Uncertainty and technical efficiency in Finnish agriculture

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Abstract: In this article, we present one of the first real-world empirical applications of state-contingent production theory. Our state-contingent behavioural model allows us to analyze production under both inefficiency and uncertainty without regard to the nature of producer risk preferences. Using farm data for Finland, we estimate a flexible production model that permits substitutability between state-contingent outputs. We test empirically, and reject, an assumption that has been implicit in almost all efficiency studies conducted in the last three decades, namely that the production technology is output-cubical, i.e., that outputs are not substitutable between states of nature.

Keywords: uncertainty, state-contingent production function, technical efficiency, Finland.

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1 Introduction

There is a large literature on comparisons of productive efficiency, beginning with the work of Farrell (1957). Assessments of the relative efficiency of agricultural producers have been of particular interest for a number of reasons. First, because agricultural producers typically own land and live on their farms, the standard assumption that market competition will ensure that only efficient producers remain in a given industry is unlikely to be applicable, and the process of adjustment is likely to cause social problems. Second, there exist a wide range of policy interventions, such as education, training and extension programs, which may be interpreted as attempts to increase the efficiency of agricultural production. Third, policy questions relating to the existence and estimation of an optimal size, or minimum efficient size, for farms have been debated in many countries.

All production is subject to uncertainty, but the risks associated with agricultural production are particularly salient. Crop yields may be affected by the amount and timing of rainfall, temperatures during the growing season, pests, diseases, hailstorms and fire among many other factors. Hence, observed differences in outputs and inputs may reflect differences in efficiency, differences in the outcomes of risky decisions, or both.

One common method for dealing with production uncertainty in efficiency comparisons has been the estimation of stochastic frontier models (see among others, Battese, Rambaldi and Wan, 1997; Kumbhakar 2002; Karagiannis, Tzouvelekas and Xepapadeas, 2003; Morrison Paul and Nehring, 2005). In the standard stochastic frontier model, maximum likelihood estimation is used to partition deviations from an estimated production frontier into two components: a one-sided stochastic term representing technical efficiency and a two-sided term representing exogenous stochastic shocks. Implicitly, the production technology being modelled is stochastic.

In general equilibrium theory and finance theory, among other fields, it is more common to model uncertainty in terms of a state-contingent technology. The origins of state-contingent production theory, which considers that outputs are conditional on the states of Nature (each state representing a particular uncertain event) can be traced back to Arrow and Debreu (1954). More recently, Chambers and Quiggin (2000) have shown that all the tools of modern production theory, including cost and distance functions, may be applied to state-contingent production technologies.

Chambers and Quiggin (2000) describe several different types of state-contingent production technologies, including technologies they refer to as *state-allocable*. A feature of state-allocable technologies is that producers can manage uncertainty through the allocation of productive inputs to different states of Nature. This concept is best illustrated by a simplified example (Chambers and Quiggin, 2000, pp. 36–39). Consider a producer who makes a pre-season allocation of a fixed amount of effort to construction of irrigation infrastructure and/or flood-control facilities. If the producer allocates his pre-season effort to the development of irrigation facilities instead of flood control, output will be relatively high if there happens to be a drought (state 1) and low in the event of a flood (state 2). Conversely, if pre-season effort is allocated mainly to flood control, output will be relatively high in state 2 and low in state 1. In this simple example, different pre-season allocations of the input imply a trade-off between output realized in state 1 and output realized in state 2. That is, the producer allocates the input to different states of Nature in order to effect a substitution between state-contingent outputs.

The state-contingent approach, by permitting the allocation of productive inputs to different states of Nature, recognizes that actions (input choices) can have different consequences in different states of Nature. This is not a property of conventional stochastic production theory, in which the role that inputs play remains the same regardless of which state occurs, and which does not permit substitutability between state-contingent outputs. The different types of state-contingent technology described by Chambers and Quiggin allow for more or less substitutability between state-contingent outputs. A technology that does not permit any substitutability between state-contingent outputs is referred to as *output-cubical* (such a technology is Leontief in state-contingent outputs).

Whereas, on the one hand, the theory of state-contingent production is now well established, on the other hand, empirical implementation of the state-contingent approach is still in its infancy. The most notable applications to efficiency analysis are O'Donnell and Griffiths (2006), O'Donnell, Chambers and Quiggin (2006), and Chavas (2008). O'Donnell and Griffiths (2006) have used a Bayesian approach to estimate an output-cubical state-contingent production frontier for rice farmers from the Philippines. They show that, where state-contingent uncertainty plays a major role, the stochastic frontier approach may lead to significant overestimation of the inefficiency of some producers. Indeed, the part of the deviation from the frontier that was due to risk was misinterpreted as inefficiency in the conventional stochastic frontier model. Chavas (2008) estimates a state-contingent cost function using aggregated data from the United States (1949–1999 annual series). The results generated using this data provide empirical support for an output-cubical technology.

O'Donnell, Chambers and Quiggin (hereafter OCQ) have used simulated data to estimate a stochastic frontier which allows for state-allocable inputs. They show that, where technically efficient producers make state-contingent production plans under conditions of uncertainty, standard techniques of efficiency analysis such as Stochastic Frontier Analysis (SFA) and Data Envelopment Analysis (DEA), may produce spurious findings of inefficiency. An overly restrictive feature of the single-input model of OCQ is that the (single) input is *state-specific* in the sense that output realized in a particular state of Nature will be zero if none of the input has been allocated to that state.

Overall, this small set of empirical studies indicates that, in uncertain decision environments, conventional stochastic production frontier models can provide a restrictive and often unrealistic representation of the production process, and can lead to significantly biased estimates of measures of technical efficiency. However, in a state-contingent framework, such producers are judged to have merely encountered a state of Nature that is unfavorable, given their state-contingent production plan, and need not necessarily be inefficient. For example, a producer may choose to use a low level of pesticides because the expected return is negative. In states of Nature leading to a severe pest infestation, output will be low.[1]

In this article, we propose to i) generalize the state-allocable model of OCQ so that output in a particular state of Nature can still be non-zero even when none of the input has been allocated to that state (such an input is said to be *state-general*), ii) allow the OCQ model to accommodate additional inputs that cannot be allocated to different states, iii) show how this multiple-input state-allocable model can be estimated within a frontier

[1]Kumbhakar (2002) shows the importance of controlling for both risk and inefficiency in an expected utility framework.

framework, and iv) use the methodology to estimate levels of input-allocability and technical efficiency using farm data from Finland.

The paper is organized as follows. The theoretical model, which is an extension of OCQ (2006), is described in Section 2. In Section 3, we present the empirical application, including a discussion of model specification, description of data, and discussion of estimation results. Section 4 concludes.

2 Theoretical model

2.1 The technology

In OCQ (2006), the technology of production is modeled as follows:

$$\ln q_s = b^{-1}(\ln x_s - \ln a_s) \tag{1}$$

where q_s denotes output realized in state $s \in \Omega = (1, 2, \ldots, S)$ and x_s is the amount of input x allocated to state s. OCQ assume that the producer chooses x_s for all values of s before the uncertainty is resolved (that is, before s is known). The unknowns satisfy $b \geq 1$ and $a_s \geq 0$ for all s. The input is state-specific in the sense that output in state s is zero if no input has been allocated to that state.

The parameters a_s can be thought of as technical parameters that are specific to the production of output in state s. The parameter b is interpretable as the cost flexibility associated with production in state s (OCQ, 2006) and will thus indicate the extent to which the state-contingent outputs are substitutable: as $b \to 1$, the state-contingent production transformation curve tends to a linear function which corresponds to perfect substitutability between state-contingent outputs. As $b \to \infty$, the state-contingent transformation curve is Leontief in outputs, indicating that no substitution is possible and that the production technology is output-cubical. The restriction that $b \geq 1$ implies that the technology always exhibits non-increasing returns to scale (OCQ, 2006). As explained in the introduction, this model is likely to be too restrictive if estimated using real-world data, if only because pure state-specific inputs seldom exist. Accordingly, we consider the following more flexible model:

$$\ln q_s = b^{-1}[\ln(\theta x - \theta x_s + x_s) - \ln a_s] \tag{2}$$

where $0 \leq \theta \leq 1$. The parameter θ is a measure of how output in state s responds to input allocations to states other than s. Three cases are of special interest. First, if $\theta = 0$ the model collapses to the state-specific model of OCQ, as described in (1). Second, if $\theta \neq 0$ then the input is state-general in the sense that output in state s is non-zero for any non-zero level of total input, even if none of the input has been allocated to state s. Finally, if $\theta = 1$ then the technology is output-cubical for all values of $b \geq 1$. Specifically, if $\theta = 1$ then

$$\ln q_s = b^{-1}[\ln x - \ln a_s]. \tag{3}$$

In this restrictive model, the parameter b is also the inverse of the elasticity of output with respect to x.

2.2 Firm behavior

This section is incidental to the empirical application, but provides some insights into the properties of our assumed technology. For related details and discussion, see OCQ (2006).

The technology (2) can be written in the following equivalent form:

$$q_s = \left(\frac{\theta x - \theta x_s + x_s}{a_s}\right)^{1/b}. \tag{4}$$

The *ex post* net return in state of Nature s is defined as

$$y_s = q_s - w\sum_{s=1}^{S} x_s \tag{5}$$

where w is the normalized price of input x. We assume the firm chooses $x_1, \ldots, x_S$ and $q_1, \ldots, q_S$ to

$$\max W(y) \text{ s.t. } q_s = \left(\frac{\theta x - \theta x_s + x_s}{a_s}\right)^{1/b} \quad \forall s \in \Omega \tag{6}$$

where $W(.)$ is a benefit function and $y = (y_1, \ldots, y_S)'$ is the vector of state-contingent net returns. The benefit function is assumed to be strictly increasing in y and suitably smooth to allow differential changes in its arguments (OCQ, 2006).[2] The first-order conditions for an

[2] An expected utility function is one example of such an objective function.

interior solution can be solved to yield the following system of S equations in S unknowns:

$$bw = \sum_{s=1}^{S} \Big[\pi_s \Big(\frac{\theta x - \theta x_s + x_s}{a_s}\Big)^{1/b} \Big(\frac{\theta - \theta\delta_{ms} + \delta_{ms}}{\theta x - \theta x_s + x_s}\Big)\Big] \qquad \text{for } m = 1, \dots, S, \tag{7}$$

where δ_{ms} is the Kronecker delta and $\pi_s \equiv W_s(y)/\sum_{s=1}^{S} W_m(y)$ is a risk-neutral probability. This result says that any efficient choice for a rational firm with an objective function defined over net-returns can be viewed as though it were generated by a risk-neutral firm with subjective probabilities given by $\pi = (\pi_1, \dots, \pi_S)'$. This means it is possible to analyze the behavior of any firm *as if* it were risk-neutral.

In the case where $\theta = 1$, the technology is output-cubical and the system (7) becomes:

$$x = \Big(\frac{\sum_{s=1}^{S}[\pi_s a_s^{-1/b}]}{bw}\Big)^{\frac{b-1}{b}} \qquad \text{for } m = 1, \dots, S. \tag{8}$$

Thus, the firm only chooses the amount of total input. In the case where $\theta = 0$, the technology is state-allocable and the inputs are state-specific. OCQ (2006) discuss the nature of input choices in this case.

2.3 The case of multiple inputs

Let $z = (z_1, \dots, z_K)'$ denote a vector of K non-negative exogenous variables, including inputs that are not state-allocable. A simple way of incorporating these variables into the analysis is to simply replace a_s in (2) with $a_s f(z)$. Then the technology takes the form:

$$\ln q_s = b^{-1}[\ln(\theta x - \theta x_s + x_s) - \ln a_s - \ln f(z)]. \tag{9}$$

3 Empirical illustration

3.1 Specification of the model

Using equation (9), observed log-output is given by

$$\ln q = b^{-1} \sum_{s=1}^{S} e_s \ln(\theta x - \theta x_s + x_s) - \sum_{s=1}^{S} e_s b^{-1} \ln a_s - b^{-1} \ln f(z) \tag{10}$$

where e_s is a dummy variable that takes the value 1 when Nature chooses state s (and 0 otherwise). In our empirical illustration we assume $\ln f(z)$ is linear:

$$\ln f(z) = \sum_{k=1}^{K} \alpha_k z_k \tag{11}$$

where α_k $(k = 1, \ldots, K)$ a set of unknown parameters to be estimated. Combining and then embedding (10) and (11) in a stochastic frontier framework gives:

$$\ln q = \sum_{s=1}^{S} \beta_s e_s + \beta \sum_{s=1}^{S} e_s \ln(\theta x - \theta x_s + x_s) + \sum_{k=1}^{K} \gamma_k z_k + v - u \tag{12}$$

where $\beta = b^{-1}$; $\beta_s = -b^{-1} \ln a_s$; $\gamma_k = -\beta\alpha_k$; v is a symmetric random error representing noise; and $u \geq 0$ is a one-sided random variable representing technical inefficiency. We assume that the v's are independently and identically distributed normal random variables with mean zero and variance σ_v^2, and that the u's are independently and identically distributed half-normal random variables with scale parameter σ_u^2. In what follows, we will adopt the following notation: $\sigma^2 = \sigma_v^2 + \sigma_u^2$ and $\lambda = \sigma_u^2/(\sigma_v^2 + \sigma_u^2)$. Recall that the unknowns in (10) satisfy $b \geq 1$, $a_s \geq 0$ and $0 \leq \theta \leq 1$. Thus, the parameters in (12) must satisfy $0 \leq \beta \leq 1$ and $0 \leq \theta \leq 1$. Equation (12) is in the form of a conventional stochastic frontier model except that it is nonlinear in the parameters. Thus, estimation is straightforward in any nonlinear sampling theory or Bayesian framework.

3.2 Data

The data have been taken from the Finnish profitability bookkeeping records (which serve as a basis for the European Commission's Farm Accountancy Data Network survey) and cover the 1998–2003 period. The data comprise annual farm-level observations on acreage allocated to each crop, crop output, and expenditures on labor, pesticides and fertilizers.[3] The sample used in our analysis considers specialized grain farmers from southern regions in Finland, the main grain production area in the country. These data were complemented by weather data (rainfall, temperature, and the starting date of the growing season) for each province produced by the Finnish Meteorological Institute. Data on input and output

[3] As is often the case with agricultural data sets, input data are not available by crop.

prices have been collected from Finnish Agriculture and Rural Industries, an annual report of Finnish agriculture. Our sample is an unbalanced panel of 274 farmers from 17 provinces over the 1998–2003 period, making a total of 1,020 observations. For greater details on the data, see Koundouri et al. (forthcoming).

In our model, the output variable is an implicit quantity index obtained by dividing the total value of production of wheat, barley and oats by an output price index. We consider five inputs: land (x), labor (which corresponds to total working hours in crop production, including both hired labor and family labor) (z_1), capital (defined as the total value of fixed assets on the farm) (z_2), fertilizers (z_3) and plant protection (z_4). We also control for technical change by including a time trend variable (z_5).

Finnish farmers face different types of risk but production risk due to unstable weather conditions (frost may occur in the middle of the summer) is recognized as the main source of risk for cereal producers in Finland.[4] Cereal producers have been found to be risk-averse before Finland's European Union (EU) accession in 1995 and risk-lovers after, due to the increase in the non-random part of farm income generated by the policy change after application of the Common Agricultural Policy (Koundouri et al., forthcoming).[5] For the period under consideration in this article (1998–2003), the risk premium has been estimated between -1 and -2 percent of farmer's profit (see Koundouri et al., Table 2). In this context, we believe that the assumption of farmers' risk-neutrality over the 1998-2003 period is reasonable.

Because of the primary role of production risk, we define (based on our discussions with Finnish grain specialists) the states of Nature in terms of two meteorological variables: the starting date of the growing season and the sum of rainfall in June.[6] The comparison of crop yields under different conditions (early, average, and late start of the growing season, and low, average and high sum of rainfall) permits identification of three states: a state of

[4] Liu and Pietola (2005) showed that yield volatility is large and dominates price volatility in the hedging decisions of Finnish wheat producers.

[5] After entering the EU, target prices were replaced by substantially lower intervention prices while direct area payments became the corner stone of agricultural support.

[6] The starting date of the growing season (measured as a number of days from January 1st) is defined as the period of each year with daily mean temperatures above +5 Celsius degrees, which is the temperature at which soil is sufficiently thawed for root activity to begin.

Nature that is most favorable to the growing of wheat ($s = 1$), a state of Nature that is most favorable to the growing of barley ($s = 2$), and a state of Nature that is most favorable to the growing of oats ($s = 3$), see Table 1.[7]

[Table 1 around here]

Table 1 reads as follows: an early start of the growing season combined with a low [respectively average, and high] rainfall in June is most favorable to barley [resp. oats, and barley]. That is, the highest yields are observed on average for barley [respectively oats, and barley]. An average starting date of the growing season is always favorable to wheat production. A late start of the growing season combined with a low [respectively average, and high] rainfall is most favorable to barley [respectively wheat, and wheat]. Hence, for each observation (a farmer in a specific year), based on the observation of the starting date of the growing season and the sum of rainfall in June in the province (we have 17 such provinces), we know whether the realized state of Nature was wheat-favorable, barley-favorable or oats-favorable. In Table 2, we report the number of farmers experiencing each of the three states, for each year covered by our sample.

[Table 2 around here]

In our model, only land (x) is regarded as state-allocable. Specifically, farmers are assumed to allocate the land input to the production of wheat, barley and/or oats, in line with subjective risk-neutral probabilities attached to states of Nature that are considered favorable to the production of each of those crops. Land allocated to wheat, barley and oats is denoted x_1, x_2, and x_3, respectively. For each farmer and each year, we have $x = x_1 + x_2 + x_3$, with $x_k \geq 0$ for $k = 1, 2, 3$. Basic statistics of the main variables of interest are shown in Table 3.

[Table 3 around here]

[7] The comparison of crop yields has been made on a sub-sample of observations since information on yields is missing for some farmers.

3.3 Estimation results

The most flexible production technology (FLEX model) that we estimate is given by

$$\ln q = \sum_{s=1}^{3} \beta_s e_s + \beta \sum_{s=1}^{3} e_s \ln(\theta x - \theta x_s + x_s) + \sum_{k=1}^{5} \gamma_k z_k + v - u \tag{13}$$

where $\beta = b^{-1}$; $\beta_s = -b^{-1} \ln a_s$; $\gamma_k = -\beta \alpha_k$. We then compare the FLEX model to three more restrictive models. The first of these is a state-allocable model in which inputs are state-specific (OCQ model) - this model corresponds to the FLEX model with θ constrained to be 0:

$$\ln q = \sum_{s=1}^{3} \beta_s e_s + \beta \sum_{s=1}^{3} e_s \ln(x_s) + \sum_{k=1}^{5} \gamma_k z_k + v - u. \tag{14}$$

We then estimate an output-cubical (OC model) model - this model corresponds to the FLEX model with θ constrained to be 1:

$$\ln q = \sum_{s=1}^{3} \beta_s e_s + \beta \sum_{s=1}^{3} e_s \ln(x) + \sum_{k=1}^{5} \gamma_k z_k + v - u. \tag{15}$$

Finally, we estimate the conventional frontier model (CF model):

$$\ln q = \beta_0 + \beta \ln(x) + \sum_{k=1}^{5} \gamma_k z_k + v - u. \tag{16}$$

Estimation results are shown in Table 4.

[Table 4 around here]

When estimating the FLEX model, the parameters β and θ were constrained to lie in the unit interval, but only the constraint on β was ever binding. The Maximum Likelihood estimate of θ was found using a combination of grid search and gradient methods and all t-ratios are thus conditional on θ (to conduct valid finite sample inference we would need to use a Bayesian approach). The parameters of interest θ and b have been estimated at 0.8573 and $1/0.9889 = 1.011$ respectively. As expected, θ is found to be different from 0, which indicates that land in our model is state-general, in the sense that output in state s is non-zero even if none of the land has been allocated to that state. For example, output will be strictly positive even for a farmer who planted only wheat and barley in an oats-favorable state. The value of b (close to one) indicates substitutability between state-contingent outputs.

The coefficients of the non-allocable inputs (labor, capital, fertilizers and plant protection) are all found to be positive but vary across specifications. In the FLEX model, the rate of technical change is estimated at 1.35 per cent per year. In the OCQ model, θ is constrained to be zero and b is found equal to 288. This model is however likely to be inappropriate for this data set, since we observe output to be non-zero in state s even when no land has been allocated to that state. In the OC and CF models, θ is constrained to be 1 and b is estimated at values very close to 1.

We compare the FLEX model to the three restricted specifications using a Likelihood-Ratio (LR) test: $LR = -2[\ln L_R - \ln L_U] \sim \chi^2(J)$ where $\ln L_R$ and $\ln L_U$ denote the maximized values of the restricted and unrestricted log-likelihood functions and J is the number of restrictions. The outcome of these tests indicates that the FLEX model is preferred to the three (restricted) models (OCQ, OC and CF) - see Table 5.

[Table 5 around here]

3.4 Elasticities of output and technical efficiency scores

The elasticity of output in state s with respect to the amount of input allocated to state k can be computed as follows:

$$\eta_{sk} \equiv \frac{\partial \ln q_s}{\partial \ln x_k} = b^{-1} \frac{\partial \ln(\theta x - \theta x_s + x_s)}{\partial x_k} \frac{\partial x_k}{\partial \ln x_k} = \frac{x_k(\theta - \theta\delta_{ks} + \delta_{ks})}{b(\theta x - \theta x_s + x_s)} \tag{17}$$

where δ_{ks} is the Kronecker delta. If $x_k > 0$ and $\theta > 0$ then $\eta_{sk} > 0$ for all s and k. The elasticity of total output with respect to the amount of input allocated to state k is $\eta_k \equiv \sum_{s=1}^{S} e_s \eta_{sk}$. The elasticity of total output with respect to z_k is:

$$\varepsilon_k \equiv \frac{\partial \ln q}{\partial \ln z_k} = \gamma_k z_k. \tag{18}$$

In Table 6, we report (estimated) elasticities of output in the three states with respect to the amount of land allocated to each of those states (η_{sk} for $s, k = 1, 2, 3$) as well as the elasticity of output with respect to the four non-allocable inputs (ε_k for $k = 1$ to 4). The elasticities have been evaluated at the sample means of x, z_1, z_2, z_3 and z_4 (see Table 3 for mean values). As for land allocated to wheat, barley, and oats, we report elasticities computed at $x_1 = x_2 = x_3 = \frac{1}{3}\bar{x}$ where $\bar{x}$ is the sample average of total land, in order to allow for a simpler discussion of elasticities.

[Table 6 around here]

Elasticities of output computed from the FLEX model have expected signs and reasonable magnitude. Elasticities of output with respect to land vary between 0.31 and 0.36, with higher values when land has been allocated to the crop for which conditions are the most favorable (η_{11}, η_{22}, η_{33}). More precisely, if the input is equally allocated between states then, for the FLEX model, ceteris paribus, a 1 per cent increase in x_s leads to a 0.36 increase in q_s and a 0.31 per cent increase in q_k for $k \neq s$; whereas in the more restrictive OC model, a 1 per cent increase in x_s leads to a 0.3312 per cent increase in q_k for all k and s.

The OCQ model produces elasticities that are quite far from elasticities obtained with the FLEX and OC models. Elasticities from the OCQ model may be low because the OCQ model considers that there is no output in state s if $x_s = 0$. In other words, this model forces $\partial \ln q_s / \partial \ln z_k = 0$ whenever $x_s = 0$. It means that there can be no output response to an increase in capital, for example, if no land was allocated to the realized state (and this happens a lot in our sample). In the FLEX model, we find that elasticities of output with respect to labor, capital, fertilizers, and plant protection are respectively 0.02, 0.11, 0.03 and 0.05.

We then compute individual technical efficiency (TE) scores in the four models and report some summary statistics in Table 7. In the last three columns of the table, we report summary statistics for the difference (in absolute value) between TE scores obtained with the FLEX model and TE scores obtained using the OCQ, OC and CF models, respectively. The average TE score in the FLEX model is 0.63, varying between from 0.04 to 0.94. The average TE in the FLEX model is only slightly higher than the average TE estimated in the OC and CF models, but estimated TEs do vary across models on an individual basis. The OCQ model appears to overestimate farmers' inefficiency, on average.

[Table 7 around here]

4 Conclusions

In this article, we present one of the first real-world empirical applications of state-contingent production theory. Our state-contingent behavioural model allows us to analyze production under both inefficiency and uncertainty without regard to the nature of producer risk preferences. Using farm data for Finland, we estimate a flexible production model that permits substitutability between state-contingent outputs. Our model extends the theoretical model described in OCQ (2006) by allowing for a state-general input as well as multiple non-allocable inputs. In our application, we treat land as a state-allocable input, and we specify four non-allocable inputs (labor, capital, fertilizers and pesticides). Uncertainty is represented by three states of Nature, defined in terms of climatic conditions (rainfall and start of the growing season): a wheat-favorable state, a barley-favorable state, and an oats-favorable state.

We test empirically, and reject, an assumption that has been implicit in almost all efficiency studies conducted in the last three decades, namely that the production technology is output-cubical. Our results indicate that a state-allocable state-contingent production model is preferred to the more restrictive output-cubical state-contingent model, as well as a conventional stochastic frontier.

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Tables

Table 1: Definition of crop-favorable states

	Starting date		
	Early	Average	Late
Low rainfall	barley	wheat	barley
Average rainfall	oats	wheat	wheat
High rainfall	barley	wheat	wheat

Table 2: Distribution of farmers across states, by year

Year	Wheat-favorable state (s=1)	Barley-favorable state (s=2)	Oats-favorable state (s=3)	Total
1998	123	16	28	167
1999	20	124	13	157
2000	26	33	102	161
2001	170	0	0	170
2002	150	0	20	170
2003	126	48	21	195
Total	615	221	184	1,020

Table 3: Descriptive statistics of the main variables

Variable	Unit	Mean	Std. Dev.	Min	Max
land (x)	*ha*	38.58	30.94	1.61	233.78
land to wheat (x_1)	*ha*	11.53	20.53	0	157.07
land to barley (x_2)	*ha*	18.55	23.27	0	211.76
land to oats (x_3)	*ha*	8.49	10.42	0	89.15
labor (z_1)	*hours/year*	876	789	0	12319
capital (z_2)	*quantity index*	199,219	155,220	4,989	1,022,397
fertilizers (z_3)	*quantity index*	3,968	4185	0	27,837
plant protection (z_4)	*quantity index*	1,837	2,422	0	25,027
trend (z_5)		3.59	1.74	1	6

Table 4: Estimation results for the four models

	FLEX		OCQ		OC		CF	
Parameter	Est.	t-ratio	Est.	t-ratio	Est.	t-ratio	Est.	t-ratio
β_1	6.2636	75.32	8.9998	107.36	6.1270	69.95	6.1331	69.42
β_2	-0.1714	-5.04	-0.0458	-0.95	-0.1302	-3.81	.	.
β_3	0.1066	3.03	0.0698	1.37	0.0874	2.46	.	.
β	0.9889	36.00	0.0035	2.37	0.9935	35.32	0.9836	34.62
γ_1	2.48E-05	0.96	2.00E-04	5.18	2.24E-05	0.86	1.90E-05	0.72
γ_2	5.30E-07	3.77	2.22E-06	12.08	5.65E-07	4.01	6.08E-07	4.23
γ_3	7.37E-06	1.35	6.94E-05	9.69	7.18E-06	1.30	7.74E-06	1.39
γ_4	2.60E-05	2.95	8.71E-05	7.34	2.82E-05	3.14	3.09E-05	3.42
γ_5	0.0135	1.63	0.0230	1.97	0.0147	1.76	0.0185	2.27
σ^2	0.5336	16.50	1.3085	17.20	0.5389	16.52	0.5598	16.60
γ	0.9066	56.21	0.9397	84.91	0.9050	55.50	0.9098	58.30
b	1.0113		288.1080		1.0065		1.0167	
θ	0.8573		0.0000		1.0000		0.0000	

Table 5: LR tests - FLEX model against OCQ, OC and CF

	FLEX	OCQ	OC	CQ
Log-L	-640.0708	-1055.0253	-647.2202	-660.9343
LR against FLEX		829.9090	14.2988	41.7270
p-value		0.0000	0.0002	0.0000

Table 6: Elasticities of output

	FLEX	OCQ	OC	CF
η_{11}	0.3643	0.0035	0.3312	0.9836
η_{12}	0.3123	0.0000	0.3312	
η_{13}	0.3123	0.0000	0.3312	
η_{21}	0.3123	0.0000	0.3312	
η_{22}	0.3643	0.0035	0.3312	0.9836
η_{23}	0.3123	0.0000	0.3312	
η_{31}	0.3123	0.0000	0.3312	
η_{32}	0.3123	0.0000	0.3312	
η_{33}	0.3643	0.0035	0.3312	0.9836
ε_1 (labor)	0.0217	0.1752	0.0196	0.0166
ε_2 (capital)	0.1056	0.4413	0.1126	0.1212
ε_3 (fertilizers)	0.0293	0.2755	0.0285	0.0307
ε_4 (plant protection)	0.0478	0.1600	0.0518	0.0568

Table 7: Technical efficiency scores

	FLEX	OCQ	OC	CF	|FLEX-OCQ|	|FLEX-OC|	|FLEX-CF|
Mean	0.6309	0.5056	0.6298	0.6244	0.1552	0.0165	0.0281
St. Dev.	0.1828	0.2234	0.1827	0.1861	0.1313	0.0118	0.0235
Max	0.9429	0.9326	0.9483	0.9493	0.7197	0.0711	0.1164
Min	0.0381	0.0148	0.0372	0.0369	0.0011	0.0001	0.0001