Overconfidence is a Social Signal not a Judgment Bias *

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Abstract

Accurate assessments of one's abilities are important in many domains. These assessments form the basis for the choice of education, of career, of whether or not to start a business, or of the wage asked in negotiations. Evidence from psychology and economics indicates that many individuals overestimate their ability, both absolute (their real ability compared to the estimated ability) and relative (their real ranking compared to estimated ranking). These results are typically taken as evidence that individuals are overconfident in their abilities. Recent modeling efforts in economics have shown that this need not be irrational, but can be the outcome of rational Bayesian updating, thus providing a potential explanation for overconfidence [4].

In this paper, we provide three results: First, we show that Bayesian updating imposes restrictions on relative ability judgments that can be tested empirically. Using data on 1015 individuals' confidence judgments about two cognitive tests, we test these restrictions and reject the hypothesis that our data are generated by Bayesian updating and truthful statements. Second, we test whether self-image concerns contribute to overconfidence ([12, 26]). Our results suggest that individuals do not care about inferences created by signals of their ability, but rather consume the signals themselves - they like to hear that they are good. Third, we provide evidence that personality characteristics strongly affect confidence judgments: for example, subjects scoring high in the trait social dominance have more confident judgments about their ability, but they do not have in reality a better performance. Our evidence suggests that overconfidence in statements is bias induced by social concerns.

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1 Introduction

Well-calibrated judgments about one's abilities are important in many economic decisions. However, there is evidence from studies in psychology and economics suggesting that individuals have excessive confidence in their abilities. This excessive confidence may be absolute (subjects predict they will exhibit a better performance than they really do) or relative (subjects state their performance ranks higher, compared to that of others, than it really is). In this paper we will use the term overconfidence to describe relative excessive confidence. For example in a typical study few individuals rate themselves in the bottom 40 percent of a distribution, largely independent of the skill in question (see [1], [9], [23]). Individuals might err on the side of over and under confidence, but overconfidence seems the dominant behavior, although in some cases individuals may be factually incorrect in their evaluation in the opposite direction, under-confidence ([11]). Studies also link measures of overconfidence to behavior, and show that more confident judgments are associated with more daring behaviors. Malmendier and Tate ([17]) show that more confident CEOs make more daring merger decisions (see also [18]). Barber and Odean ([2]) show that men engage in more frequent trading in common stock, consistent with the evidence from psychology that men are more overconfident than women. The trading reduces their returns substantially relative to women. Thus, if overconfidence is truly a judgment bias, these studies should raise concern, as they raise the possibility that individuals act on biased beliefs.

If individuals had perfect knowledge of their abilities, results showing that, e.g., 50 percent of the individuals rate themselves in the top 25 percent of an ability distribution necessarily imply a judgment bias. [1] However, assuming perfect knowledge of one's ability may not be realistic. Rather, individuals may only vaguely know their abilities, and update their beliefs as new information arrives. A recent paper by Benoît and Dubra ([4]) shows that, if individuals have imperfect knowledge of their own ability, even perfectly rational Bayesian updaters may report overconfident beliefs in a typical study.

Benoît and Dubra point out that, in most studies, individuals are asked to indicate their most likely place in the ability distribution. They provide a general characterization of the information structure leading to results that, for example, 50 percent of the individuals put themselves in the top 25 percent of the ability distribution. Intuitively, this can arise if the signals individuals receive become more noisy the better the signals are, akin to taking an easy test: Everyone who fails the test can be sure that his ability is low. However, low-ability types sometimes also pass the test by sheer luck. But still, passing the test rationally leads individuals to believe it is more likely that they have high ability, therefore creating 'overconfidence' by this measure. Several papers [12, 26] also provide plausible psychological underpinnings, showing that such types of information structures can arise endogenously. They argue that image concerns by individuals lead individuals with high

[1] Merkle and Weber ([20]) do show overconfidence leads to bias in beliefs. Their test is based on eliciting the c.d.f. of beliefs over abilities for which it is difficult to pin down the true distribution. This allows them to reject Bayesian updating without even knowing what the true distribution of ability is.

beliefs to refrain from seeking more information, leading to an information structure that is conducive to creating overconfidence. Yet, while overconfidence in that sense may prevail in the population, the beliefs are still unbiased, as the individuals who think they are in the upper half of the distribution recognize that this is not sure and, in their model, attach the correct probability to this state. As a consequence, individuals may take actions based on these beliefs, which may not be optimal compared to the case in which the individual knew his true type. However, conditional on the information that the individual received, the judgments are consistent and their actions optimal.

In this paper, we show that if judgments are the result of Bayesian information processing from a common prior, then testable restrictions are placed on the beliefs as a function of the individual's true ability. As we show, it must be true that of all individuals placing themselves in ability quantile k, individuals of ability quantile k must be most likely to do so. Therefore, we can base a test of Bayesian overconfidence on whether this is the case. We test the model with data from 1012 subjects, judging their ability for each of two cognitive tests that we administer to them. We clearly reject the restriction: In general, individuals from an ability quantile $j < k$ are more likely to think they are in quantile k.

Out test evaluates the joint hypothesis of Bayesian updating and the common prior assumption, leaving a question unanswered: Which part of the joint hypothesis has failed? We provide an alternative test model relying on image concerns to generate overconfident beliefs, a test that is independent of the common prior assumption. In models of image concerns, individuals like to believe that they have high ability, but their beliefs are constrained by Bayesian updating [12, 26]. The core of these models is that once individuals are sufficiently certain that they are of high ability, they stop seeking information, as this only offers the downside risk of revising their beliefs downward. By contrast, individuals with a low belief seek information as long as there is a chance to find out that they may be better. We tested this prediction by offering our subjects the opportunity to find out exactly how well they did in the tests relative to the other participants. Our data strongly reject the prediction of self-image models: We find that individuals with high beliefs are *more likely* to demand information about their ability.

Thus, beliefs do play an important role in demanding information, but not in ways that is consistent with preserving self-image. Our results are more in line with a model in which individuals enjoy sending public signals, rather than the resulting belief, confirming that they are good. We further corroborate this interpretation by examining how individual personality differences affect confidence judgments. Consistent with our interpretation, we find that more socially dominant individuals make more confident judgments, holding constant their actual ability. This effect is also quantitatively large: Of those individuals with a below-median score in social dominance, only 33 percent think they are in the top 20 percent of the IQ distribution. Of the individuals with an above-median score in social dominance, 55 percent think they are in the top 20 percent, when, in fact in both groups there is no difference in the frequency of being in the top 20 percent. We also find that trait neuroticism reduces confidence by a similar magnitude.

Overall, our findings suggest that overconfidence is likely to arise in the process of communicating private beliefs to others, rather than in a process of self-deception. Our results show that overconfidence cannot arise from Bayesian updating on signals about one's ability. We also show that individuals with high confidence are more likely to seek information about their ability, which should provide them with information that undermines overconfidence. This strongly suggests that individuals interpret information in a biased way. This view receives further support from the fact that social dominance makes judgments more confident while neuroticism makes judgments less confident. Thus, these two personality traits shift confidence in the way that would be predicted if they biased the interpretation of information.

The remainder of this paper is structured as follows: Section 2 describes our empirical setup. Section 3 presents the basic findings on confidence judgments. Section 4 introduces a framework of incomplete information about one's own ability, derives restrictions that this places on confidence judgments, and tests them. Section 5 discusses image preservation as a source of overconfidence, and provides an empirical test. Section 6 presents evidence on how personality traits are related to overconfidence.

2 Design of the Study

The data was collected from 1,063 trainee truck drivers at a driver training school in the upper Midwest of the United States, on Saturdays that fell in the middle of a two-week basic training course the subjects were undertaking in order to earn a commercial drivers license. The two tests were part of a larger data collection process for the Truckers and Turnover Project ([6], [7]), which was administered to participants in groups of twenty to thirty from December, 2005 to August, 2006.

The subjects participated in two tests of cognitive ability, in which appropriate incentives were provided. The first was part of a standard non-verbal IQ test, Ravens Progressive Matrices ([22]), which involves identifying visual patterns; this was administered on notebook computers. The second was a section of the Adult Test of Quantitative Literacy (here after Numeracy), from the Educational Testing Service (ETS), which involves reading text samples and solving arithmetic problems that are based on the text; this was administered using paper and pencil. The IQ test was administered before the numeracy test. In total, 1063 subjects participated in this study; however, because we tried out a different IQ measure before switching to the Raven's, 1,016 subjects have non-verbal IQ data.

The sequence of events was the same in both tests. First, using the standard instructions that came with each test, the nature of the test was explained, directions about how to complete questions were given, and a sample question was provided and the correct solution presented. After the instructions, we recorded the first self-assessment of the subjects' abilities: the subjects were asked how well they thought they would do in this test relative to the rest of the session's participants by identifying the quintile of group performance in which their score would fall. After the test was completed, the subjects were asked to self-assess a second time, by again picking the quintile of group performance in which their own score would fall.

We paid subjects for their attendance and their performance ([5]). Each subject took part in two sessions, each two-hours long; both cognitive skills tests were in the second session. We paid an initial amount of $20 for participation at the beginning of each session. In addition, for each cognitive skill test, we randomly selected two subjects from each group after the test and paid each of these two one dollar for every correct answer in the IQ test (maximum possible payout of $48), and two dollars for every correct answer in the Numeracy test (maximum possible payout of $24). We also paid each subject $2 each time the subject correctly identified the quintile into which his or her own score actually fell (maximum possible payout of $4 per test). Payments depending on performance were offered before each test, as part of the test's instructions.

Because the payout calculations for the Numeracy task were manual, and because subjects were enrolled in a course that continued for another week, we paid out all the earnings from participation beyond the show-up fees at the beginning of the work week following the Saturday test administration. This provided us with the opportunity to also ask subjects, immediately after their second self-assessment response on each test, whether they would like to learn on payout day both their exact score, and what their actual relative performance was, i.e. which quintile they were actually in. Those who answered "no" only received their payout, and not this extra information. Thus, this answer is our measure of each subject's demand for information about their relative performance:"yes" signaled the desire to know. We added this question after data collection began, so there are 839 subjects that indicated their demand for information on the IQ test, and 889 that did so on the Numeracy test.

In addition to providing a clear measure of the demand for information about one's relative performance, this design provides incentives to truthfully report one's self-assessment of relative performance, and to make that estimate as accurate as possible. A strength of the design is that we asked subjects about their performance relative to a specific group of people, whom they have known for more than a week by the time of the experiment. Therefore, unlike the most common studies of overconfidence in the psychology literature, our design rules out that subjects were comparing themselves to groups outside the lab. Finally, it avoids the ambiguities of earlier studies that asked individuals whether they were above or below the mean.[2]

During the entire experiment we collected a variety of additional demographic and socio-economic information. Subjects also filled a Multidimensional Personality Questionnaire (MPQ) questionnaire. The MPQ is a standard personality profile test ([21], [24], [25]). It consists of questions concerning 11 different scales that represent primary trait dimensions:

[2]If, e.g., the median of abilities is significantly below the mean, a large fraction could correctly answer that they are better than average, which makes the interpretation of these studies difficult.

wellbeing, social potency, achievement, social closeness, stress reaction, alienation, aggression, control, harm avoidance, traditionalism, and absorption. In our study we used the short version ([21]), which has 154 multiple choice questions.

3 Evidence of Overconfidence

Table 1 presents some basic descriptive statistics. The first panel in the table shows the number of correct answers in the two cognitive tests. In [6], it is shown that the distribution of the score in the Raven's task in our sample is close to that of representative samples, although slightly lower: for example, the median score in our sample is 47.5, in the representative sample (reported in [22]) the median is 52. Turning to the demographics of our sample, we see that the most frequent education level in our sample is a high school degree, though some have also degrees from technical schools, and a significant fraction has at least some college education. The table shows that our sample is predominantly Caucasian, male, and relatively young. See [6, 7] for a more extensive discussion.

Confidence Judgments

In this subsection, we present the basic evidence on confidence judgments in our study. This serves two purposes: First, showing that our results are comparable to confidence judgments found in other studies, and, second, motivating the theoretical model we discuss later.

Figure 1 displays the distribution of confidence judgments across all individuals. It shows a typical pattern found in a large number of studies: Very few individuals rate their ability in the bottom 40 percent of the ability distribution. By contrast, well above 60 percent think they are in the top 40 percent. The figure shows a very similar pattern for the confidence judgments in the two tests.

Figure 2 displays confidence judgments as a function of the true ability in the IQ test, reporting under- and overconfident judgment relative to the true ability of the individual. Shadings indicate the the extent of overconfidence: Light shading indicates that the individual is just one quintile off, darker shading indicates that the individual is more than one quintile off. Panel A displays confidence judgments before the IQ test. The figure shows that overconfident judgments are pervasive across the ability spectrum, except where impossible by definition in the top ability quintile. The figure also shows that the confidence judgments are strongly asymmetric: Underconfidence is much rarer than overconfidence. Panel B in Figure 2 displays confidence judgments after the IQ test and shows essentially the same pattern. The confidence judgments for the Numeracy test are presented in Figure 3: The results are very similar to the case of IQ test.

4 The Bayesian Model

In this section we establish the benchmark model of the behavior for a population that is forming beliefs on their own ranking using belief updating based on the information they have available. Our aim is to show that such a model can produce some features of the confidence judgments that we showed in the previous section, but to also derive testable restrictions imposed by the Bayesian theory.

In the model we consider a large population of individuals, each one endowed with a type t, which is the value of a specific characteristic. For example the type of an individual might be his height, something easily determined and observed. Another more interesting example is his ability to score in an intelligence test, a quality that we briefly described as the individual's *IQ*. We are interested in types that are ordinal qualities. In what follows, we will restrict attention to judgments about the individual's position in the distribution of outcome. As in our empirical study we elicit judgments about the quintiles, we also restrict our notation in the model to quintiles.

The type of each individual is determined independently, according to a known probability measure (common prior) on the set of types. Thus, the population has a common prior on the distribution of types, which (since types are percentiles) is the uniform distribution. Individuals do not know their type, but during their life they gather information by observing private signals on it. On the basis of this information they update in a Bayesian fashion their belief on their type, which initially was the common prior, and therefore also they update the belief they have on their own relative position in the population with respect to the characteristic we are considering. For example, through their school performance, job performance, as well as occasional exchanges with other people they form an opinion on their IQ, and hence of their relative standing with respect to this characteristic within the population. Formally, we assume that individuals observe an outcome $x_i \in X, i = 1, ...n$ from some signal space X, where we assume n is larger than 5, the number of quintiles. [3] A subject participating in an experiment like ours comes to the laboratory with this posterior belief on his ability. Denote the probability that an individual receives signal x_i given that he is of ability t_k by $p_k(x_i)$. Then the individuals posterior beliefs about his ability is given by

$$\Pr(t_k|x_i) = \frac{p_k(x_i) \cdot \frac{1}{5}}{\sum_j p_j(x_i) \cdot \frac{1}{5}} = \frac{p_k(x_i)}{\sum_j p_j(x_i)} \tag{1}$$

The signal structure $p = (p_k(x_i))_{k=1,...,5,i=1,...,n}$ is the true information structure. We have very little hope of determining this object empirically, so we will have to find a way around it. Suppose we ask the individual to predict the quintile in which his IQ score will fall,

[3] Notice that we restrict attention to one draw from a signal structure, rather than, e.g., a dynamic acquisition of signals. Dynamic acquisition signals can be redefined as a single draw from a single signal structure.

and promise him a payment if his prediction is correct. Let us assume that our incentives are sufficient motivation for him to state the truth, and that he believes that our test is un-biased. Then an individual who observes the signal x_i will pick the most likely quintile given x_i, i.e. the individual will indicate that he is *most likely* in ability quintile $s(i)$, where:

$$s(i) = \arg\max_j Pr(t_j|x_i) = \arg\max_j p_j(x_i) \tag{2}$$

We call the theory that subjects follow this procedure of deriving posteriors with Bayesian updating and truthfully reporting the most likely quintile the *Bayesian model.*

A large fraction of subjects thinking that they are in the top two quintiles is consistent with this model. To illustrate, consider an example with only two types, good and bad. The top two quintiles (40 percent) are good types, and the remaining three quintiles are bad types. This is the distribution of types and the common prior. The only source of information for individuals is a test that everybody takes. Good types pass the test for sure, bad types only pass it with probability 50 percent. The posterior probability that an individual is a good type if he passes the test is:

$$\Pr(\text{good type}|pass) = \frac{1 \cdot 0.4}{1 \cdot 0.4 + 0.5 \cdot 0.6} = \frac{4}{7} \tag{3}$$

so individuals who pass the test and answering truthfully state that their most likely type is the good type. A fraction of 70 percent of the population passes the test (all the good types, plus half the bad types): Thus in this population, 70 percent truthfully report that they belong to the top 40 percent, much like we observe in the data presented in the previous section. Beliefs are on average correct: 70 percent of the population believe that they are good with probability 4/7, and 30 percent believe that they are are good with probability 0. Overconfidence in beliefs arises because the test was easy (all good types and half of the bad types pass the test). If the test were hard (for example, all bad types and half of the good fail), underconfidence would arise, and only 20 percent would state that they are good types.

Testable Restrictions on Beliefs

Incomplete information about one's abilities, and some peculiar feature of the signal structure (an easy test) may lead to overconfident beliefs. However, the Bayesian model imposes testable implications on how the distribution of confidence judgments should be related to true abilities. These are testable because the experimenter also observes the true score of the individual in the test, so he has the end of the experiment for each subject a pair of observations, (*true score, stated quintile).* The true score is not a precise measure of the IQ of an individual, of course, but that it is good enough so that we can ignore sampling error with respect to the quintiles.

Since individuals have incentive to choose the most likely quintile, the Bayesian model requires them to use (1) to form their posterior using , and (2) to select their statement. Denote the expected fraction of individuals in true ability quintile k assigning themselves to quintile j based on the signal structure provided in (1) by $q_k(s_j)$. We call the function $(q_k(s_j)_{k,j=1,\ldots,5})$ allocating each type k to five quintiles is specific proportions, the theoretical allocation function. It defines a 5-by-5 matrix of confidence judgments. Note that for every true ability quintile k, $\sum_j q_k(s_j) = 1$. The items in the diagonal denote the fraction that hold the correct beliefs about their abilities. Entries $q_k(s_j)$ with $k < j$ indicate individuals who hold overconfident beliefs, while entries with $k > j$ indicate the fraction of individuals holding underconfident beliefs. What restrictions does Bayesian updating place on this matrix? Because individuals pick the most likely quintile given the signal x_i that they received, the mode of individuals thinking they are in quintile k must actually have true ability quintile k. That is, Bayesian updating imposes that:

$$q_k(s_k) = \max_l q_l(s_k) \tag{4}$$

In the appendix, we characterize this property more fully. The theoretical allocation function allows us to sidestep a problem that has no easy direct solution: what is the true information structure p?. If the behavior we want to describe only depends on the posterior distribution on quintiles given the signal, then we may assume that the true information structure takes values in the simple signal space given by the set of quintiles. To see this, consider an information structure where individuals observe some signal x in some arbitrary signal space X, compute the posterior on their type, and state the most likely quintile. This information structure, in our environment in which the only task of the individuals is to state the most likely quintile, is equivalent to a simple information structure where individuals are directly communicated the quintile they should state (so the signal space is the set of quintiles), and they do so (because the diagonal condition (4) insures this behavior is incentive compatible). The theoretical allocation function derived from equations (1) and (2) can be considered a canonical information structure. The harder problem: "Is there an information structure that can generate the data?" has been replaced by the easier problem: "Is there a canonical information structure that can generate the data?". This problem has an answer, that we present in the next section [4].

Rejection of the Bayesian Model

We have seen that Bayesian updating implies condition (4), which we may call diagonal condition, because if the theoretical allocation function is read as a matrix, then the entries

[4] Notice that we have so far assumed that all individuals draw signals from a common signal structure. This, however, is not a crucial assumption. If different individuals drew signals from different signal structures, this can be modeled as a meta signal-structure, in which individuals first observe from which sub-structure they will draw signals.

with the largest values are on the diagonal. Table 3 presents an illustration of an allocation function satisfying this condition. But how can restrictions imposed by (4) be tested against the *empirical allocation function* $\hat{q}_k(s_j)$, i.e., the empirical distribution of confidence judgments as a function of the individuals' true ability? Intuitively strong evidence that the main diagonal condition is violated rejects the Bayesian model.

Table 2 displays the empirical allocation function for the numeracy and IQ test. The table shows that in both cases, the empirical frequencies violate the diagonal condition. For example, in the numeracy test, only 18 percent of the individuals from the third quintile put themselves into the third quintile. By contrast, 40 percent from the first quintile and 27 percent from the second quintile put themselves in the third quintile, in violation of the diagonal condition (4). But is the violation significant? Since we don't know the underlying signal structure, how likely is it that a signal structure satisfying (4) generated the data in Table 2? We propose a test that gives the Bayesian model the best chance not to be rejected.

We estimate the parameters of the theoretical signal structure by maximum likelihood subject to the constraint imposed by (4). That is, we compute the $q = (q_k(s_j)_{k,j}$ that solves:

$$\max_q \sum_{j,k} n_{kj} \log(q_k(s_j)) \tag{5}$$

subject to for every $k, j, q_k(s_j) \geq 0, \sum q_k(s_j) = 1$ and

$$\text{for every } k, q_k(s_k) = \max_l q_l(s_k)$$

where n_{kj} is the number of individuals of ability quintile k saying that they are in quintile j. This is a concave problem and maximization is straightforward with numerical methods. Denote the solution to (5) by $q_k^{ML}(s_j)$. Notice that this gives the best chance to the null hypothesis of Bayesian updating, since we pick q^{ML} as the one satisfying (4) that best fits the observed data. The constrained maximum likelihood estimator for Numeracy and IQ test are reported in table 3.

We then calculate the fit of q^{ML} to $\hat{q}$ as the mean square root error from each cell:

$$\hat{d} = \frac{1}{25}\sqrt{\sum_{j,k}(\hat{q}_k(s_j) - q_k^{ML}(s_j))^2} \tag{6}$$

The distance measure is $\hat{d}^{IQ} = 0.026$ for the IQ test, and $\hat{d}^{Num} = 0.033$ for the numeracy test. That is, the average deviation from the ML estimate of q is 2.6 percentage points in the IQ test and 3.3 percentage points in the Numeracy test. In order to assess whether the fit $\hat{d}$ is improbably bad, we generate 100,000 simulations of the same sample size as our data using q^{ML} as the data generating mechanism and calculate the distances d_n for each trial n. This provides us with an empirical distribution function for the distance measure d to calculate the probability that a draw from q^{ML} has a worse fit than the empirical allocation function $\hat{q}$. The p-values are $p = 0.005$ for the IQ test, and $p = 0.001$ for the numeracy test. Therefore, we clearly reject the hypothesis that our data is generated by imperfect information about ability and Bayesian updating.

5 Do Self-Image Concerns create Overconfidence?

The previous section tests and rejects a wide class of models that rely on Bayesian updating from a common prior after exogenous arrival of information. Other models have been developed to explain overconfidence arising endogenously as a function of individuals' choices.

Two recent papers [12] [26] have argued that a concern for self-image can lead to overconfidence. If individuals' utility depends on their belief about their ability, this can lead to an endogenous mechanism that produces results as if they were drawing signals from "easy test" signal structure in [4]. This requires that utility is sufficiently "kinked" in the belief. [12] provides an example in which an individual's utility discretely increases by some fixed amount v if the individual believes that the chance that his ability t is below some threshold $\hat{t}$ is small. Formally, utility is given by

$$U(c, \hat{t}) = u(c) + v \cdot I(F(\hat{t}) \leq x) \tag{7}$$

where F is the c.d.f. of the individual's current belief over his ability. To see how this can lead to overconfidence, assume that the individual's belief currently is that $F(\hat{t}) < x$ and that he is offered more information about his ability. Assume that the only change in utility he has from further information is, possibly, from the change in self-image. Then he will never seek more information, because more information only harbors the risk of revising his belief downward. Conversely, if $F(\hat{t}) > x$, the individual will seek more information. If his belief is further revised downward, this leaves utility unchanged. If the individual receives a positive signal, he will gain utility v if $F'(\hat{t}) < x$ where $F'()$ is the c.d.f of beliefs incorporating the new information. Thus, this model can generate a pattern in which individuals with low beliefs will seek all the information they can find, while individuals with high beliefs will have less accurate information: Of all the individuals with initially low beliefs, all individuals with high ability will find out. By contrast, some of the individuals who initially had high beliefs will have received good signals by chance, but will not find out. The result is that too many individuals will belief they have high abilities.

Demand of Information

We test the prediction of this model by testing the implication that individuals with high beliefs should be less likely to seek more information about their ability. Recall that after each test, we offered the subjects the opportunity to find out exactly how well they did relative to the others. We thus gave the individuals the chance to obtain more information, exactly as required in the model. This test also has the feature that it does not rely on the assumption of common prior. Rather, it measures the demand for information directly as a function of the individuals' beliefs.

Figure 4 displays the fraction of individuals information about their performance as a function of how well they did in the test, and their belief about their performance. Because of the small number of observation, we exclude individuals with beliefs in the bottom two quintiles.

Panel A in Figure 4 displays the results for the IQ test, while the results for the Numeracy test are displayed in Panel B. Both Panel show a strong impact of the beliefs on the demand for information. However, in contrast to what is predicted by models in which the belief about ability enters the utility function, individuals with a higher belief are more likely to ask for the performance information. The figure also controls in a rudimentary way for differences in true abilities by splitting the sample into the top and bottom half of the performers. Thus, by comparing individuals with identical beliefs in the top and bottom half of the true abilities, we can gauge the impact of true ability on the demand for information. There is, essentially, no relationship between ability and the demand for information.

To formally test the model, we estimate the following probit equation

$$Pr(seek = 1|q, x) = \Phi(\beta_0 + \gamma q + \beta x) \tag{8}$$

where *seek* is an indicator variable equal to 1 if the individuals seeks information about his performance in the test, and zero otherwise. Φ is the cumulative normal distribution. We estimate the equation separately for the IQ and numeracy test. Our variable of interest is stated belief of the individual $q \in \{1, 2, ..., 5\}$ regarding the most likely quintile. The control variables x include controls for the test performance. We estimate a five-part linear spline in test performance, with the splines defined over quintiles in order to control for test performance in a flexible way. We also include personality characteristics as measured by the Minnesota Personality Questionnaire [21]. Our estimates also include a large set of controls for socio-demographic differences across subjects: 5 dummy variables for education levels, 5 categories for ethnicity, a gender dummy, age and age squared, and household income.

The results are displayed in Tables 4 and 5 for the demand for information about one's performance in the IQ and numeracy test, respectively. The table displays marginal effects on the probability of seeking information, rather than the bare coefficient estimates. Both tables are structured the same way. In the first column, we test whether, as indicated by the figure, a higher belief increases the likelihood of demanding information. Column (1) in Table 4 controls for test performance using a flexible functional form. It shows that conditional on actual performance, the subject's belief about their performance predicts whether or not they seek information. More optimistic beliefs increase the likelihood of seeking information: a one-quintile increase in beliefs is associated with a 3 percentage point higher probability of demanding information about the test.

The results are even stronger for the numeracy test, where a one-quintile increase in the belief leads to almost a 6 percentage point increase in the likelihood of seeking information. In both cases, the effects are statistically highly significant. Column (2) adds personality traits as controls, obtained from the MPQ. The only significant trait is Harm Avoidance, a measure of the relative preference of individuals for less risky situations. The effect is small, but lends itself to a plausible interpretation that individuals who are less risk averse are more likely to seek information, preferring the extreme values to their expected value. In column (3), we add the socio-economic control variables. However, they have no effect on the coefficient of interest. Finally, in column (4), we also add the beliefs about the ability in the test as well as the beliefs about the ability in the other test as explanatory variables. Some individuals do change their evaluation over the course of the test (correlation between pre and post test beliefs: $\rho = 0.64$ for IQ and $\rho = 074$ for numeracy). Similarly, while beliefs are correlated across tests, they are not perfectly correlated ($\rho = 0.54$ for beliefs after the test). This allows us to examine the specificity of the link between beliefs and the demand for information. Our results show that the link is highly specific. In Table 4, we see that only the most recent belief significantly correlated with the demand for information. Confidence in the numeracy test is uncorrelated with the demand for information, and so is confidence before the test, ceteris paribus. Our results are slightly weaker for numeracy, where we find a weak effect of confidence in IQ on the demand for information.

Overall these results clearly reject the driving force for overconfidence postulated by models of self-image concerns ([12], [26]). In fact, we find the opposite of what these models predict: More confident individuals are more likely to seek information. This is consistent with a model in which individuals value the signals about their ability, not the resulting belief. However, this mechanism also tends to undermine overconfidence, as individuals with high confidence judgments are more likely to seek information, thus begging the question how overconfidence comes about in our subjects. One possibility is that individuals do not process information in a Bayesian manner. This interpretation is consistent with our evidence from the previous section, rejecting overconfidence as a consequence of incomplete information and learning. In particular, this explanation suggests that personality characteristics may be related to the misinterpretation of information. We explore this explanation in the next section.

6 Personality Traits affect Overconfidence

An additional insight into the motivation of overconfidence may be provided by information on personality traits of individuals. Personality traits of an individual can affect his confidence in two different ways. First, they can affect the information that an individual collects during his life. This is true even if we consider choices on information gathering as part of a single player problem. In this case an individual in general should want to be as well informed as he can: however, different personality traits may influence the choice of signal structure he uses (for example the information that he is gathering, or he is paying attention to) among several incomparable ones. Second, personality traits may affect the way in which individual process the same information, and signal their opinion to the outside world: they affect what people state about themselves, rather than what people think of themselves.

The interpretation we gave of our data so far suggests that an individual's personality characteristics related to his desire for status and dominance, as well as his susceptibility to negative feedback should be influencing confidence judgments. We focus on two dimensions that can readily be measured using personality scales: the first is an individual's desire to be in a dominant position relative to others. One of the MPQ traits, Social Potency, provides a good measure of the strength of this preference. We predict that individuals who score high on Social Potency will have more confident beliefs, as they may interpret information in a more self-serving way. The MPQ also allows us to distinguish this from a more general desire to be connected to others, which is measured in the Social Closeness scale. It also allows us to distinguish the desire to dominate from general drive to achieve, using the Achievement scale in the MPQ.

The second important dimension is how individuals respond to negative social feedback. We hypothesize that if individuals are worry-prone and feel vulnerable, this may moderate their beliefs about themselves to make it less likely to experience these negative social emotions. The MPQ also allows us to control for other aspects of risk preferences, such as a more general tendency towards prudence, as measured by the Harm Avoidance scale, and general pessimism captured by the Alienation scale.

Figure 5 provides a first summary of the evidence. It shows confidence judgments and actual abilities for individuals who have different scores in personality traits. Each panel reflects a different personality trait. In each case, we cut the sample by the median trait score. For example, in Panel A, the first graph shows that about 30 percent of the individuals scoring below the median in social potency think they belong to the top 20 percent in the IQ distribution. By contrast, 55 percent of the individuals scoring above the median in social potency think they are in the top 20 percent. Each graph also contains the actual fraction of individuals scoring in the top 20 percent for each subsample.

The graph shows virtually no difference between high- and low- social potency individuals in terms of actual ability. The results for confidence judgments in the numeracy test are very similar. Thus, social dominance appears to pick up quantitatively important differences in confidence judgments, while being unrelated to differences in actual abilities. Turning to the graph that cuts the sample by social closeness, we see no differences in confidence judgments. Thus, it is not the case that individuals who care more about sociability individuals are more confident in general, it is limited to the aspect of dominance relative to others. The third graph cuts the sample by the median of the stress reaction score. Individuals who are highly sensitive to social stress have substantially more timid judgments about their ability, as can be seen in the graph, while this is again not related to differences in actual abilities. Again, a very similar pattern emerges when we examine confidence judgments regarding the numeracy test in Panel B.

In order to examine these hypotheses using a formal statistical test, we estimate an ordered probit model of the form

$$Pr(q = k|MPQ, x) = \Phi(\alpha_k - \beta_0 - \gamma MPQ - \beta x) - \Phi(\alpha_{k-1} - \beta_0 - \gamma MPQ - \beta x) \qquad (9)$$

where MPQ is the full set of 11 dimensions of personality characteristics and x contains the same control variables as in the previous section. Tables 6 and 7 present the results. We report the marginal effects on the probability of believing that the individual thinks he is in the highest quintile.

In both tables, a range of personality characteristics are significant. Consistent with our interpretation, social potency is a highly significant predictor of confidence. A one-point increase in the scale leads to a 1.1 percent higher probability that the individual ranks himself in the top 20 percent. Given that the interquartile range on this index is 8, it predicts large and important differences in confidence judgments. Similarly, stress reaction predicts differences in confidence. Moving from the 25th to the 75th percentile in the stress reaction distribution (a 9-point increase) predicts a decrease in the highest confidence judgments by 8 percentage points. As we move to more restrictive specifications, using more flexible controls for cognitive ability and include our standard set of control variables, two personality characteristics remain significant: social potency strongly increases confidence judgments, while high scores in stress reactions reduces confidence. Thus, personality traits have a strong and significant impact on confidence judgments, in line with our interpretation that individuals interpret information in a biased way.

7 Conclusions

We have examined in an experimental setup evidence for overconfidence of individuals on their intelligence and its possible motivation. We reported three main findings. First, we rejected the Bayesian model, that is the hypothesis that overconfidence results from incomplete information about one's own ability, Bayesian updating and truthful revelation, together with some peculiar characteristic of the information structure used by individuals. The test we use is general, and may be used to probe the same hypothesis in similar studies. In our data overconfidence of the statements is beyond what can occur in a Bayesian world.

Second, we rejected the hypothesis that optimistic beliefs about one's abilities lead individuals not to demand information about relative performance. As a consequence we reject the central prediction of models of self-image that can lead to overconfidence in beliefs, that individuals with better belief are more reluctant to search for information on their skill. The opposite is true: We find a positive and highly significant association between optimism of beliefs and demand for information about one's relative performance. This relationship is, as we have shown, specific to the belief about one's relative performance in test at hand. Further, it is the belief after the test, not the belief about one's ability before the test that predicts the demand for information. Overall, we take this as evidence that it is the signal about good relative performance that the individual derives utility from.

Third, we show that specific measures of personality traits are good predictors of stated overconfidence, that is of difference between the stated position in the ranking and the real ranking. The personality traits that are affecting this relevant position, and the direction of the effect, are consistent with the idea that the motivation of confidence is the social signal that confidence inspires. Personality traits do not significantly affect the demand for information. This is consistent with additional finding that personality traits which should affect the accuracy of self-evaluation (control) do not appear to affect neither demand or over-confidence, whereas traits that measure motivation for ranking (social potency) significantly affect statements but not demand.

These experimental findings are consistent with the current evaluation of the importance of self-esteem on individual performance and success. In the recent years, a re-examination of the correlation between self-esteem and, for example, school performance has been found to be consistently weak ([8], [13] for school performance, and [10] for IQ). In addition, the causal direction is likely to go from performance to self-esteem as much as it is going in the opposite direction. The 2003 survey of Baumeister et al. ([3]) is a thorough discussion of the evidence in favor of a positive effect of self-esteem on a range of performance measures, including happiness and healthy lifestyle, and the overall conclusion is that the evidence of a causal relation is weak at best. Similar results are reported in other surveys ([19], [14]). If the utility from positive self-image has no functional basis and offers no improvement in any significant performance index, and is instead due to an arbitrary preference, then it is more natural to consider the possibility that over-confidence is a social signal ([15], [16]).

A Tables and Figures

Figure 1: Distribution of beliefs about ability in IQ test and Numeracy test.

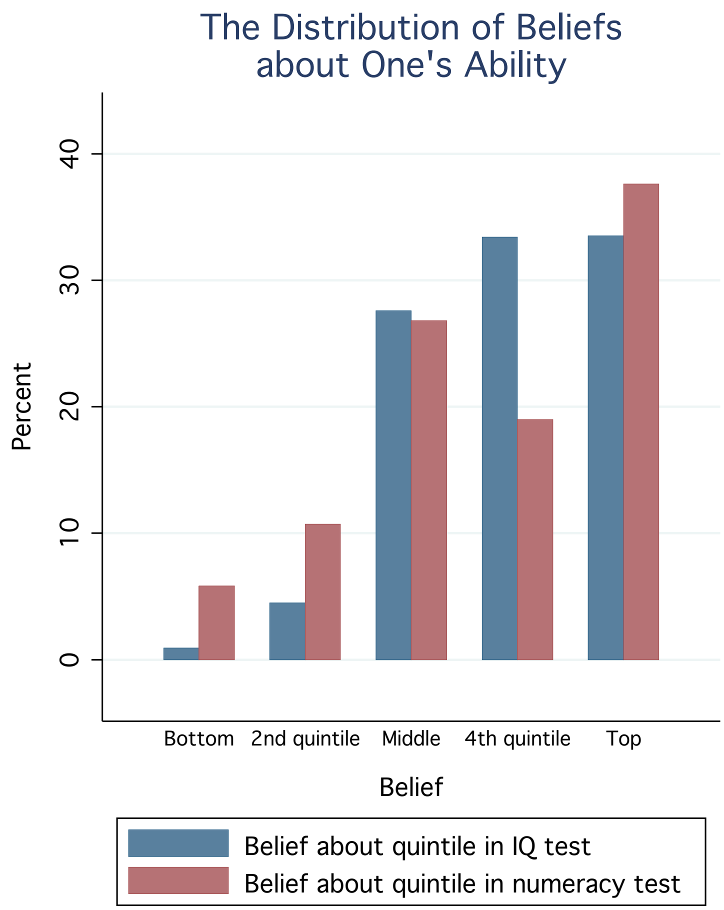


Figure 2: Confidence judgments as a function of actual ability: IQ.

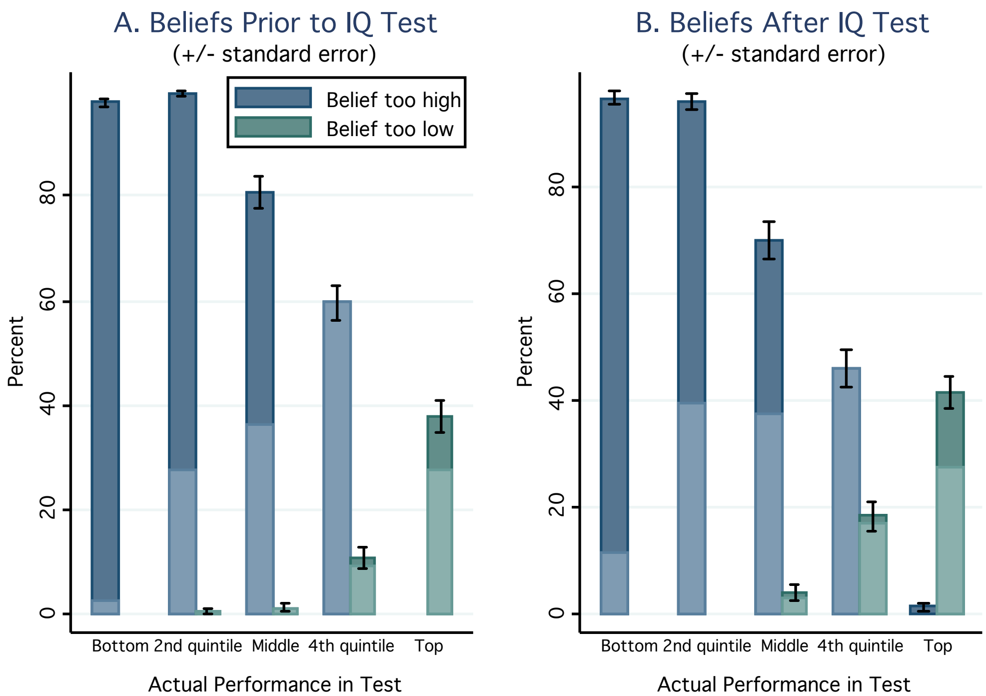


Figure 3: Confidence judgments as a function of actual ability: Numeracy.

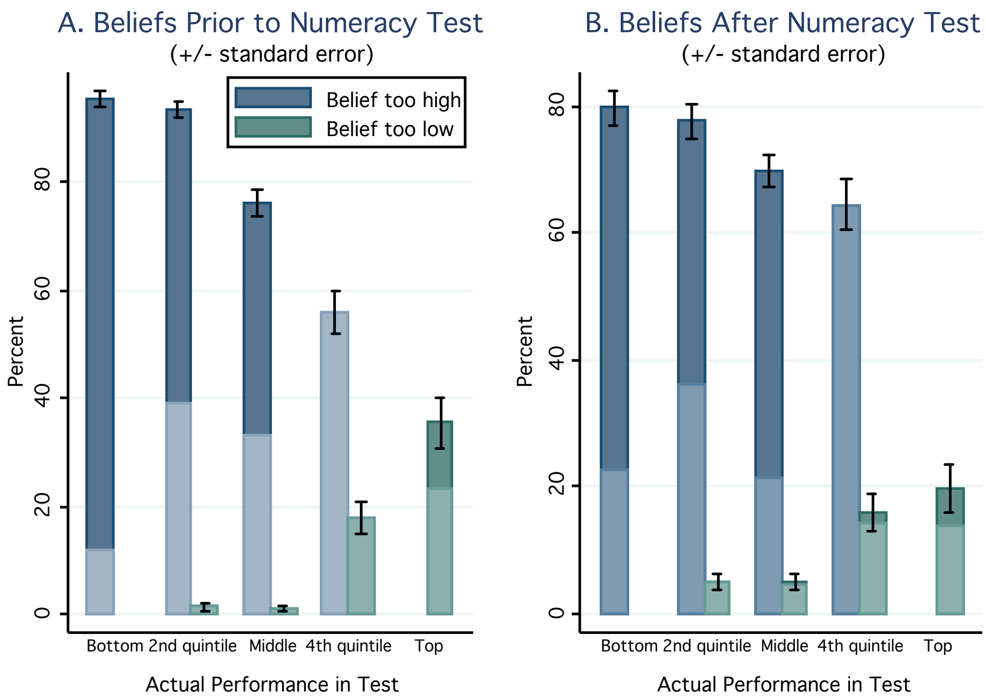
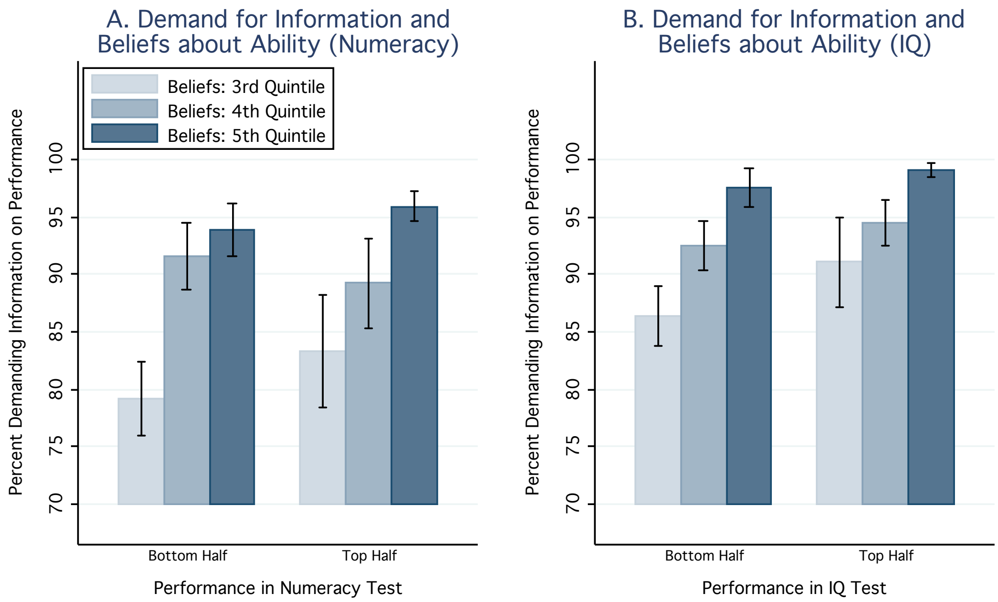


Figure 4: The demand for information



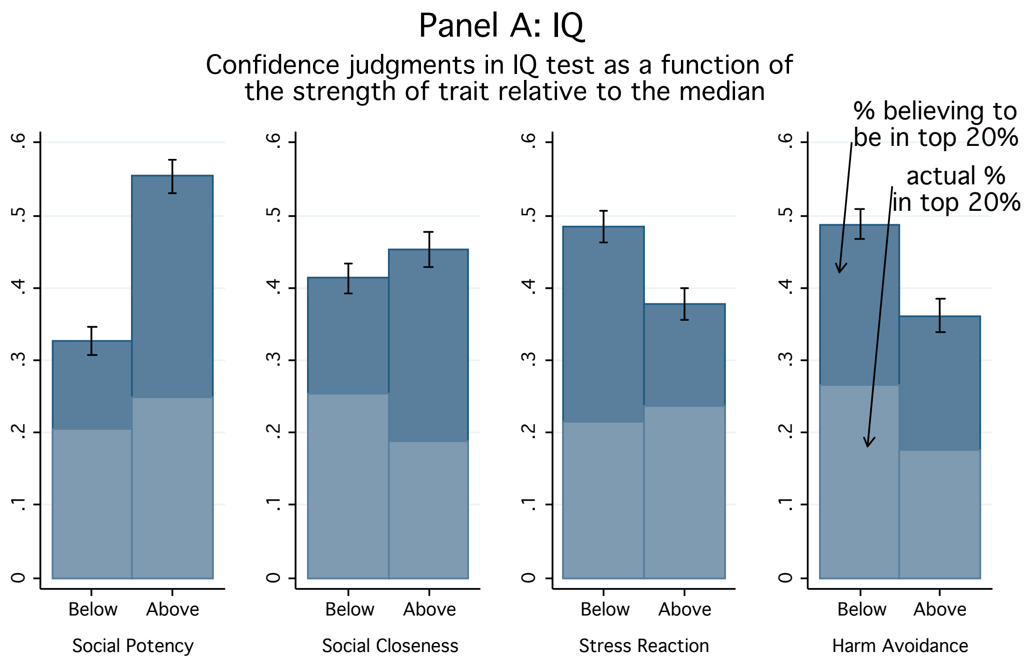
Notes: Caps indicate standard error of the mean.

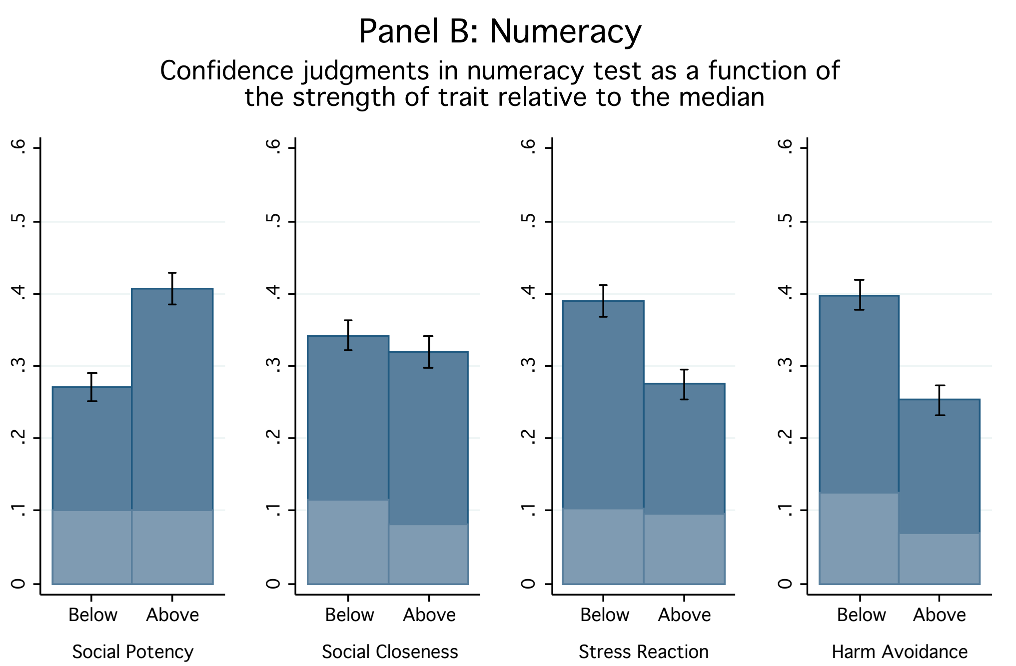
Table 1: Descriptive Statistics

	Mean	Standard Deviation	Min	Max
Test Scores: Number of correct answers.				
Numeracy Test	8.54	2.55	1	12
IQ Test	45.33	8.15	1	60
Education: Highest level attained				
Middle School	3.9%			
High School	39.3%			
Technical School	14.9%			
Some College	33.2%			
College	6.5%			
Graduate School	2.3%			
Ethnic Categories:				
Caucasian	82.7%			
African-American	14.1%			
Indian	2.8%			
Asian	0.7%			
Latino	1.8%			
Other	1.0%			
Other Demographics:				
Age	37.43	10.90	21	69
Male	88.7%			
Household income (in thousands of US Dollars)	52.66	27.07	10	150

Notes: $N = 888$ individuals.

Figure 5: Personality characteristics and confidence judgments





Notes: Caps indicate standard error of the respective mean.

Table 2: The Empirical Allocation functions $\hat{q}_k(s_j)$

Numeracy Test					
	s_1	s_2	s_3	s_4	s_5
t_5	0.0	0.0	0.1	0.27	**0.62**
t_4	0.004	0.009	0.091	**0.298**	0.59
t_3	0.0	0.0125	**0.181**	0.362	0.443
t_2	0.004	**0.0**	0.272	0.377	0.345
t_1	**0.02**	0.02	0.401	0.376	0.175

IQ Test					
	s_1	s_2	s_3	s_4	s_5
t_5	0.004	0.016	0.121	0.271	**0.579**
t_4	0.0	0.014	0.168	**0.355**	0.461
t_3	0.006	0.031	**0.262**	0.375	0.325
t_2	0.0	**0.04**	0.39	0.363	0.204
t_1	**0.033**	0.11	0.42	0.322	0.104

Notes: The empirical allocation function indicates for each ability quintile k, what fraction of individual put themselves in ability quintile j.

Table 3: Constrained Maximum Likelihood estimators of the allocation function $q_k^{ML}(s_j)$.

Numeracy Test

	s_1	s_2	s_3	s_4	s_5
t_5	0	0	0.121	0.232	**0.646**
t_4	0	0	0.159	**0.335**	0.504
t_3	0	0.007	**0.364**	0.275	0.352
t_2	0.012	**0.106**	0.364	0.335	0.180
t_1	**0.071**	0.106	0.364	0.335	0.122

IQ Test

	s_1	s_2	s_3	s_4	s_5
t_5	0	0	0.101	0.277	**0.621**
t_4	0.004	0.008	0.080	**0.378**	0.528
t_3	0	0.01	**0.343**	0.290	0.355
t_2	0.004	**0.015**	0.269	0.373	0.337
t_1	**0.012**	0.015	0.343	0.378	0.251

Notes: The Maximum likelihood estimator is the solution of the problem described by equation (5). It indicates for each ability quintile k, what fraction of individual receives a signal that would induce him to choose the quintile j as most likely.

Table 4: The Demand for Information: IQ Test

Dependent Variable: Demand Information (=1)
Marginal Effects from Probit Estimates

	(1)	(2)	(3)	(4)
q_i^{IQ} after test	0.031***	0.029***	0.030***	0.030***
	(0.009)	(0.009)	(0.009)	(0.011)
q_i^{IQ} before test				– 0.004
				(0.011)
q_i^{NT} after test				0.005
				(0.007)
Piece-wise linear profile in test score				
first quintile	0.000	0.000	0.000	0.000
	(0.002)	(0.002)	(0.002)	(0.002)
second quintile	0.001	0.000	– 0.000	0.000
	(0.005)	(0.005)	(0.005)	(0.005)
third quintile	0.017	0.016	0.015	0.014
	(0.012)	(0.012)	(0.011)	(0.011)
fourth quintile	– 0.008	– 0.006	– 0.007	– 0.007
	(0.011)	(0.010)	(0.010)	(0.010)
fifth quintile	0.006	0.006	0.006	0.005
	(0.011)	(0.010)	(0.010)	(0.010)
Harm Avoidance		– 0.003**	– 0.003**	– 0.003**
		(0.002)	(0.002)	(0.002)
Social Closeness		0.002*	0.002	0.002
		(0.001)	(0.001)	(0.001)
Social Potency		– 0.001	– 0.001	– 0.001
		(0.002)	(0.002)	(0.002)
Stress Reaction		0.000	0.000	0.000
		(0.001)	(0.001)	(0.001)
Demographic controls?	No	No	Yes	Yes
p	0.000	0.001	0.003	0.005
N	838	838	826	825

Table 5: The Demand for Information: Numeracy Test

Dependent Variable: Demand Information (=1)
Marginal Effects from Probit Estimates

	(1)	(2)	(3)	(4)
q_i^{NT} after test	0.06***	0.057***	0.058***	0.039***
	(0.01)	(0.011)	(0.011)	(0.013)
q_i^{NT} before test				0.018
				(0.017)
q_i^{IQ} after test				0.028**
				(0.014)
Piece-wise linear profile in test score				
first quintile	0.022*	0.022*	0.023*	0.023*
	(0.014)	(0.013)	(0.014)	(0.013)
second quintile	0.011	0.001	0.002	0.002
	(0.020)	(0.020)	(0.020)	(0.019)
third quintile	0.038*	0.009	0.010	0.008
	(0.021)	(0.020)	(0.020)	(0.020)
fourth quintile	0.014	0.008	0.015	0.015
	(0.041)	(0.040)	(0.039)	(0.039)
fifth quintile	0.014	0.009	0.013	0.009
	(0.047)	(0.045)	(0.044)	(0.044)
Harm Avoidance		– 0.005**	– 0.005**	– 0.004*
		(0.002)	(0.002)	(0.002)
Social Closeness		0.003	0.002	0.002
		(0.002)	(0.002)	(0.002)
Social Potency		– 0.000	0.001	0.001
		(0.002)	(0.002)	(0.002)
Stress Reaction		0.002	0.002	0.002
		(0.002)	(0.002)	(0.002)
Demographic controls?	No	No	Yes	Yes
p	0.000	0.001	0.003	0.005
N	888	886	873	873

Table 6: Personality Characteristics and Confidence Judgments: IQ Test

Marginal Effects from Ordered Probit Model

	(1)	(2)	(3)	(4)
Absoption	0.004	0.002	0.001	
	(0.003)	(0.004)	(0.004)	
Achievement	0.010***	0.007*	0.007*	0.005
	(0.003)	(0.004)	(0.004)	(0.003)
Aggression	0.002	0.001	0.002	
	(0.003)	(0.003)	(0.003)	
Alienation	-0.007**	-0.003	-0.002	
	(0.003)	(0.003)	(0.003)	
Control	0.002	0.002	0.003	
	(0.003)	(0.004)	(0.004)	
Harm Avoidance	-0.009***	-0.008**	-0.007**	-0.007**
	(0.003)	(0.003)	(0.003)	(0.003)
Social Closeness	-0.004	-0.004	-0.001	-0.002
	(0.003)	(0.003)	(0.003)	(0.003)
Social Potency	0.014***	0.012***	0.012***	0.012***
	(0.003)	(0.003)	(0.003)	(0.003)
Stress Reaction	-0.006**	-0.006**	-0.007**	-0.006***
	(0.003)	(0.003)	(0.003)	(0.002)
Traditionalism	-0.010***	-0.006*	-0.005	-0.005*
	(0.003)	(0.003)	(0.003)	(0.003)
Wellbeing	-0.005	-0.004	-0.004	
	(0.003)	(0.004)	(0.004)	
All 11 MPQ traits?	Yes	Yes	Yes	No
Control for performance	linear	linear	spline	spline
Demographics?	No	Yes	Yes	Yes
N	1063	1014	1014	1014

Table 7: Personality Characteristics and Confidence Judgments: Numeracy Test

Marginal Effects from Ordered Probit Model

	(1)	(2)	(3)	(4)
Absoption	0.006**	0.006*	0.004	
	(0.003)	(0.003)	(0.003)	
Achievement	0.004	0.001	0.001	0.000
	(0.003)	(0.003)	(0.003)	(0.003)
Aggression	-0.001	-0.001	0.000	
	(0.003)	(0.003)	(0.003)	
Alienation	-0.006**	-0.002	0.000	
	(0.003)	(0.003)	(0.003)	
Control	-0.001	-0.001	0.002	
	(0.003)	(0.003)	(0.003)	
Harm Avoidance	-0.008***	-0.004	-0.004	-0.004
	(0.003)	(0.003)	(0.003)	(0.003)
Social Closeness	-0.006**	-0.006**	-0.003	-0.004
	(0.003)	(0.003)	(0.003)	(0.002)
Social Potency	0.011***	0.008***	0.007**	0.007***
	(0.003)	(0.003)	(0.003)	(0.003)
Stress Reaction	-0.007***	-0.008***	-0.009***	-0.007***
	(0.002)	(0.003)	(0.003)	(0.002)
Traditionalism	-0.007**	-0.003	-0.002	-0.002
	(0.003)	(0.003)	(0.003)	(0.003)
Wellbeing	-0.006*	-0.005*	-0.005	
	(0.003)	(0.003)	(0.003)	
All 11 MPQ traits?	Yes	Yes	Yes	No
Control for performance	linear	linear	spline	spline
Demographics?	No	Yes	Yes	Yes
N	1063	1014	1014	1014

B Restrictions imposed by the Bayesian Model

We provide here the conceptual structure to set up the empirical test of the Bayesian hypothesis, that that statements of individuals about their most likely percentile are produced by truthful reporting of Bayesian updating on the basis of private information.

Private Information

Prior to the experimental session, each individual has observed in his lifetime possibly complex signal on his intellectual abilities. These signals may include all sorts of different personal experiences: their success in school, on the job, in day to day comparison with others, including their speed in solving Sudoku games. All these signals are summarized in our model by a single observation. This signal is his private information, and is produced by an experiment (in the sense of statistical theory), which is a function from the set of types to distribution on signals. We take as set of signals the real line, X, endowed with the Borel σ-algebra $\mathcal{B}(X)$.

So the private experiment is:

$$(X, \mathcal{B}(X), (P_\theta)_{\theta \in \Theta}) \tag{10}$$

where for every θ, $P_\theta \in \Delta(X, \mathcal{B}(X))$, the set of probability measures on X.

We do not know or observe the experiment P, so we are trying to estimate the most likely experiment given our data; and to test whether the overall hypothesis that the data are produced by Bayesian updating is supported or rejected by the data.

In the Bayesian model, a subject with a type θ observes a signal x with probability induced by P_θ, and then computes the posterior given the signal, which we denote

$$m(\cdot|x) \in \Delta(\Theta, \mathcal{B}(\Theta)) \tag{11}$$

Over and under confidence

Let $S \equiv \{s^i : i = 1, \dots, 5\}$ be the set of statements that the subject can make, where s^i is interpreted as *"I am in the i^{th} quintile"*. Given the signal x he has observed, the subject determines which of the 5 quintiles has the largest probability according to his posterior, that is he solves:

$$\max_{i=1,\dots,5} m(R^i|x) \tag{12}$$

and then states s^k if k is the solution of the problem (12).

Definition 1. *A subject in the quintile R^i stating s^j is overconfident if $j > i$ and underconfident if $i > j$.*

The model implicitly describes a function giving for every θ a probability over the set of quintiles. Note that only we, the experimenters, observe θ, although with some noise due to the imprecision of the task.

Allocation functions

An allocation function is a function $q : \Theta \to \Delta(S)$. An allocation function is induced by an experiment P with the distribution m over the type space Θ if it can be obtained from Bayesian updating according to P. Formally:

Definition 2. *An allocation function q is induced by an experiment P with m the prior distribution over the type space Θ if there exists a choice function $C : X \to \Delta(S)$ such that*

$$\text{if } C(x, s^j) > 0 \text{ then } m(R^j|x) = \max_k m(R^k|x) \tag{13}$$

and such that for every θ and s^j,

$$q_\theta(s^j) = \int_X P_\theta\,(dx)\, C(x, s^j) \tag{14}$$

We denote by $A(P)$ the set of allocation functions induced by an experiment P.

The allocation function of an experiment is not unique because the choice function C is not unique. Note that $(S, \mathcal{P}(S), (q_\theta)_{\theta\in\Theta})$, where $\mathcal{P}(S)$ is the set of all the subsets of S, is an experiment on Θ, dominated by P in the Blackwell order, since it is obtained from P though the Markov kernel C. The function q depends on the experiment P (and is a coarsening of P): we may use the notation q^P when we want to emphasize this dependence.

We denote $X^i \equiv \{x :\ \text{argmax}_j m(R^j|x) = i\}$. We can also define the average theoretical allocation function

$$A_q(R^i, s^j) = \int_{R^i} q_\theta(s^j) dm(\theta). \tag{15}$$

An allocation function displays overconfidence (respectively underconfidence) at $\theta \in R^i$ if $q_\theta(s^j) > 0$ for $j > i$ (respectively $j < i$).

Finite types

For our intended application, providing a test of the Bayesian model in our experimental data, a finite type space is enough. We consider a type space where a quintile coincides with a type. An individual has type θ^i if his IQ score in the Raven's matrices task is in the the i^{th} quintile. So formally we have:

$$\Theta \equiv \{\theta^i : i = 1, \ldots, 5\} \tag{16}$$

From the point of view of our more general model with a continuum of types, this simplification ignores the problem of aggregation of the different types within a quintile and simply assumes that all the individuals in a quintile are identical. We lose some information (for example, it seem natural that people with higher IQ score have more optimistic beliefs that those with lower score in the same quintile), but we gain in simplicity in the analysis of the data.

Experiments and allocation functions

To make the search for the experiment P more systematic we may proceed as follows. First we pose the problem: in our simple environment (with finite types, signals and states), when can an observed empirical allocation function possibly be produced as the allocation function of some experiment, when the prior is uniform over the types? The answer turns out to be simple: if and only if each quintile considers itself more likely than any other quintile does. Formally:

Theorem 3. *Let q be an allocation function. The following conditions are equivalent:*

1. *There exists an experiment $(X, \mathcal{X}, (P_\theta)_{\theta\in\Theta})$ over some signal space X such that q is one of its allocation functions;*
2. *For every i*
$$q_{\theta^i}(s^i) = \max_k q_{\theta^k}(s^i) \tag{17}$$

Proof

Let $(X, \mathcal{X}, (P_\theta)_(\theta \in \Theta)$ be the experiment and C the choice function inducing q. Then for every i,

$$q_{\theta^i}(s^i) = \int_X P_{\theta^i}(dx) C(x, s^i)$$

By the definition of choice function, if $C(x, s^i) > 0$ then

$$m(R^i|x) = \max_k m(R^k|x). \tag{18}$$

But in the present case $R^k = \{\theta^k\}$, and the m is uniform, so 18 is equivalent to

$$P_{\theta^i}(x) = \max_k P_{\theta^k}(x) \tag{19}$$

and therefore for every k:

$$\begin{array}{rcl} q_{\theta^i}(s^i) & = & \int_{\{x:C(x,s^i)>0\}} P_{\theta^i}(dx)C(x,s^i) \\ & \geq & \int_{\{x:C(x,s^i)>0\}} P_{\theta^k}(dx)C(x,s^i) \\ & \equiv & q_{\theta^k}(s^i) \end{array}$$

Conversely, let q be an allocation function that satisfies (17). We construct an experiment inducing q as its allocations function. Let $X = S$, and for every i and j let $P_{\theta^i}(s^j) = q_{\theta^i}(s^j)$. This is an experiment: we only need to construct a choice function for this experiment that induces q. Let $C(s, s^j) = \delta_s(s^j)$ (that is, $= 1$ if and only if $s = s^j$ and $=0$ otherwise). The condition (13) on the choice function follows from the assumption (17), and the induced allocation is

$$\sum_s q_{\theta^i}(s)\delta_s(s^j) = q_{\theta^i}(s^j).$$

QED

References

[1] Alicke, M. D., M. L. Klotz, D. L. Breitenbecher, T. J. Yurak, and D.S. Vredenburg, (1995), Personal contact, individuation, and the better-than-average effect, *Journal of Personality and Social Psychology*, 68, 5, 804-825.

[2] Barber, B. and Odean, T. (2001), Boys Will be Boys: Gender, Overconfidence, and Common Stock Investment, *Quarterly Journal of Economics*, 116, 1, 261-292.

[3] Baumeister, R. F., Campbell, J. D., Krueger, J. I. and Vohs, K. D., (2003), Does High Self-Esteem Cause Better Performance, Interpersonal Success, Happiness or Healtier Lifestyles?, *Psychological Science in the Public Interest*, 4, 1, 1-44.

[4] Benoit, J.-P. and Dubra, J. (2007), Overconfidence?, MPRA Paper 6017, University Library of Munich, Germany;

[5] Borghans, L., H. Meijers and B. ter Weel, (2008), The role of noncognitive skills in explaining cognitive test scores, *Economic Inquiry*, 46, 1, 2-12.

[6] Burks S, et al. (2008), "Using Behavioral Economic Field Experiments at a Firm: the Context and Design of the Truckers and Turnover Project", in *The Analysis of Firms and Employees: Quantitative and Qualitative Approaches*, eds Bender S, Lane J, Shaw K, Andersson F., & von Wachter, T. (NBER and University of Chicago)

[7] Burks, S., Carpenter, J., Goette, L. and Rustichini, A. (2009), 'Cognitive skills affect economic preferences, strategic behavior, and job attachment', *Proceedings of the National Academy of Sciences*, 106, 19, 7745.

[8] Davies, J. and Brember, I., (1999)., Reading and mathematics attainments and self-esteem in years 2 and 6an eight-year cross-sectional study. *Educational Studies*, 25, 145157.

[9] Dunning, D. (1989), Ambiguity and Self-Evaluation: The Role of Idiosyncratic Trait De.n- itions in Self-Serving Assessments of Ability, *Journal of Personality and Social Psychology*, 57, 6, 1082-1090.

[10] Gabriel, M.T., Critelli, J.W., and Ee, J.S., (1994), Narcissistic illusions in selfevaluations of intelligence and attractiveness, *Journal of Personality*, 62, 143155.

[11] Hoelzl, E. and A. Rustichini, (2005), Overconfident: do you put your money on it? *The Economic Journal*, 115, 305-18.

[12] Koszegi, B., (2006), Ego utility, overconfidence, and task choice, *Journal of the European Economic Association*, 4, 4, 673-707.

[13] Kugle, C.L., Clements, R.O., and Powell, P.M., (1983), Level and stability of self-esteem in relation to academic behavior of second graders, *Journal of Personality and Social Psychology*, 44, 201207.

[14] Leary, M.R. (1999). The social and psychological importance of self-esteem. In R.M. Kowalski and M.R. Leary (Eds.), *The social psychology of emotional and behavioral problems: Interfaces of social and clinical psychology*, 197221, Washington, DC, American Psychological Association.

[15] Leary, M.R. and Downs, D.L., (1995)., Interpersonal functions of the self-esteem motive: The selfesteem system as a sociometer, In M.H. Kernis, editor, *Efficacy, agency, and self-esteem*, New York, Plenum Press, 123144.

[16] Leary, M.R., Tambor, E.S., Terdal, S.J., and Downs, D.L., (1995), Self-esteem as an interpersonal monitor: The sociometer hypothesis. *Journal of Personality and Social Psychology*, 68, 518530.

[17] Malmendier, U. and Tate, G. (2005), CEO overconfidence and corporate investment, *Journal of Finance*, 2661-2700.

[18] Malmendier, U. and Tate, G. (2008), 'Who makes acquisitions? CEO overconfidence and the market's reaction', *Journal of Financial Economics*, 89, 1, 20-43.

[19] Mecca, A.M., Smelser, N.J., and Vasconcellos, J. (Eds.), (1989), *The social importance of self-esteem*, Berkeley, University of California Press.

[20] Merkle, C. and Weber, M. (2009), True overconfidence, Working paper, University of Mannheim.

[21] Patrick, C., Curtin, J., and Tellegan, A., (2002), "Development and Validation of a Brief Form of the Multidimensional Personality Questionnaire", *Psychological Assessment*, 14, 2, 150-163

[22] Raven, J.C., (2000), *Ravens Standard Progressive Matrices (SPM) and Ravens Standard Progressive Matrices Plus (SPM Plus)*, Pearson

[23] Svenson, O., (1981), Are we all less risky and more skillful than our fellow drivers?, *Acta Psychologica*, 94, 143-148.

[24] Tellegen, A., (1988), The analysis of consistency in personality assessment, *Journal of Personality*, 56, 621-663.

[25] Tellegen, A. and Waller, N.G., (1992), Exploring personality through test construction: Development of the Multi-dimensional Personality Questionnaire (MPQ), *The Sage Handbook of Personality Theory and Assessment*, Boyle, G. J., Matthews, G., Saklofske, D. H. editors, SAGE Publications, Los Angeles, CA, Vol. 2, 261-292.

[26] Weinberg, B. (2006), A model of overconfidence, Working paper, Ohio State University