)=v\left(y_{s}^{*}+Rh\right)-v\left(y_{s}^{*}\right)$ for all $h\in[0,+\infty)$ is concave, strictly increasing and continuous with $v_{s}\left(0\right)=0$ and $\left(v_{s}\right)_{+}'\left(0\right)=Rv_{+}'\left(y_{s}^{*}\right)$, thus $D^{-}W\left(x^{*}\right)\leq U_{-}'\left(x^{*}\right)\mathbin{\dot{-}}\beta\sum_{s\in S}p_{s}D_{+}\left(\gamma\circ v_{s}\right)\left(0\right)$. The assumption $D_{+}\gamma\left(0\right)>0$ allows to apply a suitable chain rule so that

$$
D^{-}W\left(x^{*}\right)\leq U_{-}'\left(x^{*}\right)\mathbin{\dot{-}}\beta R\sum_{s\in S} p_{s}v_{+}'\left(y_{s}^{*}\right)D_{+}\gamma\left(0\right).
$$

Summing up:

$$
\begin{aligned}
& D_{+}W\left(x^{*}\right)\geq U_{+}'\left(x^{*}\right)+v_{+}'\left(x^{*}\right)D_{+}\gamma\left(0\right) \quad \forall x^{*}\in[0,\bar{x}_{0}) \text{ and} \\
& D^{-}W\left(x^{*}\right)\leq U_{-}'\left(x^{*}\right)\mathbin{\dot{-}}\beta R\sum_{s\in S} p_{s}v_{+}'\left(y_{s}^{*}\right)D_{+}\gamma\left(0\right) \quad \forall x^{*}\in(0,\bar{x}_{0}].
\end{aligned}
$$

- If $x^* \in (0, \bar{x}_0)$, then, since u is differentiable on $(0, +\infty)$, U is differentiable at x^* and $U'_+(x^*) = U'_-(x^*) = U'(x^*)$. Thus
  - either $U'(x^*) \geq 0$, then $D_+W(x^*) > 0$ and x^* is not a maximizer.
  - either $U'(x^*) < 0$, then $D^-W(x^*) < 0$ and x^* is not a maximizer.
- If $x^* = 0$ then $U'_+(0) > 0$ and $D_+W(x^*) > 0$, thus x^* is not a maximizer.
- If $x^* = \bar{x}_0$ then $U'_-(\bar{x}_0) < 0$ and $D^-W(x^*) < 0$, thus x^* is not a maximizer.

(ii) Notice that if $c^* \in L$ is a social equilibrium, then, setting $m_0^* = \int_I v(c_\iota)\, d\lambda(\iota)$ and $m_s^* = \int_I v(\bar{x}_s + R(\bar{x}_0 - c_\iota)) d\lambda(\iota)$ for all $s \in S$, c_i^* is a solution of problem

$$\max_{x \in [0, \bar{x}_0]} U(x) + \gamma(v(x) - m_0^*) + \beta \sum_{s \in S} p_s [\gamma(v(\bar{x}_s + R(\bar{x}_0 - x)) - m_s^*)] \tag{25}$$

for λ-almost all $i \in I$.

Let $c^* : I \to \mathbb{R}$ be an asymmetric social equilibrium and $I^* \in \Lambda$ be such that $\lambda(I^*) = 1$ and c_i^* is a solution of problem (25) for all $i \in I^*$.

There is at least one agent, call him $i_0 \in I^*$, such that $v\left(c_{i_0}^*\right) < m_0^*$. For, assume per contra $v(c_i^*) \geq m_0^*$ for all $i \in I^*$. Then, $\left(v \circ c^* - \int_I v(c_\iota^*)\, d\lambda(\iota)\right) \geq 0$ λ-a.e. and

$$\int_I \left(v(c_i^*) - \int_I v(c_\iota^*)\, d\lambda(\iota) \right) d\lambda(i) = 0.$$

Therefore, $v \circ c^* = \int_I v(c_\iota^*)\, d\lambda(\iota)$ λ-a.e. and $c^* = v^{-1}\left(\int_I v(c_\iota^*)\, d\lambda(\iota)\right)$ λ-a.e., which is absurd since c^* is asymmetric. Analogously, for all $s \in S$ there exists $i_s \in I^*$ such that $v(\bar{x}_s + R(\bar{x}_0 - c_{i_s}^*) < m_s^*$ for all $s \in S$.

Suppose agent i_1 is such that $c_{i_1}^* = \max_{s \in S} c_{i_s}^*$. Then, $v(\bar{x}_s + R(\bar{x}_0 - c_{i_1}^*)) \leq v(\bar{x}_s + R(\bar{x}_0 - c_{i_s}^*)) < m_s^*$ for all $s \in S$. Since v is concave, by the Jensen inequality,

$$v\left(c_{i_0}^*\right) < m_0^* < v\left(c_{i_1}^*\right) \text{ and } v(\bar{x}_s + R(\bar{x}_0 - c_{i_1}^*)) < m_s^* < v(\bar{x}_s + R(\bar{x}_0 - c_{i_0}^*)) \quad \forall s \in S.$$

In particular $0 \leq c_{i_0}^* < c_{i_1}^* \leq \bar{x}_0$. Since v is continuous and increasing, there is $\varepsilon > 0$ small enough so that

$$0 \leq c_{i_0}^* < c_{i_0}^* + \varepsilon < c_{i_1}^* - \varepsilon < c_{i_1}^* \leq \bar{x}_0, \tag{26}$$

$$v(x) < m_0^* \text{ and } m_s^* < v(\bar{x}_s + R(\bar{x}_0 - x)) \quad \forall s \in S, \forall x \in \left[0, c_{i_0}^* + \varepsilon\right), \tag{27}$$

$$m_0^* < v(x) \text{ and } v(\bar{x}_s + R(\bar{x}_0 - x)) < m_s^* \quad \forall s \in S, \forall x \in \left(c_{i_1}^* - \varepsilon, \bar{x}_0\right]. \tag{28}$$

Since $c_{i_0}^*$ is a maximizer for W on $[0, \bar{x}_0]$, the first order conditions guarantee that

$$\begin{aligned}
0 \geq D_+W\left(c_{i_0}^*\right) &\geq \liminf_{h \to 0^+} \frac{U\left(c_{i_0}^* + h\right) - U\left(c_{i_0}^*\right)}{h} \\
&+ \liminf_{h \to 0^+} \frac{\gamma\left(v\left(c_{i_0}^* + h\right) - m_0^*\right) - \gamma\left(v\left(c_{i_0}^*\right) - m_0^*\right)}{h} \\
&+ \beta \sum_{s \in S} p_s \liminf_{h \to 0^+} \frac{\gamma\left(v\left(\bar{x}_s + R(\bar{x}_0 - c_{i_0}^*) - Rh\right) - m_s^*\right) - \gamma\left(v\left(\bar{x}_s + R(\bar{x}_0 - c_{i_0}^*)\right) - m_s^*\right)}{h} \\
&= U'_+\left(c_{i_0}^*\right) + \liminf_{h \to 0^+} \frac{\gamma\left(v\left(c_{i_0}^* + h\right) - m_0^*\right) - \gamma\left(v\left(c_{i_0}^*\right) - m_0^*\right)}{h}.
\end{aligned}$$

In fact, the third summand is null because of $\gamma_{|[0,+\infty)} \equiv 0$ and (27). Consider the function $v_0 : \mathbb{R}_+ \to \mathbb{R}$ defined by $v_0(h) = v\left(c^*_{i_0} + h\right) - m^*_0$ for all $h \in [0,+\infty)$, v_0 is concave, strictly increasing and continuous with $v_0(0) = v\left(c^*_{i_0}\right) - m^*_0$ and $(v_0)'_+(0) = v'_+\left(c^*_{i_0}\right)$. The condition $D_+\gamma_{|(-\infty,0)} > 0$, equation (27), and a suitable chain rule deliver $0 \geq D_+W\left(c^*_{i_0}\right) \geq U'_+\left(c^*_{i_0}\right) + \liminf_{h\to 0^+} \frac{\gamma(v_0(h))-\gamma(v_0(0))}{h} = U'_+\left(c^*_{i_0}\right) + v'_+\left(c^*_{i_0}\right) D_+\gamma\left(v\left(c^*_{i_0}\right) - m^*_0\right)$, that is,

$$U'_+\left(c^*_{i_0}\right) \leq -v'_+\left(c^*_{i_0}\right) D_+\gamma\left(v\left(c^*_{i_0}\right) - m^*_0\right) < 0. \tag{29}$$

Analogously, the first order conditions at $c^*_{i_1}$ guarantee that

$$\begin{aligned}
&0 \leq D^-W\left(c^*_{i_1}\right) \leq \\
&\leq \limsup_{h\to 0^-} \frac{U\left(c^*_{i_1}+h\right) - U\left(c^*_{i_1}\right)}{h} \\
&\dot{+} \limsup_{h\to 0^-} \frac{\gamma\left(v\left(c^*_{i_1}+h\right) - m^*_0\right) - \gamma\left(v\left(c^*_{i_1}\right) - m^*_0\right)}{h} \\
&\dot{+}\beta \sum_{s\in S} p_s \limsup_{h\to 0^-} \frac{\gamma\left(v\left(\left(\bar{x}_s + R\left(\bar{x}_0 - c^*_{i_1}\right)\right) - Rh\right) - m^*_s\right) - \gamma\left(v\left(\bar{x}_s + R\left(\bar{x}_0 - c^*_{i_1}\right)\right) - m^*_s\right)}{h} = \\
&U'_-\left(c^*_{i_1}\right) \dot{+}\beta \sum_{s\in S} p_s \inf_{\delta>0} \sup_{h\in(0,\delta)} \frac{\gamma\left(v\left(\left(\bar{x}_s + R\left(\bar{x}_0 - c^*_{i_1}\right)\right) + Rh\right) - m^*_s\right) - \gamma\left(v\left(\bar{x}_s + R\left(\bar{x}_0 - c^*_{i_1}\right)\right) - m^*_s\right)}{-h}
\end{aligned}$$

since the second summand is null because of $\gamma_{|[0,+\infty)} \equiv 0$ and (28). Consider, for all $s \in S$ the function $v_s : \mathbb{R}_+ \to \mathbb{R}$ defined by $v_s(h) = v\left(\left(\bar{x}_s + R\left(\bar{x}_0 - c^*_{i_1}\right)\right) + Rh\right) - m^*_s$ for all $h \in [0,+\infty)$ is concave, strictly increasing and continuous with $v_s(0) = v\left(\bar{x}_s + R\left(\bar{x}_0 - c^*_{i_1}\right)\right) - m^*_s$ and $(v_s)'_+(0) = Rv'_+\left(\bar{x}_s + R\left(\bar{x}_0 - c^*_{i_1}\right)\right)$. Equation (28) implies $v_s(0) = v(\bar{x}_s + R(\bar{x}_0 - c^*_{i_1}) - m^*_s < 0$, then $D_+\gamma_{|(-\infty,0)} > 0$ and a suitable chain rule deliver

$$\begin{aligned}
0 \leq D^-W\left(c^*_{i_1}\right) &\leq U'_-\left(c^*_{i_1}\right) \dot{+}\beta \sum_{s\in S} p_s \inf_{\delta>0} \sup_{h\in(0,\delta)} \frac{\gamma(v_s(h)) - \gamma(v_s(0))}{-h} \\
&= U'_-\left(c^*_{i_1}\right) \dot{-}\beta \sum_{s\in S} p_s (v_s)'_+(0)\, D_+\gamma(v_s(0)) \\
&= U'_-\left(c^*_{i_1}\right) \dot{-}\beta R \sum_{s\in S} p_s v'_+\left(\bar{x}_s + R\left(\bar{x}_0 - c^*_{i_1}\right)\right) D_+\gamma(v_s(0)).
\end{aligned}$$

That is,

$$U'_-\left(c^*_{i_1}\right) \geq \beta R \sum_{s\in S} p_s v'_+\left(\bar{x}_s + R\left(\bar{x}_0 - c^*_{i_1}\right)\right) D_+\gamma(v_s(0)) > 0. \tag{30}$$

Denoting by $\hat{c}$ the unique (internal) asocial equilibrium it follows, from the differentiability of u on $(0,+\infty)$, that $U'_+(\hat{c}) = U'_-(\hat{c}) = U'(\hat{c}) = 0$. Thus, since U'_+ and U'_- are decreasing, from (29) and (30) it follows that

$$\begin{aligned}
U'_+\left(c^*_{i_0}\right) < 0 = U'_+(\hat{c}) \Rightarrow c^*_{i_0} > \hat{c}, \\
U'_-\left(c^*_{i_1}\right) > 0 = U'_-(\hat{c}) \Rightarrow c^*_{i_1} < \hat{c},
\end{aligned}$$

and $c^*_{i_0} > c^*_{i_1}$, which contradicts (26). Therefore, c^* is not a social equilibrium.

Finally suppose $D^- \gamma(0) = 0$ and $c^* : I \to \mathbb{R}$ is a symmetric equilibrium with $c^*_i = x^*$ for λ-almost all $i \in I$. Then

$$\begin{aligned} 0 \geq D_+ W(x^*) \geq{} & U'_+(x^*) + \sup_{\delta>0} \inf_{h\in(0,\delta)} \frac{\gamma(v(x^*+h) - v(x^*)) - \gamma(0)}{h} \\ & + \beta \sum_{s\in S} p_s \sup_{\delta>0} \inf_{h\in(0,\delta)} \frac{\gamma(v(y^*_s - Rh)) - v(y^*_s)) - \gamma(0)}{h} \end{aligned}$$

if $x^* \in [0, \bar{x}_0)$, and

$$\begin{aligned} 0 \leq D^- W(x^*) \leq{} & U'_-(x^*) \dotplus \inf_{\delta>0} \sup_{h(0,\delta)} \frac{\gamma(v(x^*-h) - v(x^*)) - \gamma(0)}{-h} \\ & \dotplus \beta \sum_{s\in S} p_s \inf_{\delta>0} \sup_{h(0,\delta)} \frac{\gamma(v(y^*_s + Rh)) - v(y^*_s)) - \gamma(0)}{-h} \end{aligned}$$

if $x^* \in (0, \bar{x}_0]$. Then, $\gamma_{|[0,+\infty)} \equiv 0$ delivers

$$\begin{aligned} 0 \geq D_+ W(x^*) & \geq U'_+(x^*) + \beta \sum_{s\in S} p_s \sup_{\delta>0} \inf_{h\in(0,\delta)} \frac{\gamma(v(y^*_s - Rh)) - v(y^*_s)) - \gamma(0)}{h} \\ & = U'_+(x^*) - \beta \sum_{s\in S} p_s \limsup_{h\to 0^-} \frac{\gamma(v(y^*_s + Rh)) - v(y^*_s)) - \gamma(0)}{h} \end{aligned}$$

if $x^* \in [0, \bar{x}_0)$, and

$$\begin{aligned} 0 \leq D^- W(x^*) & \leq U'_-(x^*) \dotplus \inf_{\delta>0} \sup_{h(0,\delta)} \frac{\gamma(v(x^*-h) - v(x^*)) - \gamma(0)}{-h} \\ & = U'_-(x^*) \dotplus \limsup_{h\to 0^-} \frac{\gamma(v(x^*+h) - v(x^*)) - \gamma(0)}{h} \end{aligned}$$

if $x^* \in (0, \bar{x}_0]$. By a suitable chain rule, $0 \geq D_+ W(x^*) \geq U'_+(x^*) - \beta R \sum_{s\in S} p_s v'_-(y^*_s) D^- \gamma(0)$, that is, $0 \geq D_+ W(x^*) \geq U'_+(x^*)$ if $x^* \in [0, \bar{x}_0)$, and $0 \leq D^- W(x^*) \leq U'_-(x^*) \dotplus v'_-(x^*) D^- \gamma(0)$, that is, $0 \leq D^- W(x^*) \leq U'_-(x^*)$ if $x^* \in (0, \bar{x}_0]$. Therefore x^* is a maximizer for U. ■

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