THE ENVIRONMENT AND DIRECTED TECHNICAL CHANGE

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ABSTRACT

This paper introduces endogenous and directed technical change in a growth model with environmental constraints and limited resources. A unique final good is produced by combining inputs from two sectors. One of these sectors uses "dirty" machines and thus creates environmental degradation. Research can be directed to improving the technology of machines in either sector. We characterize dynamic tax policies that achieve sustainable growth or maximize intertemporal welfare, as a function of the degree of substitutability between clean and dirty inputs, environmental and resource stocks, and cross-country technological spillovers. We show that: (i) in the case where the inputs are sufficiently substitutable, sustainable long-run growth can be achieved with temporary taxation of dirty innovation and production; (ii) optimal policy involves both "carbon taxes" and research subsidies, so that excessive use of carbon taxes is avoided; (iii) delay in intervention is costly: the sooner and the stronger is the policy response, the shorter is the slow growth transition phase; (iv) the use of an exhaustible resource in dirty input production helps the switch to clean innovation under laissez-faire when the two inputs are substitutes. Under reasonable parameter values (corresponding to those used in existing models with exogenous technology) and with sufficient substitutability between inputs, it is optimal to redirect technical change towards clean technologies immediately and optimal environmental regulation need not reduce long-run growth. We also show that in a two-country extension, even though optimal environmental policy involves global policy coordination, when the two inputs are sufficiently substitutable environmental regulation only in the North may be sufficient to avoid a global disaster.

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1 Introduction

How to control and limit climate change caused by our growing consumption of fossil fuels and to develop alternative energy sources to these fossil fuels are among the most pressing policy challenges facing the world today. While climate scientists have focused on various aspects of the damage that our current energy consumption causes to the environment,[1] economists have emphasized both the benefits—in terms of limiting environmental degradation—and costs—in terms of reducing economic growth—of different policy proposals. More importantly, while a large part of the discussion among climate scientists focuses on the effect of various policies on the development of alternative—and more "environmentally friendly"—energy sources, the response of technological change to environmental policy has until very recently been all but ignored by leading economic analyses of environment policy, which have mostly focused on computable general equilibrium models with exogenous technology.[2] This omission is despite the fact that existing empirical evidence indicates that changes in the relative price of energy inputs have an important effect on the types of technologies that are developed and adopted. For example, Newell, Jaffe and Stavins (1999) show when energy prices were stable, innovations in air-conditioning reduced the prices faced by consumers, but following the oil price hikes, air conditioners became more energy efficient. Popp (2002) provides more systematic evidence on the same point by using patent data from 1970 to 1994; he documents the impact of energy prices on patents for energy-saving innovations.

A satisfactory framework for the study of the costs and benefits of different environmental policies must therefore include at its centerpiece the endogenous response of different types of technologies to proposed policies. Our purpose is to take a first step towards the development of such a framework. We propose a simple two-sector model of directed technical change. The unique final good is produced by combining the inputs produced by these two sectors. One of them uses "dirty" machines and creates environmental degradation. Profit-maximizing researchers build on previous innovations ("build on the shoulders of giants") and direct their research to improving the quality of machines in one or the other sector. We first focus on a single (and closed) economy.

Our framework highlights the central roles played by the *market size* and the *price effects* on the direction of technical change (Acemoglu, 1998, 2002). The market size effect encourages innovation towards the larger input sector, while the price effect directs innovation towards the sector with higher price. The relative magnitudes of these effects in our framework are, in turn, determined by three factors: (1) the elasticity of substitution between the two sectors;[3]

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[1] See, for instance, Stott et al. (2004) on the contribution of human activity to the European heatwave of 2003, Emanuel (2005) and Landsea (2005) on the increased impact and destructiveness of tropical cyclones and Atlantic hurricanes over the last decades, and Nicholls and Lowe (2006) on sea-level rise.

[2] See, e.g., Nordhaus (1994), MacCracken et al. (1999), Nordhaus and Boyer (2000).

[3] The degree of substitution, which plays a central role in the model, has a clear empirical counterpart. For

(2) the relative levels of development of the technologies of the two sectors; (3) whether dirty inputs are produced using an exhaustible resource. Because of the environmental externality, the decentralized equilibrium is not optimal. Moreover, the laissez-faire equilibrium typically leads to an "environmental disaster," where the quality of the environment falls below a critical threshold.

More interesting are the results concerning the types of policies that can prevent such disasters, the structure of optimal environmental regulation and its long-run growth implications, and the costs of delay in implementing environmental regulation. Approaches based on exogenous technology lead to three different types of answers to (some of) these questions depending on their assumptions. Simplifying existing approaches and assigning colorful labels, we can summarize these as follows. The *Nordhaus* answer is that only limited and gradual interventions are necessary. Optimal regulations should only reduce long-run growth by a modest amount. The *Stern/Al Gore* answer is less optimistic. It calls for more extensive and immediate interventions, and argues that these interventions need to be in place permanently and will likely reduce long-run growth as the price for avoiding environmental disaster. The more pessimistic *Greenpeace* answer is that essentially all growth needs to come to an end in order to save the planet.

Against this background, our analysis suggests a very different answer. In the empirically plausible case where the two sectors (clean and dirty inputs) are highly substitutable (i.e., are "strong substitutes"), immediate and decisive intervention is indeed necessary. Without intervention, the economy would rapidly head towards an environmental disaster, in particular, because the market size effect and the initial productivity advantage of dirty inputs would direct innovation and production to that sector, contributing to environmental degradation. However, optimal environmental regulation, or even simple suboptimal policies just using carbon taxes or profit taxes/research subsidies, would be sufficient to *redirect* technical change and avoid an environmental disaster. Moreover, these policies only need to be in place for a *temporary* period, because once clean technologies are sufficiently advanced, research would be directed towards these technologies without further government intervention. Consequently, environmental goals can be achieved without permanent intervention and without sacrificing (much or any) long-run growth. While this conclusion is even more optimistic than Nordhaus answer, as in the Stern/Al Gore or Greenpeace perspectives delay costs are significant, not simply because of the direct environmental damage, but because delay increases the gap be-

example, renewable energy, provided it can be stored and transported efficiently, would be highly substitutable with energy derived from fossil fuels. This reasoning would suggest a (very) high degree of substitution between dirty and clean inputs, since the same production services can be obtained from alternative energy with less pollution. In contrast, if the "clean alternative" were to reduce our consumption of energy permanently, for example by using less effective transport technologies, this would correspond to a low degree of substitution, since greater consumption of non-energy commodities would increase the demand for energy. Moreover, this parameter, though not systematically investigated by existing research, can be estimated in future empirical work and should become a crucial input into the design of environmental policy.

tween clean and dirty sectors, thus calling for higher taxes (and for a more extended period of economic slowdown) in the future.

Notably, our model also nests the Stern/Al Gore and Greenpeace answers. When the two sectors are substitutable, but not sufficiently so, preventing an environmental disaster requires a permanent policy intervention (even though, in this case, an environmental disaster develops less rapidly). When the two sectors are complementary, then the only way to stave off a disaster is to stop long-run growth.

A simple but important implication of our analysis is that optimal environmental regulation should always use both an input tax ("carbon tax") to control current emissions and research subsidies or profit taxes to influence the direction of research. Even though a carbon tax would by itself discourage research in the dirty sector, using this tax both to reduce current emissions and to influence the path of research would lead to excessive distortions. Instead, optimal policy relies less on a carbon tax and more on direct encouragement to the development of clean technologies.

As a first step towards a quantitative analysis of environmental policy in the presence of endogenous and directed technical change, we also perform a simple calibration exercise. We relate our environmental quality variable to temperature and atmospheric concentration of carbon. We find that, in the presence of directed technical change, for high (but reasonable) elasticities of substitution between clean and dirty inputs (nonfossil and fossil fuels), the optimal policy involves an immediate switch of all R&D effort to clean technologies, even though in our baseline case it takes about seven decades for 90% of production to switch to clean technologies. The general quantitative structure of optimal environmental policy appears broadly robust to whether one uses a low or medium discount rate (which is the main source of the different conclusions on optimal environmental policy in the Stern report or in Nordhaus's research), when the clean and dirty inputs are sufficiently substitutable.

Our framework also illustrates the effects of exhaustibility of resources on the laissez-faire equilibrium and on the structure of optimal policy. An environmental disaster is less likely when the dirty sector uses an exhaustible resource (and the two sectors have a high degree of substitution), because the increase in the price of the resource as it is depleted reduces its use, and this encourages research to be redirected towards clean technologies. Therefore, an environmental disaster could be avoided without government intervention. Nevertheless, the structure of optimal environmental regulation looks broadly similar to the case without an exhaustible resource and again relies both on carbon taxes and research subsidies.

Finally, we briefly discuss whether in a multi-country world an environmental disaster can be avoided by policies in the "North" alone, that is, without global policy coordination imposing similar environmental regulations in the South (i.e., in developing countries such as India and China). Our framework suggests that when there are international technology linkages

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and no international trade, and when the two sectors are highly substitutable, environmental regulation only in the North may be sufficient to stave off an environmental disaster, because once these policies induce a sufficient improvement in the technology of the clean sector, the South will also adjust its production and technology choices. However, free international trade, without global policy coordination, may lead to increased environmental damage by creating a "pollution haven" in the South and thus increase the need for global policy coordination.

Our paper relates to the literature on growth, resources, and the environment. Nordhaus' (1994) pioneering study proposed a dynamic integrated model of climate change and the economy (the DICE model), which extends the neoclassical Ramsey model with equations representing emissions and climate change, and their interactions with economic outcomes.[4] In our calibration exercise we build on Nordhaus' study and results. Another branch of the literature focuses on the measurement of the costs of climate change, particularly stressing issues related to risk, uncertainty and discounting.[5] Based on the assessment of discounting and related issues, this literature has prescribed either decisive and immediate governmental action (for example, Stern, 2006) or a more gradualist approach, with modest control in the short-run followed by sharper emissions reduction in the medium and the long run.[6] Recent work by Golosov, Hassler, Krusell and Tsyvinski (2009) characterizes the structure of optimal policies in a model with exogenous technology and exhaustible resources, where oil suppliers set prices to maximize discounted profits. They show that the optimal resource tax should be decreasing over time. Finally, some authors have built on Weitzman's (1974) analysis on the use of price or quantity instruments to study climate change policy and the choice between taxes and quotas.[7]

The response of technology to environmental degradation and environmental policy, our main focus in this paper, has received much less attention in the economics literature, however. Early work by Stokey (1998) highlighted the tension between growth and the environment, and showed that degradation of the environment can create an endogenous limit to growth. Recent research by Jones (2009) provides a systematic analysis of conditions under which environmental and other costs of growth will outweigh its benefits. Aghion and Howitt (1998, Chapter 5) introduced environmental constraints in a Schumpeterian growth model and em-

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[4] Nordhaus and Boyer (2000) extend the DICE model to include eight regions making decisions independently (the "RICE" model, or Regional Dynamic Integrated model of Climate and the Economy). The analysis of economic activity and its consequences in terms of climate change using this type of approach has been the subject of an extensive report conducted by Stern (2006).

[5] For example, Stern (2006), Weitzman (2007, 2008), Dasgupta (2007, 2008), Nordhaus (2007), von Below and Persson (2008), Mendelsohn (2007), and Tol and Yohe (2006).

[6] See, for example, the work by Nordhaus and coauthors (1994, 2000, 2002). A survey of the results of greenhouse-gas stabilization policy in several climate-change models can be found in Energy Modeling Forum Study 19 (2004).

[7] See for example Hepburn (2006) and Pizer (2002). In addition, several studies address the importance of internationally coordinated policy, such as Stern (2006) and Watson (2001). Aldy et al. (2003) provide a comparison of the different architectures for global climate policy.

phasized that environmental constraints may not prevent sustainable long-run growth when environment-saving innovations are allowed. Neither of these early contributions allowed technological change to be directed to clean or dirty technologies.

Subsequent work by Popp (2004) allowed for directed innovation in the energy sector.[8] Popp presents a calibration exercise and establishes that models that ignore the directed technical change effects can significantly overstate the cost of environmental regulation. While Popp's work is highly complementary to ours, neither his work nor others develop a systematic framework for the analysis of the impact of environmental regulations on the direction of technological change. We develop a general and tractable framework, extending the models in Acemoglu (1998, 2002), that allows us:[9] (i) to perform systematic comparative analyses for the effects of different types of policies on innovation, growth and environmental resources both with and without directed technical change; (ii) to study the implications of dirty inputs using exhaustible resources; (iii) to characterize dynamic optimal policy; and (iv) to study the role of international linkages in technology and trade on the effects of environmental regulations.

The remainder of the paper is organized as follows. Section 2 introduces our basic framework without exhaustible resources and presents the majority of our main results. In particular, it characterizes the laissez-faire equilibrium and shows how this can lead to an environmental disaster. It then shows how simple policy interventions can prevent environmental disasters and clarifies the role of directed technical change in these results. Section 3 characterizes the structure of optimal environmental policy in this setup. Section 4 provides a preliminary quantitative assessment of how directed technical change affects the structure of optimal policy under reasonable parameter values. Section 5 studies the economy with exhaustible resources. Section 6 discusses global policy coordination. Section 7 concludes. The main appendices contain the proofs of some of the key results stated in the text, and the Supplementary Appendix contains the remaining proofs and additional quantitative exercises.

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[8]Nordhaus (2002) also extends the R&DICE model by including a simple form of induced technical change. In particular, he uses a variant of his previous framework with fixed proportions, in which R&D is modeled as shifting the minimum level of carbon/energy inputs required for production. However, since factor substitution is not allowed in the model, it is not possible to compare the role of induced innovation with that of factor substitution in reducing greenhouse emissions. Popp's (2004) ENTICE model allows for both endogenous technological change and factor substitution.

[9]First attempts at introducing endogenous directed technical change in models of growth and the environment also build on Acemoglu (1998, 2002) and include Grubler and Messner (1998), Manne and Richels (2002), Messner (1997), Buonanno et al (2003), Nordhaus (2002), Sue Wing (2003), and Di Maria and Valente (2006). Grimaud and Rouge (2008) and Aghion and Howitt (2009, Chapter 16) are more closely related to the approach followed in this paper.

More recently, Gans (2009) develops a two-period model based on Acemoglu (2009b) to discuss the Porter hypothesis, that environmental regulation can lead to faster technological progress (see also Rauscher, 2009). In particular he shows that this would require a high degree of substitutability between clean and dirty inputs. We abstract from this channel in the current paper by assuming that the total R&D resources in the economy are constant, focusing instead on long-run growth sustainability and the characterization of dynamic optimal policies.

2 Baseline Model: Non-Exhaustible Resource

In this section, we introduce the baseline framework (without an exhaustible resource). We identify the market size and price effects on the direction of technical change and characterize the equilibrium of the economy under laissez-faire. We then discuss how policy interventions may be necessary to avoid "environmental disasters", and the costs of delayed intervention.

2.1 Preferences, Production and the Environment

We consider an infinite-horizon discrete-time economy inhabited by a continuum of households comprising workers, entrepreneurs and scientists. We assume that all households have preferences (or that the economy admits a representative household with preferences):

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u(C_t, S_t) \tag{1}$$

where C_t is consumption of the unique final good at time t, S_t denotes the *quality of the environment* at time t, and $\rho > 0$ is the discount rate.[10] We assume that $S_t \in [0, \overline{S}]$, where $\overline{S}$ is the quality of the environment absent any human pollution, and to simplify the notation, we also assume that this is also the initial level of quality, that is, $S_0 = \overline{S}$.

The instantaneous utility function $u(C, S)$ is increasing both in C and S, twice differentiable and jointly concave in (C, S). Moreover, we impose the following Inada-type conditions:

$$\lim_{C \downarrow 0} \frac{\partial u(C, S)}{\partial C} = \infty, \; \lim_{S \downarrow 0} \frac{\partial u(C, S)}{\partial S} = \infty, \text{ and } \lim_{S \downarrow 0} u(C, S) = -\infty. \tag{2}$$

The last two conditions imply that the quality of the environment reaching its lower bound has severe utility consequences.[11] Finally we assume that

$$\lim_{S \uparrow \overline{S}} \frac{\partial u(C, S)}{\partial S} \equiv \frac{\partial u(C, \overline{S})}{\partial S} = 0, \tag{3}$$

which implies that as S approaches $\overline{S}$, the value of the marginal increase in environmental quality is small. This assumption is adopted to simplify the characterization of optimal environmental policy in Section 3, and we discuss below how relaxing it affects the results.

There is a unique final good, produced competitively using "clean" and "dirty" inputs Y_c and Y_d, according to the aggregate production function

$$Y_t = \left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{4}$$

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[10] For now, S can be thought of as a measure of general environmental quality. In our quantitative exercise in Section 4, we will explicitly relate S to the increase in temperature since pre-industrial times and to carbon concentration in the atmosphere.

[11] Alternatively, the negative consequences of environmental degradation could have been incorporated into the production structure with equivalent results.

where $\varepsilon \in (0, +\infty)$ is the elasticity of substitution between the two sectors. Throughout, we say that the two sectors are (gross) *substitutes* when $\varepsilon > 1$ and (gross) *complements* when $\varepsilon < 1$ (throughout we ignore the "Cobb-Douglas" case of $\varepsilon = 1$).[12] The case of substitutes $\varepsilon > 1$ (in fact, an elasticity of substitution significantly greater than 1) appears as the more empirically relevant benchmark, since we would expect successful clean technologies to substitute for the functions of dirty technologies. Nevertheless, since the relevant elasticity of substitution has not yet been carefully estimated, and because the case of complements both highlights a variety of different and novel economic forces and is theoretically interesting, throughout we discuss both cases, though we place more emphasis on the case of substitutes.

Both Y_{ct} and Y_{dt} are produced using labor and a continuum of sector-specific machines (intermediates) according to the production functions

$$Y_{ct} = L_{ct}^{1-\alpha} \int_0^1 A_{cit}^{1-\alpha} x_{cit}^{\alpha} di \text{ and } Y_{dt} = L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha} x_{dit}^{\alpha} di, \tag{5}$$

where $\alpha \in (0, 1)$, A_{jit} is the quality of machine of type i used in sector $j \in \{c, d\}$ at time t and x_{jit} is the quantity of this machine. This setup is similar to Acemoglu (1998), except that employment in the two sectors is endogenously determined and the distribution parameters have been dropped in (4) to simplify the algebra. We also define

$$A_{jt} \equiv \int_0^1 A_{jit} di \tag{6}$$

as the aggregate productivity in sector $j \in \{c, d\}$. This specification implies that A_d corresponds to "dirty technologies," while A_c represents "clean technologies". None of our results depend on a complete separation between dirty and clean technologies. In fact, the production side could be alternatively written without the inputs Y_{ct} and Y_{dt}, directly as

$$Y_t = \left(\left(L_{ct}^{1-\alpha} \int_0^1 A_{cit}^{1-\alpha} x_{cit}^{\alpha} di \right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha} x_{dit}^{\alpha} di \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

so that changes in A_c and A_d correspond to the fraction of "tasks" performed using clean vs. dirty technologies.

Market clearing for labor requires labor demand to be less than total labor supply, which is normalized to 1, i.e.,

$$L_{ct} + L_{dt} \leq 1. \tag{7}$$

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[12] As mentioned in the Introduction, renewable energy that can be stored and transported efficiently would correspond to a high degree of substitution between dirty and clean inputs, since the same production services can be obtained from alternative energy with less pollution. Similarly, cars using gasoline versus cars using clean energy sources would be examples of highly substitutable dirty and clean inputs. In contrast, if "clean alternatives" involved reductions in our consumption of energy or transportation services, this would correspond to a low degree of substitution. Similarly, if "green cars" were produced using components that require other dirty inputs, the relevant elasticity of substitution between clean and dirty sectors would be smaller.

In line with the literature on endogenous technical change, machines (for both sectors) are supplied by monopolistically competitive firms. Regardless of the quality of machines and of the sector for which they are designed, producing one unit of any machine costs ψ units of the final good. Without loss of generality, we normalize $\psi \equiv \alpha^2$.

The innovation possibilities frontier is as follows. At the beginning of every period, each scientist decides whether to direct her research to clean or dirty technology. She is then randomly allocated to at most one machine (without any congestion; so that each machine is also allocated to at most one scientist) and is successful in innovation with probability $\eta_j \in (0,1)$ in sector $j \in \{c,d\}$, where innovation increases the quality of a machine by a factor $1+\gamma$ (with $\gamma > 0$), that is, from A_{jit} to $(1+\gamma)A_{jit}$.[13] A successful scientist (who has invented a better version of machine i in sector $j \in \{c,d\}$) obtains a one-period patent and becomes the entrepreneur for the current period in the production of machine i. In sectors where innovation is not successful, monopoly rights are allocated randomly to an entrepreneur drawn from the pool of potential entrepreneurs who then uses the old technology.[14] Our innovation possibilities frontier where scientists can only target a sector (rather than a specific machine) ensures that scientists are allocated across the different machines in a sector.[15] We also normalize the measure of scientists s to 1 and denote the mass of scientists working on machines in sector $j \in \{c,d\}$ at time t by s_{jt}. Hence market clearing for scientists takes the form

$$s_{ct} + s_{dt} \leq 1. \tag{8}$$

Finally, the quality of the environment, S_t, evolves according to the difference equation.

$$S_{t+1} = -\xi Y_{dt} + (1+\delta) S_t, \tag{9}$$

whenever the right hand side of (9) is in the interval $(0, \overline{S})$. Whenever the right hand side is negative, $S_{t+1} = 0$, and whenever the right hand side is greater than $\overline{S}$, $S_{t+1} = \overline{S}$.[16]

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[13] Our model therefore imposes that all technical change takes a "factor-augmenting" form, increasing A_{ct} or A_{dt} (see Acemoglu, 2009). In practice, non-factor-augmenting improvements are also possible, though more difficult to incorporate into a growth model. Acemoglu (2007) provides a comprehensive analysis of the effects of changes in factor supplies on the endogenous bias of technology without restricting productivity improvements to take a factor-augmenting form (and thus allowing endogenous changes in the elasticity of substitution, ε).

[14] The assumptions here are adopted to simplify the exposition and mimic the structure of equilibrium in continuous time models as in Acemoglu (2002) (see also Aghion and Howitt, 2009, for this approach). We adopt a discrete time setup throughout to simplify the analysis of dynamics. The Supplementary Appendix shows that the qualitative results are identical in an alternative formulation with patents and free entry (instead of monopoly rights being allocated to entrepreneurs).

[15] As highlighted further by equation (15) below, this structure implies that innovation builds on the existing level of quality of a machine, and thus incorporates the "building on the shoulders of giants" feature. In terms of the framework in Acemoglu (2002), this implies that there is "state dependence" in the innovation possibilities frontier, in the sense that advances in one sector make future advances in that sector more profitable or more effective. This is a natural feature in the current context, since improvements in fossil fuel technology should not (and in practice do not) directly translate into innovations in alternative and renewable energy sources. Nevertheless, one could allow some spillovers between the two sectors, that is, "limited state dependence" as in Acemoglu (2002).

[16] Or equivalently, $S_{t+1} = \max\left\{\min\langle -\xi Y_{dt} + (1+\delta) S_t;\rangle 0; \overline{S}\right\}$.

The parameter ξ measures the rate of environmental degradation resulting from the production of dirty inputs,[17] and δ is the rate of "environmental regeneration". Recall that $\overline{S}$ is the maximum level of environmental quality corresponding to zero pollution. This equation introduces the major externality in our model, from the production of the dirty input to environmental degradation. Note that if $S_t = 0$, then S_τ will remain at 0 for all $\tau > t$.

While other papers in the environment literature typically use more detailed descriptions of environmental dynamics, in this paper we take a "reduced-form" approach and concentrate instead on identifying the new economic forces that arise in the presence of directed technical change. Nevertheless, equation (9) captures several important features of environmental change in practice. First, we assume an exponential regeneration rate δ because greater environmental degradation is typically presumed to lower the regeneration capacity of the globe. For example, part of the carbon in the atmosphere is absorbed by the ice cap; as the ice cap melts because of global warming, more carbon is released into the atmosphere and the melting of the ice cap decreases the albedo of the planet further contributing to global warming. Similarly, the depletion of forests reduces carbon absorption, contributing further to global warming. Second, as already mentioned above, the upper bound $\overline{S}$ captures the idea that environmental degradation results from pollution, and that pollution cannot be negative. We discuss below how our results change under alternative laws of motion for the quality of the environment.

2.2 The laissez-faire equilibrium

In this subsection we characterize the *laissez-faire equilibrium* outcome, that is, the decentralized equilibrium without any policy intervention. We first characterize the equilibrium production and labor decisions for given productivity parameters. We then analyze the direction of technical change.

An *equilibrium* is given by sequences of wages (w_t), prices for inputs (p_{jt}), prices for machines (p_{jit}), demands for machines (x_{jit}), demands for inputs (Y_{jt}), labor demands (L_{jt}) by input producers $j \in \{c, d\}$, research allocations (s_{dt}, s_{ct}), and quality of environment (S_t) such that, in each period t: (i) (p_{jit}, x_{jit}) maximizes profits by the producer of machine i in sector j; (ii) L_{jt} maximizes profits by producers of input j; (iii) Y_{jt} maximizes the profits of final good producers; (iv) (s_{dt}, s_{ct}) maximizes the expected profit of a researcher at date t; (v) the wage w_t and the prices p_{jt} clear the labor and input markets respectively; and (vi) the evolution of S_t is given by (9).

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[17] Equation (9) can be equivalently written as

$$S_{t+1} = - \left(\xi Y_{dt} / Y_t \right) Y_t + \left(1 + \delta \right) S_t,$$

where $\xi Y_{dt}/Y_t$ represents the rate of emissions per unit of final good production. This alternative form highlights both that clean technologies enable a reduction in emissions per unit of final good and that technological changes reducing ξ would have similar results to those increasing A_c.

To simplify the notation, we define $\varphi \equiv (1-\alpha)(1-\varepsilon)$ and impose the following assumption, which is adopted throughout the text (often without explicitly specifying it).

Assumption 1

$$\frac{A_{c0}}{A_{d0}} < \min\left((1+\gamma\eta_c)^{-\frac{\varphi+1}{\varphi}}\left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}, (1+\gamma\eta_d)^{\frac{\varphi+1}{\varphi}}\left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}\right).$$

This assumption imposes the reasonable condition that initially the clean sector is sufficiently backward relative to the dirty (fossil fuel) sector, so that under laissez-faire and with $\varepsilon > 1$, the economy starts innovating in the dirty sector. This assumption enables us to focus on the more relevant part of the parameter space (see Appendix A for the case in which this assumption does not hold).

We first consider the equilibrium at time t for given technology levels A_{cit} and A_{dit}. For this particular part we drop the subscript t. As the final good is produced competitively the ratio of relative price satisfies

$$\frac{p_c}{p_d} = \left(\frac{Y_c}{Y_d}\right)^{-\frac{1}{\varepsilon}}. \tag{10}$$

This equation implies that the relative price of clean inputs (compared to dirty inputs) is decreasing in their relative supply, and moreover, that the elasticity of the relative price response is the inverse of the elasticity of substitution between the two inputs. Our normalization of the final good price at 1 then also implies that

$$\left[p_c^{1-\varepsilon} + p_d^{1-\varepsilon}\right]^{1/(1-\varepsilon)} = 1.$$

The profit-maximization problem of the producer of machine i in sector $j \in \{c, d\}$ can be written as

$$\max_{x_{ji}, L_j}\left\{p_j L_j^{1-\alpha}\int_0^1 A_{ji}^{1-\alpha}x_{ji}^\alpha di - wL_j - \int_0^1 p_{ji}x_{ji}di\right\},$$

and leads to the following iso-elastic inverse demand curve:

$$x_{ij} = \left(\frac{\alpha p_j}{p_{ji}}\right)^{-\frac{1}{1-\alpha}}A_{ji}L_{ji}. \tag{11}$$

Thus the demand for machine i in sector j increases with the price p_j of input j and with employment L_j in that sector, since both increase the profitability of all machines used in that sector, encouraging producers to use more of each. It is also increasing in the quality of such machines, A_{ji}, and decreasing in their price, p_{ji}.

The monopolist producer of machine i in sector j chooses p_{ji} and x_{ji} to maximize profits $\pi_{ji} = (p_{ji} - \psi)x_{ji}$, subject to the inverse demand curve (11). Given this iso-elastic demand, the profit-maximizing price is a constant markup over marginal cost, thus $p_{ji} = \psi/\alpha$. Recalling

the normalization $\psi \equiv \alpha^2$, this implies that $p_{ji} = \alpha$ and thus the equilibrium demand for machines i in sector j is obtained as

$$x_{ji} = p_j^{\frac{1}{1-\alpha}} L_j A_{ji}. \tag{12}$$

Equilibrium profits for the monopolist are then given by

$$\pi_{ji} = (1-\alpha)\,\alpha p_j^{\frac{1}{1-\alpha}} L_j A_{ji}. \tag{13}$$

Next combining equation (12) with the first-order condition with respect to labor, $(1-\alpha)\,p_j L_j^{-\alpha} \int_0^1 A_{ji}^{1-\alpha} x_{ji}^\alpha di = w$ and using (6) gives the relative prices of clean and dirty inputs as

$$\frac{p_c}{p_d} = \left(\frac{A_c}{A_d}\right)^{-(1-\alpha)}. \tag{14}$$

This equation formalizes the natural idea that the input produced with more productive machines will be relatively cheaper.

We next endogeneize productivity by linking productivity growth to R&D in clean and dirty technologies (for clarity, we now reintroduce the time subscript t). If a scientist succeeds in innovation, she discovers a new machine that is $(1+\gamma)$ times more productive than its previous vintage, A_{jit-1}. Therefore, denoting the mass of scientists directing their effort to sector j by s_{jt}, and recalling that scientists targeting sector j are randomly allocated across machines in that sector, the average productivity in sector j at time t evolves over time according to the difference equation

$$A_{jt} = \left(1 + \gamma\eta_j s_{jt}\right) A_{jt-1}. \tag{15}$$

To determine the evolution of average productivities in the two sectors, we need to characterize the profitability of research in these sectors, which will determine the direction of technical change. Taking into account the probability of success, the expected profit Π_{jt} for a scientist engaging in research in sector j is

$$\begin{aligned}
\Pi_{jt} &= \eta_j \int_0^1 (1-\alpha)\,\alpha p_{jt}^{\frac{1}{1-\alpha}} L_{jt}\,(1+\gamma)\,A_{jit-1} di \\
&= \eta_j\,(1+\gamma)\,(1-\alpha)\,\alpha p_{jt}^{\frac{1}{1-\alpha}} L_{jt} A_{jt-1}, \tag{16}
\end{aligned}$$

where the second line simply uses (6). Consequently, the relative benefit from undertaking research in sector c relative to sector d is governed by the ratio:

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d} \quad\times\quad \underbrace{\left(\frac{p_{ct}}{p_{dt}}\right)^{\frac{1}{1-\alpha}}}_{\text{price effect}} \quad\times\quad \underbrace{\frac{L_{ct}}{L_{dt}}}_{\text{market size effect}} \quad\times\quad \underbrace{\frac{A_{ct-1}}{A_{dt-1}}}_{\text{direct productivity effect}}. \tag{17}$$

When this ratio is higher, R&D directed towards the clean technologies becomes more profitable. This equation shows that incentives to innovate in the clean versus the dirty sector

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machines are shaped by three forces: (i) the *direct productivity effect* (captured by the term A_{ct}/A_{dt}), which pushes towards innovating in the sector with higher productivity; this force results from the presence of the "building on the shoulders of giants" effect highlighted in (15); (ii) the *price effect* (captured by the term $(p_{ct}/p_{dt})^{\frac{1}{1-\alpha}}$), encouraging innovation towards the sector with higher prices, which from (14) is the relatively backward sector; (iii) the *market size effect* (captured by the term L_{ct}/L_{dt}), encouraging innovation in the sector with greater employment, which has the larger market for machines. Which sector has greater employment and a larger market is in turn determined by relative productivities and the elasticity of substitution between the two inputs. The more substitutable the two inputs are, the more important is the market size effect compared to the price effect. We next explore this issue.

Equation (12) together with (5) gives the equilibrium production level of input j as

$$Y_{jt} = (p_{jt})^{\frac{\alpha}{1-\alpha}} A_{jt} L_{jt}. \tag{18}$$

Combining (18) with (10), then using (14) and the definition of $\varphi \equiv (1-\alpha)(1-\varepsilon)$, we obtain the relationship between relative productivities and relative employment as:

$$\frac{L_{ct}}{L_{dt}} = \left(\frac{p_{ct}}{p_{dt}}\right)^{-\frac{\varphi-1}{1-\alpha}} \frac{A_{dt}}{A_{ct}} = \left(\frac{A_{ct}}{A_{dt}}\right)^{-\varphi}. \tag{19}$$

Thus the market size effect creates a force towards innovation in the more backward sector when $\varepsilon < 1$, and in the more advanced sector when $\varepsilon > 1$. More specifically, combining (14), (17) and (19), we obtain

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d} \left(\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d s_{dt}}\right)^{-\varphi-1} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi}, \tag{20}$$

which yields the following lemma:

Lemma 1 *In the laissez-faire equilibrium, innovation at time t occurs in the clean sector only when $\eta_c A_{ct-1}^{-\varphi} > \eta_d (1+\gamma\eta_c)^{\varphi+1} A_{dt-1}^{-\varphi}$, in the dirty sector only when $\eta_c (1+\gamma\eta_d)^{\varphi+1} A_{ct-1}^{-\varphi} < \eta_d A_{dt-1}^{-\varphi}$, and in both sectors when $\eta_c (1+\gamma\eta_d s_{dt})^{\varphi+1} A_{ct-1}^{-\varphi} = \eta_d (1+\gamma\eta_c s_{ct})^{\varphi+1} A_{dt-1}^{-\varphi}$ (with $s_{ct} + s_{dt} = 1$).*

Proof. See Appendix A, where we also present a complete characterization of the equilibrium allocations of scientists and equilibrium innovation. ∎

The noteworthy conclusion of this lemma is that innovation will favor the more advanced sector when $\varepsilon > 1$. In particular, in this case $\varphi \equiv (1-\alpha)(1-\varepsilon) < 0$, and thus the direct productivity and market size effects are stronger than the price effect. In contrast, when $\varepsilon < 1$, innovation will favor the less advanced sector because $\varphi > 0$ and therefore the direct productivity effect is weaker than the price effect and the market size effect, which now reinforce each other.

Finally, output of the two inputs and the final good in the laissez-faire equilibrium can be written as (again dropping time subscripts to simplify notation):

$$Y_c = \left(A_c^\varphi + A_d^\varphi\right)^{-\frac{\alpha+\varphi}{\varphi}} A_c A_d^{\alpha+\varphi}, \qquad Y_d = \left(A_c^\varphi + A_d^\varphi\right)^{-\frac{\alpha+\varphi}{\varphi}} A_c^{\alpha+\varphi} A_d, \qquad (21)$$
$$\text{and } \quad Y = \left(A_c^\varphi + A_d^\varphi\right)^{-\frac{1}{\varphi}} A_c A_d.$$

Using these expressions and Lemma 1, we establish:

Proposition 1 *Suppose Assumption 1 holds. Then there exists a unique laissez-faire equilibrium, which takes the following form:*

- *If $\varepsilon > 1$, innovation always occurs in the dirty sector only, and the long-run growth rate of dirty input production is $\gamma\eta_d$.*

- *If $\varepsilon < 1$ innovation first occurs in the clean sector, then occurs in both sectors and asymptotically the share of scientists devoted to the clean sector is given by $s_c = \eta_d/\left(\eta_c + \eta_d\right)$; the long-run growth rate of dirty input production in this case is $\gamma\widetilde{\eta}$, where $\widetilde{\eta} \equiv \eta_c\eta_d/\left(\eta_c + \eta_d\right)$.*

Proof. See Appendix B. ■

The intuition for this proposition follows from Lemma 1. When the two inputs are substitutes ($\varepsilon > 1$), innovation starts in the dirty sector, which is more advanced initially (Assumption 1). This increases the gap between the dirty and the clean sectors and the initial pattern of equilibrium is reinforced. In this case, only A_d grows (at the rate $\gamma\eta_d$) and A_c remains constant; moreover, since φ is negative in this case, in the long run Y_d also grows at the rate $\gamma\eta_d$. In contrast, in the empirically less relevant case where the two inputs are complements ($\varepsilon < 1$), the price effect dominates and innovation initially takes place in the more backward—in this case, the clean—sector. This reduces the technology gap between the two sectors and ultimately the equilibrium must involve innovation in both sectors; in particular, the share of scientists allocated to the clean sector converges towards $s_c = \eta_c/\left(\eta_c + \eta_d\right)$, which ensures that both sectors grow at the same rate (see Appendix B). In particular, in this case average quality levels in both sectors, A_c and A_d, grow at the same asymptotic rate $\gamma\widetilde{\eta}$, and thus so does Y_d.

2.3 Directed technical change and environmental disasters

A major concern by climate scientists is that the environment may deteriorate so much that it reaches a "point of no return". In our environment equation (9), this notion is captured by the fact that if environmental quality S_t reaches 0 in finite time, it remains at 0 forever thereafter. Motivated by this feature, we define the notion of an *environmental disaster*, which will be useful for developing the main intuitions implied by our framework, before we provide a more complete characterization of optimal environmental policy.

Definition 1 *An* environmental disaster *occurs if $S_t = 0$ for some $t < \infty$.*

Our assumptions on the utility function, in particular, that $u(C, 0) = -\infty$, imply that an environmental disaster cannot be part of a welfare-maximizing allocation (for any $\rho < \infty$). In this subsection, we show that a simple policy of "redirecting technical change" can avoid an environmental disaster (which would otherwise occur in the laissez-faire equilibrium). We will then highlight the role of directed technical change by comparing the results to a model in which scientists cannot direct their research to different sectors.

We first note that the economy under laissez-faire will necessarily end up in a disaster. This follows both from the fact (Proposition 1) that dirty input production Y_d always grows without bound, and that a level of production of dirty input greater than $(1 + \delta)\,\xi^{-1}\overline{S}$ necessarily leads to a disaster next period. We thus have (proof omitted):

Proposition 2 *Suppose Assumption 1 holds. Then the laissez-faire equilibrium always leads to an environmental disaster.*

Remark 1 Equation (21) implies that the long-run growth rate of dirty input production in the substitutable case, $\gamma\eta_d$, is greater than its long-run growth rate in the complementarity case, $\gamma\widetilde{\eta}$, since in the latter case R&D resources are spread across the two sectors. Then for given initial technological levels A_{c0} and A_{d0}, the production of dirty input is always higher in the substitutability case than in the complementarity case. This implies that the disaster occurs sooner in the substitutability case than in the complementarity case (see Supplementary Appendix for a proof).

Remark 2 Our analysis can be extended to different laws of motion of the environmental stock, with similar results. For example, we could dispense with the upper bound on environmental quality, so that $\overline{S} = \infty$. In this case, the results are similar, except that a disaster can be avoided even if dirty input production grows at a positive rate, provided that this rate is lower than the regeneration rate of the environment, δ. An alternative is to suppose $S_{t+1} = -\xi Y_{dt} + S_t + \Delta$, so that the regeneration of the environment is additive rather than proportional to current quality. With this alternative law of motion, it is straightforward to show that the results are essentially identical to the baseline formulation because a disaster can only be avoided if Y_{dt} does not grow at a positive exponential rate in the long run. Consequently, in this case, Proposition 2 continues to apply. Finally, one could assume that environmental degradation is caused by dirty machines, the x_{jit}'s, rather than by dirty input production, Y_{dt}. Given our other assumptions, the results in this case are also similar to those in Proposition 2.

Proposition 2 implies that some type of intervention is necessary to avoid a disaster. For a preliminary investigation of the implications of such intervention, suppose that the government

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can subsidize scientists to work in the clean sector, for example, using a proportional profit subsidy (financed through a lump-sum tax on the representative household).[18] Denoting this subsidy rate by q_t, the expected profit from undertaking research in the clean sector becomes

$$\Pi_{ct} = (1 + q_t)\, \eta_c \, (1 + \gamma)\, (1 - \alpha)\, \alpha p_{ct}^{\frac{1}{1-\alpha}} L_{ct} A_{ct-1},$$

while Π_{dt} is still given by (16). This immediately implies that a sufficiently high subsidy to clean research can redirect innovation towards the clean sector.[19] Moreover, while this subsidy is implemented, the ratio A_{ct}/A_{dt} will grow at the rate $\gamma\eta_c$. The implications of the tax then depend on the degree of substitutability between the two inputs. When the two inputs are substitutes ($\varepsilon > 1$), a temporary subsidy (maintained for the necessary number of periods, D) is sufficient to redirect all research to the clean sector. More specifically, while the subsidy is being implemented, the ratio A_c/A_d will increase, and when it has become sufficiently high, it will be profitable for scientists to direct their research to the clean sector even without the subsidy.[20] Then (21) implies that Y_d will grow asymptotically at the same rate as $A_c^{\alpha+\varphi}$.

If $\varepsilon \geq 1/(1-\alpha)$ so that the two inputs are strong substitutes, then $\alpha + \varphi \leq 0$ and Y_d will not grow in the long-run. In this case, provided that the initial environmental quality is sufficiently high, a temporary subsidy is sufficient to avoid an environmental disaster. This case thus gives the most *optimistic* implications of our analysis, where a temporary intervention is sufficient to redirect technical change, and avoid an environmental disaster without preventing long-run growth or even creating long-run distortions. This contrasts with the Nordhaus, the Stern/Al Gore, and the Greenpeace answers discussed in the Introduction.

If, instead, $\varepsilon \in (1, 1/(1-\alpha))$ (or $\alpha + \varphi > 0$) so that we have weak but not strong substitutes, then temporary intervention is sufficient to redirect all research to the clean sector, but equation (21) implies that even after this happens, Y_d will grow at rate $(1 + \gamma\eta_c)^{\alpha+\varphi} - 1 > 0$. Intuitively, since $\varepsilon > 1$, as the average quality of clean machines increases, workers get reallocated towards the clean sector (because of *the market size effect*). At the same time the increase of the relative price of the dirty input over time encourages production of the dirty input (*the price effect*). As shown in the previous paragraph, in the strong substitutes case the first effect dominates. In contrast, in the weak substitutes case, where $\varepsilon < 1/(1-\alpha)$, the second effect

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[18] The results are identical if we focus on profits taxes on the dirty sector or on other types of research subsidies.

[19] In particular, following the analysis in Appendix A, to implement a unique equilibrium where all scientists direct their research to the clean sector, the subsidy rate q_t must satisfy

$$q_t > (1 + \gamma\eta_d)^{-\varphi-1} \frac{\eta_d}{\eta_c} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^\varphi - 1 \text{ if } \varepsilon \geq \frac{2-\alpha}{1-\alpha} \text{ and } q_t \geq (1 + \gamma\eta_c)^{(\varphi+1)} \frac{\eta_d}{\eta_c} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^\varphi - 1 \text{ if } \varepsilon < \frac{2-\alpha}{1-\alpha}.$$

[20] The temporary tax needs to be imposed for D periods where D is the smallest integer such that:

$$\frac{A_{ct+D-1}}{A_{dt+D-1}} > (1 + \gamma\eta_d)^{\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}} \text{ if } \varepsilon \geq \frac{2-\alpha}{1-\alpha} \text{ and } \frac{A_{ct+D-1}}{A_{dt+D-1}} \geq (1 + \gamma\eta_c)^{-\frac{\varphi+1}{\varphi}} \left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}} \text{ if } 1 < \varepsilon < \frac{2-\alpha}{1-\alpha}$$

dominates,[21] and Y_d increases even though A_d is constant. In this case, we thus obtain the less optimistic conclusion that a temporary subsidy redirecting research to the clean sector will not be sufficient to avoid an environmental disaster; instead, similar to the Stern/Al Gore position, permanent government regulation is necessary to avoid environmental disaster.

Finally, when the two inputs are complements ($\varepsilon < 1$), our model delivers the most pessimistic conclusion. With or without a temporary subsidy to clean research, the more backward sector always catches up with the more advanced sector.[22] Thus in the long run, innovation will take place in both sectors, and production of the dirty input will continue to grow asymptotically. Therefore, an environmental disaster becomes unavoidable, unless long-run growth is halted entirely as in the Greenpeace position. Nevertheless, as noted above, we would plausibly expect a relatively high degree of substitution between clean and dirty inputs, so our analysis also highlights why the Greenpeace position rests on empirically less plausible assumptions.

This discussion establishes the following proposition (proof in the text):

Proposition 3 *When the two inputs are strong substitutes ($\varepsilon \geq 1/(1-\alpha)$) and $\overline{S}$ is sufficiently high, a temporary subsidy to clean research will prevent an environmental disaster. In contrast, when the two inputs are complements or weak substitute ($\varepsilon < 1/(1-\alpha)$), a temporary subsidy to clean research cannot prevent an environmental disaster.*

Thus, when the two inputs are *strong* substitutes, *redirecting* technical change using a temporary policy intervention can be sufficient to avoid a disaster. This shows the importance of directed technical change: temporary incentives are sufficient to induce research to be directed to clean technologies, and once clean technologies are sufficiently advanced, innovation and production will shift sufficiently towards those technologies so that environmental disaster can be avoided without further intervention.

Remark 3 It is useful to note that all of the main results in this section are a consequence of endogenous and directed technical change. We can envisage an environment without directed technical change by considering the same model with scientists randomly allocated across sectors. Suppose, for simplicity, that this is done so as to ensure equal growth in the qualities of clean and dirty machines (at the rate $\gamma\widetilde{\eta}$). Recall that when the two inputs are strong substitutes ($\varepsilon \geq 1/(1-\alpha)$), dirty input production grows at the higher rate $\gamma\eta_d$. Thus, when the two inputs are strong substitutes, under laissez-faire a disaster will occur sooner with directed

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[21] A different intuition for the case in which $\varepsilon \in (1, 1/(1-\alpha))$ is that improvements in the technology of the clean sector also correspond to improvements in the technology of the final good, which uses them as inputs; the final good, in turn, is an input for the dirty sector because machines employed in this sector are produced using the final good; hence, technical change in the clean sector creates a force towards the expansion of the dirty sector.

[22] The proof of this claim follows closely the proof of Proposition 1. In particular, regardless of which sector innovation is first directed at, innovation in the long run must take place in both sectors, which in turn implies that the long-run growth rate must be $\gamma\widetilde{\eta}$.

technical change than without. But also while with directed technical change a temporary subsidy can redirect innovation towards the clean sector, preventing an environmental disaster, without directed technical change such redirecting is not possible and thus temporary interventions cannot prevent an environmental disaster.

2.4 Costs of delay

Policy intervention is costly in our framework because during the period of adjustment (while productivity in the clean sector is catching up with that in the dirty sector), final output increases more slowly than had innovation been directed towards the dirty sector (in the absence of intervention). We will study the welfare costs of intervention in Section 3. Before doing this, it is instructive to look at a simple measure of the (short-run) cost of intervention, defined as the number of periods T necessary for the economy under the policy intervention to reach the same level of output as it would have done within one period in the absence of the intervention: in other words, this is the length of the transition period or the number of periods of "slow growth" in output growth. We focus here on the substitutability case $\varepsilon > 1$.

This measure T (starting at time t) is then the smallest integer such that:

$$\frac{(1 + \gamma\eta_c)^T}{\left((1 + \gamma\eta_c)^{T\varphi} A_{ct-1}^{\varphi} + A_{dt-1}^{\varphi}\right)^{\frac{1}{\varphi}}} \geq \frac{(1 + \gamma\eta_d)}{\left(A_{ct-1}^{\varphi} + (1 + \gamma\eta_d)^{\varphi} A_{dt-1}^{\varphi}\right)^{\frac{1}{\varphi}}}$$

or equivalently,

$$T = \left\lceil \frac{\ln\left(\left((1 + \gamma\eta_d)^{-\varphi} - 1\right)\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{\varphi} + 1\right)}{-\varphi \ln(1 + \gamma\eta_c)} \right\rceil \tag{22}$$

It can be verified that starting at any $t \geq 1$, we have $T \geq 2$ (in the equilibrium in Proposition 3 and with $\varepsilon \geq 1/(1 - \alpha)$). Thus once innovation is directed towards the clean sector it will take more than one period for the economy to achieve the same output growth as it would have achieved in just one period in the laissez-faire equilibrium of Proposition 1 (with innovation still directed at the dirty sector). The following corollary then follows immediately from equation (22), in particular, recalling that $\varphi \equiv (1 - \alpha)(1 - \varepsilon)$ (proof omitted):

Corollary 1 *For $A_{dt-1}/A_{ct-1} \geq 1$, the short-run cost of intervention, T, is nondecreasing in the technology gap A_{dt-1}/A_{ct-1} and the elasticity of substitution ε. Moreover, T increases more with A_{dt-1}/A_{ct-1} when ε is greater.*

The (short-run) cost of intervention, T, is increasing in A_{dt-1}/A_{ct-1} because a larger gap between the initial quality of dirty and clean machines leads to a longer transition phase, and thus to a longer period of low growth. In addition, T is also increasing in the elasticity of substitution ε. Intuitively, if the two inputs are close substitutes, final output production relies

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mostly on the more productive input, and therefore, productivity improvement in the clean sector (taking place during the transition phase) will have less impact on overall productivity until the clean technologies surpass the dirty ones.

The corollary shows that delaying intervention is costly, not only because of the continued environmental degradation that will result, but also because during the period of delay A_{dt}/A_{ct} will increase further, and thus when the intervention is eventually implemented, the temporary subsidy to clean research will need to be imposed for longer and there will be a longer period of slow growth (higher T). This result is clearly related to the "building on the shoulders of giants" feature of the innovation process. Furthermore, the result that the effects of ε and A_{dt-1}/A_{ct-1} on T are complementary implies that delaying the starting date of the intervention is more costly when the two inputs are more substitutable. These results imply that even though for the strong substitutes case the implications of our model are more optimistic than those of Nordhaus, in contrast to the implications of his analysis, gradual and delayed intervention would have significant costs.

Overall, the analysis in this subsection has established that a simple policy intervention that "redirects" technical change towards environment friendly technologies can help prevent an environmental disaster. Our analysis also highlights that delaying intervention may be quite costly, not only because it further damages the environment (an effect already recognized in the climate science literature), but also because it widens the gap between the dirty and clean technologies, thereby inducing a longer period of catch-up with slower growth.

3   Optimal environmental policy

We have so far studied the behavior of the laissez-faire equilibrium and discussed how environmental disaster may be avoided. In this subsection, we characterize the optimal allocation of resources in this economy and discuss how it can be decentralized by using "carbon" taxes and research subsidies. The socially-planned (optimal) allocation will "correct" for two externalities: (1) the environmental externality exerted by dirty input producers, and (2) the knowledge externalities from R&D (the fact that in the laissez-faire equilibrium scientists do not internalize the effects of their research on productivity in the future). In addition, the planner can and will correct for the standard static monopoly distortion in the price of machines, encouraging more intensive use of existing machines (see, for example, Aghion and Howitt, 1998, or Acemoglu, 2009). Throughout this section, we assume that the social planner (government) has access to lump-sum taxes and transfers to complement the other policy instruments (and thus raise or redistribute revenues as required). A key conclusion of the analysis in this section is that optimal policy must use both a "carbon" tax (i.e., a tax on dirty input production) and a subsidy to clean research, the former to control carbon emissions and the latter to influence the path of future research. Relying only on carbon taxes would be excessively distortionary.

3.1 The social planner's problem

The social planner's problem is one of choosing a dynamic path of final good production Y_t, consumption C_t, input productions Y_{jt}, expected machines production x_{jit}, labor share allocation L_{jt}, scientists allocation s_{jt}, environmental quality S_t, and quality of machines A_{jit}, that maximizes the intertemporal utility of the representative consumer, (1), subject to (4), (5), (7), (8), (9), (15), and

$$C_t = Y_t - \psi \left(\int_0^1 x_{cit} di + \int_0^1 x_{dit} di \right). \tag{23}$$

Let λ_t denote the Lagrange multiplier for (4), which is naturally also the shadow value of one unit of final good production. The first-order conditions with respect to Y_t imply that this shadow value is also equal to the Lagrange multiplier for (23), so that it is also equal to the shadow value of one unit of consumption. Then the first-order condition with respect to C_t yields

$$\lambda_t = \frac{1}{(1+\rho)^t} \frac{\partial u(C_t, S_t)}{\partial C}, \tag{24}$$

so that, again naturally, the shadow value of the final good is equal to the marginal utility of consumption.

The ratio λ_{jt}/λ_t can then be interpreted as the shadow price of input j at time t (relative to the price of the final good). To emphasize this interpretation, we will denote this ratio by $\widehat{p}_{jt}$. We can now combine the first-order condition with respect to x_{ji} with (5) to obtain:[23]

$$Y_{jt} = \left(\frac{\alpha}{\psi} \widehat{p}_{jt} \right)^{\frac{\alpha}{1-\alpha}} A_{jt} L_{jt} \tag{25}$$

so that for given price, average technology and labor allocation, the production of each input is scaled up by a factor $\alpha^{\frac{-\alpha}{1-\alpha}}$ compared to the laissez-faire equilibrium (this results from the more intensive use of machines in the socially-planned allocation).

Next, letting ω_t denote the Lagrange multiplier for the environmental equation (9), the first-order condition with respect to S_t gives

$$\omega_t = \frac{1}{(1+\rho)^t} \frac{\partial u(C_t, S_t)}{\partial S} + (1+\delta) I_{S_t < \overline{S}} \omega_{t+1}, \tag{26}$$

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[23] This first-order condition with respect to x_{ji} is

$$x_{jit} = \left(\frac{\alpha}{\psi} \widehat{p}_{jt} \right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt} = \left(\frac{1}{\alpha} \widehat{p}_{jt} \right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt},$$

which can be compared to the equilibrium inverse demand, (11), and highlights that existing machines will be used more intensively in the socially-planned allocation. This is a natural consequence of the monopoly distortions and can also be interpreted as the socially-planned allocation involving a subsidy of $1-\alpha$ in the use of machines, so that their price should be identical to the marginal cost, i.e., $(1-(1-\alpha))\psi/\alpha = \psi \equiv \alpha^2$.

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where $I_{S_t < \overline{S}}$ is equal to 1 if $S_t < \overline{S}$ and to 0 otherwise. This implies that the shadow values of environmental quality at time t is equal to the marginal utility that it generates in this period plus the shadow value of $(1 + \delta)$ units of environmental quality at time $t + 1$ (as one unit of environmental quality at time t generates $1 + \delta$ units at time $t + 1$). This equation also implies that if for all $\tau > T$, $S_\tau = \overline{S}$, then $\omega_t = 0$ for all $t > T$.[24]

The first-order conditions with respect to Y_{ct} and Y_{dt} then give

$$Y_{ct}^{\frac{-1}{\varepsilon}} \left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{1}{\varepsilon-1}} = \widehat{p}_{ct}, \tag{27}$$

$$Y_{dt}^{\frac{-1}{\varepsilon}} \left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{1}{\varepsilon-1}} - \frac{\omega_{t+1}\xi}{\lambda_t} = \widehat{p}_{dt}.$$

These equations imply that compared to the laissez-faire equilibrium, the social planner introduces a wedge of $\omega_{t+1}\xi/\lambda_t$ between the marginal product of the dirty input in the production and its price. This wedge $\omega_{t+1}\xi/\lambda_t$ is equal to the environmental cost of an additional unit of the dirty input (evaluated in terms of units of the final good at time t; recall that one unit of dirty production at time t destroys ξ units of environmental quality at time $t + 1$). Naturally, this wedge is also equivalent to a tax of

$$\tau_t = \frac{\omega_{t+1}\xi}{\lambda_t \widehat{p}_{dt}} \tag{28}$$

on the use of dirty input by the final good producer. This tax rate will be higher when the shadow value of environmental quality is greater, when the marginal utility of consumption today is lower, and when the price of dirty input is lower.

Finally, the social planner must correct for the knowledge externality. Let μ_{jt} denote the Lagrange multiplier for equation (15) for $j = c, d$. Naturally, this variable would then correspond to the shadow value of average productivity in sector j at time t. The relevant first-order condition then gives:

$$\mu_{jt} = \lambda_t \left(\frac{\alpha}{\psi} \right)^{\frac{\alpha}{1-\alpha}} (1 - \alpha) \widehat{p}_{jt}^{\frac{1}{1-\alpha}} L_{jt} + \left(1 + \gamma\eta_j s_{jt+1} \right) \mu_{j,t+1}. \tag{29}$$

Intuitively, the shadow value of a unit increase in average productivity in sector $j \in \{c, d\}$ is equal to its marginal contribution to time-t utility plus its shadow value at time $t + 1$ times $\left(1 + \gamma\eta_j s_{jt+1} \right)$ (the number of units of productivity created out of it at time $t + 1$). This last term captures the intertemporal knowledge externality.

At the optimum, scientists will be allocated towards the sector with the higher social gain from innovation, as measured by $\gamma\eta_j\mu_{jt}A_{jt-1}$. Using (29), we then have that the social planner

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[24] This result depends on the assumption that $\partial u \left(C, \overline{S} \right)/\partial S = 0$. If instead $\partial u \left(C, \overline{S} \right)/\partial S > 0$, then we would have $\omega_t \neq 0$ even when $S_t = \overline{S}$, and the optimal carbon tax may remain positive asymptotically.

will allocate scientists to the clean sector whenever the ratio

$$\frac{\eta_c \left(1 + \gamma \eta_c s_{ct}\right)^{-1} \sum_{\tau \geq t} \lambda_\tau \widehat{p}_{c\tau}^{\frac{1}{1-\alpha}} L_{c\tau} A_{c\tau}}{\eta_d \left(1 + \gamma \eta_d s_{dt}\right)^{-1} \sum_{\tau \geq t} \lambda_\tau \widehat{p}_{d\tau}^{\frac{1}{1-\alpha}} L_{d\tau} A_{d\tau}} \tag{30}$$

is greater than 1. This contrasts with the decentralized outcome where scientists are allocated according to the private value of innovation, that is, according to the ratio $(\lambda_t \widehat{p}_{ct}^{\frac{1}{1-\alpha}} L_{ct} A_{ct} / \lambda_t \widehat{p}_{dt}^{\frac{1}{1-\alpha}} L_{dt} A_{dt})$.[25] This discussion implies that optimal environmental policy can be implemented using a simple tax/subsidy scheme, as stated in the next proposition.

Proposition 4 *The social planner can implement the social optimum through a tax on the use of the dirty input (a "carbon" tax), a subsidy to clean innovation, and a subsidy for the use of all machines (all proceeds from taxes/subsidies being redistributed/financed lump-sum).*

   Proof. The main idea of the proof is in the text. The formal proof is provided in the Supplementary Appendix. ∎

   That we need both a "carbon" tax and a subsidy to clean research to implement the social optimum (in addition to the subsidy to remove the monopoly distortions) is intuitive: the subsidy deals with future environmental externalities by directing innovation towards the clean sector, whereas the carbon tax deals more directly with the current environmental externality by reducing current production of the dirty input, which causes this externality in the first place. By reducing production in the dirty sector, the carbon tax also discourages innovation in that sector. However, using only the carbon tax to deal with both current environmental externalities and future (knowledge-based) externalities would necessitate a very high carbon tax, potentially distorting current production and reducing current consumption excessively. Thus an important implication of this result is that, without additional restrictions on policy, it would not be optimal to rely only on a carbon tax to deal with global warming; one should also use additional instruments (R&D subsidies or profit tax on the dirty sector) that direct innovation towards clean technologies, so that in the future production can be increased using alternative technologies.

3.2   The structure of optimal environmental regulation

In subsection 2.3, we showed that a temporary profit tax could prevent a disaster when the two inputs are substitutes. Here we show that, when the two inputs are sufficiently substitutable and the discount rate is sufficiently low, the optimal policy characterized in Proposition 4

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[25] The knowledge externality in our model is extreme because researchers (scientists) capture profits from innovation for only one period. Nevertheless, a similar externality exists more generally in endogenous and directed technical change models, where researchers do not fully capture the social value of innovation because of both monopoly distortions and knowledge spillovers on future innovations (e.g., Acemoglu, 2002).

also only involves temporary interventions (except for the standard subsidy that corrects for monopoly distortions).

More formally, recall that the optimal carbon tax schedule is given by $\tau_t = \omega_{t+1}\xi/\lambda_t \widehat{p}_{dt}$, where ω_{t+1}, the shadow value of one unit of environmental quality at time $t+1$, is equal to the discounted marginal utility of environmental quality as of period $t+1$, that is:

$$\omega_{t+1} = \sum_{v=t+1}^{\infty} (1+\delta)^{v-(t+1)} \frac{1}{(1+\rho)^v} \frac{\partial u(C_v, S_v)}{\partial S} I_{S_{t+1},...,S_\nu < \overline{S}},$$

where $I_{S_{t+1},...,S_\nu < \overline{S}}$ takes value 1 if $S_{t+1},...,S_\nu < \overline{S}$ and 0 otherwise. Thus, using (24), we get

$$\tau_t = \frac{\xi}{\widehat{p}_{dt}} \frac{\frac{1}{1+\rho} \sum_{v=t+1}^{\infty} \left(\frac{1+\delta}{1+\rho}\right)^{v-(t+1)} I_{S_{t+1},...,S_\nu < \overline{S}} \partial u(C_v, S_v)/\partial S}{\partial u(C_t, S_t)/\partial C}. \tag{31}$$

This expression shows that once S_t reaches the upper bound $\overline{S}$, the optimal tax on dirty input falls down to zero since $\partial u\left(C_{t+1}, \overline{S}\right)/\partial S = 0$. This, in turn, has implications on how the dynamics of the optimal tax schedule depend upon the degree of substitutability between the clean and the dirty inputs.

Proposition 5 *If $\varepsilon > 1$ and the discount rate ρ is sufficiently small, then in finite time innovation ends up occurring only in the clean sector, the economy grows at rate $\gamma\eta_c$ and the optimal subsidy on profits in the clean sector, q_t, is temporary. Moreover, the optimal carbon tax, τ_t, is temporary if $\varepsilon > 1/(1-\alpha)$ but not if $1 < \varepsilon < 1/(1-\alpha)$. Finally, if $\varepsilon < 1$, the optimal carbon tax and the clean research subsidy are both permanent, and the long-run growth rate is zero.*

Proof. See Appendix C. ∎

To obtain an intuition for this proposition, first note that an optimal policy requires avoiding a disaster, since a disaster leads to $u(C, 0) = -\infty$. This in turn implies that the production of dirty input must always remain below a fixed upper bound. But when the discount rate is sufficiently low, it is optimal to have positive long-run growth which, when $\varepsilon > 1$, can be achieved by relying increasingly more on clean input production over time. Not allocating all research to clean innovation in finite time, would slow down the increase in clean input production, and thus reduce intertemporal welfare. An appropriately-chosen subsidy to clean profits then ensures that innovation occurs only in the clean sector, and when A_c exceeds A_d by a sufficient amount, innovation in the clean sector will have become sufficiently profitable that it will continue even after the subsidy is removed (and hence there is no longer a need for the subsidy). When the two inputs are strong substitutes ($\varepsilon > 1/(1-\alpha)$), production of dirty input decreases to 0 over time, and as a result, the environmental stock S_t reaches $\overline{S}$ in finite time due to positive regeneration. This in turn ensures that the optimal carbon tax will reach

zero in finite time. Since dirty input production converges to zero, the economy will generate a long-run growth rate equal to the growth rate of A_c, namely $\gamma\eta_c$.

In the complementarity case, the long-run growth rate of final output is the minimum of the long-run growth rates of the two inputs, so it is not possible to achieve positive long-run growth while avoiding a disaster. Nevertheless, avoiding an environmental disaster is still necessary, so optimal environmental regulation will stop long-run growth.

Remark 4 It is straightforward to compare the structure of optimal policy in this model to the variant without directed technical change discussed in Remark 3. Since the allocation of scientists in that case is insensitive to policy, redirecting innovation towards the clean sector is not possible. Consequently, optimal environmental regulation must prevent an environmental disaster by imposing an ever-increasing sequence of carbon taxes. This comparison highlights that the optimistic conclusion that optimal environmental regulation can be achieved using temporary taxes/subsidies, and with little cost in terms of long-run distortions and growth, is due to the presence of directed technical change.

4   Calibration

Propositions 4 and 5 provided insights into the qualitative features of optimal environmental policy. The next step is a careful quantitative analysis to investigate how the endogenous response of the direction of technical change affects the costs and benefits of different environmental policies. Such a quantitative analysis is beyond the scope of the present paper, because it requires an estimation of the parameters of the innovation possibilities frontier and the exact degree of substitution between clean and dirty inputs. Instead, we take a first step in this direction by investigating the effects of different values of discount rates and elasticities of substitution on the form of open environmental regulation and the resulting timing of a switch (of R&D and production) to clean technology. We choose parameters to make our simple calibration exercise as similar to existing quantitative analyses as possible in order to highlight the new effects resulting from directed technical change.

We take a period in our model to correspond to 5 years. We set $\eta_c = \eta_d = 0.02$ (per annum) and $\gamma = 1$ so that the long-run annual growth rate is equal to 2% (which matches Nordhaus's assumptions in his 2007 DICE calibration). We take $\alpha = 1/3$ (so that the share of national income spent on machines is approximately equal to the share of capital). We then compute A_{ct-1} and A_{dt-1} to match the implied values of Y_{ct-1} and Y_{dt-1} to the production of nonfossil and fossil fuel production in the world primary energy supply from 2002 to 2006 (according to the Energy Information Administration data)**.** Note that in all our exercises, when ε varies, A_{ct-1} and A_{dt-1} also need to be adjusted (in particular, a higher ε leads to a higher value of the ratio A_{ct-1}/A_{dt-1}).

Estimating the elasticity of substitution that would be appropriate for this exercise is beyond the scope of our simple calibration exercise here. We simply note that since fossil and nonfossil fuels should be close substitutes (at the very least, once nonfossil fuels can be transported efficiently), reasonable values of ε should be quite high. Throughout the following calibration exercise, we consider three different values for ε: a baseline intermediate value of $\varepsilon = 5$, a high value of $\varepsilon = 10$, and a low value $\varepsilon = 3$ (all three values, together with our choice of $\alpha = 1/3$, imply strong substitutability between the two inputs). This range of values will give us a sense of which of the conclusions depend on the exact value of the elasticity of substitution.

4.1 Relating environmental quality to carbon concentration

To relate the environmental quality variable S to the atmospheric concentration of carbon, we use a common approximation to the relationship between the increase in temperature since preindustrial times (in degrees Celsius), Δ, and the atmospheric concentration of carbon dioxide (in ppm), C_{CO2}:

$$\Delta \simeq 3 \log_2 \left(C_{CO2}/280 \right).$$

This equation implies that a doubling of atmospheric concentration in CO_2 (since pre-industrial times, when the concentration was equal to 280 ppm) leads to a 3°C increase in current temperature (see, e.g., the Fourth Assessment Report of the Intergovernmental Panel on Climate Change, FARIPCC). We then express S as a decreasing function of Δ and thus of C_{CO2}, so that $S = 0$ corresponds to a level of temperature change Δ approximating "disaster temperature" $\Delta_{disaster}$ (described below). More specifically, we set

$$S = 280 \times 2^{\Delta_{disaster}/3} - \max \left\{ C_{CO_2}, 280 \right\}.$$

For the quantitative exercise here, we relax the assumption that $S_0 = \overline{S}$ and set the initial environmental quality S_0 to correspond to the current atmospheric concentration of 379 ppm ($\overline{S}$, in turn, corresponds to $C_{CO_2} = 280$ ppm, the pre-industrial value).

We then estimate parameter ξ from the observed value of Y_d and the annual emission of CO_2 (ξY_d in our model) from 2002 to 2006 according to the Energy Information Administration. Finally, we choose δ such that only half of the amount of emitted carbon contributes to increasing CO_2 concentration in the atmosphere (the rest being offset by "environmental regeneration," see again FARIPCC).

4.2 Parametrizing the utility function

Nordhaus—and much of the literature following his work—assumes that environmental quality affects aggregate productivity. We find it more reasonable that high temperature levels

and high concentrations of carbon dioxide affect utility as well as production, and we formulated our model under the assumption that environmental quality directly affects utility. Nevertheless, to highlight the similarities and the differences between our model and existing quantitative models with exogenous technology, we choose the parameters such that the welfare consequences of changes in temperature (for the range of changes observed so far) are the same in our model as in previous work. We parameterize the utility function as

$$u(C_t, S_t) = \frac{(\phi(S_t) C_t)^{1-\sigma}}{1-\sigma}, \tag{32}$$

with $\sigma = 2$, which matches Nordhaus's choice of intertemporal elasticity of substitution. In addition, this utility function contains the term $\phi(S)$ to capture the costs from the degradation of environmental quality. We choose this function as

$$\phi(S) = \varphi(\Delta(S)) \equiv \frac{(\Delta_{disaster} - \Delta(S))^{\lambda} - \lambda \Delta_{disaster}^{\lambda-1} (\Delta_{disaster} - \Delta(S))}{(1-\lambda) \Delta_{disaster}^{\lambda}}, \tag{33}$$

for $\Delta(S) \in [0, \Delta_{disaster}]$, where φ is a strictly decreasing and concave function, with $\varphi(0) = 1$, $\varphi(\Delta_{disaster}) = 0$, $\varphi'(0) = 0$ and $\lim_{\Delta \to \Delta_{disaster}} \varphi'(\Delta) = -\infty$. This functional form ensures that our assumptions on the utility function, (2) and (3), are satisfied. Note that (33) defines a flexible family of continuous functions parameterized by λ. As $\lambda \to 1$, this function converges to $\varphi_1(\Delta) = (1 - \Delta/\Delta_{disaster})(1 - \ln(1 - \Delta/\Delta_{disaster}))$ for all $\Delta \in [0, \Delta_{disaster})$ (from L' Hopital's rule) and $\varphi_1(\Delta_{disaster}) = 0$, and as $\lambda \to 1$, it converges (pointwise) to the "step function" $\varphi_0(\Delta) = 1$ for all $\Delta \in [0, \Delta_{disaster})$ and $\varphi_0(\Delta) = 0$ for $\Delta = \Delta_{disaster}$. For our baseline calibration we choose $\Delta_{disaster} = 9.2°C$, which is twice the highest estimate of the temperature increase that would eventually lead to the melting of the Greenland Ice Sheet (FARIPCC). In Nordhaus' DICE model output is affected by temperature through a multiplicative term $\Omega(\Delta) = (1 + 0.0028388\Delta^2)^{-1}$ in the aggregate production function. We compute the parameter λ so as to match our function φ with Ω over the range of temperature increases up to $3.5°C$.[26] This leads to a value of $\lambda = 0.3501$ in (33), and with this choice of φ function, our model generates effects that are very close to those obtained in Nordhaus's calibration exercises for increases in temperature less than $3.5°C$. In most of our calibrations, the temperature increases remain within this range of values under the optimal environmental regulation, though the implications of temperature increases outside this range influence the structure of optimal policy.[27] Figure 1 compares Nordhaus's Ω function with our φ function.

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[26] More precisely, we minimize the L^2-norm of the difference $\Omega - \varphi$ on the interval $[0, 3.5]$.

[27] Here, we note that in Nordhaus's quantitative exercises, the damage from temperature increases beyond $3.5°C$ still remains quite modest. We do not find this feature, which is based on out-of-sample extrapolation, plausible, and in our specification, where environmental quality directly affects utility, the cost of increases above $3.5°$, particularly those close to $9.2°$, are substantial. It seems more plausible to us that increases in temperature close to $9.2°C$ would have disastrous consequences for utility (as well as production).
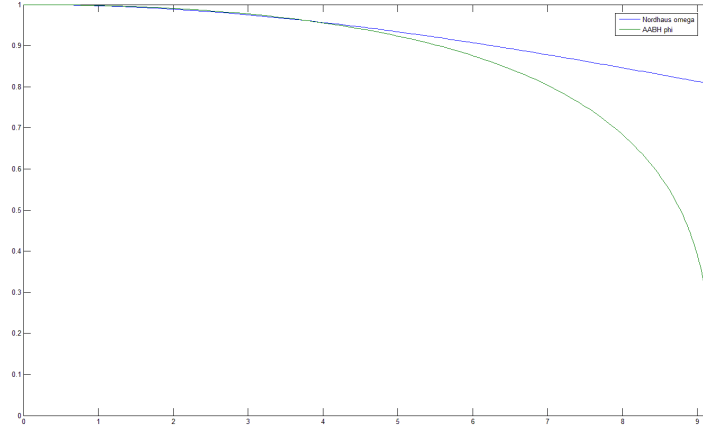
Figure 1: The impact of the environmental stock in our calibration and in Nordhaus's DICE model.

4.3 Results

The debate between Stern and Nordhaus highlighted the importance of the discount rate when determining the optimal environmental policy. Here we consider three different values for the discount rate: the Stern discount rate of 0.0.01 per annum (which we write as $\rho = 0.001$), an intermediate value of 0.01 per annum ($\rho = 0.01$), and the Nordhaus's discount rate of 0.015 per annum ($\rho = 0.015$, which, as in Nordhaus, corresponds to an annual long-run interest rate of about $r = \rho + \sigma g = 5.5\%$).

We start in Figure 2 by looking at the effects of different values of the elasticity of substitution for the baseline discount rate value of $\rho = 0.01$. The different panels show the magnitude of the subsidy to the clean sector, the allocation of scientists to clean technologies, the "carbon" tax, the share of clean inputs in total production, and the increase in temperature. Different curves correspond to different values of ε.

Figure 2B shows that for $\varepsilon = 5$ and $\varepsilon = 10$, optimal policy involves an immediate switch of all research to clean technologies. Remarkably, this can be achieved with subsidies to the clean sector that come to an end very rapidly (Figure 2A). Moreover, carbon taxes are very low in both of these cases (Figure 2C). This is because subsidies are sufficient to redirect technical change to the clean sector before temperature increases significantly; thus creating intertemporal distortions to induce large contemporaneous reductions in emissions is unnecessary.[28] The share of clean inputs in total input production increases steadily in both cases, though with

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[28] If there were restrictions on research subsidies or if such subsidies created other distortions, then the role of carbon taxes would be greater.
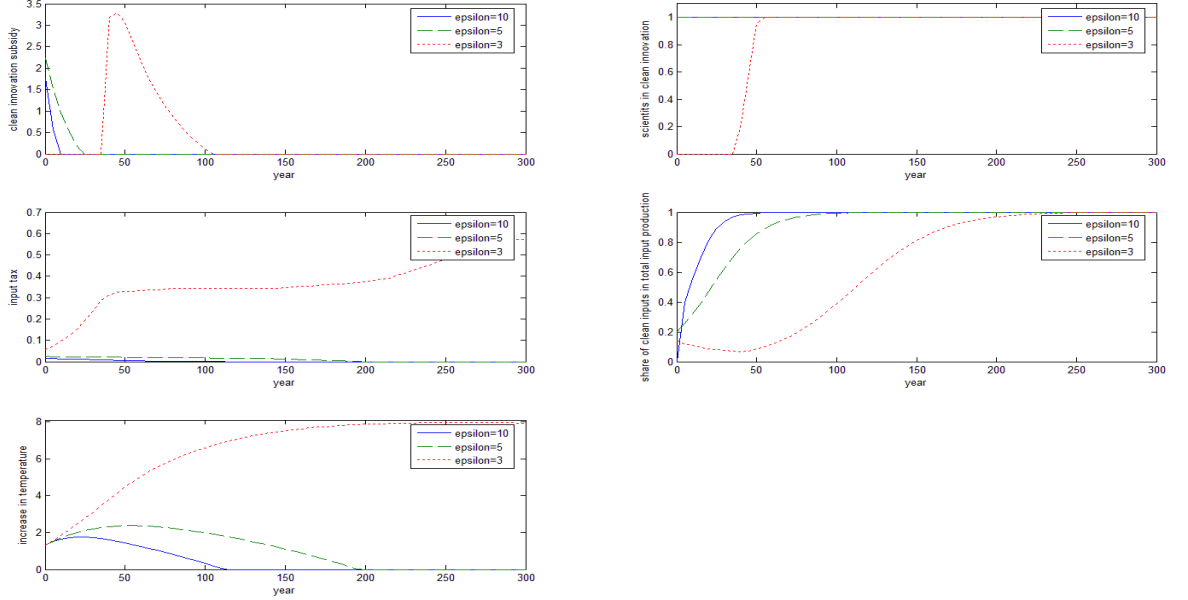
Figure 2: Optimal environmental policy for different values of ε and $\rho = 0.01$

$\varepsilon = 5$, it does not exceed 90% until year 60 (Figure 2D, and see also Table 1). Also noteworthy is the fact that because the share of clean inputs increases only slowly, temperature continues to increase after the implementation of the optimal policy. For example, with $\varepsilon = 5$, temperature increases for another 55 years (see Table 1), though it does not come close to the "disaster" levels. The pattern is different with $\varepsilon = 3$. In this case, the switch to clean technologies is delayed, and thus dirty technologies continue to improve during the first 45 years (Table 1). To compensate for this, however, the carbon tax is much higher than in the other two cases, and it continues to increase during almost the entire 300 years for which we show the results of the simulations. Temperature also increases for almost 300 years. The reason for this pattern is that with $\varepsilon = 3$, productive efficiency requires the use of both clean and dirty inputs, and thus optimal policy delays intervention and the output costs of intervention until later. However, we believe that relatively high values for the elasticity of substitution between clean and dirty inputs are much more plausible, and thus view the patterns with $\varepsilon = 5$ and $\varepsilon = 10$ as more representative.

Table 1 shows some of the key features that emerge from Figure 2, in particular, the first year in which more than 50% of research is allocated to clean technologies, both the first year in which the output of the clean sector exceeds that of the dirty sector and the first in which it becomes 10 times as large as that of the dirty sector, the maximum temperature the earth reaches, the year in which this maximum temperature is reached, and the first year in which

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the temperature is back to the baseline of $\Delta = 0$. Columns 2, 5 and 8 of this table refer to Figure 2. They show, for example, that with $\varepsilon = 10$, under optimal environmental regulation research immediately switches to clean inputs, the production of clean inputs exceeds the production of dirty input in year 10, and becomes the majority of input production by year 30. Maximum temperature is only 1.76°C above the baseline and temperatures start decreasing from year 20 onwards. In contrast, with $\varepsilon = 5$, the expansion of clean technologies is slower, and temperature increases for 55 years and reaches a level of 2.38°C above the baseline. The table also highlights the different results that emerge with a smaller elasticity of $\varepsilon = 3$.

Table 1: **Optimal policy for different values of ε and ρ**

Elasticity of substitution ε	10			5			3		
Discount rate ρ	0.001	0.01	0.015	0.001	0.01	0.015	0.001	0.01	0.015
$\geq 50\%$ clean research (1st year)	0	0	0	0	0	60	0	45	55
$\geq 50\%$ clean inputs (1st year)	10	10	10	25	25	115	40	115	125
$\geq 90\%$ clean inputs (1st year)	30	30	30	60	60	155	100	170	180
max Δ (in °C)	1.75	1.76	1.76	2.33	2.38	8.20	3.27	7.96	8.56
year when Δ is maximal	20	20	20	50	55	195	115	260	285
1st year when $\Delta = 0$	115	115	115	195	115	> 395	200	> 395	> 395

Figure 3 shows the implications of different discount rates focusing on the high value of the elasticity of substitution, $\varepsilon = 10$. The structure of optimal policy and the resulting allocations, including the path of temperature, are very similar for different values of the discount rate. The only difference is that the carbon tax is somewhat lower with higher discount rates, since sacrificing current output for future environmental quality is less attractive. Nevertheless, with all three values of the discount rate, the carbon tax always remains small and declines rapidly over time. These results are also shown in columns 1, 2 and 3 of Table 1. Overall, when the elasticity of substitution between clean and dirty inputs is sufficiently high so that directed technical change effects are pronounced, whether one uses the Nordhaus, the Stern or an intermediate discount rate has little bearing on the structure of optimal environmental regulation.

The value of the discount rate matters more when the elasticity of substitution is lower. Figure 4 and Table 1 show that with the Stern discount rate of $\rho = 0.001$, an immediate switch to clean technologies is optimal even with $\varepsilon = 3$, though with other discount rates, the switch is delayed. Figures 2 and 4 show that a delayed switch in research is compensated by higher carbon taxes.
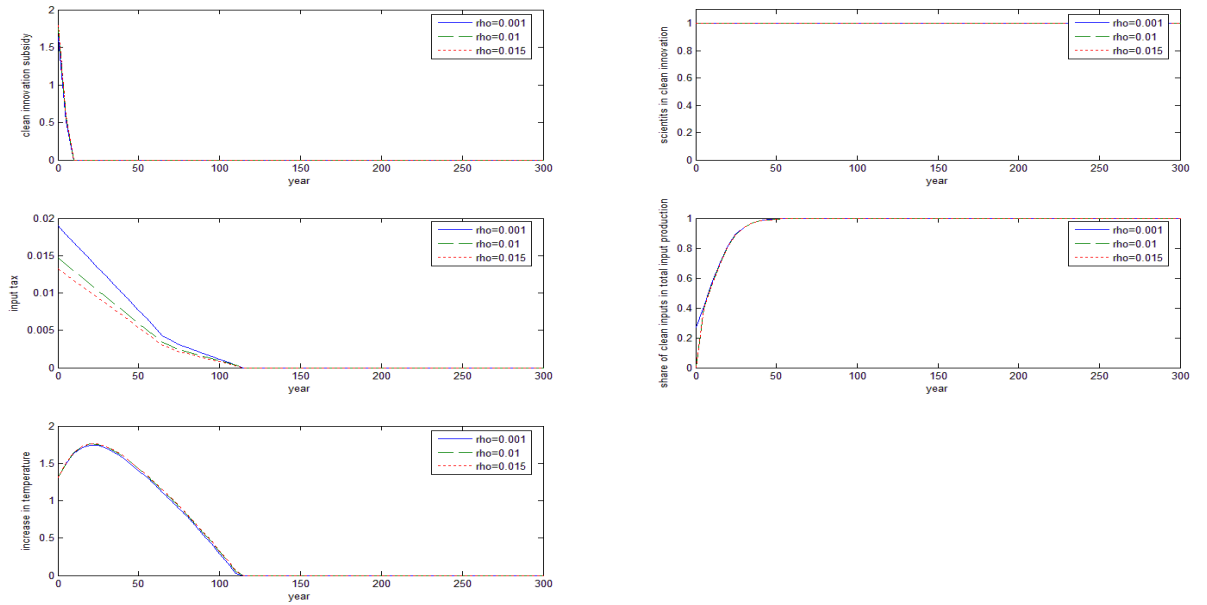
Figure 3: Optimal environmental policy for $\varepsilon = 10$ and various values of ρ.
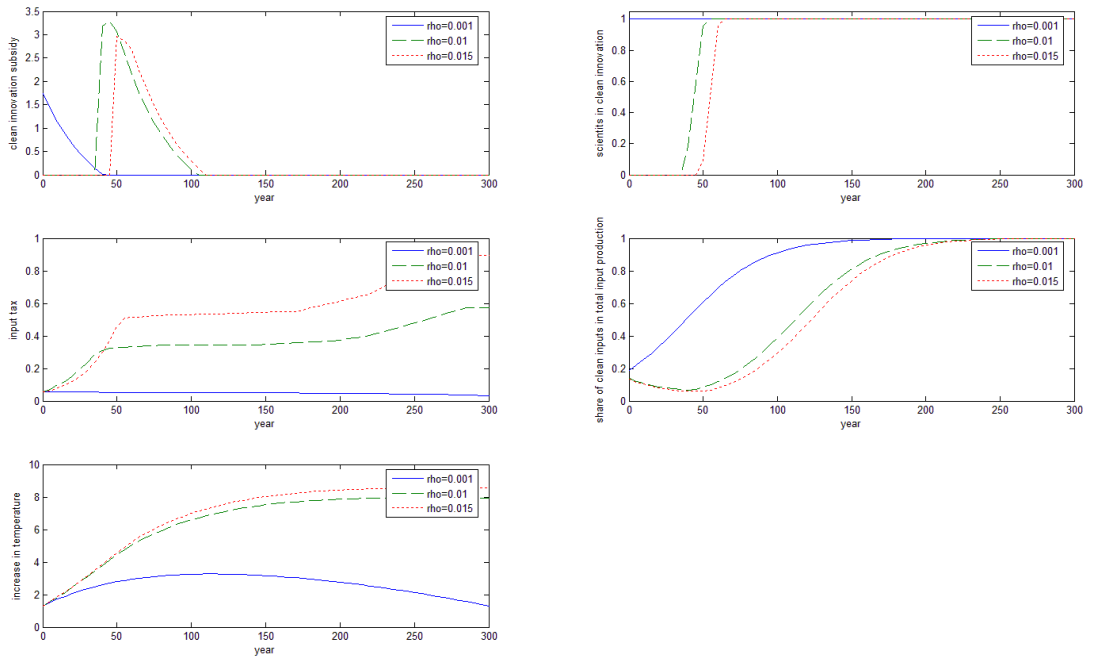

Figure 4: Optimal environmental policy for $\varepsilon = 3$ and various values of ρ.

Corollary 1 above related the costs of delayed intervention to the number of additional periods of slow growth that this delay would induce. Table 2 instead shows the welfare costs of delaying implementation of the optimal policy, that is, maintaining the clean innovation subsidy and the carbon tax at zero for a while before implementing the optimal policy, may be more informative.[29] Welfare costs are measured as the equivalent percentage reduction in per period consumption relative to the allocation with immediate intervention (we assume that when intervention starts, it takes the optimal form). The numbers in the table correspond to different values of the elasticity of substitution ε (with the initial value A_{ct-1} and A_{dt-1} being changed accordingly), the discount rate ρ and amounts of delay. The table shows that delay costs can be substantial. For example, with $\varepsilon = 10$ and $\rho = 0.01$, a delay of 10 years is equivalent to 5.99% decline in consumption. The cost of delay increases with the duration of the delay and the elasticity of substitution between the two inputs,[30] and decreases with the discount rate (since the benefit from delaying intervention, which is greater consumption early on, increases with the discount rate). Note that the variations in the delay cost are of the same order of magnitude when one varies ε or ρ; this suggests that the elasticity of substitution between clean and dirty input is as important a consideration as the discount rate when assessing the costs of delaying intervention.

Table 2: Welfare costs of delayed intervention as a function of the elasticity of substitution and the discount rate

(Percentage reductions in consumption relative to immediate intervention)

Elasticity of substitution ε	10			5			3		
Discount rate ρ	0.001	0.01	0.015	0.001	0.01	0.015	0.001	0.01	0.015
delay = 10 years	9.00	5.99	2.31	5.64	0.63	0.05	3.10	0.05	0.04
delay = 20 years	14.62	8.31	2.36	9.49	0.71	0.11	5.51	0.13	0.11
delay = 30 years	18.55	8.88	2.43	12.5	0.82	0.21	7.81	0.26	0.21

Finally, we briefly discuss the welfare costs of relying solely on a carbon (input) tax instead of combining it with the subsidy to clean research (the optimal environmental policy derived in Proposition 5). Relying only on this single instrument necessitates very high tax levels, which distort and reduce production. Consequently, Table 3 shows that there can be significant welfare losses relative to the case in which the full optimal environmental regulation is implemented. The welfare loss tends to be smaller when the elasticity of substitution is high because in this case a relatively small carbon tax is sufficient to redirect R&D to clean technologies, but the pattern is non-monotone because changes in this elasticity also affect the

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[29]To isolate the cost of delay, we maintain the optimal subsidy on machines, which corrects for the standard monopoly distortions, during the period of delay.

[30]The intuition for this result is that when the two inputs are close substitutes, further advances in dirty technologies that occur before the optimal policy is implemented do not contribute much to aggregate ouput once clean technologies have become sufficiently more advanced than dirty technologies.

optimal date of switching to clean innovation. The welfare loss also tends to increase with the discount rate, as a higher discount rate increases the weight on earlier dates where a significantly higher carbon tax is imposed under the suboptimal policy (but for the same reasons, the pattern is again non-monotone).

Table 3: Welfare costs of relying solely on carbon tax as a function of the elasticity of substitution and the discount rate

(Percentage reductions in consumption relative to the optimal policy)

Elasticity of substitution ε	10			5			3		
Discount rate ρ	0.001	0.01	0.015	0.001	0.01	0.015	0.001	0.01	0.015
Welfare cost	0.92	1.33	1.55	1.38	2.10	4.35	1.57	3.49	2.84

5 Directed technical change with exhaustible resources

Polluting activities often make use of exhaustible resources such as oil or coal. In this section we analyze a variant of our basic model where dirty input production uses an exhaustible resource. Exhaustibility of polluting resources may help prevent an environmental disaster because it increases the cost of using the dirty input even without policy intervention. In particular, we will show that the presence of an exhaustible resource can prevent a disaster in the laissez-faire equilibrium when the two inputs are sufficiently substitutable.

More formally, we amend our basic model by assuming that the dirty input is now produced according to the technology:

$$Y_{dt} = R_t^{\alpha_2} L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha_1} x_{dit}^{\alpha_1} di, \tag{34}$$

where R_t is the flow consumption of the exhaustible resource at time t, and $\alpha_1 + \alpha_2 = \alpha$ (so the labor share in the production of intermediary input remains $1 - \alpha$). The basic model is then simply a subcase of this extended model with $\alpha_2 = 0$. We assume that the exhaustible resource can be directly extracted at a cost $c(Q_t)$ in terms of units of final good, where Q_t denotes the resource stock at date t, and c is a decreasing function of Q. This specification also makes the exhaustible resource subject to the "tragedy of the commons": the price of the exhaustible resource does not reflect its scarcity value. This assumption is adopted to simplify the exposition. When we characterize optimal environmental regulation below, this scarcity value will feature in the social planner's optimization problem. Given the amount of extraction, the evolution of the exhaustible resource is given by the difference equation:

$$Q_{t+1} = Q_t - R_t \tag{35}$$

In the first subsection we analyze the laissez-faire equilibrium of this augmented model, and in the second subsection we derive the socially optimal environmental regulation and compare its structure to environmental regulation without exhaustible resources.

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5.1 The laissez-faire equilibrium

The structure of equilibrium remains mostly unchanged, particularly the equilibrium demands for the two types of inputs, and the production of clean inputs. The equilibrium demands for machines used in the dirty sector become:

$$x_{dit} = \left(\frac{(\alpha_1)^2 p_{dt} R_t^{\alpha_2} L_{dt}^{1-\alpha}}{\psi} \right)^{\frac{1}{1-\alpha_1}} A_{dit}.$$

Profits of monopolists and expected profits from research in the dirty sector are also modified accordingly.

The relative profitabilities of innovation in the clean or in the dirty sector reflect the same three effects as before: the direct productivity effect, the price effect and the market size effect identified above. The only change relative to the baseline model is that the resource stock now affects the magnitude of the price and market size effects. In particular, as the resource stock declines, the effective productivity of the dirty input also declines and its price increases. We show in the Supplementary Appendix that the price ratio of dirty to clean input is now given by:

$$\frac{p_{ct}}{p_{dt}} = \frac{\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2} A_{dt}^{1-\alpha_1}}{c(Q_t)^{\alpha_2} \alpha^{2\alpha} A_{ct}^{1-\alpha}}, \tag{36}$$

and the relative employment in the two sectors becomes

$$\frac{L_{ct}}{L_{dt}} = \left(\frac{c(Q_t)^{\alpha_2} \alpha^{2\alpha}}{\psi^{\alpha_2} \alpha_1^{2\alpha_1} (\alpha_2)^{\alpha_2}} \right)^{(\varepsilon-1)} \frac{A_{ct}^{-\varphi}}{A_{dt}^{-\varphi_1}} \tag{37}$$

(where $\varphi_1 \equiv (1-\alpha_1)(1-\varepsilon)$) so that the share of labor allocated to the dirty sector decreases with the extraction cost only when the two inputs are substitutes. Using these expressions, we obtain the ratio of expected profits from research in the two sectors, which will determine the direction of equilibrium research, as (see the Supplementary Appendix):

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \kappa \frac{\eta_c c(Q_t)^{\alpha_2(\varepsilon-1)}}{\eta_d} \frac{(1+\gamma\eta_c s_{ct})^{-\varphi-1}}{(1+\gamma\eta_d s_{dt})^{-\varphi_1-1}} \frac{A_{ct-1}^{-\varphi}}{A_{dt-1}^{-\varphi_1}} \tag{38}$$

where $\kappa \equiv \frac{(1-\alpha)\alpha}{(1-\alpha_1)\alpha_1^{(1+\alpha_2-\alpha_1)/(1-\alpha_1)}} \left(\frac{\alpha^{2\alpha}}{\psi^{\alpha_2} \alpha_1^{2\alpha_1} \alpha_2^{\alpha_2}} \right)^{(\varepsilon-1)}$.

The main difference from the profit ratio in the baseline model is the term $c(Q_t)^{\alpha_2(\varepsilon-1)}$ in the right hand side of (38). This new term, together with the assumption that $c(Q_t)$ is decreasing in Q_t, implies the following proposition (proof in the text):

Proposition 6 *As the resource stock gets depleted over time, innovation incentives in the clean sector increase when the two inputs are substitutes ($\varepsilon > 1$) but decrease when the two inputs are complements ($\varepsilon < 1$).*

Intuitively, resource depletion increases the relative price of the dirty input, and thus reduces the market for the dirty input. In the substitutability case this encourages innovation in the clean sector. In fact, in the laissez-faire equilibrium, innovation will ultimately occur in the clean sector only (either because the extraction cost increases sufficiently rapidly, inducing all innovation to be directed at clean machines, or because the resource stock gets fully depleted in finite time). In this case, the dirty input is not essential to final production and therefore, provided that initial environmental quality is sufficiently high, an environmental disaster can be avoided while the economy achieves positive long-run growth at the rate $\gamma\eta_c$. In contrast, in the complementarity case the increase in the relative price of the dirty input encourages innovation in the dirty sector. In addition, in this case the dirty input remains essential for final production. Thus positive growth requires an ever increasing rate of extraction, which in turn leads to the exhaustion of the natural resource in finite time. This in turn prevents positive long-run growth. This discussion establishes the following proposition (see the Supplementary Appendix for a proof).

Proposition 7   *1. When the two inputs are substitutes ($\varepsilon > 1$), innovation in the long-run will be directed towards the clean sector only and the economy will grow at rate $\gamma\eta_c$. Provided that $\overline{S}$ is sufficiently high, an environmental disaster is avoided under laissez-faire.*

   2. When the two inputs are complements ($\varepsilon < 1$), economic growth is not sustainable in the long-run.

The most important result in this proposition is that when an exhaustible resource is necessary for production of the dirty input, the market generates incentives for research to be directed towards the clean sector, and these market-generated incentives may be sufficient for the prevention of environmental disaster. This contrasts with the result that an environmental disaster was unavoidable under laissez-faire without the exhaustible resource. Therefore, in practice to the extent that the increasing price of oil and the higher costs of oil extraction will create a natural move away from dirty inputs (and other activities creating environmental degradation), the implications of growth are not as damaging to the environment as our baseline model suggests. Nevertheless, because of the environmental and the knowledge externalities, even though an environmental disaster can be averted, the equilibrium is still Pareto suboptimal (even if it avoids an environmental disaster); the next subsection discusses the structure of optimal environmental regulation with an exhaustible resource.

5.2   Optimal environmental regulation with exhaustible resources

We now briefly discuss the structure of optimal policy with exhaustible resource. The social planner again maximizes (1), now subject to subject to the constraints: (4), (7), (8), (9), (15),

(34) (which replaces (5)), the resource constraint $Q_t \geq 0$,

$$C_t = Y_t - \psi \left(X_{ct} + X_{dt} \right) - c(Q_t)R_t, \tag{39}$$

and the law of motion of the resource stock given by (35).

As in Section 3, the social planner will correct for the monopoly distortion by subsidizing the use of machines and will again introduce a wedge between the shadow price of the dirty input and its marginal product in the production of the final good, equivalent to a tax $\tau_t = \omega_{t+1}\xi / \left(\lambda_t p_{dt} \right)$ on dirty input production. In addition, as noted above, we have assumed that the private cost of extraction is $c\left(Q \right)$ and does not incorporate the scarcity value of the exhaustible resource. The social planner will naturally recognize this scarcity value and will use a "resource tax" to create a wedge between the cost of extraction and the social value of the exhaustible resource. We can establish:

Proposition 8 *The social planner can implement the social optimum through a "carbon" tax (i.e., a tax on the use of the dirty input), a subsidy to clean research, a subsidy on the use of all machines and a resource tax (all proceeds from taxes/subsidies being redistributed/financed lump-sum). The resource tax must be maintained forever.*

Proof. See the Supplementary Appendix. ■

In the Supplementary Appendix, we also report a calibration exercise for the model with exhaustible resources. This exercise shows that the presence of an exhaustible resource does not change the structure of the optimal policy. As in the baseline case, higher discount rates or smaller elasticities of substitution push towards delaying the switch to clean innovation. The presence of an exhaustible resource appears not to have an unambiguous effect on the date of the switch to clean innovation (for instance, when $\rho = 0.015$, switch occurs immediately for $\varepsilon = 5$ whereas it did not without the exhaustible resource; in contrast, when $\rho = 0.015$ and $\varepsilon = 3$, the switch occurs 15 years later in the exhaustible resource case than in the non-exhaustible resource case). With exhaustible resources, the capability of the economy to grow using the dirty input diminishes as the resource is depleted and its price increases; counteracting this, the increase in the price of the resource already lowers the production of the dirty input and slows down environmental degradation. Finally, the calibration exercise in this case also shows that the increasing price of the resource implies that lower subsidies are sufficient to induce a switch to clean technologies, and for a given switching time to clean innovation, production switches faster towards the clean sector, since dirty inputs are still becoming more expensive because of resource depletion; as a consequence, there is less need for a high carbon tax in this case.

6   Global environmental externalities and policy coordination

We now study a two-country extension of the baseline model in order to investigate the implications of environmental and knowledge externalities on the need for global policy coordination. We ask whether environmental regulation in one set of countries (the more advanced North) can be sufficient to avoid environmental disasters and how this conclusion is affected by the presence or absence of international trade.

The world economy consists of two "countries," North and South, and we index all variables (except the quality of the environment, which is common to all countries) with a superscript $k \in \{N, S\}$. We think of the North as the technological leader and of the South as representing countries behind the world technology frontier, benefiting from technological spillovers from the North.

The environmental quality S enters the utility of households in both countries in the same way as in (1). Most importantly, environmental externalities are global, thus the law of motion of the quality of the environment is a function of the total dirty input production in the two countries. In particular, equation (9) is now replaced with

$$S_{t+1} = -\xi \left(Y_{dt}^N + Y_{dt}^S \right) + (1 + \delta) S_t, \tag{40}$$

again with S_{t+1} taking the value 0 or $\overline{S}$ at the boundaries of the set $\left(0, \overline{S}\right)$.

The North is identical to the economy described in the baseline model of Section 2. To simplify the discussion, we assume that the South has exactly the same production structure (the same technology for final production, (4), the same technology for the production of dirty and clean inputs, (5), and the same marginal cost of producing machines, $\psi \equiv \alpha^2$ units of the final good), and also has $s = 1$ scientists, but Southern scientists work only on imitating already developed technologies in the North (e.g., Grossman and Helpman, 1991). This assumption captures the intuitive notion that the South is technologically less advanced and adopts the innovations developed in the North (perhaps with some delay).[31]

Southern scientists direct their (imitation) research towards dirty or clean machines in the same way that Northern scientists did. In particular, once they choose a particular sector, as with the Northern scientists, they are randomly allocated to a single machine in the sector of their choice, and in sector $j \in \{c, d\}$ they have a probability $\kappa_j \in (0, 1)$ of successfully imitating this machine (again without congestion, so there is at most one scientist per machine). If they are successful, they will have imitated the frontier machine in the North, thus for machine i in sector $j \in \{c, d\}$, they will have access to the machine of quality A_{jit}^N. Moreover, they will be given a one-period monopoly right over this successfully imitated machine (for use in the

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[31] Naturally, we could allow scientists in the South to also choose whether to work towards original innovations or to imitate Northern technologies. We do not introduce this choice to simplify the exposition and focus on the effects of Northern technological advances on technology adoption decisions in the South, which is a crucial global interaction that has not been highlighted by previous analyses.

South only). For a machine type for which there has not been a successful innovation, monopoly rights are allocated to a Southern entrepreneur drawn at random, and this entrepreneur will use the technology from the previous period A_{jit-1}^S. Therefore, the structure of technological advances in the South is very similar to that in the North, with the only difference being that "success" brings a machine of quality equal to the frontier quality in the North rather than an incremental improvement over the current machine quality in the South.

Given these assumptions, when s_{jt}^S scientists in the South undertake research in sector $j \in \{c, d\}$ at time t, the law of motion of average technology of sector $j \in \{c, d\}$ in the South evolves according to:

$$A_{jt}^S = \kappa_j s_{jt}^S A_{jt}^N + \left(1 - \kappa_j s_{jt}^S\right) A_{jt-1}^S. \tag{41}$$

This equation, together with the law of motion of productivity in the North, (15), gives the evolution of productivity levels in the two sectors in the North and the South.

In the remainder of this section, we investigate this global economy without trade in inputs, and then turn to the implications of international trade for the environment and the need for global policy coordination.

6.1 Preventing environmental disaster without global policy coordination

Suppose to start with that the North follows an environmental policy (sequences of taxes/subsidies) denoted by $\left\{\tau_t^N, q_t^N\right\}$, where τ_t^N is a carbon tax at time t and q_t^N is the subsidy on clean profits (both of those applied only in the North). There is no global policy coordination, so that these policies do not apply to producers in the South. Instead, to capture the relevant situation in which environmental regulations are more lax in developing countries, we assume that the Southern firms operate under laissez-faire.

Since in the South all machines are also supplied by monopolists, the static equilibrium in both the South and the North, given technology levels, is the same as that given by our analysis in subsection 2.2. In addition, as with the researchers in the North, the decision of Southern scientists to direct their (imitation) activity towards dirty or clean inputs will be determined by the relative profitability of having access to monopoly rights (for one period) in the two sectors. The expected profits from these monopoly rights in the South are denoted by Π_{jt}^S for sector $j \in \{c, d\}$ at time t and are given by an equation very similar to (16), adapted only to the different innovation possibilities frontier facing Southern scientists:

$$\Pi_{jt}^S = \kappa_j \left(1 - \alpha\right) \alpha (p_{jt}^S)^{\frac{1}{1-\alpha}} L_{jt}^S A_{jt}^N.$$

The crucial difference here from (16) is that successful imitation will lead to the imitation of the currently available technology in the North, which explains the term A_{jt}^N at the end. Consequently, the profitability of imitating clean relative to dirty technologies in the South is

determined by the ratio

$$\frac{\Pi_{ct}^S}{\Pi_{dt}^S} = \frac{\kappa_c (p_{ct}^S)^{\frac{1}{1-\alpha}} L_{ct}^S A_{ct}^N}{\kappa_d (p_{dt}^S)^{\frac{1}{1-\alpha}} L_{dt}^S A_{dt}^N} = \frac{\kappa_c \left(A_{ct}^S\right)^{-\varphi-1} A_{ct}^N}{\kappa_d \left(A_{dt}^S\right)^{-\varphi-1} A_{dt}^N}. \tag{42}$$

If this ratio is greater than 1, then imitation will be directed to the clean sector only; and if it is smaller than one, imitation will be directed towards the dirty sector only (finally, if it is equal to 1 imitation can occur in both sectors simultaneously).[32]

Equation (42) shows that the relative profitability of imitation of different types of machines is shaped by the same market size and price effects that determined innovation in the North. However, there is also a different type of *knowledge externality*, reflected by the term (A_{ct}^N/A_{dt}^N) on the right hand side of (42), now resulting from the innovation decisions in the North. Intuitively, profits from imitation are proportional to the target productivity level, which here is the technology in the North, and thus, this knowledge externality favors imitation in the sector that is relatively more advanced in the North. In particular if the quality of clean machines becomes much higher than the quality of dirty machines in the North, this will create an incentive for the South to imitate in the clean sector. This last observation is important for understanding why, under certain circumstances, environmental disaster can be avoided without global policy coordination.

A key implication of (42) is that if $\varepsilon > 1$ and the North devotes all its research effort to innovation on clean machines, firms in the South will also switch to clean imitation activities in the long run, and A_{ct}^S will grow at the same rate as A_{ct}^N, i.e., at rate $\gamma\eta_c$.[33] In particular, suppose that indeed the North undertakes an environmental policy that redirects all innovation towards the clean sector, but there is no global policy coordination, so that the South remains under laissez-faire. The equilibrium production of dirty inputs in the South is then given by the equivalent of (21) from our analysis in subsection 2.3, and thus:

$$Y_{dt}^S = \frac{\left(A_{ct}^S\right)^{\varphi+\alpha} A_{dt}^S}{\left(\left(A_{ct}^S\right)^\varphi + \left(A_{dt}^S\right)^\varphi\right)^{\frac{\alpha+\varphi}{\varphi}}}. \tag{43}$$

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[32] In terms of time-$t-1$ productivity levels, this can be written as

$$\frac{\Pi_{ct}^S}{\Pi_{dt}^S} = \frac{\kappa_c \left(\left(1 - \kappa_c s_{ct}^S\right) A_{ct-1}^S + \kappa_c s_{ct}^S A_{ct}^N\right)^{-\varphi-1} A_{ct}^N}{\kappa_d \left(\left(1 - \kappa_d s_{dt}^S\right) A_{dt-1}^S + \kappa_d s_{dt}^S A_{dt}^N\right)^{-\varphi-1} A_{dt}^N}.$$

[33] To see this, note that: (i) A_{ct}^S cannot grow faster than A_{ct}^N since the South cannot do better than imitating the North; (ii) in the long run, A_{ct}^S will in fact grow at the same rate as A_{ct}^N; to obtain a contradiction suppose it grew more slowly; then (42), together with the fact that A_{dt}^N and therefore A_{dt}^S remain bounded as the North innovates in the clean sector only, would imply that Π_{ct}^S/Π_{dt}^S goes to infinity as $t \to \infty$; but then imitation in the South would end up occurring in the clean sector only in finite time; this in turn implies that A_{ct}^S and A_{ct}^N must grow at the same rate in the long-run, yielding a contradiction; (iii) the fact that A_{ct}^S and A_{ct}^N grow at the same rate in the long-run in turn implies that Π_{ct}^S/Π_{dt}^S must exceed 1, and consequently, in finite time all imitation in the South will switch to the clean sector.

This expression highlights that in the long run, output of the dirty input in the South will be approximately equal to $Y_{dt}^S \approx \left(A_{ct}^S/A_{dt}^S\right)^{\varphi+\alpha}$, which does not grow if $\varphi + \alpha \leq 0$ (that is if $\varepsilon \geq 1/(1-\alpha)$), and otherwise grows at rate $(1+\gamma\eta_c)^{\varphi+\alpha} - 1$.

Given this observation, the main insights here parallel those in subsection 2.3. In particular, as in subsection 2.3, when $\varepsilon < 1/(1-\alpha)$, the global production of dirty input will grow to infinity (since production over the dirty input in the South grows steadily over time). Consequently, an environmental disaster is unavoidable. In contrast, when $\varepsilon \geq 1/(1-\alpha)$, i.e. when the two inputs are strong substitutes, environmental disaster can be avoided if $\overline{S}$ is sufficiently large. As all innovation in the North is directed to clean inputs, in this case, the production of dirty inputs in the North stops growing. The analysis in this subsection shows that Southern scientists will ultimately imitate clean technologies in the North. Moreover, equation (43) then implies that the production of dirty inputs in the South will also stop growing. Thus with a sufficiently high level of initial environmental quality, a global environmental disaster can be prevented even without global policy coordination. The role of directed technical change in this result is clear, since it is directed technical change that allows the North to redirect innovation towards clean technologies, and it is also the ability of Southern scientists to redirect their imitation activity towards clean technologies that enables Southern firms to switch to clean frontier technologies once these have become sufficiently advanced.

We summarize this result in the next proposition (proof omitted):

Proposition 9 *In the two-country case when $\varepsilon \geq 1/(1-\alpha)$, a policy $\left\{\tau_t^N, q_t^N\right\}$ in the North that would direct innovation towards clean technologies only, is sufficient to avoid a disaster without taxation in the South provided that $\overline{S}$ is sufficiently high. If $1 < \varepsilon < 1/(1-\alpha)$, then such a policy cannot prevent a disaster.*

Proposition 9 shows that a global environmental disaster can be avoided without global policy coordination. Clearly, optimal environmental regulation will be more complex in this case and will involve global policy coordination. We characterize the structure of optimal environmental regulation in the Supplementary Appendix (from the point of view of a global social planner interested in maximizing the sum of the utilities of households in both countries given by (1)). The following proposition summarizes the results:

Proposition 10 *The global social optimum can be implemented through the combination of research subsidies, carbon taxes both in the North and in the South, and a subsidy to machine consumers (against the standard monopoly distortion). If $\varepsilon > 1/(1-\alpha)$, then all optimal environmental regulation (taxes) are temporary.*

6.2 International trade and pollution havens

The argument that knowledge spillovers should induce the South to follow the North in its switch to clean technologies may be counteracted by international trade, which would create a greater need for global policy coordination in environmental policies. In particular, free international trade between the North and the South, combined with environmental regulation in the North, creates a comparative advantage in dirty input production in the South. Loosely speaking, in this case, the South may become a "pollution haven". This in turn may make an environmental disaster more likely, as we illustrate below.

In the presence of international trade in inputs between the North and the South, our model has a Ricardian structure, and the country with the higher relative productivity in dirty inputs, given technology and policies, will specialize in the production of these inputs. To highlight the main issues, let us focus on the case where $\varepsilon > 1$. As in subsection 6.1, we assume (i) that the North follows an environmental policy $\left\{\tau_t^N, q_t^N\right\}$ that redirects innovation towards the clean technologies (here, as before, τ_t^N is a tax on the production of the dirty input in the North, and q_t^N is subsidy on profits in the clean sector in the North); (ii) that the South remains under pure laissez-faire. Free trade in inputs implies that the post-tax price for each input ($j = c, d$) must be equalized in the North and the South, so that:

$$p_{ct}^N = p_{ct}^S \text{ and } (1 + \tau_t^N)p_{dt}^N = p_{dt}^S. \tag{44}$$

The ratio of marginal products of labor in sectors c and d in country k is then equal to

$$\frac{MPL_c^k}{MPL_d^k} = \left(\frac{p_c^k}{p_d^k}\right)^{\frac{1}{1-\alpha}} \frac{A_c^k}{A_d^k}, \tag{45}$$

and the country with a higher ratio in (45) will have a comparative advantage in the clean sector; the other country will have a comparative advantage in the dirty sector. This implies that the South will have a comparative advantage in dirty inputs at time t if

$$\left(1 + \tau_t^N\right)^{\frac{1}{1-\alpha}} \frac{A_{ct}^N}{A_{dt}^N} > \frac{A_{ct}^S}{A_{dt}^S}. \tag{46}$$

This expression encapsulates the "pollution haven hypothesis". It implies that a higher carbon tax rate τ_t^N and a higher relative quality of clean machines in the North create a comparative advantage for the South in dirty input production. In this case, environmental policy in the North alone, without global policy coordination, may be insufficient to avoid an environmental disaster because it may indirectly increase dirty production by inducing these activities to move to the South. A full analysis of this case is beyond the scope of the current paper. The Supplementary Appendix provides a simple example where a policy that avoids environmental disaster under autarky fails to do so under free trade (without global policy coordination or environmental regulations in the South).

7 Conclusion

In this paper we introduced endogenous directed technical change in a growth model with environmental constraints and limited resources. We characterized the structure of equilibria and the dynamic tax/subsidy policies that achieve sustainable growth or maximize intertemporal welfare. Both the long-run properties of the equilibrium and optimal policies (or the necessary policies to avoid environmental disaster) are related to the degree of substitutability between clean and dirty inputs, to whether dirty input production uses exhaustible resources, and to initial environmental and resource stocks.

The main implications of factoring in the importance of directed technical change are as follows: (i) when the inputs are sufficiently substitutable, sustainable long-run growth can be achieved using temporary policy intervention (e.g., a temporary research subsidy to the clean sector), and need not involve long-run distortions; (ii) optimal policy involves both "carbon taxes" and research subsidies, so that excessive use of carbon taxes can be avoided; (iii) delay in intervention is costly: the sooner and the stronger is the policy response, the shorter is the slow growth transition phase; (iv) the use of an exhaustible resource in dirty input production helps the switch to clean innovation under laissez-faire when the two inputs are substitutes. Thus the response of technology to policy leads to a more optimistic scenario than that emerges from models of with exogenous technology; in particular, environmental problems can be solved with only temporary intervention and without causing major long-run distortions. However, directed technical change also calls for immediate and decisive action in contrast to the implications of several exogenous technology models used in previous economic analyses.

A simple quantitative evaluation suggests that, provided that elasticity of substitution between clean and dirty inputs is sufficiently high, optimal environmental regulation should involve an immediate switch of R&D resources to clean technology, followed by essentially all production switching to clean inputs. This conclusion appears robust to the range of discount rates used in the Stern report and in Nordhaus's work (which lead to very different policy conclusions in models with exogenous technology). Interestingly, in most cases, optimal environmental regulation involves small carbon taxes because research subsidies are able to redirect innovation to clean technologies before there is more extensive environmental damage.

Our paper is a first step towards a comprehensive framework that can be used for theoretical and quantitative analysis of environmental regulation with endogenous technology. Several directions of future research appear fruitful. First, it would be useful to develop a more detailed multi-country model with endogenous technology and environmental constraints, which can be used to discuss issues of global policy coordination and the degree to which international trade should be linked to environmental policies. Second, an interesting direction is to incorporate "environmental risk" into this framework, for example, because of the ex ante uncertainty

on the regeneration rate, δ, or on future costs of environmental damage. Another line of important future research would be to exploit macroeconomic and microeconomic (firm- and industry-level) data to estimate the relevant elasticity of substitution between clean and dirty inputs.

Appendix A: Equilibrium allocations of scientists

In this Appendix, we characterize the equilibrium allocation(s) of innovation effort across the two sectors and provide a proof of Lemma 1. Recall from (17) that

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d} \left(\frac{1 + \gamma\eta_c s_{ct}}{1 + \gamma\eta_d s_{dt}}\right)^{-\varphi-1} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi},$$

where $\varphi \equiv (1-\alpha)(1-\varepsilon)$ and $s_{dt} = 1 - s_{ct}$. Define:

$$f(s) \equiv \frac{\eta_c}{\eta_d} \left(\frac{1 + \gamma\eta_c s}{1 + \gamma\eta_d(1-s)}\right)^{-\varphi-1} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi},$$

where $s = s_c = 1 - s_d \in [0,1]$. Clearly, if $f(1) > 1$, then $s = 1$ is an equilibrium; if $f(0) < 1$, then $s = 0$ is an equilibrium; and finally if $f(s) = 1$ for some $s \in (0,1)$, then s is an equilibrium. Given these observations, we have:

1. If $1 + \varphi > 0$ (or equivalently $\varepsilon < (2-\alpha)/(1-\alpha)$), then $f(s)$ is strictly decreasing in s. Then it immediately follows that: (i) if $f(1) > 1$, then $s = 1$ is the unique equilibrium (we only have a corner solution in that case); (ii) if $f(0) < 1$, then $s = 0$ is the unique equilibrium (again a corner solution); (iii) if $f(0) > 1 > f(1)$, then by continuity there exists a unique $s \in (0,1)$ such that $f(s) = 1$, which is the unique (interior) equilibrium.

2. If $1 + \varphi < 0$ (or equivalently $\varepsilon > (2-\alpha)/(1-\alpha)$), then $f(s)$ is strictly increasing in s. In that case: (i) if $1 < f(0) < f(1)$, then $s = 1$ is the unique equilibrium; (ii) if $f(0) < f(1) < 1$, then $s = 0$ is the unique equilibrium; (iii) if $f(0) < 1 < f(1)$, then there are three equilibria, an interior one $s = s^* \in (0,1)$ where s^* is such that $f(s^*) = 1$, $s = 0$ and $s = 1$.

3. If $1 + \varphi = 0$, then $f(s) \equiv f$ is a constant. If f is greater than 1, then $s = 1$ is the unique equilibrium; if it is less than one, then $s = 0$ is the unique equilibrium.

This characterizes the allocation of scientists and implies the results in Lemma 1. ∎

Appendix B: Proof of Proposition 1

We consider the cases where the two inputs are gross substitutes ($\varepsilon > 1$) and complements ($\varepsilon < 1$) separately.

Case $\varepsilon > 1$: Assumption 1 together with the characterization of equilibrium allocation of scientists in Appendix A implies that initially innovation will occurs in the dirty sector only

($s_{dt} = 1$ and $s_{ct} = 0$). From (15), this widens the gap between clean and dirty technologies and ensures that $s_{dt+1} = 1$ and $s_{ct+1} = 0$, and so on in subsequent periods. This shows that under Assumption 1, the equilibrium is uniquely defined under laissez-faire and involves $s_{dt} = 1$ and $s_{ct} = 0$ for all t.

Case $\varepsilon < 1$: In this case the result follows from the following lemma:

Lemma 2 *When $\varepsilon < 1$, long-run equilibrium innovation will be in both sectors so that the equilibrium share of scientists in the clean sector converges to $s_c = \eta_d/(\eta_c + \eta_d)$.*

Proof. Suppose that at time t innovation occurred in both sectors so that $\Pi_{ct}/\Pi_{dt} = 1$. Then from (17), we have

$$\frac{\Pi_{ct+1}}{\Pi_{dt+1}} = \left(\frac{1 + \gamma\eta_c s_{ct+1}}{1 + \gamma\eta_d s_{dt+1}}\right)^{-\varphi-1} \left(\frac{1 + \gamma\eta_c s_{ct}}{1 + \gamma\eta_d s_{dt}}\right).$$

Innovation will therefore occur in both sectors at time $t+1$ whenever the equilibrium allocation of scientists (s_{ct+1}, s_{dt+1}) at time $t+1$ is such that

$$\frac{1 + \gamma\eta_c s_{ct+1}}{1 + \gamma\eta_d s_{dt+1}} = \left(\frac{1 + \gamma\eta_c s_{ct}}{1 + \gamma\eta_d s_{dt}}\right)^{\frac{1}{\varphi+1}}. \tag{47}$$

This equation defines $s_{ct+1}(= 1 - s_{dt+1})$ as a function of $s_{ct}(= 1 - s_{dt})$. We next claim that this equation has an interior solution $s_{ct+1} \in (0,1)$ when $s_{ct} \in (0,1)$ (i.e., when s_{ct+1} is itself interior). First, note that when $\varphi > 0$ (that is, $\varepsilon < 1$), the function $z(x) = x^{1/(\varphi+1)} - x$ is strictly decreasing for $x < 1$ and strictly increasing for $x > 1$. Therefore, $x = 1$ is the unique positive solution to $z(x) = 0$. Second, note also that the function

$$X(s_{ct}) = \frac{1 + \gamma\eta_c s_{ct}}{1 + \gamma\eta_d s_{dt}} = \frac{1 + \gamma\eta_c s_{ct}}{1 + \gamma\eta_d(1 - s_{ct})},$$

is a one-to-one mapping from $(0,1)$ onto $((1 + \gamma\eta_d)^{-1}, 1 + \gamma\eta_c)$. Finally, it can be verified that whenever $X \in ((1 + \gamma\eta_d)^{-1}, 1 + \gamma\eta_c)$, we also have $X^{1/(\varphi+1)} \in ((1 + \gamma\eta_d)^{-1}, 1 + \gamma\eta_c)$. This, together with (47), implies that if $s_{ct} \in (0,1)$, then $s_{ct+1} = X^{-1}(X(s_{ct})^{1/(\varphi+1)}) \in (0,1)$, proving the claim at the beginning of this paragraph.

From Appendix A, when $\varphi > 0$, the equilibrium allocation of scientists is unique at each t. Thus as $t \to \infty$, this allocation must converge to the unique fixed point of the function $Z(s) = X^{-1} \circ (X(s))^{\frac{1}{\varphi+1}}$, which is

$$s_c = \frac{\eta_d}{\eta_c + \eta_d}.$$

This completes the proof of the lemma. ∎

Now given the characterization of the equilibrium allocations of scientists in Appendix A, under Assumption 1 the equilibrium involves $s_{dt} = 0$ and $s_{ct} = 1$, i.e., innovation occurs

initially in the clean sector only. From (15), A_{ct}/A_{dt} will grow at a rate $\gamma\eta_c$, and in finite time, it will exceed the threshold $(1 + \gamma\eta_c)^{-(\varphi+1)/\varphi} (\eta_c/\eta_d)^{1/\varphi}$. Lemma 2 implies that when this ratio is in the interval $\left((1 + \gamma\eta_c)^{-(\varphi+1)/\varphi} (\eta_c/\eta_d)^{1/\varphi}, (\eta_c/\eta_d)^{1/\varphi} (1 + \gamma\eta_d)^{(\varphi+1)/\varphi} (\eta_c/\eta_d)^{1/\varphi} \right)$, equilibrium innovation occurs in both sectors, i.e., $s_{dt} > 0$ and $s_{ct} > 0$, and from this point onwards, innovation will occur in both sectors and the share of scientists devoted to the clean sector converges to $\eta_d/(\eta_d + \eta_c)$. This completes the proof of Proposition 1. $\blacksquare$

Appendix C: Proof of Proposition 5

Using (27), the shadow values of clean and dirty inputs satisfy

$$\widehat{p}_{ct}^{1-\varepsilon} + (\widehat{p}_{dt}(1 + \tau_t))^{1-\varepsilon} = 1. \tag{48}$$

This, together with (61), yields

$$\widehat{p}_{dt} = \frac{A_{ct}^{1-\alpha}}{\left(A_{ct}^{\varphi}(1 + \tau_t)^{1-\varepsilon} + A_{dt}^{\varphi} \right)^{\frac{1}{1-\varepsilon}}} \text{ and } \widehat{p}_{ct} = \frac{A_{dt}^{1-\alpha}}{\left(A_{ct}^{\varphi}(1 + \tau_t)^{1-\varepsilon} + A_{dt}^{\varphi} \right)^{\frac{1}{1-\varepsilon}}}. \tag{49}$$

Using (7), (25), (62) and (49), we obtain the optimal production of each input at time t as:

$$Y_{ct} = \left(\frac{\alpha}{\psi} \right)^{\frac{\alpha}{1-\alpha}} \frac{(1 + \tau_t)^{\varepsilon} A_{ct} A_{dt}^{\alpha+\varphi}}{\left(A_{dt}^{\varphi} + (1 + \tau_t)^{1-\varepsilon} A_{ct}^{\varphi} \right)^{\frac{\alpha}{\varphi}} \left(A_{ct}^{\varphi} + (1 + \tau_t)^{\varepsilon} A_{dt}^{\varphi} \right)} \tag{50}$$

$$Y_{dt} = \left(\frac{\alpha}{\psi} \right)^{\frac{\alpha}{1-\alpha}} \frac{A_{ct}^{\alpha+\varphi} A_{dt}}{\left(A_{dt}^{\varphi} + (1 + \tau_t)^{1-\varepsilon} A_{ct}^{\varphi} \right)^{\frac{\alpha}{\varphi}} \left(A_{ct}^{\varphi} + (1 + \tau_t)^{\varepsilon} A_{dt}^{\varphi} \right)} \tag{51}$$

so that the production of dirty input is decreasing in τ_t. Moreover, clearly as $\tau_t \to \infty$, we have $Y_{dt} \to 0$.

We next characterize the behavior of this tax rate and the research subsidy, q_t, in the three cases separately covered in the proposition. Recall that to avoid an environmental disaster, the optimal policy must always ensure that Y_{dt} remained bounded, in particular, $Y_{dt} \leq (1 + \delta) \overline{S}/\xi$.

Substitutability case: Assume $\varepsilon > 1$. The proof consists of six parts: (1) We show that, for a discount rate ρ sufficiently low, the optimal allocation cannot feature a bounded Y_{ct} thus Y_{ct} must become unbounded as t goes to infinity; (2) We show that this implies that A_{ct} must tend towards infinity (3) We show that if the optimal allocation involves Y_{ct} unbounded (i.e $\limsup Y_{ct} = \infty$) then it must be the case that at the optimum $Y_{ct} \to \infty$ as t goes to infinity; (5) We prove that the economy switches towards clean research, that is, $s_{ct} \to 1$, (6) we prove that the switch in research to clean technologies occurs in finite time, that is, there exists $\tilde{T}$ such that $s_{ct} = 1$ for all $t \geq \tilde{T}$. (6) We then derive the implied behavior of τ_t and q_t.

Part 1: To obtain a contradiction, suppose that the optimal allocation features Y_{ct} remaining bounded as t goes to infinity. If Y_{dt} was unbounded then there would be an environmental disaster, but then the allocation cannot be optimal in view of the assumption that $\lim_{S \downarrow 0} u(C, S) = -\infty$ (equation (2)). Thus Y_{dt} must also remain bounded as t goes to infinity. But if both Y_{ct} and Y_{dt} remain bounded, so will Y_t and C_t. We use the superscript ns (ns for "no switch") to denote the variables under this allocation.

Consider an alternative (feasible) allocation, featuring all research being directed to clean technologies after some date $\hat{t}$, i.e., $s_{ct} = 1$ for all $t > \hat{t}$ and no production of dirty input (by taking an infinite carbon tax τ_t). This in turn implies that S_t reaches $\overline{S}$ in finite time because of regeneration at the rate δ in (9). Moreover, (21) implies that $Y_t/A_{ct} \to$ constant and thus $C_t/A_{ct} \to$ constant. Let us use superscript a to denote all variables under this alternative allocation. Then there exists a consumption level $\overline{C} < \infty$, and a date $T < \infty$ such that for $t \geq T$, $C_t^{ns} < \overline{C}$, $C_t^a > \overline{C} + \theta$ (where $\theta > 0$) and $S_t^a = \overline{S}$. Now using the fact that u is strictly increasing in C and S, for all $t \geq T$ we have

$$u\left(C_t^a, S_t^a\right) - u\left(C_t^{ns}, S_t^{ns}\right) \geq u\left(C_t^a, \overline{S}\right) - u\left(\overline{C}, \overline{S}\right) > 0$$

which is positive and strictly increasing over time. Then the welfare difference between the alternative and the no-switch allocations can be written as

$$
\begin{aligned}
W^a - W^{ns} &= \sum_{t=0}^{T-1} \frac{1}{(1+\rho)^t} \left(u\left(C_t^a, S_t^a\right) - u\left(C_t^{ns}, S_t^{ns}\right)\right) + \sum_{t=T}^{\infty} \frac{1}{(1+\rho)^t} \left(u\left(C_t^a, S_t^a\right) - u\left(C_t^{ns}, S_t^{ns}\right)\right) \\
&\geq \sum_{t=0}^{T-1} \frac{1}{(1+\rho)^t} \left(u\left(C_t^a, S_t^a\right) - u\left(C_t^{ns}, S_t^{ns}\right)\right) + \frac{1}{(1+\rho)^T} \sum_{t=T}^{\infty} \frac{1}{(1+\rho)^{t-T}} \left(u\left(C_t^a, \overline{S}\right) - u\left(\overline{C}, \overline{S}\right)\right).
\end{aligned}
$$

Since the utility function is continuous in C, and C_t^{ns} is finite for all $t < T$ (for all ρ), then as ρ decreases the first term remains bounded above by a constant, while the second term tends to infinity. This establishes that $W^a - W^{ns} > 0$ for ρ sufficiently small, yielding a contradiction and establishing that we must have Y_{ct} unbounded when t goes to infinity.

Part 2: Now (21) directly implies that

$$A_{ct} \geq g\left(Y_{ct}\right) = \left(\frac{\alpha}{\psi}\right)^{\frac{-\alpha}{1-\alpha}} Y_{ct} \left(1 + \left(\frac{Y_{ct}}{M}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right)^{\frac{\alpha}{\varphi}}$$

where M is an upper-bound on Y_{dt}. g is an increasing function and $\limsup Y_{ct} = \infty$, so $\limsup A_{ct} = \infty$ and as A_{ct} is weakly increasing, $\lim A_{ct} = \infty$.

Part 3: Now suppose by contradiction that $\liminf Y_{ct} \neq \infty$, then by definition if must be the case that $\exists M'$ such that $\forall T, \exists t > T$ with $Y_{ct} < M'$. Let us consider such an M' and note that we can always choose it to be higher than the upper bound on Y_{dt}. Then we can define a sequence $t(n)$ with $t(n) \geq n$ and $Y_{ct(n)} < M'$ for all n. Since $Y_{dt} < M'$ as well, we have, for all n: $C_{t(n)} <$

$Y_{t(n)} < 2^{\varepsilon/(\varepsilon-1)} M'$. But we also know that A_{ct} converges towards infinity with t, thus there exists a integer θ such that for any $t > \theta$, $A_{ct} > (\alpha/\psi)^{-\alpha/(1-\alpha)} 2^{\varepsilon/(\varepsilon-1)} M'/(1-\alpha)$. Consequently, for $n \geq \theta$ we have: $C_{t(n)} < Y_{t(n)} < 2^{\varepsilon/(\varepsilon-1)} M'$ and $A_{ct(n)} > (\alpha/\psi)^{-\alpha/(1-\alpha)} 2^{\varepsilon/(\varepsilon-1)} M'/(1-\alpha)$.

Consider now the alternative policy that mimics the initial policy, except that in all periods $t(n)$ for $n \geq \theta$ the social planner chooses the carbon tax $\tau_{t(n)}^a$ to be sufficiently large (the superscript a designates "alternative") that $Y_{dt(n)}^a = 0$. Then we have: $Y_{t(n)}^a = Y_{ct(n)}^a = (\alpha/\psi)^{\alpha/(1-\alpha)} A_{ct(n)}$, which yields $S_t^a \geq S_t$ for all $t \geq t(n)$ since the alternative policy either reduces or maintains dirty input production relative to the original policy. Moreover, we have: $C_{t(n)}^a = (1-\alpha)Y_{t(n)}^a \geq (1-\alpha)(\alpha/\psi)^{\alpha/(1-\alpha)} A_{ct(n)} > 2^{\varepsilon/(\varepsilon-1)} M' > C_{t(n)}$, whereas consumption in periods $t \neq t(n)$ remains unchanged. Thus the alternative policy leads to (weakly) higher consumption and environmental quality in all periods, and to strictly higher consumption in periods $t = t(n)$, thus overall to strictly higher welfare, than the original policy. Hence the original policy is not optimal, using a contradiction. This in turn establishes that on the optimal path $\liminf Y_{ct} = \infty$ and therefore $\lim Y_{ct} = \infty$.

Part 4: From Part 3 we know that on the optimal path $Y_{ct}/Y_{dt} \to \infty$, that is $(1+\tau_t)^{1-\varepsilon} (A_{ct}/A_{dt})^\varphi \to 0$. Now from (50) and (51), one can reexpress consumption as a function of the carbon tax and technologies:

$$C_t = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{A_{ct} A_{dt}}{\left((1+\tau_t)^{1-\varepsilon} A_{ct}^\varphi + A_{dt}^\varphi\right)^{\frac{1}{\varphi}}} \left(1 - \alpha + \frac{\tau_t A_{ct}^\varphi}{A_{ct}^\varphi + (1+\tau_t)^\varepsilon A_{dt}^\varphi}\right); \qquad (52)$$

Since $(1+\tau_t)(A_{ct}/A_{dt})^{1-\alpha} \to \infty$, we get

$$\lim \frac{C_t}{A_{ct}} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha)$$

Now by contradiction let us suppose that $\liminf s_{ct} = s < 1$. Then for any $\tilde{T}$ there exists $\theta > \tilde{T}$, such that $s_{c\theta} < (1+s)/2$. Now, as $\lim(C_t/A_{ct}) = (\alpha/\psi)^{\alpha/(1-\alpha)} (1-\alpha)$, there exists some T such that for any $t > T$, we have $C_t < (\alpha/\psi)^{\alpha/(1-\alpha)} (1-\alpha) A_{ct} (1+\gamma\eta_c)/(1+\gamma\eta_c(1+s)/2)$. Then take θ sufficiently large that $\theta > T$ and $s_{c\theta} < (1+s)/2$, and consider the following alternative policy: the alternative policy is identical to the original policy up to time $\theta - 1$, then at θ, the alternative policy allocates all research to the clean sector, and for $t > \theta$, the allocation of research is identical to the original policy, and for $t \geq \theta$, the carbon tax is infinite. Then under the alternative policy, there is no pollution for $t \geq \theta$ so the quality of the environment is weakly better than under the original policy. Moreover: $A_{ct}^a = (1+\gamma\eta_c) A_{ct}/(1+\gamma\eta_c s_{c\theta})$, for all $t \geq \theta$ (where the superscript a indicates the alternative policy schedule). Thus for $t \geq \theta$:

$$\begin{aligned}
C_t^a &= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) A_{ct}^a > \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) \frac{1+\gamma\eta_c}{1+\gamma\eta_c s_{c\theta}} A_{ct} \\
&> \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) \frac{1+\gamma\eta_c}{1+\gamma\eta_c\left(\frac{1+s}{2}\right)} A_{ct} > C_t,
\end{aligned}$$

so that the alternative policy brings higher welfare. This in turn contradicts the optimality of the original policy. Hence $\liminf s_{ct} = 1$, so $\lim s_{ct} = 1$, and consequently, $\lim(A^\varphi_{ct}/A^\varphi_{dt}) = 0$.

Part 5: First note that (51) and (52) can be rewritten as:

$$\ln(C_t) - \ln\left(\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\right) = \ln(A_{ct}) + \ln(A_{dt}) - \frac{1}{\varphi}\ln\left(\left((1+\tau_t)^{1-\varepsilon}A^\varphi_{ct} + A^\varphi_{dt}\right)\right) + \ln\left(1 - \alpha + \frac{\tau_t A^\varphi_{ct}}{A^\varphi_{ct} + (1+\tau_t)^\varepsilon A^\varphi_{dt}}\right),$$
(53)

$$\ln(Y_{dt}) - \ln\left(\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\right) = (\alpha + \varphi)\ln(A_{ct}) + \ln(A_{dt}) - \frac{\alpha}{\varphi}\ln\left(\left(A^\varphi_{dt} + (1+\tau_t)^{1-\varepsilon}A^\varphi_{ct}\right)\right) - \ln\left(\left(A^\varphi_{ct} + (1+\tau_t)^\varepsilon A^\varphi_{dt}\right)\right)$$
(54)

Now, suppose that s_{ct} does not reach 1 in finite time. Then for any T, there exists $\theta > T$, such that $s_{c\theta} < 1$. For T arbitrarily large $s_{c\theta}$ becomes arbitrarily close to 1, so that $1 - s_{c\theta}$ becomes infinitesimal and is accordingly denoted ds. We then consider the following thought experiment: let us increase the allocation of researchers to clean innovation at θ from $s_{c\theta} < 1$ to 1, but leave this allocation unchanged in all subsequent periods. Meanwhile, let us adjust the tax τ_t in all periods after θ in order to leave Y_{dt} unchanged. Then using superscript a to denote the value of technologies under the alternative policy, we have for $t \geq \theta$:

$$A^a_{ct} = \frac{1 + \gamma\eta_c}{1 + \gamma\eta_c s_{c\theta}}A_{ct}.$$

A first-order Taylor expansion of the logarithm of the productivity around $s_{c\theta} = 1$ yields:

$$d(\ln(A_{ct})) = \frac{\gamma\eta_c ds}{1 + \gamma\eta_c} + o(ds),$$
(55)

and similarly,

$$d(\ln(A_{dt})) = -\gamma\eta_d ds + o(ds).$$

Using the fact that that $d(\ln(A_{ct}))$ and $d(\ln(A_{dt}))$ are of the same order as ds, first-order Taylor expansions of (53) and (54) give:

$$d(\ln(C_t)) \tag{56}$$
$$= d(\ln(A_{ct})) + d(\ln(A_{dt})) - \frac{(1+\tau)^{1-\varepsilon}A^\varphi_{ct}\left(\varphi d(\ln(A_{ct})) + (1-\varepsilon)d(\ln(1+\tau_t))\right) + \varphi A^\varphi_{dt}d(\ln(A_{dt}))}{\varphi\left((1+\tau_t)^{1-\varepsilon}A^\varphi_{ct} + A^\varphi_{dt}\right)}$$
$$+ \frac{1}{1 - \alpha + \frac{\tau_t A^\varphi_{ct}}{A^\varphi_{ct} + (1+\tau_t)^\varepsilon A^\varphi_{dt}}}\frac{(1+\tau_t)A^\varphi_{ct}d(\ln(1+\tau_t)) + \varphi\tau_t A^\varphi_{ct}d(\ln(A_{ct}))}{A^\varphi_{ct} + (1+\tau_t)^\varepsilon A^\varphi_{dt}}$$
$$- \frac{\tau_t A^\varphi_{ct}}{1 - \alpha + \frac{\tau_t A^\varphi_{ct}}{A^\varphi_{ct} + (1+\tau_t)^\varepsilon A^\varphi_{dt}}}\frac{\varphi A^\varphi_{ct}d(\ln(A_{ct})) + (1+\tau_t)^\varepsilon A^\varphi_{dt}\left(\varphi d(\ln(A_{dt})) + \varepsilon d(\ln(1+\tau_t))\right)}{\left(A^\varphi_{ct} + (1+\tau_t)^\varepsilon A^\varphi_{dt}\right)^2}$$
$$+ o(ds) + o(d(\ln(1+\tau_t))),$$

and

$$d\left(\ln\left(Y_{dt}\right)\right) = \left(\alpha + \varphi\right) d\left(\ln\left(A_{ct}\right)\right) + d\left(\ln\left(A_{dt}\right)\right)$$
$$- \frac{\left(1+\tau_t\right)^{1-\varepsilon} A_{ct}^{\varphi}\left(\varphi d\left(\ln\left(A_{ct}\right)\right) + \left(1-\varepsilon\right) d\left(\ln\left(1+\tau_t\right)\right)\right) + \varphi A_{dt}^{\varphi} d\left(\ln\left(A_{dt}\right)\right)}{\varphi\alpha^{-1}\left(\left(1+\tau_t\right)^{1-\varepsilon} A_{ct}^{\varphi} + A_{dt}^{\varphi}\right)}$$
$$- \frac{\varphi A_{ct}^{\varphi} d\left(\ln\left(A_{ct}\right)\right) + \left(1+\tau_t\right)^{\varepsilon} A_{dt}^{\varphi}\left(\varphi d\left(\ln\left(A_{dt}\right)\right) + \varepsilon d\left(\ln\left(1+\tau_t\right)\right)\right)}{A_{ct}^{\varphi} + \left(1+\tau_t\right)^{\varepsilon} A_{dt}^{\varphi}} + o\left(ds\right) + o\left(d\left(\ln\left(1+\tau_t\right)\right)\right).$$

Then, using the fact that in the variation in question taxes are adjusted to keep production of the dirty input constant, the previous equation gives

$$\left(\frac{\varepsilon\left(1+\tau_t\right)^{\varepsilon} A_d^{\varphi}}{A_{ct}^{\varphi} + \left(1+\tau_t\right)^{\varepsilon} A_{dt}^{\varphi}} + \frac{\alpha}{\varphi}\frac{\left(1-\varepsilon\right)\left(1+\tau_t\right)^{1-\varepsilon} A_{ct}^{\varphi}}{\left(1+\tau_t\right)^{1-\varepsilon} A_{ct}^{\varphi} + A_{dt}^{\varphi}}\right) d\left(\ln\left(1+\tau_t\right)\right)$$
$$= \left(\alpha + \varphi\right) d\left(\ln\left(A_{ct}\right)\right) + d\left(\ln\left(A_{dt}\right)\right) - \frac{\alpha}{\varphi}\frac{\varphi\left(1+\tau\right)_t^{1-\varepsilon} A_{ct}^{\varphi} d\left(\ln\left(A_{ct}\right)\right) + \varphi A_{dt}^{\varphi} d\left(\ln\left(A_{dt}\right)\right)}{\left(1+\tau_t\right)^{1-\varepsilon} A_{ct}^{\varphi} + A_{dt}^{\varphi}}$$
$$- \frac{\varphi A_{ct}^{\varphi} d\left(\ln\left(A_{ct}\right)\right) + \varphi\left(1+\tau\right)^{\varepsilon} A_{dt}^{\varphi} d\left(\ln\left(A_{dt}\right)\right)}{A_{ct}^{\varphi} + \left(1+\tau_t\right)^{\varepsilon} A_{dt}^{\varphi}} + o\left(ds\right) + o\left(d\left(\ln\left(1+\tau_t\right)\right)\right).$$

Now recall the following: (i) $\lim_{t\to\infty} A_{ct}^{\varphi}/A_{dt}^{\varphi} = 0$; (ii) the term $\frac{\varepsilon\left(1+\tau_t\right)^{\varepsilon} A_d^{\varphi}}{A_{ct}^{\varphi} + \left(1+\tau_t\right)^{\varepsilon} A_{dt}^{\varphi}} + \frac{\alpha}{\varphi}\frac{\left(1-\varepsilon\right)\left(1+\tau_t\right)^{1-\varepsilon} A_{ct}^{\varphi}}{\left(1+\tau_t\right)^{1-\varepsilon} A_{ct}^{\varphi} + A_{dt}^{\varphi}}$ is bounded and bounded away from 0; (iii) the terms in front of $d\left(\ln\left(A_{dt}\right)\right)$ and $d\left(\ln\left(A_{ct}\right)\right)$ are bounded. Therefore, we can rewrite (56) as:

$$d\left(\ln\left(C_t\right)\right) = d\left(\ln\left(A_{ct}\right)\right) + \frac{\left(1+\tau_t\right)^{1-\varepsilon} A_{ct}^{\varphi} A_{dt}^{-\varphi}}{\left(1+\tau_t\right)^{1-\varepsilon} A_{ct}^{\varphi} A_{dt}^{-\varphi} + 1}\left(d\left(\ln\left(A_{dt}\right)\right) - d\left(\ln\left(A_{ct}\right)\right) - \left(1-\alpha\right)^{-1} d\left(\ln\left(1+\tau_t\right)\right)\right)$$
$$+ \frac{1}{1-\alpha + \frac{\tau_t\left(1+\tau_t\right)^{-\varepsilon} A_{ct}^{\varphi} A_{dt}^{-\varphi}}{A_{ct}^{\varphi} A_{dt}^{-\varphi}\left(1+\tau_t\right)^{-\varepsilon}+1}}\frac{\left(1+\tau_t\right)^{1-\varepsilon} A_{ct}^{\varphi} A_{dt}^{-\varphi}}{\left(1+\tau_t\right)^{-\varepsilon} A_{ct}^{\varphi} A_{dt}^{-\varphi} + 1}\left(d\left(\ln\left(1+\tau_t\right)\right) + \varphi\frac{\tau_t}{1+\tau_t} d\left(\ln\left(A_{ct}\right)\right)\right)$$
$$- \frac{\tau_t\left(1+\tau_t\right)^{-\varepsilon} A_{ct}^{\varphi} A_{dt}^{-\varphi}}{1-\alpha + \frac{\tau_t\left(1+\tau_t\right)^{-\varepsilon} A_{ct}^{\varphi} A_{dt}^{-\varphi}}{A_{ct}^{\varphi} A_{dt}^{-\varphi}\left(1+\tau_t\right)^{-\varepsilon}+1}}\frac{\varphi A_{ct}^{\varphi} A_{dt}^{-\varphi}\left(1+\tau_t\right)^{-\varepsilon} d\left(\ln\left(A_{ct}\right)\right) + \varphi d\left(\ln\left(A_{dt}\right)\right) + \varepsilon d\left(\ln\left(1+\tau_t\right)\right)}{\left(\left(1+\tau_t\right)^{-\varepsilon} A_{ct}^{\varphi} A_{dt}^{-\varphi} + 1\right)^2} + o\left(ds\right)$$

Using again the fact that $\lim_{t\to\infty} A_{ct}^{\varphi}/A_{dt}^{\varphi} = 0$ and (55), the previous expression becomes

$$d\left(\ln\left(C_t\right)\right) = \left(\frac{\gamma\eta_c}{1+\gamma\eta_c} + O\left(\frac{A_{ct}^{\varphi}}{A_{dt}^{\varphi}}\right)\right) ds + o\left(ds\right),$$

which implies that for T sufficiently large, $O\left(A_{ct}^{\varphi}/A_{dt}^{\varphi}\right)$ will be smaller than $\gamma\eta_c/\left(1+\gamma\eta_c\right)$, and thus consumption increases. This implies that the alternative policy raises consumption for all periods after θ, and does so without affecting the quality of the environment, hence the original policy cannot be optimal. This contradiction establishes that s_{ct} reaches 1 in finite time.

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Part 6: Thus the optimal allocation must involve $s_{ct} = 1$ for all $t \geq \tilde{T}$ (for some $\tilde{T} < \infty$) and $A_{ct}/A_{dt} \to \infty$. Then, note that (64) implies that even if $\tau_t = q_t = 0$, the equilibrium allocation of scientists involves $s_{ct} = 1$ for all $t \geq T$ for some T sufficiently large. This is sufficient to establish that $q_t = 0$ for all $t \geq T$ is consistent with an optimal allocation. Finally, equation (51) implies that when $\varepsilon > 1/(1 - \alpha)$, $Y_{dt} \to 0$, which together with (9), implies that S_t reaches $\overline{S}$ in finite time. But then the assumption that $\partial u\left(C, \overline{S}\right)/\partial S = 0$ combined with (28) and (26) implies that the optimal input tax reaches 0 in finite time. On the contrary, when $\varepsilon \leq 1/(1 - \alpha)$, even when all research ends up being directed towards clean technologies, (21) shows that without imposing a positive input tax we have $Y_{dt} \to \infty$ and therefore $S_t = 0$ in finite time which cannot be optimal. So in this case, taxation must be permanent at the optimum.

Complementarity case: If $\varepsilon < 1$, from (21), the growth rate of the economy is the minimum of the growth rates of Y_{dt} and Y_{ct}. Positive asymptotic growth then implies that $Y_{dt} \to \infty$ and $S_t \to 0$, which cannot be optimal. Thus optimal allocations involve zero long-run growth. Moreover, the carbon tax must also be positive in the long run, since the environmental externality remains first order as $t \to \infty$. This completes the proof of Proposition 5. ∎

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Supplementary Appendix

Speed of disaster in laissez-faire

From the expressions in (21), dirty input production is given by:

$$Y_{dt} = \left(A_{ct}^{\varphi} + A_{dt}^{\varphi}\right)^{-\frac{\alpha+\varphi}{\varphi}} A_{ct}^{\alpha+\varphi} A_{dt} = \frac{A_{dt}}{\left(1 + \left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi}\right)^{\frac{\alpha+\varphi}{\varphi}}}.$$

When the two inputs are gross substitutes ($\varepsilon < 1$), we have $\varphi = \varphi^{su} < 0$, whereas when they are complements ($\varepsilon > 1$), we have $\varphi = \varphi^{co} > 0$. Since all innovations occur in the dirty sector in the substitutability case, but not in the complementarity case, if we start with the same levels of technologies in both cases, at any time $t > 0$ we have $A_{dt}^{su} > A_{dt}^{co}$ and $A_{ct}^{su} < A_{ct}^{co}$, where A_{kt}^{su} and A_{kt}^{co} denote the average productivities in sector k at time t respectively in the substitutability and in the complementarity case, starting from the same initial productivities $A_{k0}^{su} = A_{k0}^{co}$.

Assumption 1 implies that

$$\left(\frac{A_{dt}^{su}}{A_{ct}^{su}}\right)^{\varphi^{su}} < \frac{\eta_d}{\eta_c} \leq \left(\frac{A_{dt}^{co}}{A_{ct}^{co}}\right)^{\varphi^{co}}$$

so that

$$\begin{aligned}
Y_{dt}^{su} &= \frac{A_{dt}^{su}}{\left(1 + \left(\frac{A_{dt}^{Su}}{A_{ct}^{su}}\right)^{\varphi^{su}}\right)^{\frac{\alpha}{\varphi^{su}}+1}} > \frac{A_{ct}^{su}}{\left(1 + \left(\frac{A_{ct}^{co}}{A_{dt}^{co}}\right)^{\varphi^{co}}\right)} \left(1 + \left(\frac{A_{dt}^{Su}}{A_{ct}^{su}}\right)^{\varphi^{su}}\right)^{-\frac{\alpha}{\varphi^{su}}} \\
&> \frac{A_{ct}^{su}}{\left(1 + \left(\frac{A_{ct}^{co}}{A_{dt}^{co}}\right)^{\varphi^{co}}\right)} > \frac{A_{ct}^{su}}{\left(1 + \left(\frac{A_{ct}^{co}}{A_{dt}^{co}}\right)^{\varphi^{c0}}\right)^{\frac{\alpha}{\varphi_{c0}}+1}} > Y_{dt.}^{co}
\end{aligned}$$

Repeating the same argument for $t+1$, $t+2$,..., we have that $Y_{dt}^{su} > Y_{dt}^{co}$ for all t. This establishes that, under Assumption 1, there will be a greater amount of dirty input production for each t when $\varepsilon > 1$ than when $\varepsilon < 1$, implying that an environmental disaster will occur sooner when the two sectors are gross substitutes.

Equilibrium profit ratio with exhaustible resources

We first analyze how the static equilibrium changes when we introduce the limited resource constraint. Thus here we drop subscript t for notational simplicity. The description of clean sectors remains exactly as before. Profit maximization by producers of machines in the dirty sector now leads to the equilibrium price $p_{di} = \frac{\psi}{\alpha_1}$ (as α_1 is the share of machines in the production of dirty input). The equilibrium output level for machines is then given by:

$$x_{di} = \left(\frac{(\alpha_1)^2 p_d R^{\alpha_2} L_d^{1-\alpha}}{\psi}\right)^{\frac{1}{1-\alpha_1}} A_{di} \tag{57}$$

Profit maximization by the dirty input producer leads to the following demand equation for the resource:

$$p_d \alpha_2 R^{\alpha_2-1} L_d^{1-\alpha} \int_0^1 A_{di}^{1-\alpha_1} x_{di}^{\alpha_1} di = c(Q)$$

Plugging in the equilibrium output level of machines (57) yields:

$$R = \left(\frac{(\alpha_1)^2}{\psi} \right)^{\frac{\alpha_1}{1-\alpha}} \left(\frac{\alpha_2 A_d}{c(Q)} \right)^{\frac{1-\alpha_1}{1-\alpha}} p_d^{\frac{1}{1-\alpha}} L_d \tag{58}$$

which in turn, together with (34), leads to the following expression for the equilibrium production of dirty input:

$$Y_d = \left(\frac{(\alpha_1)^2}{\psi} \right)^{\frac{\alpha_1}{1-\alpha}} \left(\frac{\alpha_2 A_d}{c(Q)} \right)^{\frac{\alpha_2}{1-\alpha}} p_d^{\frac{\alpha}{1-\alpha}} L_d A_d. \tag{59}$$

The equilibrium profits from producing machine i in the dirty sector becomes:

$$\pi_{di} = (1-\alpha_1) \alpha_1^{\frac{1+\alpha_1}{1-\alpha_1}} \left(\frac{1}{\psi^{\alpha_1}} \right)^{\frac{1}{1-\alpha_1}} p_d^{\frac{1}{1-\alpha_1}} R^{\frac{\alpha_2}{1-\alpha_1}} L_d^{\frac{1-\alpha}{1-\alpha_1}} A_{di}.$$

The production of the clean input and the profits of the producer of machine i in the clean sector are still given by (18), that is:

$$Y_c = \left(\frac{\alpha^2}{\psi} p_c \right)^{\frac{\alpha}{1-\alpha}} L_c A_c \tag{60}$$

and profits from producing machines ci are

$$\pi_{ci} = (1-\alpha) \alpha^{\frac{1+\alpha}{1-\alpha}} \left(\frac{1}{\psi} \right)^{\frac{\alpha}{1-\alpha}} p_c^{\frac{1}{1-\alpha}} L_c A_{ci}.$$

Next, labor market clearing requires that the marginal product of labor be equalized across sectors; this, together with (59) and (60), leads to the equilibrium price ratio (36): thus a higher extraction cost will bid up the price of the dirty input. Profit maximization by final good producer still yields (10) which, combined with (36), (59) and (60) yields the equilibrium labor share (equation (37)). Hence, the higher the extraction cost, the higher the amount of labor allocated to the clean industry when $\varepsilon > 1$, but the opposite holds when $\varepsilon < 1$.

The ratio of expected profits from undertaking innovation at time t in the clean versus the dirty sector, is then equal to (we reintroduce the time subscript):

$$
\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d} \frac{(1-\alpha_1)\,\alpha_1^{\frac{1+\alpha_1}{1-\alpha_1}}\left(\frac{1}{\psi^{\alpha_1}}\right)^{\frac{1}{1-\alpha_1}}}{(1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}\left(\frac{1}{\psi}\right)^{\frac{\alpha}{1-\alpha}}} \frac{p_{ct}^{\frac{1}{1-\alpha}} L_{ct}}{p_{dt}^{\frac{1}{1-\alpha_1}} R_t^{\frac{\alpha_2}{1-\alpha_1}} L_{dt}^{\frac{1-\alpha}{1-\alpha_1}}} \frac{A_{ct-1}}{A_{dt-1}}
$$

$$
= \kappa \frac{\eta_c}{\eta_d} \frac{c(Q_t)^{\alpha_2(\varepsilon-1)}\left(1+\gamma\eta_c s_{ct}\right)^{-\varphi-1} A_{ct-1}^{-\varphi}}{\left(1+\gamma\eta_d s_{dt}\right)^{-\varphi_1-1} A_{dt-1}^{-\varphi_1}}
$$

where we let $\kappa \equiv \dfrac{(1-\alpha)\alpha}{(1-\alpha_1)\alpha_1^{\frac{1+\alpha_2-\alpha_1}{1-\alpha_1}}}\left(\dfrac{\alpha^{2\alpha}}{\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}}\right)^{(\varepsilon-1)}$. This establishes (38). ∎

Proof of Proposition 4

The analysis in the text implies that the social optimum can be implemented through a combination of a carbon tax τ_t, a subsidy to clean research q_t, and a subsidy to the use of machines. In particular, τ_t is given by (28), and the subsidy to the use of all machines is chosen to induce (25). To determine the subsidy q_t to clean research, first note that in the optimal allocation the shadow values of the clean and dirty inputs satisfy

$$
\widehat{p}_{ct}^{\frac{1}{1-\alpha}} A_{ct} = \widehat{p}_{dt}^{\frac{1}{1-\alpha}} A_{dt}. \tag{61}
$$

Then, using (25), (27) and (61), we obtain:

$$
\frac{L_{ct}}{L_{dt}} = (1+\tau_t)^{\varepsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{-\varphi}. \tag{62}
$$

Next, incorporating the subsidy to the use of machines,

$$
x_{jit} = \left(\frac{\alpha}{\psi}\widehat{p}_{jt}\right)^{\frac{1}{1-\alpha}} A_{jit}L_{jt}, \tag{63}
$$

so that pre-tax profits are

$$
\pi_{jit} = (1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \widehat{p}_{jt}^{\frac{1}{1-\alpha}} A_{jit}L_{jt}.
$$

Therefore, for given research subsidy q_t on profits in sector c, the ratio of expected profits from innovation in sectors c and d, the equivalent of (20) becomes

$$
\frac{\Pi_{ct}}{\Pi_{dt}} = (1+q_t)\frac{\eta_c}{\eta_d}\left(\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d s_{dt}}\right)^{-\varphi-1}(1+\tau_t)^{\varepsilon}\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi}. \tag{64}
$$

Clearly, when the optimal allocation involves $s_{ct}=1$, we (the planner) can choose q_t to make this expression greater than one. Or more explicitly, set

$$
q_t \geq \hat{q}_t \equiv \frac{\eta_d}{\eta_c}(1+\gamma\eta_d)^{-\varphi-1}(1+\tau_t)^{-\varepsilon}\left(\frac{A_{dt-1}}{A_{ct-1}}\right)^{-\varphi} - 1.
$$

When the optimal value of $s_{ct}\in(0,1)$, then setting $q_t = \hat{q}_t$ achieves the desired objective. ∎

Proof of Proposition 7

First, we derive the equilibrium production of R and Y_d.

Using both, the fact that the final good is chosen as numeraire and the expression for the equilibrium price ratio (36), we get:

$$p_c = \frac{\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2} A_d^{1-\alpha_1}}{\left((\alpha^{2\alpha} c \, (Q)^{\alpha_2})^{1-\varepsilon} A_c^{\varphi} + \left(\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2} \right)^{1-\varepsilon} A_d^{\varphi_1} \right)^{\frac{1}{1-\varepsilon}}}$$

$$p_d = \frac{\alpha^{2\alpha} (c(Q))^{\alpha_2} A_c^{1-\alpha}}{\left((\alpha^{2\alpha} c \, (Q)^{\alpha_2})^{1-\varepsilon} A_c^{\varphi} + \left(\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2} \right)^{1-\varepsilon} A_d^{\varphi_1} \right)^{\frac{1}{1-\varepsilon}}}$$

Similarly, using the expression for the equilibrium labor ratio (37), and labor market clearing (7), we obtain:

$$L_d = \frac{\left(c(Q)^{\alpha_2} \alpha^{2\alpha} \right)^{(1-\varepsilon)} A_c^{\varphi}}{(c(Q)^{\alpha_2} \alpha^{2\alpha})^{(1-\varepsilon)} A_c^{\varphi} + \left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} (\alpha_2)^{\alpha_2} \right)^{(1-\varepsilon)} A_d^{\varphi_1}}$$

$$L_c = \frac{\left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} (\alpha_2)^{\alpha_2} \right)^{(1-\varepsilon)} A_d^{\varphi_1}}{(c(Q)^{\alpha_2} \alpha^{2\alpha})^{(1-\varepsilon)} A_c^{\varphi} + \left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} (\alpha_2)^{\alpha_2} \right)^{(1-\varepsilon)} A_d^{\varphi_1}}$$

Next, using the above expressions for equilibrium prices and labor allocation, and plugging them in (59) and (58), we obtain:

$$Y_d = \frac{\left(\frac{\alpha_1^2}{\psi} \right)^{\frac{\alpha_1}{1-\alpha}} \alpha_2^{\frac{\alpha_2}{1-\alpha}} \alpha^{2\alpha\left(\frac{1}{1-\alpha} - \varepsilon \right)} c(Q)^{-\varepsilon\alpha_2} A_c^{\alpha+\varphi} A_d^{\frac{1-\alpha_1}{1-\alpha}}}{\left((c(Q)^{\alpha_2} \alpha^{2\alpha})^{(1-\varepsilon)} A_c^{\varphi} + \left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} (\alpha_2)^{\alpha_2} \right)^{(1-\varepsilon)} A_d^{\varphi_1} \right)^{\frac{\alpha+\varphi}{\varphi}}}$$

and

$$R = \frac{\alpha^{2\alpha\left(\frac{1}{1-\alpha}+1-\varepsilon \right)} \alpha_1^{2\frac{\alpha_1}{1-\alpha}} \alpha_2^{\frac{1-\alpha_1}{1-\alpha}} \psi^{-\frac{\alpha_1}{1-\alpha}} (c(Q))^{\alpha_2-1-\alpha_2\varepsilon} A_c^{1+\varphi} A_d^{\frac{1-\alpha_1}{1-\alpha}}}{\left((c(Q)^{\alpha_2} \alpha^{2\alpha})^{(1-\varepsilon)} A_c^{\varphi} + \left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} (\alpha_2)^{\alpha_2} \right)^{(1-\varepsilon)} A_d^{\varphi_1} \right)^{\frac{1+\varphi}{\varphi}}},$$

so that:

$$\frac{R}{Y_d} = \frac{\alpha_2 \alpha^{2\alpha} (c(Q))^{\alpha_2-1}}{\left((\alpha^{2\alpha} c \, (Q)^{\alpha_2})^{1-\varepsilon} + \left(\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2} \right)^{1-\varepsilon} \frac{A_d^{\varphi_1}}{A_c^{\varphi}} \right)^{\frac{1}{1-\varepsilon}}}.$$

In the remaining part of the proof, we again separately consider the case in which the two inputs are complements and the case where they are substitutes.

Substitutability case: When $\varepsilon > 1$, production of the dirty input is not essential to final good production. Thus, even if the stock of exhaustible resource gets fully depleted, it is still possible to achieve positive long-run growth. For a disaster to occur for any initial value of the environmental quality, it is necessary that Y_d grow at a positive rate while R must converge to 0. This implies that R/Y_d must converge to 0. This in turn means that the expression

$$\left(\alpha^{2\alpha} c\left(Q\right)^{\alpha_2}\right)^{1-\varepsilon} + \left(\psi^{\alpha_2}\left(\alpha_1\right)^{2\alpha_1}\left(\alpha_2\right)^{\alpha_2}\right)^{1-\varepsilon} \frac{A_d^{\varphi_1}}{A_c^{\varphi}}$$

must be equal to zero, which is impossible since $c\left(Q\right)$ is bounded above. Therefore, for sufficiently high initial quality of the environment, a disaster will be avoided.

Next, one can show that innovation will always end up occurring in the clean sector only. This is obvious if the resource gets depleted in finite time, so let us consider the case where it never gets depleted. The ratio of expected profits in clean versus dirty innovation is given by

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \kappa \frac{\eta_c}{\eta_d} \frac{c(Q_t)^{\alpha_2(\varepsilon-1)}\left(1+\gamma\eta_c s_{ct}\right)^{-\varphi-1} A_{ct-1}^{-\varphi}}{\left(1+\gamma\eta_d s_{dt}\right)^{-\varphi_1-1} A_{dt-1}^{-\varphi_1}},$$

so that to prevent innovation from occurring asymptotically in the clean sector only it must be the case that $A_c^{-\varphi}$ does not grow faster then $A_{dt}^{-\varphi_1}$. In this case $R = O\left(A_d^{\frac{1-\alpha_1}{1-\alpha}}\right)$. But $A_d^{\frac{1-\alpha_1}{1-\alpha}}$ grows at a positive rate over time, so that the resource gets depleted in finite time after all.

Complementarity case: When $\varepsilon < 1$, Y_d is now essential for production and thus so is the resource flow R. Consequently, it is necessary that Q does not get depleted in finite time in order to get positive long-run growth. Recall that innovation takes place in both sectors if and only if $\kappa \frac{\eta_c}{\eta_d} \frac{c(Q_t)^{\alpha_2(\varepsilon-1)}(1+\gamma\eta_c s_{ct})^{-\varphi-1} A_{ct-1}^{-\varphi}}{(1+\gamma\eta_d s_{dt})^{-\varphi_1-1} A_{dt-1}^{-\varphi_1}} = 1$, and positive long-run growth requires positive growth of both dirty input and clean input productions. This requires that innovation occurs in both sectors, so $A_d^{(1-\alpha_1)}$ and $A_c^{(1-\alpha)}$ should be of same order.

But then:
$$R = O\left(A_d^{\frac{1-\alpha_1}{1-\alpha}}\right),$$

so that R grows over time. But this in turn leads to the resource stock being fully exhausted in finite time, thereby also shutting down the production of dirty input, which here prevents positive long-run growth. This completes the proof of Proposition 7.

Proof of Proposition 8

We denote the Lagrange multiplier for the equation (35) by m_t. Then, the first-order condition with respect to R_t implies:

$$\alpha_2 \widehat{p}_{dt} R^{\alpha_2-1} L_d^{1-\alpha} \int_0^1 A_{di}^{1-\alpha_1} x_{di}^{\alpha_1} di = \frac{m_t}{\lambda_t} + c\left(Q\right),$$

where recall that $\widehat{p}_{jt} = \lambda_{jt}/\lambda_t$. Here, the wedge m_t/λ_t is the value, in time t units of final good, of one unit of resource at time t.

The shadow value of one unit of natural resource at time t is then determined by the first-order condition with respect to Q_t, which is

$$m_t = m_{t-1} + \lambda_t c'(Q_t) R_t$$

and thus implies

$$m_t = m_\infty + \sum_{v=t+1}^{\infty} \lambda_v \left(-c'(Q_v)\right) R_v.$$

(where m_∞ is the limit of m_t when $t \to \infty$).

Thus achieving the social optimum requires a resource tax equal to

$$\theta_t = \frac{m_t}{\lambda_t c(Q_t)} = \frac{m_\infty - \sum_{v=t+1}^{\infty} \frac{1}{(1+\rho)^{v-t}} c'(Q_v) R_v \partial u(C_v, S_v)/\partial C}{c(Q_t) \partial u(C_t, S_t)/\partial C}. \tag{65}$$

In particular, the optimal resource tax is always positive.

Characterization of global optimal environmental policy with no trade case

We now characterize the optimal policy from the point of view of a global social planner interested in maximizing the sum of the utilities of households in both countries (both given by (1)). This social planner will choose a dynamic path of final good production Y_t^k, consumption C_t^k, intermediary input productions Y_{jt}^k, machines production x_{jit}^k, labor share allocation L_{jt}^k, scientists allocation s_{jt}^k and quality of machines A_{jit}^k for each country $k = N, S$ and environmental quality S_t to maximize the Social Welfare Function

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \left(L^N u \left(\frac{C_t^N}{L^N}, S_t \right) + L^S u \left(\frac{C_t^S}{L^S}, S_t \right) \right)$$

under the same constraints as for the baseline model, except that equation (9) becomes (40), and the productivity growth in the South equation (41).

Thus the maximization problem is very similar to that analyzed in subsection 3. One difference is that the shadow value of an environmental unit, which is identical in the two countries, now includes the marginal benefit to the utility of households in both countries so that:

$$\omega_t = \frac{1}{(1+\rho)^t} \left(L^N \frac{\partial u^N}{\partial S} + L^S \frac{\partial u^S}{\partial S} \right) + (1+\delta) \omega_{t+1}.$$

The social planner will still introduce a wedge $\omega_{t+1}\xi/\lambda_t^k$ between the price of the dirty input and its marginal product in the production of the final good. This wedge has the same

interpretation as in the one country case; and thus it will be the higher (in absolute value) in the country with the lowest value λ_t^k , that is, the country with the lowest marginal utility of consumption (the rich country). This wedge translates into an optimal tax on the dirty input in country k :

$$\tau_t^k = \frac{\omega_{t+1}\xi}{\lambda_{dt}^k} = \frac{\omega_{t+1}\xi}{\lambda_t^k \widehat{p}_{dt}^k}$$

This expression is identical to that in the one-country case, and one can similarly establish that the optimal carbon tax will be temporary if the clean and dirty inputs are sufficiently close substitutes.

Using the equivalent of (49) and (28) in the two-country model, one can show that the comparison between the optimal carbon tax in the North and the South, is governed by the following proposition:

Proposition 11 *The global optimal carbon tax schedule* (τ_t^N, τ_t^S) *satisfies:*

$$\left(\frac{\lambda_t^k}{\omega_{t+1}\xi} \tau_t^k \right)^{1-\varepsilon} = 1 + \left(\frac{\left(A_{dt}^k\right)^{(1-\alpha)}}{\left(1+\tau_t^k\right)\left(A_{ct}^k\right)^{(1-\alpha)}} \right)^{1-\varepsilon} .$$

In particular an increase in the relative productivity in the dirty sector in country k (A_{dt}^k/A_{ct}^k), a decrease in the marginal value of consumption λ_t^k or an increase in the shadow value of environment ω_{t+1}, either of these increases the tax τ_t^k in country k . The second effect will push towards a higher tax in the North, whereas (as long as the dirty sector is more advanced relative to the clean sector in the South than in the North), the first effect will push for a higher tax in the South. Without further assumptions either of these two effects may dominate, and in particular if the South lags far behind with respect to productivity in the clean sector, the dirty carbon tax may end up being higher in the South.

Define μ_{jt}^k as the Lagrange multiplier at time t for the growth equation for sector j in country k . The first-order condition with respect to A_{jt}^N now gives:

$$\mu_{jt}^N = \lambda_t^N \left(\frac{\alpha}{\psi} \right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) \left(p_{jt}^N\right)^{\frac{1}{1-\alpha}} L_{jt}^N + \left(1 + \gamma\eta_j s_{jt+1}^N\right) \mu_{jt+1}^N + \kappa_j s_{jt}^S \mu_{jt}^S \qquad (66)$$

In words: the shadow value of one more unit of clean productivity is equal to its marginal product at time t (corresponding to the first term), plus its shadow value at time $t+1$ times $\left(1 + \gamma\eta_j s_{jt+1}^N\right)$ - the rate of productivity growth in the North between t and $t+1$- (corresponding to the second term), plus an additional term $\kappa_c s_{jt}^S$ times the value of one unit of clean productivity in the South. This term did not exist in the closed economy, because it

represents the international knowledge spillover: each additional unit of productivity in sector j in country N creates $\kappa_c s_{jt}^S$ units of productivity in sector j in country S.[34]

The optimal allocation of scientists in the South will be governed by the comparison between the social gains from imitation in clean versus dirty technologies, namely $\mu_{ct}^S \kappa_c A_{ct}^N$ versus $\mu_{dt}^S \kappa_d A_{dt}^N$, and in the North it will be governed by the comparison between $\mu_{ct}^N \eta_c A_{ct-1}^N$ and $\mu_{dt}^N \eta_d A_{dt-1}^N$.

This analysis, combined with the same reasoning as for Proposition 5, establishes the following result (proof omitted):

Proposition 12 *The social optimum can be implemented through a combination of profits and carbon taxes both in the North and in the South, and a subsidy to machine consumers (to remove the monopoly distortion). If $\varepsilon > 1/(1-\alpha)$ and the discount rate is sufficiently low, the optimal environmental taxes are temporary.*

Perfect competition in the absence of innovation

Here we show how our results are slightly modified if, instead of having monopoly rights randomly attributed to "entrepreneurs" when innovation does not occur, machines are produced competitively. There are two types of machines. Those where innovation occurred at the beginning of the period are produced monopolistically with demand function $x_{ji} = x_{ji}^m = \left(\frac{\alpha^2 p_j}{\psi}\right)^{\frac{1}{1-\alpha}} L_j A_{ji}$. Those for which innovation failed are produced competitively. In this case, machines are priced at marginal cost ψ, which leads to a demand for competitively produced machines equal to $x_{ji} = x_{ji}^c = \left(\frac{\alpha p_j}{\psi}\right)^{\frac{1}{1-\alpha}} L_j A_{ji}$. The number of machines produced under monopoly, is simply given by $\eta_j s_j$ (the number of successful innovation).

---

[34]The shadow value μ_{jt}^S is itself determined by first order conditions with respect to A_{jt}^S, which is

$$\mu_{jt}^S = \lambda_t^S \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) \left(p_{jt}^S\right)^{\frac{1}{1-\alpha}} L_{jt}^S + \left(1 - \kappa_j s_{jt+1}^S\right) \mu_{jt+1}^S \qquad (67)$$

The interpretation is basically the same as for μ_{jt}^N: the shadow value of a unit of clean productivity is equal to its marginal product at time t, plus $1 - \kappa_j s_{jt+1}^S$ times its shadow value at time $t+1$ ($\kappa_j s_{jt+1}^S$ machines will adopt the technology in the North at time $t+1$: thus, the decision to allocate scientists to imitation in clean technologies in the South, is more "short sighted", that is with a higher weight on current profits, than if the North did not exist and the South had to innovate without benefiting from knowledge spillovers from the North). Here, there is no technological spillover from South to North, hence the absence of a third term on the RHS of this equation (unlike in the previous equation for μ_{jt}^N).

Hence the equilibrium production of input j is given by

$$
\begin{aligned}
Y_j &= L_j^{1-\alpha} \int_0^1 A_{ji}^{1-\alpha} \left(\eta_j s_j x_{ji,m}^\alpha + (1 - \eta s_j) x_{ji,c}^\alpha \right) di \\
&= \left(\frac{\alpha p_j}{\psi} \right)^{\frac{\alpha}{1-\alpha}} \left(\eta_j s_j \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1 \right) + 1 \right) A_j L_j \\
&= \left(\frac{\alpha p_j}{\psi} \right)^{\frac{\alpha}{1-\alpha}} \widetilde{A_j} L_j
\end{aligned}
$$

where s_j is the number of scientists employed in clean industries and $\widetilde{A_j} = \left(\eta_j s_j \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1 \right) + 1 \right) A_j$ is the average corrected productivity level in sector j (taking into account that some machines are produced by monopolists and others are not).

The equilibrium price ratio is now equal to:

$$
\frac{p_c}{p_d} = \left(\frac{\widetilde{A_c}}{\widetilde{A_d}} \right)^{-(1-\alpha)},
$$

and the equilibrium labor ratio becomes:

$$
\frac{L_c}{L_d} = \left(\frac{\widetilde{A_c}}{\widetilde{A_d}} \right)^{-\varphi}.
$$

The ratio of expected profits from innovation in clean versus dirty sector now becomes

$$
\begin{aligned}
\frac{\Pi_{ct}}{\Pi_{dt}} &= \frac{\eta_c}{\eta_d} \left(\frac{p_{ct}}{p_{dt}} \right)^{\frac{1}{1-\alpha}} \frac{L_{ct}}{L_{dt}} \frac{A_{ct-1}}{A_{dt-1}} \\
&= \frac{\eta_c}{\eta_d} \left(\frac{\left(\eta_c s_{ct} \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1 \right) + 1 \right) (1 + \gamma \eta_c s_{ct})}{\left(\eta_d s_{dt} \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1 \right) + 1 \right) (1 + \gamma \eta_d s_{dt})} \right)^{-\varphi-1} \left(\frac{A_{ct-1}}{A_{dt-1}} \right)^{-\varphi}
\end{aligned}
$$

This yields the modified lemma:

Lemma 3 *In the decentralized equilibrium, innovation at time t can occur in the clean sector only when $\eta_c A_{ct-1}^{-\varphi} > \eta_d \left((1 + \gamma \eta_c) \left(\left(\eta_c \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1 \right) + 1 \right) \right) \right)^{\varphi+1} A_{dt-1}^{-\varphi}$, in the dirty sector only when $\eta_c \left((1 + \gamma \eta_d) \left(\left(\eta_d \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1 \right) + 1 \right) \right) \right)^{\varphi+1} A_{ct-1}^{-\varphi} < \eta_d A_{dt-1}^{-\varphi}$ and can occur in both when $\eta_c \left(\left(\eta_d s_{dt} \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1 \right) + 1 \right) (1 + \gamma \eta_d s_{dt}) \right)^{\varphi+1} A_{ct-1}^{-\varphi} = \eta_d \left(\left(\eta_c s_{ct} \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1 \right) + 1 \right) (1 + \gamma \eta_c s_{ct}) \right)^{\varphi+1} A_{dt-1}^{-\varphi}.$*

This modified lemma can then be used to prove the analogs of Propositions 1, 2 and 3 in the text. The results with exhaustible resource can similarly be generalized to this case.

Calibration for the exhaustible resource case

We perform a similar calibration exercise as in the subsection 4; as before, a time period corresponds to $dt = 5$ years, $\gamma = 1$ and $\alpha = 1/3$. and Y_{c0} and Y_{d0} are still identified with the world production of energy from non-fossil and from fossil fuel origins respectively between 2002 and 2006. The definitions of S, ξ, and δ, and the utility function $u(C, S)$ are also unchanged from the baseline calibration. Now to map our extended model with only one exhaustible resource to reality where we find at least 3 exhaustible resources (oil, natural gas and coal), we assume that each of the resource is used in constant proportion over time and we focus on oil use. More specifically, we compute the share of world energy produced from crude oil in the total amount of energy produced from fossil fuels from 2002 to 2006 (still according to the EIA). We then convert units of crude oil production and stock into units of total fossil production and stock by dividing the former by the share of world energy produced with oil relative to the world energy produced by any fossil fuel. We take the price for the exhaustible resource to be measured by the refiner acquisition cost of imported crude oil in the US (measured in 2000 chained dollars), all these data are taken from the EIA. We detrend the price series from 1970 to 2007 and restrict attention to the period 1997 to 2007 (during which the filtered real price of oil increases). We then parametrize this price as a quadratic function of the estimated reserves of fossil resource. The estimated price of the fossil resource in 2002, combined with the consumption of fossil resource between 2002 and 2006, together with the value of world GDP from 2002 to 2006 from the World Bank,[35] and the initial values of Y_{c0} and Y_{d0}, then allow us to compute α_2, A_{c0} and A_{d0} and the cost function $c(Q)$ as the price of the exhaustible resource in units of the final good. Finally η_c is still taken to be 2% per year, but η_d needs to be rescaled. Indeed, if innovation occurs in the dirty sector only, output in the long-run -abstracting from the exhaustion of the natural resource- will be proportional to $A_d^{\frac{1-\alpha_1}{1-\alpha}}$ instead of A_d, so we compute η_d such that innovation in the dirty sector still corresponds to the same long-run annual growth rate of 2% after making this correction.

We now show how the optimal policy with exhaustible resource compares with that in the baseline case for three configurations of (ε, ρ): the case ($\varepsilon = 3$ and $\rho = 0.015$) is one where the social planner is most inclined to adopt a more gradual approach, the case ($\varepsilon = 10$ and $\rho = 0.001$) is one where the social planner is most inclined to act promptly, and ($\varepsilon = 5$ and $\rho = 0.01$) corresponds to a medium case.

As illustrated by Figure 5B, the switch towards clean innovation occurs immediately when we calibrate the model with exhaustible resource as in the baseline case for ($\varepsilon = 10, \rho = 0.0.01$) and ($\varepsilon = 5, \rho = 0.01$), however the switch to clean innovation occurs slightly later in the exhaustible resource case when ($\varepsilon = 3, \rho = 0.015$) : as already stressed above, the reason is that while the growth prospects in the dirty sector are hampered by the depletion of the resource

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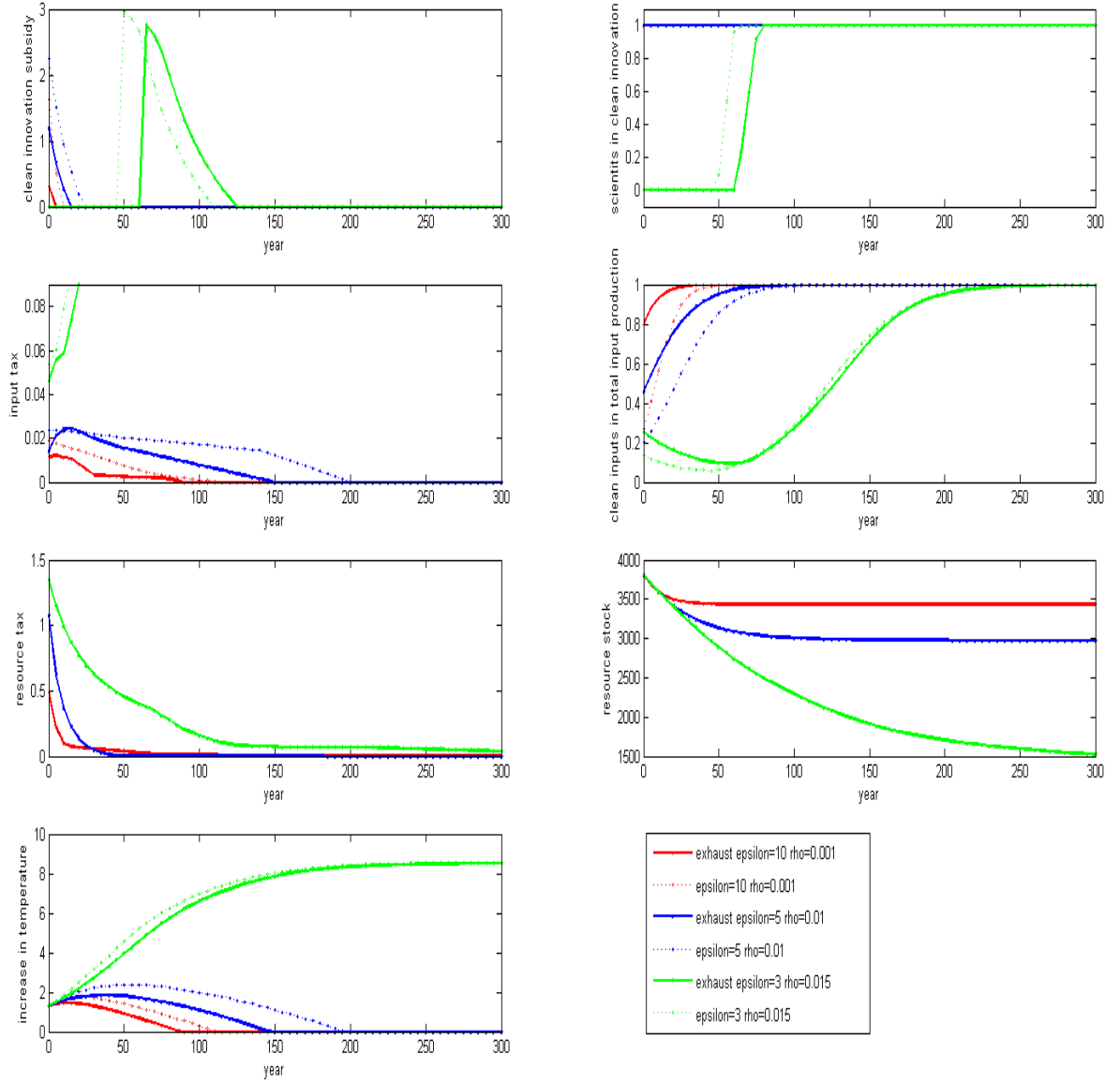[35] The dollar value of world GDP typically allows us to convert 2000$ into our price normalization

Figure 5: Optimal environmental policy with and without exhaustible resource for $(\varepsilon = 10, \rho = 0.001)$, $(\varepsilon = 5, \rho = 0.01)$ and $(\varepsilon = 3, \rho = 0.015)$.

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(this pushes towards an earlier switch to clean innovation), on the other hand, less dirty input is being produced in the exhaustible resource case, which in turn can accommodate a later switch to clean innovation. Which effect dominates depend upon the parameters: for example, when $(\varepsilon = 5, \rho = 0.015)$, switch to clean innovation occurs immediately with the exhaustible resource whereas it occurred after year 60 or so without. However, the clean research subsidy does not need to be as high as in the baseline case to induce the switch: the resource tax and the cost of extraction also do their part of the job, and this is shown in Figure 5A. Figure 5C shows that the dirty carbon tax does not need to be as high as in the baseline case (for the same reason), whereas the switch to clean production occurs earlier than in the baseline case as shown by Figure 5D—except when $(\varepsilon = 3, \rho = 0.015)$ where the later switch in innovation goes along with a slightly later switch to clean production. Figure 5E shows that when ε is lower the resource tax needs to be higher, as more of the resource ends up being extracted at any time (as shown in Figure 5F). Finally, Figure 5G shows that temperature increases less over time with the exhaustible resource.

Equilibrium disasters under free trade without global policy coordination

In this appendix, we illustrate the possibility that a policy that would avoid a disaster under autarky may nonetheless lead to an environmental disaster under free trade because it would induce the South to become a "pollution haven". We illustrate this possibility by providing a numerical example. We identify the North with the world economy of Section 4 where we let $\varepsilon = 5$, and we assume that for the South, $L^S = 3$, $A_{d-1}^S = 0.33 \times A_{d-1}^N$ and $A_{c-1}^S = 0.1 \times A_{c-1}^S$, and $\kappa_c = \kappa_d = 0.1$. We assume that the North follows the environmental policy that was optimal in Section 4 when $\rho = 0.01$ (per year), with all research in the North being directed towards clean innovation and with the carbon tax evolving over time as shown in Figure 2C for $\varepsilon = 5$. We then simulate the dynamics of this world economy both, under free trade and under autarky. Under free trade, there may be multiple equilibria in the innovation decisions in the South. For this simulation exercise, we assume that whenever there are multiple equilibria, Southern researchers will be able to coordinate on the "better" equilibrium, with coordination on research in clean technologies. This makes an equilibrium disaster under free-trade less likely. The results of the simulation are shown in Figure 6.

Figure 6A shows that the switch towards clean imitation in the South occurs later under free trade than under autarky. This will be the main cause of disaster under free trade. Indeed, we see in Figure 6B that dirty production per se does not increase much when moving from autarky to free trade (in fact world dirty production decreases only slightly at the beginning), as long as the imitation pattern is the same under the two regimes. But Figure 6C shows
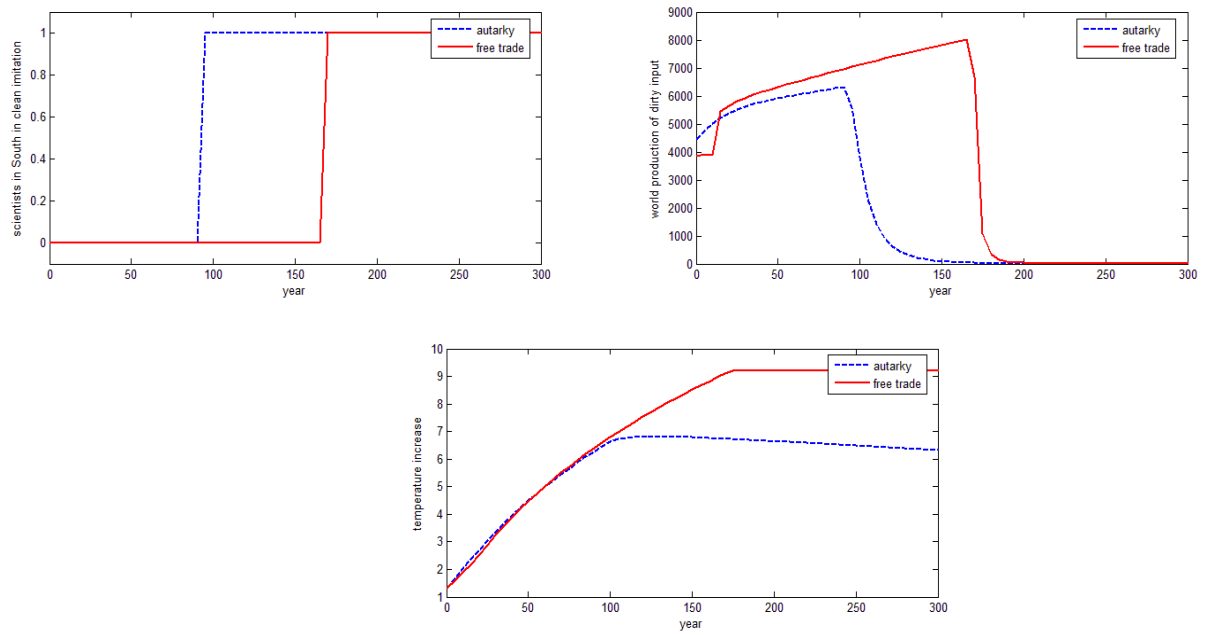
Figure 6: 2 countries case: simulation of a policy that avoids a disaster under autarky but not under free trade

that as soon as the South has switched to clean imitation under autarky, temperature follows a very different path under the two regimes: it starts declining under autarky, whereas under free trade it keeps increasing until it reaches the disaster level of 9.2°C. Intuitively, under free trade before researchers in the South switch to clean technologies developed in the North, these technologies need to becomes sufficiently more advanced (relative to autarky) so as to compensate for the profits that Southern producers can earn by producing in the dirty sector. This prolongs the duration of the period during which the world is still producing significant pollution and makes an environmental disaster more likely.