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Abstract

We look at the consequences of allowing public health insurance (PuHI) to be voluntary when its coverage can be supplemented in the market. PuHI redistributes with respect to risk and income, and the market is affected by adverse selection. We argue that making PuHI voluntary does not lead to its collapse since there are always individuals participating in it. Additionally, in some cases, a voluntary PuHI scheme creates an increase in market efficiency because participation in it becomes a sign of an individual's type. The welfare consequences depend on the status quo. If in the status quo there is no political support for a compulsory PuHI, making it voluntary constitutes a Pareto improvement, and in some cases all individuals are strictly better off. If, instead, the status quo implements compulsory PuHI, making it voluntary then results in less redistribution.

JEL classification: H23, H42, H50, D72

Keywords: Public health insurance, adverse selection, majority voting

1 Introduction

Many health care systems rely on health insurance as a form of financing health care. Health insurance may be public, if financed through general taxation and/or social security contributions, or private, if financed through private non-income related premiums. Therefore, most often, public health insurance involves redistribution across income and risk levels, which is not the case for private health insurance. Both types of insurance also diverge in terms of their degree of obligation. Public health insurance is usually mandatory, whereas in most OECD countries individuals may choose whether or not they want to insure privately, but the opposite may exist as well. Additionally, some countries mandate the purchase of health insurance irrespective of its nature, while others allow individuals to choose whether or not they wish to be insured against health accidents.[1]

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[1]Since there exist many different health insurance schemes across countries and as many nomenclature, we follow the taxonomy of health insurance types as proposed by OECD 2004.

It is often claimed the unfeasibility of a voluntary yet redistributive PuHI scheme co-existing with a voluntary private health insurance (PHI) market. If individuals were given the option to opt out of PuHI, those contributing to its redistribution would leave and join the PHI market. The result would be that no one would be willing to participate in public insurance.

Whilst in reality most PuHI schemes are indeed compulsory, there nevertheless exist some examples of voluntary schemes, such as, for example, the Chilean reform of the 1980s, the programs of some US states prior to the Affordable Care Act, and the schemes for the self-employed in Europe. These cases of voluntary PuHI (discussed in more detail in Section 1) naturally ask the question as to how their existence can be theoretically justified.

In this paper we explicitly analyze the coexistence of a voluntary and redistributive PuHI scheme whose coverage can be supplemented by the market.[2] As in many countries we assume that PuHI pools risk and is financed by payroll taxes. Additionally, public coverage is decided by majority voting. On the other hand, the PHI market is competitive, discriminates in risk and is affected by adverse selection, thus offering incomplete coverage to low-risk individuals (see Cawley and Philipson 1999, Cutler and Zeckhauser 1997, and Finkelstein and Poterba 2004 for empirical tests of adverse selection in health-related insurance markets). Adverse selection is indeed an example often cited of market failure that a mandatory public scheme can overcome (Arrow 1963, Johnson 1977). We instead focus on the implications of a voluntary PuHI scheme for coverage available in both the public and private sectors.

Our paper arguably makes a number of important contributions to the relevant literature. Firstly, we provide an analytical framework for the study of voluntary PuHI and show that its existence is theoretically guaranteed since there are always individuals willing to participate in it. Other authors have already looked at the interactions between the public and private provision of health insurance, although they have always assumed a public mandatory scheme (see Epple and Romano 1996, Gouveia 1997, Breyer 2001, and De Donder and Hindriks 2003).

Secondly, we realize that the change in nature of the public policy has implications not only for itself but also for the private market. Golosov and Tsyvinski 2007 and Chetty and Saez 2010 show that the welfare gains of PuHI are smaller when markets respond endogenously, thereby

[2]Our analysis could easily be extended to other sorts of public insurance as insurance against income loss due to disability or unemployment.

stressing that markets should not be taken as given. Our contribution is in the same vein. More precisely, we demonstrate that in some cases a voluntary PuHI scheme can increase the efficiency of the PHI market. In this respect Sapelli and Vial 2003 find evidence of self-selection against Chilean public insurance. However, what has not yet been asserted in the literature is the market use of this information, implying an increase in private coverage when compared to the exclusive PHI system.

Thirdly, we draw conclusions about the welfare implications of reforming PuHI into a voluntary scheme. In our setup, the status quo can be a society with either some level of compulsory PuHI or no PuHI at all. In the former case, PHI market coverage is increased at the cost of less redistribution. In the latter case there is room for a Pareto improvement: private coverage may be increased to the benefit of more redistribution.

Finally, in our setup public coverage is decided by majority voting and thus we are able to illustrate how the non-separability of preferences, with respect to public and private provision of insurance, affects the political outcome. While this mechanism has already been noted for the top-up of compulsory public insurance,[3] we now present evidence for the top-up of voluntary PuHI. The political consequence is that there is no opposition to voluntary public coverage. This point has already been raised by Congleton 2007 who uses voluntarism to explain why individuals with different insurance needs adopt, and politically support, states with different PuHI schemes. In our setup, voluntarism allows individuals to signal their type and therefore get a higher coverage in the market. Consequently, a voluntary PuHI scheme not only never collapses but, furthermore, has stronger political support than a compulsory scheme.

Our paper is set out as follows. Section 1 describes two examples of voluntary PuHI schemes, Section 2 introduces the setup, and in Section 3 we analyze the timing of decisions. We show, in Section 3.3, how private coverage can be improved if contracts are conditioned to participation in PuHI, and, in Section 3.4, how there might exist unanimous political support for full PuHI. Section 4 focuses on the welfare implications of making PuHI voluntary, and Section 5 concludes.

[3]See Epple and Romano 1996, Gouveia 1997, Fernandez and Rogerson 1999, Casamatta et al. 2000, Breyer 2001, Besley and Coate 2003, and De Donder and Hindriks 2003 among others.

2 Voluntary public health insurance cases

In 1981, Chile turned its compulsory PuHI system into a voluntary one, where dependent workers are required to insure a minimum of their income (currently 7%), but can do so with either public insurance through the *Fondo Nacional de Salud* (FONASA) or in the PHI market through the *Instituiciones de Salud Previsional* (ISAPRES). The FONASA is financed both by the social contributions of the enrollees and by general taxation. The first years following the reform were characterized by an increase in the size of the private sector, although this tendency was reversed after the mid-1990s. Indeed, in 2009 PuHI covered almost 79% of the population. The scale of Chilean PuHI after 28 years demonstrates the feasibility of a voluntary, yet redistributive, PuHI scheme.[4]

Another high-profile case of voluntary PuHI can be found in the health insurance programs developed in some US states before the implementation of the Affordable Care Act. These programs, means tested and not universal, aimed to provide low-priced insurance to uninsured or deficiently insured individuals. Examples of such programs are the Maryland Health Insurance Plan, Minnesota Care, New Jersey Health Insurance Plans, the Family Health Plus and Healthy NY in the state of New York, Adult Basic in Pennsylvania, and Vermont Health Access Plan.[5] Despite there being important differences between the programs, for the purpose of our analyses we stress their common features:*(i)* comprehensive coverage; and *(ii)* premiums often increase with the enrollee's (family) income and they do not discriminate on risk.

The features of these programs have been expanded to all states by the Affordable Care Act (March 2010). However, a remarkable difference is that while the above state insurance programs were voluntary, the Affordable Care Act *mandates* the purchase of insurance for the majority of the population, even if individuals may choose from amongst public and private insurance suppliers. This paper aims to illustrate a possible foundation for the emergence and sustainability of voluntary health insurance schemes, such as the ones cited above. The reality

[4]Cid et al. 2008 point out the introduction in 1994 of the Universal Access with Explicit Rights Plan (AUGE plan) as one of the reasons that have led to the latest decrease in the market share of PHI. The AUGE plan promoted uniform health care provision among the public and the private sectors and therefore insurance choices are not so driven by quality of care concerns. Statistics available at the Chilean *Ministério de Planificación* webpage, accessed in June 2011: *http://www.mideplan.gob.cl/casen/Estadisticas/salud.html*.

[5]This list is not exhaustive and aims to underline the programs most illustrative of our argument.

of the Affordable Care Act is of a different nature, and is not captured by the model we put forward.

3 The Setup

The economy is composed of individuals, insurance companies, and the government. Individuals choose the level of public coverage through majority voting, whether or not to participate in PuHI, and which (if any) PHI contract to buy. Insurance companies offer insurance contracts. Lastly, the government sets the public premium so that the budget is balanced in expected terms and implements the PuHI coverage elected. Each agents' role is discussed in detail in Section 3.

Individuals face two states of nature. They can be in good health or incur the same health damage d, with $d = 1$. Individuals differ both in terms of their income and probability of incurring the health damage. There are two levels of risk $\{\theta_L, \theta_H\}$ and two levels of income $\{w_L, w_H\}$, with $0 < \theta_L < \theta_H < 1$ and $1 < w_L < w_H < \infty$.[6] The set $\mathcal{U} = \{(\theta_L, w_L), (\theta_L, w_H), (\theta_H, w_L), (\theta_H, w_H)\}$ is composed of all possible types of individuals, being $\lambda_{ij} > 0$ the share of the population of risk θ_i and income w_j, with $\sum_i \sum_j \lambda_{ij} = 1$, with $i = \{L, H\}$ and $j = \{L, H\}$.

The information structure is as follows. Individuals, insurance companies and the government know the distribution of types in the economy (λ_{ij}), and individuals' incomes are verifiable.[7] However individual probabilities of accident are private information, neither observed by insurance companies nor the government, leading to adverse selection in the PHI market, as shown in Section 3.3.

Individuals are risk averse, and we model their utility using Yaari 1987's Dual Theory (DT). For Expected Utility (EU) the attitude towards risk is modelled with a transformation of wealth (keeping linearity in probabilities), whereas under DT it is by means of a transformation of

[6] As we want to focus specifically on the interactions of the public and the private sector as insurers, we assume exogenous labor supply and abstract from the discussion the relevance of health insurance when taxation is available. Blomqvist and Horn 1984, Rochet 1991, and Cremer and Pestieau 1996 have studied this problem and conclude that health insurance can improve upon taxation since it does not have an associated deadweight loss (in the absence of moral hazard) as does income taxation. Conversely, Breyer and Haufler 2000 argue in favor of leaving income redistribution to the tax system with a view to creating efficient insurance.

[7] This assumption enables us to design social contributions proportional to individuals' income and does not bring implications to the insurance market.

probabilities (keeping linearity in wealth). Risk aversion is represented by magnifying the perceived risk of bad outcomes.

For analytical tractability we adopt DT instead of EU.[8] Under DT the utility of an uninsured individual with income w_j, facing the loss of $d = 1$ with probability θ_i can be represented by:

$$\begin{aligned} V(\theta_i, w_j) &= \phi(\theta_i)(w_j - 1) + (1 - \phi(\theta_i))(w_j) \\ &= w_j - \phi(\theta_i), \end{aligned} \tag{1}$$

where $\phi(\theta_i) > \theta_i$ in order to capture individuals' risk aversion. De Donder and Hindriks 2003 define $\phi(\theta_i) = (1+\alpha)\theta_i$ with $0 \leq \alpha \leq (1 - \theta_H)/\theta_H$, where α denotes the degree of risk aversion, the same for all individuals. Assuming α to be independent of w, the authors are thus able to dissociate attitude towards risk from the marginal value of wealth, which simplifies the analysis greatly. The utility of an individual without insurance is thus:

$$V(\theta_i, w_j) = w_j - \theta_i(1 + \alpha). \tag{2}$$

Individuals pay the premium π in both states of nature, and in the bad state of nature they receive a benefit proportional to their loss: the insurance coverage δ, with $0 \leq \delta \leq 1$. PHI and PuHI differ with respect to both coverage and premium in the terms detailed in the next section. For the time being it should be noted that any insurance contract $\{\pi, \delta\}$ affects individuals' utility in the following way:

$$V(\pi, \delta; \theta_i, w_j) = w_j - \pi - \theta_i(1 + \alpha)(1 - \delta). \tag{3}$$

While we will discuss this point in detail in Section 3.4, it can already be noted that due to DT individuals have corner preferences with respect to δ. Finally, we use the superscript u to distinguish PuHI $\{\pi^u, \delta^u\}$ from a PHI contract $\{\pi, \delta\}$, and use the shortcut $V_{ij}(\cdot)$ for $V(\cdot; \theta_i, w_j)$.

[8] It is beyond the scope of this paper to contribute to the discussion as to which theory is most suitable to represent individuals' preferences under uncertainty. We refer the reader to Appendix 6.1 where DT is briefly presented. Also, it is of interest to look at Machina 2000 who test the robustness of EU exploring a non-expected utility general analysis, and Doherty and Eeckhoudt 1995, who compare EU with DT in particular, and stress that insurance is never a Giffen good under DT, in contrast to EU.

4 The Timing

We next analyze the timing of decisions in four stages. In **stage 1** individuals vote on the level of public coverage δ^u that provides them with the highest utility (see e.g. Caplan 2007 for a departure from rationality and, in particular, the introduction of voters' misconceptions and irrational beliefs). They take as given the structure of social contributions which increase with income. Implicitly we are assuming that the redistributive role of PuHI is not questioned but that individuals have a political say on the level of public coverage. This assumption can be justified by noting that while the level of public coverage is more often subject to political discussion and changes, the redistributive degree of PuHI is decided at earlier stages, often referred to as the constitutional stage, and once implemented is costly to change.[9]

We consider only a simple majority voting rule. We do not discuss which type of political model best mimics the political choice of public coverage and simply focus on the political support for a given level of PuHI. Our choice is mainly driven by the fact that under DT individuals have corner preferences with respect to δ^u. Consequently our results hold for other models of political decision as, for example, an indirect voting ruled by two parties.

Knowing the level of public coverage elected at stage 1, in **stage 2** insurance companies compete in the offering of insurance contracts. This timing assumes that the market adjusts to the elected value of public coverage. Unlike PuHI, PHI discriminates on risk. Indeed, since an individual's probability of accident is private information, adverse selection arises. Therefore the contract intended for low-risk individuals is of incomplete coverage in order to prevent high-risk individuals from buying it (Rothschild and Stiglitz 1976). The novelty of our approach is the market use of the information concerning participation in PuHI that minimizes the adverse selection problem. We find that the market coverage of low-risk individuals does not need to be reduced as much as when compared to the one arising under an exclusive private system.

In **stage 3** individuals choose simultaneously whether or not to participate in PuHI and which private contract to buy, knowing the level of public coverage and private contracts. These are simultaneous decisions because contracts may be conditional on participation in PuHI. We

[9]For other authors using the same assumption see Gouveia 1997, Epple and Romano 1996, Casamatta et al. 2000, Breyer 2001, and De Donder and Hindriks 2003. Also of interest is the work by Cutler and Johnson 2004 which examines the factors at the origin of different degrees of redistribution across PuHI schemes.

assume that similar individuals act as a group and take a single decision as regards participation in PuHI and the market, given others' optimal participation choices. Also, we assume that type (θ_i, w_j) individuals realize the effect of their participation on the public pool of risks and incomes. For future reference, let the indicator p_{ij} take the value one if (θ_i, w_j) individuals decide to participate in PuHI, and zero otherwise. Additionally, let $\mathbf{p} = (p_{LL}, p_{LH}, p_{HL}, p_{HH})$ represent the vector of all individuals' decisions.[10]

Stage 4 is a verification stage. At this stage δ^u elected in stage 1 and participation in PuHI, vector $\mathbf{p}$ of stage 3, are observable. Therefore the government is able to charge contributions to those participating in PuHI and thus implement the public policy. Additionally, the market is able to sell contracts conditional on individuals' participation in PuHI.

We adopt the equilibrium concept of subgame-perfect Nash Equilibrium. Thus, we now solve the problem from the last stage to the first.

4.1 Stage 4: Implementation of PuHI and market contracts

We emphasize that this is simply a verification stage. PHI contracts are sold conditional on individuals' choices with respect to participation in PuHI, which is observable at this stage. Additionally, the government implements the public policy. It charges social contributions paid in both states of nature to those individuals participating in PuHI. In exchange they are entitled to the insurance coverage $\delta^u(\mathbf{p})$ in case of health damage, as elected at stage 1.

As in most real world systems, we assume PuHI to be financed by payroll taxes (see Casamatta et al. 2000, and De Donder and Hindriks 2003, among others). The premium is thus proportional to individuals' income and not related to individual risk. The public policy $\{\pi^u, \delta^u\}$ satisfies the budget balance rule such that the social contributions equal expected payout. The social contribution of an individual with income w_j is given by:

$$\pi_j^u(\mathbf{p}) = \frac{w_j}{w_{\mu|\mathbf{p}}} \theta_{\mu|\mathbf{p}} \delta^u(\mathbf{p}), \tag{4}$$

with $w_{\mu|\mathbf{p}}$ being the mean income of those participating in PuHI, i.e., conditional on the choices

[10] A group of similar individuals can be viewed as a union in which individuals sharing the same characteristics act as a unit. Alternatively, we could assume that decisions are taken individually. Depending on the parameters of the model, under this assumption we also expect equilibria in which all similar individuals choose the same action. In order to support our argument, we would focus on such symmetric equilibria. This is the reason why, for ease of understanding, that we prefer to focus from the beginning on the decision process of a unit of similar individuals.

of participation given by $\mathbf{p} = (p_{LL}, p_{LH}, p_{HL}, p_{HH})$. For example, if $\mathbf{p} = \mathbf{p}^1 \equiv (1, 0, 1, 1)$, $w_{\mu|\mathbf{p}^1}$ is the mean income conditional on all individuals participating in PuHI except low-risk-high-income ones. The same applies to $\theta_{\mu|\mathbf{p}}$. Unconditional risk and income means are denoted by, respectively, θ_μ and w_μ.

Besides the inherent insurance redistribution from those in the good state of nature to those in the bad state, PuHI redistributes with respect to both income and risk. It redistributes with respect to income because the rich pay more than the poor whereas coverage is uniform. It redistributes with respect to risk, because social contributions do not depend on risk and high-risk individuals receive the insurance benefit more often (θ_H of the times) than low-risk ones (only θ_L of the times). Obviously, redistribution occurs only among participants in PuHI.[11]

At previous stages, individuals anticipate the structure of social contributions as given by (4), and that the public policy is implemented and private contracts are sold, conditional on their decision to participate in PuHI.

4.2 Stage 3: Participation in PuHI and in the market

At stage 3 the four groups decide, simultaneously, whether or not to participate in PuHI and the PHI market. Participating in PuHI means that they can supplement it on the market. Opting out of PuHI means relying solely on private coverage.

The private contracts are as follows. In exchange for a premium π, insurance companies offer the coverage $\delta - \delta^u(\mathbf{p})p_{ij}$, where δ represents total coverage from which public coverage δ^u is deducted for those individuals who benefit from it, i.e., indicator $p_{ij} = 1$ (otherwise $p_{ij} = 0$). Given PuHI $\{\pi^u_j(\mathbf{p}), \delta^u(\mathbf{p})\}$ and the private contracts $\{\pi, \delta - \delta^u(\mathbf{p})p_{ij}\}$, the utility of a (θ_i, w_j) individual is:

$$V_{ij}\Big(\pi^u_j(\mathbf{p}), \delta^u(\mathbf{p}), \pi, \delta - \delta^u(\mathbf{p})p_{ij}\Big) = w_j - \pi - \pi^u_j(\mathbf{p})p_{ij} - (1+\alpha)\theta_i(1-\delta). \tag{5}$$

Following Rothschild and Stiglitz 1976, the contract $\{\pi, \delta - \delta^u p_{ij}\}$ depends on the risk type θ_i. Additionally, we argue that all individuals' choices regarding participation in PuHI, i.e., the whole vector $\mathbf{p}$ and not only p_{ij}, provide valuable information for the design of private contracts. Competition and separating contracts imply that the contract intended to (θ_i, w_j)

[11] In reality the government can use either social contributions or the benefits to balance the budget constraint. We opted for the first possibility.

individuals, $\{\pi_{ij}, \delta_{ij} - \delta^u(\mathbf{p})p_{ij}\}$, is actuarially fair:

$$\pi_{ij}(\mathbf{p}) \;=\; \theta_i\Big(\delta_{ij}(\mathbf{p}) - \delta^u(\mathbf{p})p_{ij}\Big). \tag{6}$$

Hence, taking the public policy and the contracts as given, individuals choose the combination of whether to participate in PuHI p_{ij} and the private contract $\{\pi_{ij}, \delta_{ij} - \delta^u(\mathbf{p})p_{ij}\}$ that delivers them more utility and satisfies the rationality constraint (RC). They solve the problem:

$$\max_{p_{ij},\{\pi_{ij},\delta_{ij}\}} V_{ij}\Big(\pi^u_j(\mathbf{p}), \delta^u(\mathbf{p}), \pi_{ij}(\mathbf{p}), \delta_{ij}(\mathbf{p}) - \delta^u(\mathbf{p})p_{ij}\Big)$$
$$s.t. \quad V_{ij}\Big(\pi^u_j(\mathbf{p}), \delta^u(\mathbf{p}), \pi_{ij}(\mathbf{p}), \delta_{ij}(\mathbf{p}) - \delta^u(\mathbf{p})p_{ij}\Big) \geq V_{ij}\Big(\pi^u_j(\mathbf{p}), \delta^u(\mathbf{p}), 0, 0\Big), \tag{RC}$$

where $\pi^u_j(\mathbf{p})$ is given by (4) and the two zeros on the RHS of the RC mean the non-purchasing of PHI. Insurance companies anticipate this behavior at Stage 2. The maximum premium an individual is willing to pay, the reservation premium, is defined when (RC) is binding:

$$w_j - \pi_{ij}(\mathbf{p}) - \pi^u_j(\mathbf{p})p_{ij} - (1+\alpha)\theta_i(1 - \delta_{ij}(\mathbf{p})) = w_j - \pi^u_j(\mathbf{p})p_{ij} - (1+\alpha)\theta_i(1 - \delta^u(\mathbf{p})p_{ij}).$$

The reservation premium is thus $\pi^R_{ij}(\mathbf{p}) = (1+\alpha)\theta_i\Big(\delta_{ij}(\mathbf{p}) - \delta^u(\mathbf{p})p_{ij}\Big)$. The premium π^R_{ij} is the product among private coverage and $(1+\alpha)\theta_i$, i.e., above the actuarially fair premium, as given by (6). Hence, any risk averse individual ($\alpha > 0$) is willing to buy insurance above the actuarially fair premium.

When evaluating whether or not to participate in PuHI, a group of similar individuals takes as given the others' optimal choices. It incorporates the effect of its own choice on the public premium, because it changes the pooling of incomes and risks of the population covered by PuHI, respectively $w_{\mu|\mathbf{p}}$ and $\theta_{\mu|\mathbf{p}}$ in (4), and on the public coverage elected in stage 1. Moreover, it also affects the structure of private contracts, as shown in Section 3.3. With these assumptions in mind, we present the definition of the Nash equilibrium of groups' simultaneous decision regarding participation in PuHI.

Definition 1 *Given δ^u elected at stage 1 and the private contracts $\{\pi_{ij}, \delta_{ij} - \delta^u p_{ij}\}$ offered at stage 2 with $i, j = \{L, H\}$, let $\mathbf{p}^* = (p^*_{LL}, p^*_{LH}, p^*_{HL}, p^*_{HH})$ represent the vector composed by all groups' choices regarding participation in PuHI. Also, let $\tilde{\mathbf{p}}_{ij}$ represent the vector of the same choices except that (θ_i, w_j) group chooses $1 - p^*_{ij}$ instead of p^*_{ij}. The strategy profile $\mathbf{p}^*$ constitutes a **Nash equilibrium** in the choice of whether or not to participate in PuHI if for* *every* $i, j \in \{L, H\}$*:*

$$\begin{aligned} & V_{ij}\Big(\pi_j^u(\mathbf{p}^*), \delta^u(\mathbf{p}^*), \pi_{ij}(\mathbf{p}^*), \delta_{ij}(\mathbf{p}^*) - \delta^u(\mathbf{p}^*)p_{ij}^*\Big) \\ \geq \; & V_{ij}\Big(\pi_j^u(\tilde{\mathbf{p}}_{ij}), \delta^u(\tilde{\mathbf{p}}_{ij}), \pi_{ij}(\tilde{\mathbf{p}}_{ij}), \delta_{ij}(\tilde{\mathbf{p}}_{ij}) - \delta^u(\tilde{\mathbf{p}}_{ij})(1 - p_{ij}^*)\Big). \end{aligned} \tag{7}$$

Otherwise, group type (θ_i, w_j) *instead chooses* $1 - p_{ij}^*$ *and the Nash equilibrium is* $\tilde{\mathbf{p}}_{ij}$*.*

Given other groups' choices, (θ_i, w_j) group considers participating in PuHI acknowledging that the public policy and the private contract adjust to the vector of decisions $\mathbf{p}^*$.[12] Assume that (θ_i, w_j) group decides to participate in PuHI, i.e. $p_{ij}^* = 1$. Therefore, it must be the case that they are better off than if they did not participate, given other groups' choices. Condition (7) can be written as

$$w_j - \pi_{ij}(\mathbf{p}^*) - \pi_j^u(\mathbf{p}^*) - (1+\alpha)\theta_i(1 - \delta_{ij}(\mathbf{p}^*)) \geq w_j - \pi_{ij}(\tilde{\mathbf{p}}_{ij}) - (1+\alpha)\theta_i(1 - \delta_{ij}(\tilde{\mathbf{p}}_{ij})),$$

where we can substitute $\pi_j^u(\mathbf{p}^*)$, given by (4), and the actuarially fair premia $\pi_{ij}(\mathbf{p}^*)$ and $\pi_{ij}(\tilde{\mathbf{p}}_{ij})$ as defined by (6). Considering the specific case of *low-risk* individuals, (7) is further reduced to (8) below:

$$\frac{\theta_L}{\theta_{\mu|\mathbf{p}^*}} \left(1 + \alpha \frac{\delta_{Lj}(\mathbf{p}^*) - \delta_{Lj}(\tilde{\mathbf{p}}_{Lj})}{\delta^u(\mathbf{p}^*)}\right) \geq \frac{w_j}{w_{\mu|\mathbf{p}^*}}, \tag{8}$$

given $\delta_{Lj}(\mathbf{p}^*)$ and $\delta_{Lj}(\tilde{\mathbf{p}}_{Lj})$ defined at stage 2. In order to interpret (8) consider that individuals are risk neutral ($\alpha = 0$). They decide to participate in PuHI if their *relative risk*, defined as the ratio of their own risk to the mean risk of the participants in PuHI, is higher than their *relative income*, defined analogously. PuHI is thus attractive to low-risk individuals if what they lose from risk redistribution is less than what they gain from income redistribution. However, since individuals are risk averse ($\alpha > 0$) and are affected by incomplete coverage in the PHI market, even low-risk-high-income individuals may be willing to participate in PuHI. Indeed, besides redistribution, the (potential) difference in total coverage arising from participating in PuHI also plays a role in individuals' decisions. Additionally, the greater α, the bigger this term.

As regards the decision of *high-risk* individuals, an analogous condition is found. Nevertheless, we can already incorporate the presumption that in stage 2 the market is offering

[12] It is important to remember that despite the fact that risk is private information, the proportion λ_{ij}, which is the size of each group, is common knowledge. Thus, this subgame is of complete information since what matters for the groups' decisions is the size of each group and not the type of each individual.

full coverage to high-risk individuals, regardless of their participation in PuHI, i.e., $\delta_{Hj}(\mathbf{p}^*) = \delta_{Hj}(\tilde{\mathbf{p}}_{Hj}) = 1$ (Rothschild and Stiglitz 1976 separating equilibrium). Condition (7) becomes:

$$\frac{\theta_H}{\theta_{\mu|\mathbf{p}^*}} \geq \frac{w_j}{w_{\mu|\mathbf{p}^*}}. \tag{9}$$

Since they are fully insured, what matters to high-risk individuals is whether what they gain in risk redistribution is greater than what they may lose in income redistribution. We can already state that high-risk-low-income individuals are always willing to participate in PuHI because they are always above the average risk and always below the average income of those participating in PuHI.

For future reference, if instead one considers $p^*_{ij} = 0$ then the analogous conditions to (8) and (9) for low- and high-risk individuals not be willing to participate in PuHI are, respectively:

$$\frac{\theta_L}{\theta_{\mu|\tilde{\mathbf{p}}_{Lj}}} \left(1 + \alpha \frac{\delta_{Lj}(\tilde{\mathbf{p}}_{Lj}) - \delta_{Lj}(\mathbf{p}^*)}{\delta^u(\tilde{\mathbf{p}}_{Lj})}\right) \leq \frac{w_j}{w_{\mu|\tilde{\mathbf{p}}_{Lj}}}, \tag{10}$$

$$\frac{\theta_H}{\theta_{\mu|\tilde{\mathbf{p}}_{Hj}}} \leq \frac{w_j}{w_{\mu|\tilde{\mathbf{p}}_{Hj}}}. \tag{11}$$

With two risk and two income levels, and each of the four groups with two possible actions, i.e., to participate in or to opt out of PuHI, there are then 16 combinations of groups' decisions as regards participation in PuHI, given $\delta^u \in [0, 1]$ elected at stage 1. Yet, not all these 16 combinations are equilibria satisfying Definition 1. For example, it is not possible to have high-risk-low-income individuals opting out of PuHI since they always benefit from risk and income redistribution. In other words, (11) never holds for (θ_H, w_L) individuals. Moreover, some of the Nash equilibria of this subgame never arise as an outcome of the game because they are not on the equilibrium. For instance, some combinations at stage 3 would only arise for insurance contracts that are never offered in equilibrium at stage 2. For this reason, we postpone to stage 2 the characterization of the equilibria of the subgame of stage 3. Still, one example to be used in the following is case 1, in which low-risk-high-income individuals opt out and all the others participate in PuHI, i.e., $\mathbf{p}^* = \mathbf{p}^1 \equiv (1, 0, 1, 1)$. For future reference the necessary conditions for case 1 to be an equilibrium are presented. These are just the case-specific conditions (8)-(11).

$$\frac{\theta_L}{\theta_{\mu|\mathbf{p}^1}}\left(1+\alpha\frac{\delta_{LL}(\mathbf{p}^1)-\delta_{LL}(\tilde{\mathbf{p}}^1_{LL})}{\delta^u(\mathbf{p}^1)}\right) \geq \frac{w_L}{w_{\mu|\mathbf{p}^1}}, \tag{12}$$

$$\frac{\theta_L}{\theta_{\mu}}\left(1+\alpha\frac{\delta_{LH}(\tilde{\mathbf{p}}^1_{LH})-\delta_{LH}(\mathbf{p}^1)}{\delta^u(\tilde{\mathbf{p}}^1_{LH})}\right) \leq \frac{w_H}{w_{\mu}}, \tag{13}$$

$$\frac{\theta_H}{\theta_{\mu|\mathbf{p}^1}} \geq \frac{w_L}{w_{\mu|\mathbf{p}^1}}, \tag{14}$$

$$\frac{\theta_H}{\theta_{\mu|\mathbf{p}^1}} \geq \frac{w_H}{w_{\mu|\mathbf{p}^1}}. \tag{15}$$

4.3 Stage 2: The market designs menus of insurance contracts

At this stage insurance companies design insurance contracts. The basic assumptions are that the insurance market is competitive, individuals can buy at most one PHI contract and risk is not observable.

To illustrate the role of information regarding participation in public health insurance in the design of PHI contracts and, in particular, how it affects the coverage offered, we focus on case 1. Case 1, with $\mathbf{p}=\mathbf{p}^1\equiv(1,0,1,1)$, illustrates how p_{ij} *is informative*. Since only low-risk-high-income individuals opt out of PuHI, the market can use this information to improve contracts. We contrast private contracts offered in the exclusive private solution to those offered in case 1 and conclude that private coverage offered to (θ_L, w_H) individuals is greater in case 1.

In case 1 when the market observes $p_{ij}=1$ it faces the same problem as in an exclusive private system. Indeed insurance companies are not able to distinguish types since both low- and high-risk individuals participate in PuHI. Following Rothschild and Stiglitz 1976, contracts are designed so that θ_H individuals are at least as well off with the contract $\{\pi_{Hj}, \delta_{Hj}-\delta^u\}$ than with the contract intended for (θ_L, w_L) individuals, $\{\pi_{LL}, \delta_{LL}-\delta^u\}$, conditional on participation in PuHI. Incentive compatibility implies:

$$\begin{aligned} &V_{Hj}\Big(\pi^u_j(\mathbf{p}^1), \delta^u(\mathbf{p}^1), \pi_{Hj}(\mathbf{p}^1), \delta_{Hj}(\mathbf{p}^1)-\delta^u(\mathbf{p}^1)\Big) \\ \geq\ &V_{Hj}\Big(\pi^u_j(\mathbf{p}^1), \delta^u(\mathbf{p}^1), \pi_{LL}(\mathbf{p}^1), \delta_{LL}(\mathbf{p}^1)-\delta^u(\mathbf{p}^1)\Big). \end{aligned} \tag{16}$$

We solve the (binding) incentive problem for δ_{LL}, substituting π_{Hj} and π_{LL} as given by (6), imposing no distortion at the top of total coverage, i.e., $\delta_{Hj}=1$, and noting that the public premium π_j^u, as defined by (4), is $(w_j/w_{\mu|\mathbf{p}^1})\theta_{\mu|\mathbf{p}^1}\delta^u(\mathbf{p}^1)$.[13] This leads to the solution:

$$\delta^*_{Hj}(\mathbf{p}^1) = 1 \quad j = \{L, H\}, \quad \text{and} \tag{17}$$

$$\delta^*_{LL}(\mathbf{p}^1) = \frac{\alpha\theta_H + \delta^u(\mathbf{p}^1)(\theta_H - \theta_L)}{\alpha\theta_H + (\theta_H - \theta_L)}, \text{or equivalently,} \tag{18}$$

$$\delta^*_{ij}(\mathbf{p}^1) = \frac{\alpha\theta_H + \delta^u(\mathbf{p}^1)(\theta_H - \theta_i)}{\alpha\theta_H + (\theta_H - \theta_i)}, \quad ij = LL, HL, HH, \quad \forall\delta^u(\mathbf{p}^1) \in [0,1]. \tag{19}$$

Private coverage is obtained by deducting δ^u from total coverage δ^*_{ij} and is offered at the actuarially fair premium. The two private contracts offered to participants in PuHI are: $\{\pi_{Hj}(\mathbf{p}^1), \delta^*_{Hj}(\mathbf{p}^1) - \delta^u(\mathbf{p}^1)\}$ intended for high-risk individuals and $\{\pi_{LL}(\mathbf{p}^1), \delta^*_{LL}(\mathbf{p}^1) - \delta^u(\mathbf{p}^1)\}$ intended for (θ_L, w_L) individuals. These contracts satisfy the rationality constraints and are thus actually purchased, as seen at stage 3.[14] Total coverage increases in δ^u, private coverage is always non-negative for $\delta^u \leq 1$, and PHI is partially crowded out by PuHI (see Eckstein et al. 1985, Cutler and Gruber 1996, De Donder and Hindriks 2003, and Gruber and Simon 2008, among others). Also, the coverage rate is increasing in risk aversion (α) and in the risk level.

The equilibrium coverage rates (17) and (18) hold for any $\delta^u \in [0,1]$. It is easy to check that (17) and (18) are also the equilibrium coverage rate of the exclusive private solution, with $\delta^u = 0$ (see Appendix 6.2). Hence, (18) with $\delta^u = 0$ is the benchmark to which we shall compare the equilibrium coverage rate offered to (θ_L, w_H) individuals, which we analyze next.

In this respect note that opting out of PuHI in case 1 becomes a perfect signal of being a low-risk-high-income individual, and the market can use this information. The contract intended for (θ_L, w_H) individuals must be incentive compatible so that all other groups prefer to participate in PuHI rather than opting out of PuHI, and to purchase the contract $\{\pi_{LH}(\mathbf{p}^1), \delta_{LH}(\mathbf{p}^1)\}$ intended for (θ_L, w_H) individuals. Note how this differs from guaranteeing that high-risk individuals prefer to top up public coverage with the contract intended for them, rather than with

[13] It should be made clear that despite individuals being characterized by two dimensions, screening takes place only over the probability of illness. Moreover, separation occurs with respect to total coverage, and not only private, because it is only effective if it takes into consideration the effect of total insurance on the individual's utility.

[14] The pair of contracts $\{\theta_i(\delta^*_{ij}(\mathbf{p}^*) - \delta^u(\mathbf{p}^*)), \delta^*_{ij}(\mathbf{p}^*) - \delta^u(\mathbf{p}^*)\}$ is undominated by other pair of separating contracts, in the sense that there is no other contract attractive to only one of the types inducing non-negative profits. Yet, as Rothschild and Stiglitz 1976 have shown, a separating equilibrium does not always exist, in which case there is no equilibrium in the competitive market. In what follows we assume that the share of high-risk individuals is high enough so that there is no profitable pooling insurance contract attractive to both high and low-risk individuals.

the contract intended for (θ_L, w_L) individuals. Or, as in an exclusive private system, from guaranteeing that high-risk individuals prefer the contract intended for them, rather than the one intended for low-risk individuals. In point of fact, the contract intended for (θ_L, w_H) individuals must satisfy a looser incentive compatibility constraint (IC) because high-risk individuals compare supplementing PuHI with relying solely on such private contract. Finally, note that given $\mathbf{p}^* = \mathbf{p}^1$, high-risk-high-income individuals are the first high-risk group willing to opt out of PuHI because they contribute to income redistribution, while high-risk-low-income individuals benefit from it. In the insurance company problem, the IC (16) is replaced by:

$$V_{HH}\Big(\pi_H^u(\mathbf{p}^1), \delta^u(\mathbf{p}^1), \pi_{HH}(\mathbf{p}^1), \delta_{HH}(\mathbf{p}^1) - \delta^u(\mathbf{p}^1)\Big) \geq V_{HH}\Big(0, 0, \pi_{LH}(\mathbf{p}^1), \delta_{LH}(\mathbf{p}^1)\Big). \tag{20}$$

Solving the binding IC for $\delta_{LH}(\mathbf{p}^1)$ with $\pi_H^u(\mathbf{p}^1)$ given by (4), the actuarially fair premia $\pi_{HH}(\mathbf{p}^1)$ and $\pi_{LH}(\mathbf{p}^1)$ given by (6), and using $\delta_{HH}^*(\mathbf{p}^1) = 1$ one finds the equilibrium total coverage rate:

$$\delta_{LH}^*(\mathbf{p}^1) = \frac{\alpha\theta_H + \delta^u(\mathbf{p}^1)\theta_H - \pi_H^u(\mathbf{p}^1)}{\alpha\theta_H + (\theta_H - \theta_L)}. \tag{21}$$

Thus the contract intended for (θ_L, w_H) individuals is $\{\pi_{HL}(\mathbf{p}^1), \delta_{LH}^*(\mathbf{p}^1)\}$. We can state the increase in market efficiency resulting from a voluntary PuHI scheme in Proposition 1:

Proposition 1 *The PHI contract intended for low-risk-high-income individuals offers a higher coverage in case 1, when (θ_L, w_H) individuals opt out of PuHI, than in the exclusive PHI system.*
Proof. See Appendix 6.2. ∎

Note that the total coverage $\delta_{LH}^*(\mathbf{p}^1)$ can be smaller than $\delta_{LL}^*(\mathbf{p}^1)$. Yet it is not worthwhile for (θ_L, w_H) individuals to pay the price of PuHI (as they redistribute both in the income and risk dimensions).

To summarize, if case 1 is anticipated, the market offers three contracts at an actuarially fair premium: (i) $\{\pi_{Hj}^*(\mathbf{p}^1), 1 - \delta^u(\mathbf{p}^1)\}$, intended for high-risk individuals; $(ii)\{\pi_{LL}^*(\mathbf{p}^1), \delta_{LL}^*(\mathbf{p}^1) - \delta^u(\mathbf{p}^1)\}$, intended for low-risk-low-income individuals; and $(iii)\{\pi_{LH}^*(\mathbf{p}^1), \delta_{LH}^*(\mathbf{p}^1)\}$, intended for low-risk-high-income individuals opting out of PuHI.

Having illustrated the increase in market coverage in case 1, we now characterize in Proposition 2 all the equilibria that can emerge at the stage 2 subgame. We then provide a numerical illustration of such equilibria.

Proposition 2 *Given $\delta^u(\mathbf{p}) \in [0,1]$ elected at stage 1 and the actuarially fair premium as defined by (6), the potential equilibria that can emerge at stage 2 subgame for $i,j = \{L,H\}$ are:*

- *The market offers the three actuarially fair contracts with premium given by (6):*
  - $\{\pi^*_{LL}(\mathbf{p}^1), \delta^*_{LL}(\mathbf{p}^1) - \delta^u(\mathbf{p}^1)\}$ *and* $\{\pi^*_{Hj}(\mathbf{p}^1), 1-\delta^u(\mathbf{p}^1)\}$, *if* $p_{ij} = 1$, *with* $\delta^*_{LL}(\mathbf{p}^1)$ *given by (18);*
  - $\{\pi^*_{LH}(\mathbf{p}^1), \delta^*_{LH}(\mathbf{p}^1)\}$, *if* $p_{ij} = 0$, *and* $\delta^*_{LH}(\mathbf{p}^1)$ *given by (21);*

  and the equilibrium of stage 3 is ***case 1*** *where (θ_L, w_H) individuals opt out, i.e.,* $\mathbf{p}^* = \mathbf{p}^1 \equiv (1,0,1,1)$*;*

- *The market offers the four actuarially fair contracts:*
  - $\{\pi^*_{HL}(\mathbf{p}^2), 1-\delta^u(\mathbf{p}^2)\}$ *and* $\{\pi^*_{LL}(\mathbf{p}^2), \delta^*_{LL}(\mathbf{p}^2) - \delta^u(\mathbf{p}^2)\}$, *if* $p_{ij} = 1$, *with* $\delta^*_{LL}(\mathbf{p}^2) = \frac{\alpha\theta_H + \delta^u(\mathbf{p}^2)(\theta_H - \theta_L)}{\alpha\theta_H + (\theta_H - \theta_L)}$*;*
  - $\{\pi^*_{HH}(\mathbf{p}^2), 1\}$ *and* $\{\pi^*_{LH}(\mathbf{p}^2), \delta^*_{LH}(\mathbf{p}^2)\}$, *if* $p_{ij} = 0$, *with* $\delta^*_{LH}(\mathbf{p}^2) = \frac{\alpha\theta_H}{\alpha\theta_H + (\theta_H - \theta_L)}$*;*

  and the equilibrium of stage 3 is ***case 2*** *where high income individuals opt out, i.e.,* $\mathbf{p}^* = \mathbf{p}^2 \equiv (1,0,1,0)$*;*

- *The market offers the three actuarially fair contracts:*
  - $\{\pi^*_{HL}(\mathbf{p}^3), 1-\delta^u(\mathbf{p}^3)\}$, *if* $p_{ij} = 1$*;*
  - $\{\pi^*_{HH}(\mathbf{p}^3), 1\}$ *and* $\{\pi^*_{Lj}(\mathbf{p}^3), \delta^*_{Lj}(\mathbf{p}^3)\}$, *if* $p_{ij} = 0$, *with* $\delta^*_{Lj}(\mathbf{p}^3) = \frac{\alpha\theta_H}{\alpha\theta_H + (\theta_H - \theta_L)}$*;*

  and the equilibrium of stage 3 is ***case 3*** *where only (θ_L, w_L) individuals participate in PuHI, i.e.,* $\mathbf{p}^* = \mathbf{p}^3 \equiv (1,0,0,0)$*;*

- *The market offers the two actuarially fair contracts* $\{\pi^*_{Lj}(\mathbf{p}^4), \delta^*_{Lj}(\mathbf{p}^1) - \delta^u(\mathbf{p}^1)\}$ *and* $\{\pi^*_{Hj}(\mathbf{p}^4), 1-\delta^u(\mathbf{p}^4)\}$, *with* $\delta^*_{LL}(\mathbf{p}^4) = \frac{\alpha\theta_H + \delta^u(\mathbf{p}^4)(\theta_H - \theta_L)}{\alpha\theta_H + (\theta_H - \theta_L)}$, *and the equilibrium of stage 3 is* ***case 4*** *with all participating in PuHI, i.e.,* $\mathbf{p}^* = \mathbf{p}^4 \equiv (1,1,1,1)$.

Proof. See Appendix 6.3. ■

The parameters of the economy determine which equilibrium emerges. Table 1 summarizes the results of a numerical illustration of Proposition 2. In this example we set $\theta_H = 0.5$, $w_L = 1$, $\lambda_{LL} = 0.25$, $\lambda_{HL} = 0.25$, and in the table $0.34 \leq \alpha \leq 1$, as imposed by the upper bound

$(1-\theta_H)/\theta_H$. We let vary θ_L from 0.1 to 0.4, and w_H from 1.1 to 2 with 0.1 increments; λ_{HH} varies from 0.05 to 0.45, with 0.05 increments; α varies with 0.1 increments. Results are robust to any value of $\delta^u(\mathbf{p}^k) \in [0.1, 1]$ and $\delta^u(\tilde{\mathbf{p}}^k_{ij}) \in [0.1, 1]$ with 0.01 increments, for $k = 1, 2, 3, 4$. PHI contracts are consistent with Proposition 2.

Having all individuals participating in PuHI, case 4, arises in equilibrium if risk and income inequality are relatively low ($\theta_L = 0.4$ and $w_H = 1.1$), so that neither high-income individuals nor low-risk ones lose too much with redistribution. Additionally, relatively few (θ_H, w_H) individuals ($\lambda_{HH} \in [0.05, 0.15]$) increase the likelihood of case 4. If instead the share of (θ_H, w_H) individuals increases ($\lambda_{HH} \in [0.2, 0.25]$)) to the detriment of (θ_L, w_H) individuals ($\lambda_{LH} \in [0.25, 0.3]$) case 1 arises in equilibrium. In fact, risk redistribution becomes too important and therefore (θ_L, w_H) individuals find profitable to opt out. High income individuals opting out of PuHI, case 2, arises when income inequality becomes high ($w_H \geq 1.3$), so that (θ_L, w_H) individuals opt out, but risk inequality is fairly low ($\theta_L = 0.4$), so that (θ_H, w_H) individuals do the same. Still, for (θ_L, w_L) individuals it pays to redistribute risk towards (θ_H, w_L) individuals in exchange for additional coverage. Finally, only high-risk-low-income individuals participate in PuHI, case 3, if neither risk ($\theta_L \leq 0.3$) nor income inequality pay for one of the other groups to participate in PuHI.

Low risk aversion makes equilibria in which low-risk individuals participate in PuHI less likely. For example, if $\alpha > 0.34$ cases 1-4 can arise in equilibrium but when it takes the value 0.34 only cases 2-3 can arise. Note as well that for $\alpha < 0.33$ (not reported on the table) only case 3 arises in equilibrium. Finally, all the simulations undertaken suggest that the four cases are mutually exclusive and there is an episode of no-equilibrium.

In all possible cases of Proposition 2 PuHI redistributes with respect to risk and/or income, except in case 3. Yet, it should be noted how different case 3 is from the case in which all individuals decide to opt out, which indeed cannot arise in equilibrium (see Appendix 6.3). The intuition is that with no one participating in PuHI, both low-risk individuals would find it profitable to unilaterally deviate towards participation with a view to signaling their type and increase their market coverage. Therefore, a very important corollary that emerges from the above proposition is that it cannot happen that all individuals decide to opt out of PuHI:

Corollary 1 *If PuHI is voluntary, there are always individuals participating in it.*

4.4 Stage 1: Political equilibrium

Finally, we consider the political process. At stage 1 individuals vote for their most preferred level of public coverage $\delta^u(\mathbf{p})$. The implemented level of public coverage is the one most voted for, obeying a simple majority rule. As individuals vote anticipating the outcomes of subsequent stages, their vote may change according to the equilibrium anticipated at the subgame of stage 2. For ease of presentation we focus first on case 1.

Considering case 1, we must distinguish the preferences of those participating in PuHI from the preferences of (θ_L, w_H) individuals who opt out. Starting with those participating in PuHI, their preferred level of public coverage is the value $\delta^u(\mathbf{p}^1)$ that maximizes:

$$V_{ij}\left(\pi_j^u(\mathbf{p}^1), \delta^u(\mathbf{p}^1), \pi_{ij}(\mathbf{p}^1), \delta_{ij}^*(\mathbf{p}^1) - \delta^u(\mathbf{p}^1)\right) \quad \text{for} \quad ij = LL, HL, HH. \tag{22}$$

Substituting $\pi_j^u(\mathbf{p}^1)$, the actuarially fair premium $\pi_{ij}(\mathbf{p}^1)$, and $\delta_{ij}^*(\mathbf{p}^1)$ given by, respectively, (4), (6), and (19) into the indirect utility function one obtains:

$$\begin{aligned} w_j - \left(\frac{\alpha\theta_H + \delta^u(\mathbf{p}^1)(\theta_H - \theta_i)}{\alpha\theta_H + (\theta_H - \theta_i)} - \delta^u(\mathbf{p}^1)\right)\theta_i - \frac{w_j}{w_{\mu|\mathbf{p}^1}}\delta^u(\mathbf{p}^1)\theta_{\mu|\mathbf{p}^1} \\ -(1+\alpha)\theta_i\left(1 - \frac{\alpha\theta_H + \delta^u(\mathbf{p}^1)(\theta_H - \theta_i)}{\alpha\theta_H + (\theta_H - \theta_i)}\right). \end{aligned} \tag{23}$$

Differentiating (23) with respect to $\delta^u(\mathbf{p}^1)$ one gets the (θ_i, w_j) individuals' most preferred level for public coverage. Given that DT implies linear preferences over δ^u their most preferred value is either zero or one (De Donder and Hindriks 2003). Indeed, (θ_i, w_j) individuals vote for $\delta^u(\mathbf{p}^1) = 1$ if $\partial V_{ij}/\partial\delta^u(\mathbf{p}^1) > 0$, i.e., if

$$\frac{\theta_i}{w_j}\left(1 + \alpha\frac{\theta_H - \theta_i}{\alpha\theta_H + (\theta_H - \theta_i)}\right) > \frac{\theta_{\mu|\mathbf{p}^1}}{w_{\mu|\mathbf{p}^1}}, \quad ij = LL, HL, HH. \tag{24}$$

Condition (24) is implied by (12), (14), and (15), necessary conditions for case 1 to be an equilibrium. Therefore (24) is always satisfied in case 1 and (θ_L, w_L), (θ_H, w_L), and (θ_H, w_H) individuals vote for $\delta^u(\mathbf{p}^1) = 1$.

As regards low-risk-high-income individuals who opt out, they vote for the value $\delta^u(\mathbf{p}^1)$ that maximizes

$$V_{LH}\Big(0, 0, \pi_{LH}(\mathbf{p}^1), \delta_{LH}^*(\mathbf{p}^1)\Big) = w_H - \theta_L\delta_{LH}^*(\mathbf{p}^1) - (1+\alpha)\left(1 - \delta_{LH}^*(\mathbf{p}^1)\right),$$

where $\delta^*_{LH}(\mathbf{p}^1)$ is given by (21). It can be checked that $\partial V_{LH}/\partial\delta^u(\mathbf{p}^1) > 0$, as implied by (15). Hence, (θ_L, w_H) individuals have a strong preference for $\delta^u(\mathbf{p}^1) = 1$. In other words, by opting out of PuHI (θ_L, w_H) individuals' private coverage is increasing in PuHI coverage. Indeed, the highest $\delta^u(\mathbf{p}^1)$ the loosest the IC constraint (20), since high-risk individuals have more to lose by opting out of PuHI. Therefore (θ_L, w_H) individuals also vote for full PuHI which is unanimously supported when the outcome of stage 2 is that of case 1.

This result contrasts with the voting outcome under a compulsory scheme. Indeed, suppose individuals vote over a compulsory level of δ^u and that the parameters of the economy are such that (24) holds for all but (θ_L, w_H) individuals. All would vote for $\delta^u = 1$ except (θ_L, w_H) individuals, who would vote for $\delta^u = 0$. Society would elect either full PuHI or, (θ_L, w_H) individuals would block PuHI, if in the majority. Hence, in a compulsory system, society elects either an exclusive public system ($\delta^u = 1$) or an exclusive private system ($\delta^u = 0$) (De Donder and Hindriks 2003). In contrast, if PuHI is voluntary and case 1 is anticipated, there is unanimous support for full public coverage.

We now address the other cases. Cases 2-3 are characterized by having high risk individuals both participating in and opting out of PuHI. Proceeding analogously as for case 1 it can be checked that those who anticipate participating in PuHI vote for $\delta^u = 1$. As for those who opt out, they are indifferent to its level. The reason is that, on the one hand, low-risk individuals' market coverage is not affected by δ^u, given that they cannot distinguish from high-risk individuals. On the other hand, high-risk individuals are always fully insured and thus also they are indifferent to δ^u. Still, being indifferent, those opting out of PuHI will not block it as they would under compulsoriness. Finally, it can be checked that if case 4 is anticipated there is unanimity for $\delta^u(\mathbf{p}^1) = 1$, by proceeding analogously as for case 1. Note that if the parameters of the economy are such that the outcome of stage 2 is that of case 4, the voluntary and compulsory PuHI schemes are equivalent and there would also be unanimity for $\delta^u(\mathbf{p}^4) = 1$ if PuHI were compulsory. Proposition 3 sums up these results.

Proposition 3 Nobody is against a voluntary PuHI coverage*, i.e., if PuHI is voluntary,* $\delta^u(\mathbf{p}^*) = 1$ *is always elected. In particular, when only the low-risk-high-income individuals opt out (case 1) and when all participate in PuHI (case 4), all individuals strongly prefer and* *vote for full PuHI* ($\delta^u(\mathbf{p}^m) = 1$, *with* $m = 1, 4$*).*

Some clarifications are required. First, despite the fact that the public sector is offering full public coverage, the PHI market is active as long as some individuals opt out of PuHI (cases 1-3). Second, the preference for corner solutions instead of interior solutions and, in particular, the fact that those participating in PuHI prefer $\delta^u(\mathbf{p}^*) = 1$, is obviously a consequence of DT. In this respect see, for example, Meltzer and Richard 1981 or Gouveia 1997 who analyze the preference for government programs in EU setups and whose voting equilibria result in interior values. However, we suspect that signalization of low-risk-high-income individuals in case 1 becoming more effective as public coverage increases is not exclusive to DT. We would also expect it to hold under EU. Third, in our setup, while PuHI is always implemented if voluntary, a majority can eventually block its implementation if compulsory.[15] Yet perhaps what most merits discussion are the welfare consequences of the compulsory and voluntary schemes. We address this question in the following section.

5 Welfare Analysis

We now investigate the impact of the mandatory and voluntary schemes on each group's welfare. To this end we contrast the two outcomes that may arise under a mandatory scheme $\delta^u = 1$ and $\delta^u = 0$, with the four possible cases of a voluntary scheme.

Suppose that in a compulsory scheme there is no political support for PuHI and thus $\delta^u = 0$ is elected. In this case, PuHI is politically feasible only if it is voluntary. Additionally no one would be worse off in any of the cases that may arise under a voluntary scheme. Indeed, those individuals relying solely on the (same) private coverage would be indifferent. The others would be better off if they benefited from income or risk redistribution in PuHI (because of a lower premium paid), and/or because private coverage increased (as in case 1 for (θ_L, w_H) individuals). Hence reforming an exclusive PHI system, i.e., a compulsory $\delta^u = 0$, into a voluntary PuHI scheme constitutes a Pareto improvement. This may serve to illustrate the emergence of the US state voluntary programs discussed in Section 2. Proposition 4 summarizes these results.

[15] Note how divided the US public opinion is with regards to the introduction of a mandate to purchase insurance for the majority of the population with the implementation of the Affordable Care Act (see for example Blendon and Benson 2009, among other works by the same authors).

Proposition 4 *If there is no political support for a compulsory level of PuHI, transforming it into a voluntary scheme constitutes a Pareto improvement. In particular, if the economy is such that only the low-risk-high-income individuals opt out of PuHI (case 1), all individuals are strictly better off under the voluntary scheme.*

Consider now that the outcome under compulsoriness is instead $\delta^u = 1$. This case may serve to illustrate the Chilean reform discussed in Section 2, in which a compulsory public scheme was reformed into a voluntary scheme. Three points should be emphasized. First, there is obviously less redistribution in a voluntary scheme as long as some individuals opt out of PuHI. The exception is when all participate in the voluntary PuHI (case 4) which is obviously equivalent to a compulsory scheme. Second, if case 1 arises, it is clear that low-risk-high-income individuals are better off and all others are worse off because they lose redistribution from (θ_L, w_H) individuals in both the risk and income dimensions. Third, cases 2 and 3 serve to illustrate the fact that even those opting out of PuHI can be worse off in the voluntary scheme rather than in the compulsory one. For instance in case 2, in which only low-income individuals participate in PuHI, (θ_H, w_H) individuals are worse off than in the compulsory scheme in which they benefit from redistribution from (θ_L, w_H) individuals. To return to the Chilean reform, it is therefore not obvious that those opting out are better off.

6 Conclusion

We provide a foundation for the sustainability of voluntary public health insurance (PuHI) schemes, which redistributes both in the risk and income dimensions, when the competitive private health insurance (PHI) market is affected by adverse selection. Additionally we contrast the welfare implications of the voluntary and compulsory schemes.

Several robustness analyses can be made. The first concerns Dual Theory (DT). The advantage of DT is that we are able to isolate the low-risk individuals' signalization mechanism, which leads to an increase of their market coverage. However, on the other hand, DT implies corner solutions with respect to insurance coverage. Although it may make matters more complex, it seems worthwhile trying to model individuals' preferences using Expected Utility (EU).

The second relates to enriching the analysis by considering other important aspects of the health system such as moral hazard (Arrow 1963), propitious selection (Hemenway 1990) or labor market effects (Buchmueller and Valletta 1996). Note that in a context of moral hazard incomplete coverage is seen as a means of increasing efficiency. Therefore it is difficult to estimate the implications of introducing moral hazard into the analysis.

Another obvious analytical extension is to consider a continuum of individuals. This would simplify the empirical testing of our predictions and facilitate the calibration of the model. In this respect, the study by Sapelli and Vial 2003 constitutes an important step in the empirical analysis of the Chilean reform. Their work could be developed to test the prediction of Proposition 1. We should find that the market menu of contracts after reform offers more coverage, understood as higher expenditures insured, to low-risk individuals. The same test could be done before and after the introduction of the US state voluntary health insurance programs.

Finally, voluntary solutions are of particular interest in developing countries, where tax systems are often more deficient, compromising compulsory public schemes, and where a major part of the (poor) population is often excluded from the market (see Pauly et al. 2006).

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7 Appendix

7.1 Dual Theory

Let wealth X be a random variable in the interval $[\underline{x}, \overline{x}]$ with distribution function $\Psi(x)$, and f a continuous real function defined on the unit interval, with $f' > 0$. Yaari 1987 shows that the DT utility function $U(X) = \int f(1-\Psi(x))d\Psi(x)$ can be written as $U(X) = \int_0^1 xf'(1-\Psi(x))d\Psi(x)$. The term $f'(1-\Psi(x))$ can be seen as weights summing up to one given to outcomes. If f is convex higher weights are given to bad outcomes. Following De Donder and Hindriks 2003 in our setup the utility of an uninsured individual with income w_j, facing the loss of $d = 1$ with probability θ_i can be represented by (1) in the text. Note that we follow Yaari 1987's presentation of DT while De Donder and Hindriks 2003 present the probability transformation on the function $\Psi(x)$, in which case the function f must be concave in order to capture risk aversion.

7.2 Proof of Proposition 1

In an exclusive PHI system incentive compatibility implies satisfying the following IC constraint (instead of (16)): $w_j - \pi_{Hj} - (1+\alpha)\theta_H(1-\delta_{Hj}) \geq w_j - \pi_{Lj} - (1+\alpha)\theta_H(1-\delta_{Lj})$, $j = \{L, H\}$, and with the actuarially fair premia π_{Hj} and π_{Lj} given by (6). Using $\delta^*_{Hj}(\mathbf{p}^1) = 1$, solving for δ_{Lj}, $\delta^*_{Lj} = \frac{\alpha\theta_H}{\alpha\theta_H + (\theta_H - \theta_L)}$ which equals (18) with $\delta^u(\mathbf{p}^1) = 0$. We now show that $\delta^*_{LH}(\mathbf{p}^1) > \delta^*_{Lj}$ is always satisfied if $\mathbf{p}^1$ is an equilibrium. Being $\delta^*_{LH}(\mathbf{p}^1)$ given by (21), $\delta^*_{LH}(\mathbf{p}^1) > \delta^*_{Lj}$ is simplified to $\frac{\alpha\theta_H + \delta^u(\mathbf{p}^1)\theta_H - \pi^u_H(\mathbf{p}^1)}{\alpha\theta_H + (\theta_H - \theta_L)} > \frac{\alpha\theta_H}{\alpha\theta_H + (\theta_H - \theta_L)}$. Replacing $\pi^u_H(\mathbf{p}^1)$ as given by (4), $\delta^*_{LH}(\mathbf{p}^1) > \delta^*_{Lj}$ is further simplified to $\frac{\theta_H}{\theta_{\mu|\mathbf{p}^1}} > \frac{w_H}{w_{\mu|\mathbf{p}^1}}$, which is implied by (15) ■

7.3 Proof of Proposition 2

Following Definition 1, (8) and (9) must hold if low-risk and high-risk individuals respectively want to participate in public health insurance. Instead, if they opt out, (10) and (11) respectively hold. We start by showing the combinations of groups' decisions that can never arise in equilibrium, out of the 16 possible combinations. **Combinations that are not equilibria:** It can be checked that, first, (11) does not hold for (θ_H, w_L) individuals for the following values of $\mathbf{p}$: $\{(0,0,0,1);(0,1,0,0);(1,0,0,0);(0,1,0,1);(1,0,0,1);(1,1,0,0);(1,1,0,1)\}$. Second, (9) does not hold for (θ_H, w_H) individuals for $\mathbf{p}=(0,0,1,1)$. Third, if (10) holds for (θ_L, w_H) individuals, it must hold as well for (θ_L, w_L) individuals. Thus, $\mathbf{p}^*$ cannot take the values $(0,1,1,0)$ and $(0,1,1,1)$. Fourth, (10) does not hold for both (θ_L, w_L) and (θ_L, w_H) individuals if $\mathbf{p}=(0,0,0,0)$. Hence the following values of $\mathbf{p}$ are not equilibria: $\{(0,0,0,1);(0,1,0,0);(1,0,0,0);(0,1,0,1);(1,0,0,1);(1,1,0,0);(1,1,0,1);(0,0,1,1);(0,1,1,0);(0,1,1,1);(0,0,0,0)\}$. We now show that the remaining combinations can arise in equilibrium. **Possible equilibria: Case 1**, (θ_L, w_H) opting out and all others participating in PuHI, with $\mathbf{p}^*=\mathbf{p}^1\equiv(1,0,1,1)$ is demonstrated in the text.

Case 2, high-income individuals opting out and low-income ones participating in PuHI, with $\mathbf{p}^*=\mathbf{p}^2\equiv(1,0,1,0)$. The following conditions must hold simultaneously (case-specific conditions (8)-(11)):

$$\frac{\theta_L}{\theta_{\mu|\mathbf{p}^2}}\left(1+\alpha\frac{\delta_{LL}(\mathbf{p}^2)-\delta_{LL}(\tilde{\mathbf{p}}^2_{LL})}{\delta^u(\mathbf{p}^2)}\right) \geq \frac{w_L}{w_{\mu|\mathbf{p}^2}}, \tag{25}$$

$$\frac{\theta_L}{\theta_{\mu|\tilde{\mathbf{p}}^2_{LH}}}\left(1+\alpha\frac{\delta_{LH}(\tilde{\mathbf{p}}^2_{LH})-\delta_{LH}(\mathbf{p}^2)}{\delta^u(\tilde{\mathbf{p}}^2_{LH})}\right) \leq \frac{w_H}{w_{\mu|\tilde{\mathbf{p}}^2_{LH}}}, \tag{26}$$

$$\frac{\theta_H}{\theta_{\mu|\mathbf{p}^2}} \geq \frac{w_L}{w_{\mu|\mathbf{p}^2}}, \tag{27}$$

$$\frac{\theta_H}{\theta_{\mu|\tilde{\mathbf{p}}^2_{HH}}} \leq \frac{w_H}{w_{\mu|\tilde{\mathbf{p}}^2_{HH}}}. \tag{28}$$

Note that $\delta_{Hj}(\mathbf{p}^2)=1$. The contract $\{\pi_{LL},\delta_{LL}\}$, intended for (θ_L, w_L) individuals, is offered conditional on participation in PuHI. Incentive compatibility implies that δ_{LL} is low enough to deter (θ_H, w_L) individuals to top up PuHI with δ_{LL}, and to deter (θ_H, w_H) individuals from participating in PuHI and topping it up with δ_{LL}. In the PHI company's problem the IC (16) is replaced by the following ICs:

$$V_{HL}\Big(\pi^u_L(\mathbf{p}^2),\delta^u(\mathbf{p}^2),\pi_{HL}(\mathbf{p}^2),\delta_{HL}(\mathbf{p}^2)-\delta^u(\mathbf{p}^2)\Big)\geq V_{HL}\Big(\pi^u_L(\mathbf{p}^2),\delta^u(\mathbf{p}^2),\pi_{LL}(\mathbf{p}^2),\delta_{LL}(\mathbf{p}^2)-\delta^u(\mathbf{p}^2)\Big), \tag{29}$$

$$V_{HH}\Big(0,0,\pi_{HH}(\mathbf{p}^2),\delta_{HH}(\mathbf{p}^2)\Big)\geq V_{HH}\Big(\pi^u_H(\tilde{\mathbf{p}}^2_{HH}),\delta^u(\tilde{\mathbf{p}}^2_{HH}),\pi_{LL}(\tilde{\mathbf{p}}^2_{HH}),\delta_{LL}(\tilde{\mathbf{p}}^2_{HH};\theta_L)-\delta^u(\tilde{\mathbf{p}}^2_{HH})\Big). \tag{30}$$

Substituting π_{HL}, π_{HH}, and π_{LL} as given by (6), imposing $\delta_{Hj}=1$, and noting that π^u_L and π^u_H are defined by (4), (29) and (30) respectively give the following upper bounds for δ_{LL}:

$$\delta_{LL}(\mathbf{p}^2) \leq \frac{\alpha\theta_H+\delta^u(\mathbf{p}^2)(\theta_H-\theta_L)}{\alpha\theta_H+(\theta_H-\theta_L)}, \tag{31}$$

$$\delta_{LL}(\mathbf{p}^2) \leq \frac{\alpha\theta_H+\delta^u(\tilde{\mathbf{p}}^2_{HH})\left(\frac{w_H}{w_{\mu|\tilde{\mathbf{p}}^2_{HH}}}\theta_{\mu|\tilde{\mathbf{p}}^2_{HH}}-\theta_L\right)}{\alpha\theta_H+(\theta_H-\theta_L)}. \tag{32}$$

Note that (31) is active whereas (32) is slack, for $\delta^u(\mathbf{p}^2)=\delta^u(\tilde{\mathbf{p}}^2_{HH})$ and given (28). This is also the case as long as $\frac{\delta^u(\mathbf{p}^2)}{\delta^u(\tilde{\mathbf{p}}^2_{HH})}\leq\frac{\frac{w_H}{w_{\mu|\tilde{\mathbf{p}}^2_{HH}}}\theta_{\mu|\tilde{\mathbf{p}}^2_{HH}}-\theta_L}{\theta_H-\theta_L}$. We anticipate that $\delta^u(\mathbf{p}^2)$ and $\delta^u(\tilde{\mathbf{p}}^2_{HH})$ defined at stage 1 satisfy this condition. Hence, (31) is the active restriction and $\delta^*_{LL}(\mathbf{p}^2)=\frac{\alpha\theta_H+\delta^u(\mathbf{p}^2)(\theta_H-\theta_L)}{\alpha\theta_H+(\theta_H-\theta_L)}$. Analogously, the contract $\{\pi_{LH},\delta_{LH}\}$, intended to (θ_L, w_H) individuals, is offered conditional on non-participation in PuHI. The active IC constraint is the one associated to (θ_H, w_H) individuals: $V_{HH}\Big(0,0,\pi_{HH}(\mathbf{p}^2),\delta_{HH}(\mathbf{p}^2)\Big)\geq V_{HH}\Big(0,0,\pi_{LH}(\mathbf{p}^2),\delta_{LH}(\mathbf{p}^2)\Big)$, resulting in the following equilibrium coverage rate $\delta^*_{LH}(\mathbf{p}^2)=\frac{\alpha\theta_H}{\alpha\theta_H+(\theta_H-\theta_L)}$.

Case 3 – (θ_H, w_L) **individuals participating in PuHI and all others opting out:** $\mathbf{p}^*=\mathbf{p}^3\equiv(0,0,1,0)$.

The following conditions must hold simultaneously:

$$
\begin{aligned}
\frac{\theta_L}{\theta_{\mu|\tilde{\mathbf{p}}^3_{LL}}}\left(1+\alpha\frac{\delta_{LL}(\tilde{\mathbf{p}}^3_{LL})-\delta_{LL}(\mathbf{p}^3)}{\delta^u(\tilde{\mathbf{p}}^3_{LL})}\right) &\leq \frac{w_L}{w_{\mu|\tilde{\mathbf{p}}^3_{LL}}}, && (33)\\
\frac{\theta_L}{\theta_{\mu|\tilde{\mathbf{p}}^3_{LH}}}\left(1+\alpha\frac{\delta_{LH}(\tilde{\mathbf{p}}^3_{LH})-\delta_{LH}(\mathbf{p}^3)}{\delta^u(\tilde{\mathbf{p}}^3_{LH})}\right) &\leq \frac{w_H}{w_{\mu|\tilde{\mathbf{p}}^3_{LH}}}, && (34)\\
\frac{\theta_H}{w_L} &\geq \frac{\theta_H}{w_L}, && (35)\\
\frac{\theta_H}{\theta_{\mu|\tilde{\mathbf{p}}^3_{HH}}} &\leq \frac{w_H}{w_{\mu|\tilde{\mathbf{p}}^3_{HH}}}. && (36)
\end{aligned}
$$

As before, $\delta_{Hj}(\mathbf{p}^3)=1$. The contract intended for low-risk individual is conditional on non-participation in PuHI. The two following IC constraints of (θ_H, w_L) and (θ_H, w_H) individuals respectively are:

$$
\begin{aligned}
V_{HL}\Big(\pi^u_{HL}(\mathbf{p}^3),\delta^u(\mathbf{p}^3),\pi_{HL}(\mathbf{p}^3),\delta_{HL}(\mathbf{p}^3)-\delta^u(\mathbf{p}^3)\Big) &\geq V_{HL}\Big(0,0,\pi_{Lj}(\tilde{\mathbf{p}}^3),\delta_{Lj}(\tilde{\mathbf{p}}^3)\Big), && (37)\\
V_{HH}\Big(0,0,\pi_{HH}(\mathbf{p}^3),\delta_{HH}(\mathbf{p}^3)\Big) &\geq V_{HH}\Big(0,0,\pi_{Lj}(\mathbf{p}^3),\delta_{Lj}(\mathbf{p}^3)\Big). && (38)
\end{aligned}
$$

Proceeding analogously as for case 2 results in the equilibrium coverage rate $\delta^*_{Lj}(\mathbf{p}^3)=\frac{\alpha\theta_H}{\alpha\theta_H+(\theta_H-\theta_L)}$.

Case 4: All Participate in PuHI: $\mathbf{p}^*=\mathbf{p}^4\equiv(1,1,1,1)$. The following conditions must hold simultaneously:

$$
\begin{aligned}
\frac{\theta_L}{\theta_\mu}\left(1+\alpha\frac{\delta_{LL}(\mathbf{p}^4)-\delta_{LL}(\tilde{\mathbf{p}}^4_{LL})}{\delta^u(\mathbf{p}^4)}\right) &\geq \frac{w_L}{w_\mu}, && (39)\\
\frac{\theta_L}{\theta_\mu}\left(1+\alpha\frac{\delta_{LH}(\mathbf{p}^4)-\delta_{LH}(\tilde{\mathbf{p}}^4_{LH})}{\delta^u(\mathbf{p}^4)}\right) &\geq \frac{w_H}{w_\mu}, && (40)\\
\frac{\theta_H}{\theta_\mu} &\geq \frac{w_L}{w_\mu}, && (41)\\
\frac{\theta_H}{\theta_\mu} &\geq \frac{w_H}{w_\mu}. && (42)
\end{aligned}
$$

PHI contracts are offered conditional on participation in PuHI. Incentive compatibility implies:

$$
V_{Hj}\Big(\pi^u_j(\mathbf{p}^1),\delta^u(\mathbf{p}^1),\pi_{Hj}(\mathbf{p}^1),\delta_{Hj}(\mathbf{p}^1)-\delta^u(\mathbf{p}^1)\Big) \geq V_{Hj}\Big(\pi^u_j(\mathbf{p}^1),\delta^u(\mathbf{p}^1),\pi_{Lj}(\mathbf{p}^1),\delta_{Lj}(\mathbf{p}^1)-\delta^u(\mathbf{p}^1)\Big). \quad (43)
$$

Proceeding analogously we get the following equilibrium coverage rate: $\delta^*_{Lj}(\mathbf{p}^1)=\frac{\alpha\theta_H+\delta^u(\mathbf{p}^1)(\theta_H-\theta_L)}{\alpha\theta_H+(\theta_H-\theta_L)}$.

Case 5 – (θ_H, w_H) opting out and all others participating in PuHI: $\mathbf{p}^*=\mathbf{p}^5\equiv(1,1,1,0)$. The following conditions must hold simultaneously:

$$
\begin{aligned}
\frac{\theta_L}{\theta_{\mu|\mathbf{p}^5}}\left(1+\alpha\frac{\delta_{LL}(\mathbf{p}^5)-\delta_{LL}(\tilde{\mathbf{p}}^5_{LL})}{\delta^u(\mathbf{p}^5)}\right) &\geq \frac{w_L}{w_{\mu|\mathbf{p}^5}}, && (44)\\
\frac{\theta_L}{\theta_{\mu|\mathbf{p}^5}}\left(1+\alpha\frac{\delta_{LH}(\mathbf{p}^5)-\delta_{LH}(\tilde{\mathbf{p}}^5_{LH})}{\delta^u(\mathbf{p}^5)}\right) &\geq \frac{w_H}{w_{\mu|\mathbf{p}^5}}, && (45)\\
\frac{\theta_H}{\theta_{\mu|\mathbf{p}^5}} &\geq \frac{w_L}{w_{\mu|\mathbf{p}^5}}, && (46)\\
\frac{\theta_H}{\theta_\mu} &\leq \frac{w_H}{w_\mu}. && (47)
\end{aligned}
$$

The contract intended for low-risk individuals is conditional on participation in PuHI and satisfies the following IC constraints:

$$
\begin{aligned}
V_{HL}\Big(\pi^u_{HL}(\mathbf{p}^5),\delta^u(\mathbf{p}^5),\pi_{HL}(\mathbf{p}^5),\delta_{HL}(\mathbf{p}^5)-\delta^u(\mathbf{p}^5)\Big) &\geq V_{HL}\Big(\pi^u_{HL}(\mathbf{p}^5),\delta^u(\mathbf{p}^5),\pi_{Lj}(\mathbf{p}^5),\delta_{Lj}(\mathbf{p}^5)\Big), && (48)\\
V_{HH}\Big(0,0,\pi_{HH}(\mathbf{p}^5),\delta_{HH}(\mathbf{p}^5)\Big) &\geq V_{HH}\Big(\pi^u_{HH}(\tilde{\mathbf{p}}^5),\delta^u(\tilde{\mathbf{p}}^5),\pi_{Lj}(\tilde{\mathbf{p}}^5),\delta_{Lj}(\tilde{\mathbf{p}}^5)\Big). && (49)
\end{aligned}
$$

Proceeding analogously as for case 2 we get the following equilibrium coverage rate: $\delta^*_{Lj}(\mathbf{p}^5) = \frac{\alpha\theta_H + \delta^u(\mathbf{p}^5)(\theta_H - \theta_L)}{\alpha\theta_H + (\theta_H - \theta_L)}$. We would expect that a sufficiently high risk aversion combined with a relatively high income inequality would increase the likelihood of this case. Although this case looks possible from the intuitive and analytical point of view, after different attempts we could not find any numerical illustration. This makes us suspect that there might be some analytical aspect of the problem preventing this case from arising in equilibrium that we have, to date, not been able to identify. For this reason case 5 is not included in Proposition 2 ■

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Tables

Equilibrium	θ_L	w_H	λ_{LH}	λ_{HH}
Case 1	0.4	1.1	[0.25, 0.3]	0.2-0.25
Case 2	0.4	[1.3-2]	[0.3, 0.45]	0.05-0.2
Case 3	[0.1,0.2]	[1.1-2]	[0.25, 0.45]	0.05-0.25
	0.3	[1.2-2]	0.05	0.45
	0.3	[1.1-2]	[0.25-0.4]	0.1-0.25
Case 4	0.4	1.1	[0.35, 0.45]	0.05-0.15
No Equilib.	0.3	1.1	0.45	0.05

Table 1: Numerical illustration of possible equilibria. In this example $\theta_H = 0.5$, $w_L = 1$, $\lambda_{LL} = 0.25$, $\lambda_{HL} = 0.25$, and $0.34 \leq \alpha \leq 1$. Increments of all other variables of 0.1 except for λ_{LH} which we have applied increments of 0.05. The share of high-risk-high-income individuals is the remaining share $1-(\lambda_{LL}+\lambda_{LH}+\lambda_{HL})$.