WORKING PAPERS

N° TSE-531

October 2014

"The World Income Distribution: The Effects of International Unbundling of Production"

Sergi Basco and Marti Mestieri

Toulouse School of Economics

The World Income Distribution: The Effects of International Unbundling of Production*

Sergi Basco
Universidad Carlos III

Martí Mestieri
Toulouse School of Economics

October 1, 2014

Abstract

We build a dynamic trade model to study how international unbundling of production and the emergence of global supply chains affect the world income distribution. We consider a world where countries only differ in their productivity. The level of productivity determines the number of varieties a country produces. To manufacture each variety a bundle of intermediates, which require capital and labor in different proportions, needs to be assembled. We characterize two trade regimes: *(i)* trade only in varieties and *(ii)* trade in both varieties and intermediates (unbundling). We show that unbundling of production generates income divergence among ex-ante identical countries (symmetry breaking). With heterogeneous countries, it increases top-bottom inequality and it has non-monotonic effects on the world income distribution (it reduces relatively more the income share of middle-productivity countries). We also show that when the South joins the global supply chain, the income share of all northern and the most productive southern countries increase, at the expense of the least productive countries. In addition, we find that the effect of a labor-saving technology, computerization, depends on the trade regime. Without unbundling, computerization has no effect on the world income distribution. With unbundling, computerization raises world inequality. Finally, we show that technology diffusion leads to income convergence under both trade regimes. However, with unbundling of production more low-productivity countries benefit from technological catch-up.

Keywords: World Income Distribution, Symmetry Breaking, Global Supply Chains.
JEL Classification: F12, F43, O11, 019, 040.

*We thank seminar participants at Calgary, CEPR-ESSIM, CEMFI, Nottingham and UC3M for useful comments and suggestions. Basco acknoweldges financial support from the Spanish Ministry of Science and Innovation (ECO2011-27014). Mestieri thanks the Agence Nationale de la Recherche for their generous financial support. E-mails: sergi.basco@gmail.com, marti.mestieri@tse-fr.eu.

1 Introduction

One of the most remarkable facts in international trade in the last twenty-five years has been the "unbundling" of production (Baldwin, 2012). Before the 1990s, the production process was much less fragmented across the globe. The unbundling of the production has made possible the emergence of "global supply chains," whereby the production of a significant fraction of the intermediate inputs required to manufacture goods is located in different countries. As a result, countries can now also specialize in different stages of the global supply chain. A paradigmatic example of this fragmentation of production is the iPod, which is designed in the United States and assembled in China from several hundred components and parts that are sourced from around the world (Dedrick et al., 2010).

Figure 1 provides new evidence consistent with this unbundling of production. It reports the ratio of the value of world exported intermediates to final goods. Before the mid-1980s this ratio was about .5, which means that for each dollar of intermediate exported there were two dollars of final goods exported. After the 1990s this ratio sharply increased and it has converged to around .8. Therefore, trade in intermediates has grown much more than trade in final goods. These findings are consistent with recent empirical work on the global supply chain. For example, Antràs (2014) shows that the average upstreamness of world exports has increased, which suggests that trade in inputs has become more important over time. Similarly, Johnson (2014) documents that the ratio of value-added to gross-value of exports fell in early 1990s, which is mostly explained by increased offshoring within manufacturing.[1]

Trade affects the income and economic growth of countries.[2] A vast and rich literature has studied the effects of trade in goods. However, the trade literature has been mostly silent about the distinctive long-run effects of trade in intermediates.[3] This paper contributes to filling this gap by providing a theory of how the unbundling of production changes the world income distribution.

The key novel aspect of our theory is the introduction of intermediates that are heterogeneous in their capital-intensity. In our framework, the unbundling of production leads countries to sort in the production of intermediates according to their productivity levels. Low-productivity countries specialize in the production of labor-intensive intermediates, while high-productivity countries sort into the production of capital-intensive intermediates. This prediction is supported by the data (see Table 1) and it is quantitatively important.[4] Note

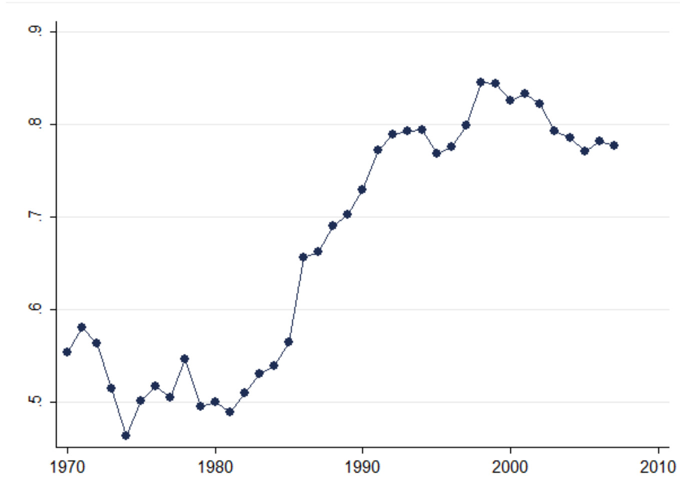
[1] Hummels et al. (2001) also document the emergence of global supply chains, which they refer as vertical specialization, whereby countries specialize in the production of different sets of intermediate inputs. Hanson et al. (2005) show that a sizeable part of this intermediate trade involves multinational firms.

[2] See, for example, Grossman and Helpman (1993) and Ventura (2005) for an overview of the channels through which trade affects economic growth.

[3] One exception is Rodríguez-Clare (2010), which emphasizes the effects of offshoring on the allocation of labor to innovation.

[4] We find that, moving from the 75th to the 25th percentile in the distribution of countries' productivity, more than doubles the value of exports of intermediates in the 75th percentile of labor-intensive intermediates

Figure 1: Ratio of Value of Exported Intermediates to Final Goods.



Source: Feenstra World Trade Database. To classify goods as intermediates, we use the end-use classification of Feenstra and Jensen (2012). Final goods also include commodities.

also that this is consistent with the existing empirical literature (e.g., Schott, 2004).[5]

We show that unbundling of production gives rise to income differences among ex-ante identical countries. For heterogeneous countries, we find that unbundling generates non-monotonic changes in the world income distribution: top-bottom inequality increases and the income share of the most productive countries rises mostly at the expense of middle-productivity countries. These predictions are broadly consistent with the change in the world income distribution in the last 25 years.

Our model features a large number of countries, which only differ in their productivity. Each country produces a certain number of varieties. These varieties are differentiated by origin (Armington assumption).[6] In order to produce a variety, a bundle of intermediates needs to be assembled. Each of these intermediates requires capital and labor in different proportions. As it is standard in the trade literature, we assume that neither labor nor capital are internationally mobile.

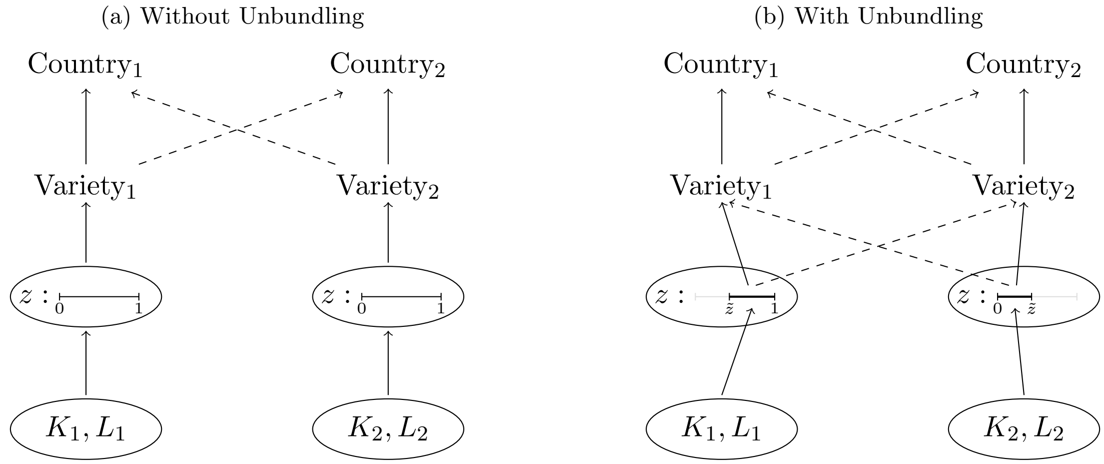
Intermediates differ in their capital-intensity requirements to be produced, while all varieties are produced with the same technology. This assumption allows us to highlight the role of heterogeneity in capital-intensity of intermediates. In fact, the dispersion in capital-intensity is larger for more disaggregated goods. Using U.S. data, Table 2 shows that the standard deviation in capital intensity at 6-digit NAICS level (which we interpret as intermediates) is larger than at 3-digit NAICS level (varieties). Therefore, our formulation is an extreme

relative to the exports of intermediates in the 25th percentile.

[5]See also Baxter and Kouparitsas (2003), Hanson (2012) and Schott (2003a,b) for similar findings.

[6]In the baseline model we assume that each country produces an exogenous number of varieties which is proportional to the productivity of the country. Section 4.4 provides an exact microfoundation to the exogenous number of varieties that we postulate in the baseline model.

Figure 2: Market Structure for the 2-country, 2-varieties case



Note: Figure represents the market structure for two countries and two varieties under the two different trade regimes. Dashed lines indicate trade flows.

representation of this fact.

We start characterizing the equilibrium without unbundling. When only trade in varieties is possible, each country needs to produce all the required intermediates within its boundaries. Once the intermediates are manufactured and bundled to produce the varieties, these varieties are traded. The structure of this economy is summarized in Figure 2a for a two-country two-variety case. We show that the country's share of world income is determined by the share of varieties the country produces, which is proportional to its productivity.For example, if a country is twice as productive as another country, its share of world income income is twice the share of the other country.

The world income distribution changes with unbundling. When intermediates can be offshored, the producer of a variety does not need to purchase all intermediates at home. Rather, it can import intermediates from the cheapest producer in the world. Therefore, the location of intermediates becomes endogenous. The structure of this economy is illustrated in Figure 2b. We show that the most productive countries have comparative advantage and specialize in capital-intensive intermediates. This endogenous selection of intermediates is important because it determines the relative income of each country in the new steady-state. We show that the world income share is determined by the mass of intermediates that a country produces and their relative capital-intensity.

The first main result of the paper is that unbundling of production generates symmetry breaking of ex-ante identical countries. To gain intuition into this result, we first consider a two-country world. In the equilibrium without unbundling, the two countries have the same income share, as they produce the same amount of varieties. This symmetric equilibrium is unstable when there is unbundling of production. To understand this result, let us assume that the first country is slightly more productive than the second one. It implies that the first country has a slight comparative advantage in capital-intensive intermediates and, thus, it specializes in more capital-intensive intermediates. By producing more capital-intensive intermediates, it accumulates more capital, thereby reinforcing the initial comparative advantage in capital-intensive intermediates. This process continues over time and the two countries end up with very different stocks of capital in the new steady-state. We show that this argument extends to an arbitrary number of ex-ante identical countries.

Our second main result characterizes the long-run change in the world income distribution with heterogeneous countries. We show that top-bottom inequality rises with unbundling: the world income share increases in high-productivity countries, while it declines in the rest. Moreover, this change is non-monotonic: the largest fall in income share is in middle-productivity countries and the largest rise is in the most productive country. Without unbundling, the stock of capital is determined by the number of varieties that a country produces. In contrast, with unbundling, the number of varieties becomes irrelevant and the stock of capital only depends on the intermediates in which the country specializes. The most productive country gains the most because it specializes in the most capital-intensive intermediates. Middle-productivity countries lose because they produce a sizeable amount of varieties, thereby accumulating substantial capital in the equilibrium without unbundling. However, when there is unbundling, they specialize in relatively low-capital-inensive intermediates and, thus, end up with less capital. In other words, there is a large mismatch between the capital accumulated during the equilibrium without unbundling and the capital needed to produce the equilibrium mass of intermediates with unbundling.

In addition to analyzing and comparing the equilibria with and without unbundling, our model is helpful to understand other substantial changes that have occurred in the process of unbundling. An important fact in international trade is the increasingly important role of emerging economies. For example, the share of world trade in developing Asia has increased from less than 15% in the early 1990s to 35% in 2011. It has been argued that unbundling of production explains the increase in the volume of trade in emerging economies (see, for example, Baldwin, 2012). Figure 5 shows that most of the growth of world trade in intermediates has come from emerging countries. Motivated by this evidence, we study how the world income distribution changes when the South joins the global supply chain. To be precise, we analyze the effect of southern countries participating in intermediates trade in a world where all countries previously traded varieties but only northern countries traded intermediates. We show that the income share increases in all northern countries and the most productive southern countries, while it declines in the rest of southern countries. Northern countries increase their income share the most because they can specialize in more capital-intensive intermediates and sell them to a larger market. For southern countries, the income share only raises in those that are productive enough to "climb up the supply chain" and specialize in relatively capital-intensive intermediates.

We also use our framework to study the role of a labor-saving technology: computerization. Computerization (or, more broadly, the Information Technologies revolution) is one important factor behind the surge of the unbundling of production.[7] Autor et al. (2003) among others have also emphasized the effects of computerization on the relative supply for labor and on the income distribution within countries. We introduce computerization into the model as a technological shift that reduces the relative demand of labor-intensive intermediates. We show that the effect of computerization depends on the trade regime. Without unbundling, computerization does not change the world income distribution. In contrast, with unbundling, computerization raises inequality. The intuition is that computerization changes the selection of intermediates. All countries specialize in more capital-intensive intermediates, thus, the average intermediate produced in each country is more capital-intensive. However, this change in the trade pattern disproportionately favors the most productive countries, which exacerbates income inequality. We also show that computerization raises the capital income share in both trade regimes.

Finally, we analyze how the diffusion of technology changes the world income distribution. In our baseline model we assume that productivity is exogenous and constant. However, in practice, technology diffuses over time and low-productivity countries learn about innovations done by the countries in the technological frontier. We show that diffusion of technology always leads to convergence of income. However, for a given amount of technology diffusion, the mass of low-productivity countries increasing their income share is larger with unbundling.

Related Literature. This paper relates to different strands of the literature on growth, trade and offshoring. There exist a large number of models that study the interaction between economic growth and trade. Our model structure for production of varieties and final good is similar to Acemoglu and Ventura (2002). The most important difference is that we introduce an additional layer of intermediates in the production process. This allows us to study the effect of unbundling on the world income distribution. In contrast to Acemoglu and Ventura (2002), we do not have long-run growth in our model because we have a collection of Cobb-Douglas countries instead of their AK countries.[8]

There exists a growing literature analyzing the unbundling of production and its effects on the pattern of specialization and the wealth of nations. For example, Baldwin and Venables (2013), Baldwin and Robert-Nicoud (2014) and Grossman and Rossi-Hansberg (2008) revisit

[7] See, for example, Basco and Mestieri (2013) and the references therein.

[8] Other papers that study how trade in goods affect economic growth include Ventura (1997), Bajona and Kehoe (2010), Baxter (1992), Cunat and Maffezzoli (2004) and Deardorff (2001b). These papers make different assumptions on the number of goods and whether factor prices equalize. However, they do not consider trade in intermediates. Yi (2003) calibrates a two-country two-stages Ricardian model to show that vertical specialization is needed to explain how small trade cost reductions resulted in the observed growth in exports.

the standard trade theorems in the presence of trade in intermediates. We model the production process as a sequential process in which intermediates are first produced and then used to assemble each variety. This is similar to, among others, Antràs and Chor (2013), Caliendo and Parro (2012), Costinot et al. (2013), Deardorff (1998, 2001a) and Kohler (2004).[9] Differently from these papers, we build a dynamic trade model and derive our main results from the interaction between the sorting of countries across intermediates of different capital-intensity and capital accumulation.

From a theoretical standpoint, as pointed out by Ethier (1984) and Costinot and Vogel (2010), general equilibrium models with an arbitrary number of countries and goods seldom provide tractable results. Our model provides a framework that accommodates a substantial amount of heterogeneity and still delivers sharp characterizations and comparative statics results. In terms of techniques, we solve for the unbundling equilibrium using assignment techniques similar to Matsuyama (2013).

Our model delivers symmetry breaking for the case of ex-ante identical countries. In this sense, it is related to Matsuyama (2004, 2013). However our mechanism does not rely on increasing returns or credit market imperfections. Matsuyama (2013) emphasizes that the share of non-traded services is heterogeneous across varieties. With increasing returns in the production of these non-traded services, this generates a two-way feedback loop that yields symmetry breaking. In our model, similar countries become different with unbundling of production because they end up specializing in different intermediates, which differ in capital-intensity and this triggers different incentives to accumulate capital across countries.

The rest of the paper is organized as follows. Section 2 presents the model and characterizes the equilibria with and without unbundling. The main results of the paper comparing the world income distribution with and without unbundling are derived in Section 3. In Section 4.1, we analyze the empirically relevant case in which southern countries join the global supply chain. Section 4.2 analyzes the role of a labor-saving technology, computerization, as a potential driver of unbundling. Section 4.3 analyzes technology diffusion under the two trade regimes and Section 4.4 provides an microfoundation for the the number of varieties produced by each country. Section 5 concludes. All proofs can be found in the Appendix.

2 The Model

This section presents the baseline model and characterizes the steady-state equilibrium without unbundling (when only trade in varieties is possible) and the equilibrium with unbundling (when trade in both varieties and intermediates is possible).

[9] There exists alternative ways to model intermediates. For example, Bems et al. (2011) and Costinot and Rodríguez-Clare (2013) assume that intermediates can either be used as inputs for production or directly consumed.

We consider a world economy with J countries, indexed by $j = 1, \ldots, J$. Countries only differ in the level of productivity θ_j. Without loss of generality, we order countries such that $\theta_1 \geq \theta_2 \geq ... \geq \theta_J$. There is a mass of varieties indexed by $v \in [0, N]$. There is one final good used for consumption and investment. There is no trade in final goods or assets.

All countries admit a representative consumer with utility

$$\int_0^\infty e^{-\rho t} \ln c_j(t) dt, \tag{1}$$

where $c_j(t)$ is consumption in country j at time t. Each country j is endowed with an initial capital stock $k_j(0) > 0$ and a fixed stock of labor, normalized to one. The budget constraint of the representative household in country j is

$$p_j(t) \left[\dot{k}_j(t) + c_j(t) \right] = p_j(t) Y_j(t) = r_j(t) k_j(t) + w_j(t). \tag{2}$$

We assume that varieties are differentiated by origin, and each country produces a measure μ_j of these differentiated varieties, so that

$$\sum_{j=1}^{J} \mu_j = N, \tag{3}$$

where N is the total number of varieties. In the baseline model we assume that the number of varieties is exogenously given by $\mu_j = \kappa \theta_j$, where $\kappa > 0$. It implies that more productive countries, produce a larger number of varieties. Section 4.4 provides an exact microfoundation of this production function of varieties.

The final good is produced according to the constant returns to scale production function

$$Y_j(t) = \exp\left(\int_0^N \frac{1}{N} \ln x_j(v, t) dv \right), \tag{4}$$

where $x_j(v, t)$ denotes the amount of varieties used in final good production in country j. The production of varieties requires a bundle of intermediates, indexed by $z \in [0, 1]$,

$$x_j(v, t) = \exp\left[\int_0^1 \beta(z) \ln a_j(z, v, t) dz \right], \tag{5}$$

where $a_j(z, v, t)$ denotes the amount of intermediate z used at time t to produce variety v in country j. $\beta(z)$ reflects the relative importance of intermediate z in the production of variety v. We assume, for simplicity, that $\beta(z) = 1$. In Section 4.2 we study the effects of computerization and make comparative statics on $\beta(z)$.

Intermediates are produced using labor l and capital k in different proportions,

$$a_j(z,t) = \theta_j \left(\frac{k_j(z,t)}{z}\right)^z \left(\frac{l_j(z,t)}{1-z}\right)^{1-z}, \qquad z \in [0,1], \tag{6}$$

where $a_j(z,t)$ denotes total production of intermediate z at time t in country j and θ_j denotes the productivity in country j.

2.1 Equilibrium Without Unbundling

This subsection analyzes the competitive equilibrium without unbundling. That is, when varieties are traded but intermediates cannot be traded between countries. We characterize the steady-state competitive equilibrium and show that the world income share of a country is determined by the share of varieties it produces.

Definition 1 *A competitive equilibrium without unbundling is defined by a sequence of prices* $\{w_j(t), r_j(t), p_j(t), p_j(v,t), p_j(z,t)\}$ *and allocations* $\{l_j(z,t), k_j(z,t), c_j(t), a_j(z,v,t), a_j(z,t), x_j(v,t)\}$ *for* $t = 0, \ldots, \infty$ *and* $j = 1, \ldots, J$*, such that for each country: (i) the representative agent maximizes utility subject to the budget constraint, (ii) final good producers maximize profits given prices, (iii) variety producers maximize profits given prices, (iv) intermediate producers maximize profits given prices, (v) labor and capital market clear and (vi) trade in varieties is balanced for each country.*

The consumer utility maximization problem (1) subject to the budget constraint (2) yields

$$\frac{\dot{c}_j(t)}{c_j(t)} = \frac{r_j(t)}{p_j(t)} - \rho, \tag{7}$$

$$\lim_{t\to\infty} e^{-\rho t} c_j(t)^{-1} \left(\frac{r_j(t)}{p_j(t)} k_j(t)\right) = 0. \tag{8}$$

Equation (7) is the Euler Equation from a standard Ramsey model, with the price p_j made explicit, as it may differ across countries. Equation (8) is the transversality condition.[10]

Omitting the time index t, the problem of the final good producer in country j is to

$$\max_{x_j(v)} p_j Y_j - \int_0^N p_j(v) x_j(v) dv,$$

[10] The optimal consumption rule can be found by integrating the budget constraint (2) and it is given by the next expression

$$c_j(0) = \rho \left(k_j(0) + \int_0^\infty e^{\int_0^t -\frac{r_j}{p_j}(s)ds} \frac{w_j}{p_j}(t) dt\right).$$

where Y_j is given by (4). It follows that

$$\frac{p_j Y_j}{N} = p_j(v) x_j(v).$$

Thus, as varieties are traded, the total demand of variety v is

$$x(v) = \sum_{i=1}^{J} x_j(v) = \frac{1}{N} \frac{\sum_{i=1}^{J} p_j Y_j}{p_j(v)}. \tag{9}$$

The problem of variety-v producer in country j is

$$\max_{a_j(z,v)} p_j(v) x_j(v) - \int_0^1 p_j(z) a_j(z, v),$$

which implies that the demand of intermediate z to produce variety v is pinned down by

$$p_j(z) a_j(z, v) = p_j(v) x_j(v) = x_j(v) \exp\left(\int_0^1 \ln p_j(z) dz \right).$$

Since there is not trade in intermediates, the aggregate demand of intermediate z in country j comes only from the production of domestic varieties,

$$a_j(z) = \mu_j a_j(z, v),$$

where μ_j is the number of varieties produced in country j.

The problem of the producer of intermediate z in country j is

$$\max_{l_j(z), k_j(z)} p_j(z) \theta_j \left(l_j(z)\right)^{1-z} k_j(z)^z - w_j l_j(z) - r_j k_j(z),$$

which implies the following labor and capital demands

$$(1-z) p_j(z) a_j(z) = w_j l_j(z), \tag{10}$$

$$z p_j(z) a_j(z) = r_j k_j(z). \tag{11}$$

Aggregating labor demand (10) across intermediates and noting that the labor supply is normalized to one, we obtain the labor market clearing condition

$$1 = \int_0^1 l_j(z) = \frac{1}{w_j} \int_0^1 (1-z) p_j(z) a_j(z) dz = \frac{1}{2} \mu_j \frac{\sum_i p_i Y_i}{N} \frac{1}{w_j}.$$

Likewise, using (11), the capital market clearing condition is given by

$$K_j = \frac{1}{r_j}\int_0^1 z p_j(z) a_j(z) dz = \frac{1}{2}\mu_j \frac{\sum_i p_i Y_i}{N}\frac{1}{r_j}.$$

To derive the trade balance equation, recall that without unbundling, only varieties are traded. Thus, the value of exported varieties $\mu_j p_j^x(v) x(v, \text{exported})$ (all varieties produced by one country are symmetric) has to be equal to the value of imported varieties,

$$\underbrace{\frac{\mu_j}{N}\left(\sum_{i=1}^{J} p_i Y_i - p_j Y_j\right)}_{\text{Exports of Varieties}} = \underbrace{\frac{N-\mu_j}{N} p_j Y_j}_{\text{Imports of Varieties}} . \tag{12}$$

All final goods are produced using the same varieties by competitive producers in all countries, thus, the prices of final goods are the same across countries $p_i = p_j$. Rewriting (12), we obtain

$$\frac{\mu_j}{\sum_{i=1}^J \mu_i} = \frac{p_j Y_j}{\sum_{i=1}^J p_i Y_i} = \frac{Y_j}{\sum_{i=1}^J Y_i}. \tag{13}$$

From the factor market clearing conditions, we can write the labor and capital income in country j as

$$\begin{aligned} w_j &= \frac{1}{2}\kappa\theta_j \frac{\sum_i p_i Y_i}{N}, \\ r_j k_j &= \frac{1}{2}\kappa\theta_j \frac{\sum_i p_i Y_i}{N}. \end{aligned}$$

Using the trade balance equation (13) and the fact that the number of varieties produced in country j is $\mu_j = \kappa\theta_j$, we can express the world income share of country j as a function of the exogenous levels of productivity[11]

$$s_j \equiv \frac{Y_j}{\sum_{i=1}^J Y_i} = \frac{\theta_j}{\sum_{i=1}^J \theta_i}. \tag{14}$$

This equation means that the relative income of country j is the relative productivity of the country.

Steady-state solution In the steady state there is no growth, $\dot{k} = \dot{c} = 0$. The Euler condition implies that the interest rate is equalized across countries (i.e., $r_j = \rho$). The consumption level is determined by the budget constraint, $c_j = p_j Y_j = w_j + \rho k_j$. Finally, note that the country ranking in income shares coincides with the welfare ranking in steady-state.

[11]Note that in Eaton and Kortum (2002), the number of varieties is also proportional to the productivity of the country. In their framework, the income share is also proportional to a re-scaled productivity measure in the zero-gravity case.

2.2 Equilibrium With Unbundling

This subsection characterizes the equilibrium with unbundling. In this case, both varieties and intermediates can be costlessly traded. This implies that countries no longer need to produce all intermediates required to produce varieties. Rather, they can specialize in a subset of these intermediates and import the rest. We show that the world income share depends on the mass of intermediates in which the country specializes.

Definition 2 *A competitive equilibrium with unbundling is defined by a sequence of prices* $\{w_j(t), r_j(t), p_j(t), p_j(v,t), p_j(z,t)\}$ *and allocations* $\{l_j(z,t), k_j(z,t), c_j(t), a_j(z,v,t), a_j(z,t), x_j(v,t)\}$ *for* $t = 0, ..., \infty$, $j = 1, ..., J$, *such that for each country: (i) the representative agent maximizes utility subject to the budget constraint, (ii) final good producers maximize profits given prices, (iii) variety producers maximize profits given prices, (iv) intermediate producers maximize profits given prices, (v) labor and capital market clear and (vi) trade in varieties* ***and*** *intermediates is balanced for each country.*

To derive the equilibrium, we repeat the same steps as in Section 2.2. The demand of varieties is given by (9), as in the previous section. The key difference is that since intermediates are now costlessly traded, the producer of variety v purchases intermediates from the cheapest location. Thus, the price of variety v is given by

$$\ln p_j(v) = \int_0^1 \ln\left(\min_{j\in\{1,\ldots,J\}}\{p_j(z)\}\right) dz.$$

This implies that the aggregate demand of intermediate z in country j, rather than coming from the domestic demand as in the equilibrium without unbundling, comes now from the entire world, provided that country j can produce z at the cheapest world price.[12] Thus, the mass of intermediates that each country produces is endogenously determined. Denoting by Z_j the mass of intermediates that country j produces in the unbundling equilibrium, we have that

$$a_j(z) = \sum_{i=1}^{J} \mu_i a_j(z,v) = N a_j(z,v), \qquad \text{if } z \in Z_j,$$

and zero otherwise. Substituting the expression for $a_j(z,v)$ into equation (9) and using that $p_j(v)x(v) = p_j(z)a_j(z,v)$, we find that the total value of intermediate z produced in country j is

$$p_j(z)a_j(z) = \sum_i p_i Y_i.$$

The expressions for the demand of labor and capital are as in the equilibrium without unbundling, (10) and (11), adjusting for the fact that each country only produces a subset Z_j

[12] We are implicitly assuming that each intermediate is done only by one country, which is indeed true almost everywhere in equilibrium.

of the intermediates,

$$1 = \int_{z\in Z_j} (1-z)dz \frac{1}{w_j}\sum_{i=1}^{J} p_i Y_i, \tag{15}$$

$$K_j = \int_{z\in Z_j} zdz \frac{1}{r_j}\sum_{i=1}^{J} p_i Y_i.$$

The trade balance changes with unbundling because now intermediates are also traded. Trade balance implies that the value of exported varieties plus the value of exported intermediates has to be equal to the value of imports of any country,[13]

$$\underbrace{\frac{\mu_j}{N}\left(\sum_{i=1}^{J} p_i Y_i - p_j Y_j\right)}_{\text{Exports of Varieties}} + \underbrace{Z_j \frac{N-\mu_j}{N}\sum_{i=1}^{J} p_i Y_i}_{\text{Exports of Intermediates}} = \underbrace{\frac{N-\mu_j}{N} p_j Y_j}_{\text{Imports of Varieties}} + \underbrace{(1-Z_j)\frac{\mu_j}{N}\sum_{i=1}^{J} p_i Y_i}_{\text{Imp. of Intermediates}}.$$

After rearranging terms, the above expression simplifies to

$$s_j = \frac{p_j Y_j}{\sum_i p_i Y_i} = Z_j.$$

This equation means that with unbundling the world income share of country j is only determined by the mass of intermediates that the country produces. Note that a country is a net exporter of intermediates when it specializes in a larger share of intermediates than the fraction of varieties it produces (i.e., $Z_j > \frac{\mu_j}{N}$). It implies that unless $Z_j = \frac{\mu_j}{N}$, there will be an imbalance in intermediates trade and the income share will change with unbundling of production.

2.2.1 Steady-state solution

The final step is to derive the equilibrium share of intermediates that each country produces, Z_j. We proceed by focusing on the steady-state equilibrium.

From the Euler equation, (7), the rental rates are equalized across countries in the steady state, $r_j = \rho$. Therefore, the cost of producing intermediate z in country j is

$$c_j(z) = \theta_j^{-1} w_j^{1-z} \rho^z.$$

This implies that the most capital-intensive intermediate ($z = 1$) is produced by the most

[13] To derive the value of exported intermediates, note that, for a given intermediate z, each producer of varieties demands $\frac{1}{N}\sum_i p_i Y_i$. Given that country j produces μ_j varieties, the value of production of a given intermediate z that goes into exporting is $\frac{N-\mu_j}{N}\sum_i p_i Y_i$. Finally, since country j produces the range Z_j of intermediates, the total value of exported intermediates is $Z_j \frac{N-\mu}{N}\sum_i p_i Y_i$. The computation for the value of imports can be done in an analogous way.

productive country, country 1, because $c_1(1) = \theta_1^{-1}\rho = \min_j\{c_j\}$.

Let $p(z) = \min_j\{c_j\}$. Consider an intermediate $\tilde{z} < 1$ with price $p(\tilde{z})$. Perfect competition implies that

$$p(\tilde{z}) - c_j(\tilde{z}) \leq 0.$$

Then, if two countries produce the same intermediate it has to be the case that

$$c_j(\tilde{z}) = c_i(\tilde{z}) \quad \Longrightarrow \quad \theta_j^{-1} w_j^{1-\tilde{z}} = \theta_i^{-1} w_i^{1-\tilde{z}},$$

which implies that

$$\frac{w_i}{w_j} = \left(\frac{\theta_i}{\theta_j}\right)^{\frac{1}{1-\tilde{z}}}.$$

Suppose that $j > i$, so that $\theta_j < \theta_i$. As $\frac{1}{1-z}$ is an increasing function of z, this implies that country j will not produce any intermediate with $z > \tilde{z}$. Thus, we have a sequence of thresholds z_j that determines the pattern of specialization in intermediates,

$$\frac{w_j}{w_{j+1}} = \left(\frac{\theta_j}{\theta_{j+1}}\right)^{\frac{1}{1-z_j}} \text{ for all } j. \tag{16}$$

We have derived two equilibrium conditions relating the equilibrium wages and the equilibrium thresholds: labor market clearing (equation 15) and the definition of the threshold intermediate (equation 16). Using the ratio of both equations, we obtain the following endogenous selection of intermediates.

Remark The endogenous selection of intermediates is given by the second order difference equation,

$$\begin{aligned} \left(\frac{\theta_j}{\theta_{j+1}}\right)^{\frac{1}{1-z_j}} &= \frac{\Delta_j}{\Delta_{j+1}}, \\ \text{where } \Delta_j &= \int_{z_j}^{z_{j-1}} (1-z)dz, \end{aligned} \tag{17}$$

with terminal conditions $z_0 = 1$ and $z_J = 0$.[14]

An implication of this endogenous selection of intermediates is that countries with relatively high-productivity have comparative advantage in high z intermediates and, thus, export capital-intensive intermediates, while low-productivity countries sort into labor-intensive intermediates. We next show that this implication is consistent with the data. Consider the

[14] Note that the left-hand side is continuous and increasing in z_j and the right-hand side is continuous and decreasing in z_j. Therefore, the solution is unique.

next equation

$$X_{ict} = \alpha + \beta \cdot \text{TFP}_c \cdot \text{Labor Intensity}_{it} + \delta_i + \delta_c + \delta_t + \varepsilon_{ict}, \tag{18}$$

where X_{ict} is the log of total exports of intermediates i of country c at time t, TFP_c is total factor productivity of country c, δ_i, δ_c and δ_t are intermediate, country and time fixed effects, respectively. Our data is for the period 1994-2008. The prediction of the model is $\beta < 0$. That is, relatively low-productivity countries have comparative advantage in labor-intensive industries.[15] Columns (1) to (4) in Table 1 report the coefficient β of the regression for different sets of fixed effects. Consistent with the model, the coefficient is negative in all specifications. Our baseline estimation (18), which includes country, time and intermediates fixed effects is significant at 5%.[16] Quantitatively, the interaction term implies that increasing TFP from the 25th percentile to the 75th, would imply a reduction in exports of the 25th percentile labor-intensive intermediates by an 8.2% and of the 75th percentile labor-intensive intermediates by 16.9%.[17] Thus, the effect of increasing TFP on the exports of the 75th percentile of labor-intensive intermediates is more than twice the effect on the exports of the 25th percentile. For example, Schott (2004) finds that richer countries specialize in low-labor-intensive goods.[18]

3 Main Results

This section compares the equilibrium with and without unbundling and derives the main results of the paper. We first consider a world of ex-ante identical countries and show that unbundling of production generates symmetry breaking. We next study a world consisting of heterogenous countries and show that unbundling raises top-bottom inequality and that middle-productivity countries experience the largest decline in income share. All omitted proofs are in the Appendix.

[15] We classify goods as intermediates using the classification in Feenstra and Jensen (2012). Our classification for intermediates is at 6-digit NAICS. We compute labor-intensity as the value added share of production workers from the NBER CES Manufacturing database. Note that we are making the standard assumption that the ranking of labor-intensive industries is stable across countries, as our labor-intensity data comes from the U.S.. We use TFP from Hall and Jones (1999), which corresponds to year 1988.Our data stops in 1994 because prior to this year we do not have the same level of disaggregation.

[16] Standard errors are clustered at country level. In specifications (2) and (3) we add country·year fixed effects, δ_{ct}, and intermediate·year fixed effects, δ_{it} and find that β remains significant at a 10% level. When we include both δ_{ct} and δ_{it} the coefficient is only significant at a 12.9%.

[17] The 25th percentile of TFP corresponds to Cameroon, with a measure of .274. The 75th percentile corresponds to Israel, with a measure of .817. Note that Hall and Jones (1999) report TFP relative to the U.S. TFP. The 25th percentile measure of intermediates corresponds to NAICS 334511 (Search, direction and navigation instrument manufacturing) with a labor-intensity measure of .127. The 75th percentile corresponds to NAICS 313312 (Textile and fabric finishing mills), with a labor intensity measure of .817.

[18] Baxter and Kouparitsas (2003), Hanson (2012) and Schott (2003a,b) find similar results. Bernard et al. (2006) find that U.S. manufacturing reallocate away from labor-intensive towards capital-intensive plants within industries, as industry exposure to imports from low-wage countries rises. At a more aggregate level, Davis and Weinstein (2001) and Romalis (2004) also provide evidence consistent with this pattern of specialization.

3.1 Symmetry Breaking of ex-ante Identical Countries

To build intuition, we start analyzing the two-country case. Then, we characterize the equilibrium for a world with an arbitrary number of countries.

3.1.1 The two-country case

Suppose that the world consists of two identical countries, $J = \{1, 2\}$ with $\theta_1 = \theta_2 = \theta$. In the equilibrium without unbundling, each country has half of the world income share,

$$\frac{s_1^{without}}{s_2^{without}} = \frac{\theta_1}{\theta_2} = 1.$$

In the equilibrium with unbundling, the endogenous selection of intermediates changes the world income shares. The difference equation (17) determining the specialization threshold becomes

$$1 = \frac{\frac{1}{2} - \left(z - \frac{z^2}{2}\right)}{\left(z - \frac{z^2}{2}\right)},$$

where we have used the terminal conditions $z_0 = 1$, $z_2 = 0$ and $\theta_1 = \theta_2 = \theta$. There exists a unique solution to this equation given by $z^* = 1 - \sqrt{1/2}$. That is, country 1 specializes in the production of intermediates $z \in (z^*, 1]$ and country 2 produces the rest of intermediates, $z \in [0, z^*)$. Thus, in the equilibrium with unbundling, the relative income share of country 1 becomes

$$\frac{s_1^{with}}{s_2^{with}} = \frac{Z_1}{Z_2} = \frac{1 - z^*}{z^*} = \frac{1}{\sqrt{2} - 1} > 1. \tag{19}$$

We have established the following result.

Proposition 1 *Consider a world with two ex-ante identical countries. Without unbundling of production, the income share of the two countries is the same. With unbundling of production, the two countries end up with strictly different world income shares.*

Equation (19) shows that the country that specializes in more capital-intensive intermediates becomes richer in the steady-state with unbundling, even though the two countries have the same productivity. The intuition is that the country that specializes in more capital-intensive intermediates accumulates more capital, which gives this country additional comparative advantage in producing capital-intensive intermediates. The symmetric equilibrium is unstable. Suppose we start with a symmetric equilibrium in which both countries produce all intermediates in the same amount. Consider a small positive perturbation to the productivity of country 1. Country 1, gains comparative advantage on the production of capital-intensive intermediates. Once country 1 starts producing more capital-intensive intermediates, it accumulates more capital, which reinforces the pattern of comparative advantage. Thus, even if the initial perturbation vanishes after a small period of time, country 1 retains the comparative advantage in capital-intensive intermediates.

Another way to understand this result is that unbundling of production changes the production function of countries. Without unbundling, all countries have the same aggregate production function because they have the same productivity and they produce the same intermediates. However, with unbundling, each country only produces a set of intermediates. Since these intermediates differ on the capital-intensity required to produce them, the capital share of the aggregate production function is larger in country 1. This causes that in the steady-state country 1 accumulates more capital. Therefore, both countries have the same capital without unbundling, but country 1 accumulates more capital than country 2 in the equilibrium with unbundling.

3.1.2 A world with a large number of ex-ante identical countries

The symmetry breaking result extends to a world with a large number of countries that are identical in terms of their productivity, $\theta(j) = \theta$. In this case, equation (17) reduces to

$$\Delta_j = \Delta_{j+1} \qquad \text{for all} \qquad j = 1, \ldots, J-1.$$

Using the boundary conditions $z_0 = 1$ and $z_J = 0$, we obtain the following result.

Proposition 2 *Consider a world with J ex-ante identical countries in terms of their productivity level θ. Without unbundling of production, the world income share of each country is identical and equal to $1/J$. With unbundling of production, symmetry breaking occurs. Country j specializes in the set of intermediates $(z_j, z_{j-1}]$ with*

$$z_j = 1 - \sqrt{\frac{j}{J}}. \tag{20}$$

Note that the threshold in equation (20) is a decreasing and convex function of j. Thus, while all countries have an equal share of the world income in the equilibrium without unbundling, inequality emerges among ex-ante identical countries in the equilibrium with unbundling. Remember that the world income share of country j is

$$s_j = Z_j,$$

where Z_j comes from the specialization in intermediates and differs across countries.

$$Z_j = z_{j-1} - z_j = \sqrt{\frac{j}{J}} - \sqrt{\frac{j-1}{J}}.$$

This term is decreasing and convex, which means that countries that specialize in capital-intensive intermediates have a higher income share.[19]

As in Matsuyama (2013), the model does not have a prediction as to which specific country will occupy rank-j in the world economy, but it shows that endogenous inequality will emerge. Notice that a symmetric equilibrium (all countries produce equal shares of all intermediates and, thus, have the same income) would also potentially be possible in this case. However, the intuition for the two-country case carries over to this general case. The symmetric equilibrium is not stable to small perturbations to productivity. As one country starts producing more capital-intensive intermediates, it accumulates more capital, which reinforces the initial comparative advantage in capital-intensive intermediates.[20]

3.2 Heterogenous Countries

In this section we study how the world income distribution changes when countries are heterogenous and differ in their productivity level. We first consider a world that consists of two countries and show that inequality increases with unbundling. Then, we show that this result extends to a large number of countries and provide the additional result that middle-productivity countries are the most likely to lose with unbundling of production.

3.2.1 The two-country case

Consider a world that consists of two countries with different productivity levels. Let us assume that $\theta_1 > \theta_2$ and, without loss of generality, $\theta_1 + \theta_2 = 1$. The threshold z^* that divides the intermediates produced by each country is given by

$$A(\theta, z) = \left(\frac{\theta_1}{\theta_2}\right)^{\frac{1}{1-z}} = \frac{\frac{1}{2} - \left(z - \frac{z^2}{2}\right)}{\left(z - \frac{z^2}{2}\right)} = B(z).$$

Note that $A(\theta, z)$ is increasing in z with $A(\theta, z = 0) = \theta_1/\theta_2 > 1$ and $\lim_{z\to 1} A(\theta, z) = \infty$. In addition, $B(z)$ is decreasing in z with $\lim_{z\to 0} B(z) = \infty$ and $B(z = 1) = 0$. This implies that the solution to this equation z^* is unique. Moreover, z^* is continuous and monotonically decreasing with θ_1/θ_2. The reason is that the larger is the productivity difference between the two countries, the larger is the share of intermediates that country 1 produces. Note that this

[19] The first derivative is proportional to $j^{-1/2} - (j-1)^{-1/2}$, which is negative for $j > 1$. The second derivative is proportional to $-j^{-3/2} + (j-1)^{-3/2}$, which is positive for $j > 1$.

[20] Note that the symmetry breaking result holds if we had assumed that varieties also differ on capital-intensity requirements, $\ln Y_j = \int_0^1 \beta_j(z) \ln a_j(z, v) dz$ provided that each country produced varieties with the same distribution of capital-intensity requirements. The reason is that we assume that varieties are differentiated by origin (Armington assumption). That is, if varieties differ also on capital-intensity, we would have that ex-ante identical countries have the same world output share in the equilibrium model without unbundling. There would still be symmetry breaking when there is unbundling of production for the same logic as in the main text.

implies that inequality in the unbundling equilibrium is greater with heterogeneous countries than with countries with the same productivity.

Proposition 3 *Inequality between countries increases with unbundling of production.*

We can write the change in the relative income of country 2 between the two equilibria as

$$s_2^{with} - s_2^{without} = z^* - \theta_2.$$

The difference in relative income share is negative, which means that unbundling of production leads to more inequality between the two countries. The reason is that the rich country specializes in more capital-intensive intermediates, thereby accumulating more capital and increasing the income gap between the two countries.

To better understand this result, we decompose the change in world income between changes in the relative labor and capital income,

$$\begin{aligned}\left(\frac{w_2}{w_1}\right)^{with} - \left(\frac{w_2}{w_1}\right)^{without} &= \left(\frac{\theta_2}{\theta_1}\right)^{\frac{1}{1-z^*}} - \frac{\theta_2}{\theta_1} < 0,\\ \left(\frac{\rho k_2}{\rho k_1}\right)^{with} - \left(\frac{\rho k_2}{\rho k_1}\right)^{without} &= \frac{z^{*2}}{1-z^{*2}} - \frac{\theta_2}{\theta_1} < 0.\end{aligned}$$

Country 2 relatively loses in both sources of income with unbundling. For relative wages, notice that unless the two countries have the same productivity, the new relative wage will be lower in country 2 (because $z^* > 0$). For capital income, country 1 specializes in more capital-intensive intermediates, thereby accumulating more capital in the steady-state.[21] Therefore, unbundling of production exacerbates the inequality between the two countries.

3.2.2 A world with a large number of countries

Equation (17) characterizes the assignment of countries to the production of intermediates. Unfortunately, equation (17) is not analytically solvable. To simplify the problem, we take the same approach as in Matsuyama (2013). We approximate the solution to the case in which the number of countries is very large, $J \to \infty$. In this case, equation (17) converges to a second-order differential equation. Making parametric assumptions on the distribution of θ_j allows us to solve the assignment problem.

Define a new country index $\omega = j\varepsilon$ for $\varepsilon > 0$ and $j = 1, 2, \ldots, J$. We proceed by taking

[21] Analytically, notice that $z^* < \theta_2$ directly implies that $\frac{z^{*2}}{1-z^{*2}} - \frac{\theta_2}{\theta_1} < 0$. Rearranging terms and noting that $\theta_1 + \theta_2 = 1$, this condition becomes ${z^*}^2 < \theta_2$, which it is true because $z^* < \theta_2 < 1$.

the limit $\varepsilon \to 0$ and $J \to \infty$ such that $\lim_{\varepsilon\to 0, J\to\infty} \varepsilon J = \bar{\omega} \leq \infty$. Equation (17) becomes

$$\left(\frac{\theta_{\omega+\varepsilon}}{\theta_\omega}\right)^{\frac{1}{1-z_\omega}} = \frac{\Delta_{\omega+\varepsilon}}{\Delta_\omega}. \tag{21}$$

Taking Taylor series expansions around $\varepsilon = 0$ for the left-hand side of equation (21) we obtain[22]

$$\left(\frac{\theta_{\omega+\varepsilon}}{\theta_\omega}\right)^{\frac{1}{1-z_\omega}} = 1 + \frac{1}{1-z(\omega)}\frac{\theta'(\omega)}{\theta(\omega)}\varepsilon + o(\varepsilon)^2.$$

Note that we are assuming that, as countries become arbitrarily close ($\varepsilon \to 0$), so do their productivities. In other words, we assume that $\theta(\omega)$ is a smooth function with a well defined derivative in its domain. For the right-hand side, we find that

$$\frac{\Delta_{\omega+\varepsilon}}{\Delta_\omega} = 1 + \left(\frac{z''(\omega)}{z'(\omega)} - \frac{z'(\omega)}{1-z(\omega)}\right)\varepsilon + o(\varepsilon)^2.$$

Taking the limit as $J \to \infty$, so that all terms of order higher than ε are negligible, we find that $z(j)$ has to satisfy the following second-order differential equation

$$(1-z(\omega))\frac{z''(\omega)}{z'(\omega)} - z'(\omega) = \frac{\theta'(\omega)}{\theta(\omega)}, \tag{22}$$

with terminal conditions $z(0) = 1$ and $z(\bar{\omega}) = 0$.

We already know from the equilibrium assignment that more productive countries specialize in capital intensive (higher index z) intermediates, $z'(\omega) < 0$. Thus, $\theta'(\omega)z'(\omega) > 0$. Rearranging (22), we find that $z(\omega)$ is convex, as $z''(\omega) = (1-z(\omega))^{-1}(\theta'(\omega)z'(\omega)/\theta(\omega) + z'^2(\omega)) > 0$.

Notation change. In what follows, we abuse notation and use j to denote the continuous country index ω.

The differential equation governing the assignment process (22) is a non-linear differential equation, which cannot be characterized in analytical form without making parametric assumptions on $\theta(j)$. To make further progress in the analysis, we specialize $\theta(j)$ to be a distribution that approximates well the data. Our theory suggests that θ_j can be obtained by looking at the distribution of TFP across countries or, alternatively, at the world income distribution without unbundling, equation (14), which is also proportional to θ_j. Figure 3 reports the distribution of TFP and income per capita shares in 1988.[23] We find that the

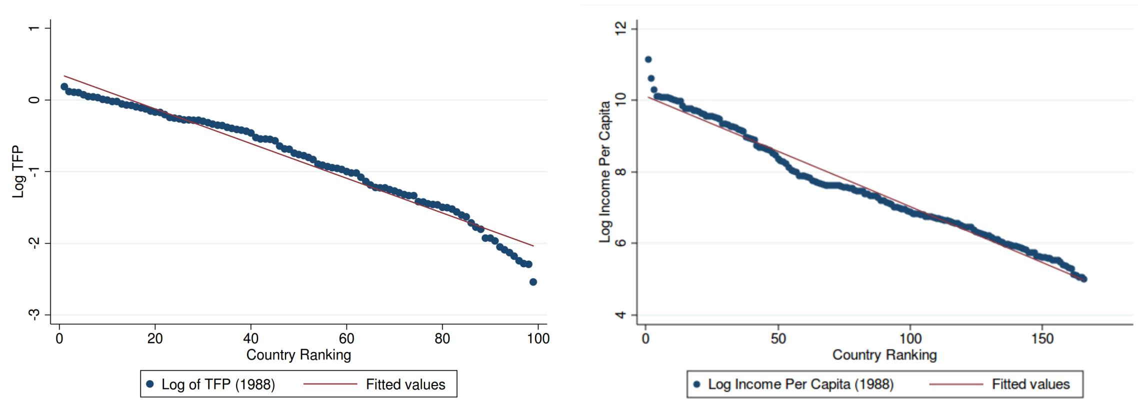
[22]Note that the ratio of productivities can be written as

$$\frac{\theta(\omega+\varepsilon)}{\theta(\omega)} = 1 + \frac{\theta'(\omega)}{\theta(\omega)}\varepsilon + o(\varepsilon)^2.$$

[23]The election of 1988 is given by our data source, Hall and Jones (1999), which report TFP data for this year. Note that it coincides with the change in trade regime documented in Figure 1.

Figure 3: Distribution of TFP and World Income Share

(a) TFP (b) World Income Shares



Notes: TFP data are obtained from Hall and Jones (1999). Income per capita data are obtained from the World Bank WDI.

exponential fit is remarkably good. The R^2 of TFP on the country ranking is .97, and .99 for income shares.[24] Thus, we proceed making the following assumption.

Assumption 1 *Countries' productivity θ is exponentially distributed,*

$$\theta(j) = \lambda \exp(-\lambda j), \qquad j \in [0, \infty).$$

Note that the most productive country, $j = 0$, has productivity level $\theta(0) = \lambda$ and productivity is decreasing in j. Given this particular functional form, the differential equation (22) becomes

$$(1 - z(j))\frac{z''(j)}{z'(j)} - z'(j) = -\lambda,$$

with terminal conditions $z(0) = 1$ and $z(\infty) = 0$. Making the change of variables

$$v(1 - z(j)) = \frac{d(1 - z(j))}{dj} = -z'(j),$$

equation (22) can be written as

$$v(1 - z)\left(\lambda + (1 - z)v'(1 - z) + v(1 - z)\right) = 0,$$

where we have used that $-z''(j) = v(1 - z)v'(1 - z)$. There are two solutions to this equation. The relevant solution is given by the terms inside the brackets (the other solution is to have

[24] This fit is better than a Pareto, which yields an R^2 of .8 and .69, respectively. We can also compute the solution of the differential equation for the Pareto distribution.

$z(j)$ being constant, so that $v(1-z)=0$). Integrating the terms inside brackets and applying the boundary conditions, we can characterize the inverse of the assignment function,

$$j(z)=\frac{z+\log z^{-1}-1}{\lambda},$$

which is monotonically decreasing in z. It is possible to invert this function and obtain $z(j)$, although, the expression involves a transcendental function,

$$z(j)=-W(-\exp(-1-\lambda j)), \tag{23}$$

where $W(z)$ is the Lambert $W-$function defined as the real solution of $z=xe^x$ for x.

Proposition 4 *The assignment function $z(j;\lambda)$ is continuously decreasing and convex in j and λ. The cross-partial derivative $z_{j,\lambda}$ is negative for all $j<\bar{\jmath}(\lambda)$ and positive for $j>\bar{\jmath}(\lambda)$.*

With a continuum of countries, the income share of country j becomes $\mu(j)/\int\mu(j)dj$ in the equilibrium without unbundling and $-z'(j)$ in the equilibrium with unbundling. Using the assumption that productivities follow an exponential distribution, the change in the world income share can be written as[25]

$$\Delta s(z)=z\lambda\left(\frac{1}{1-z}-e^{1-z}\right).$$

The change in income share is negative for $z\in(0,\bar{z})$ and positive for $z\in(\bar{z},1]$.[26] Thus, the income share declines in the countries assigned to the intermediates $z<\bar{z}$ and it increases in the rest. The next Proposition characterizes the change in the world income distribution as a function of fundamentals, rather than the endogenous variable z.

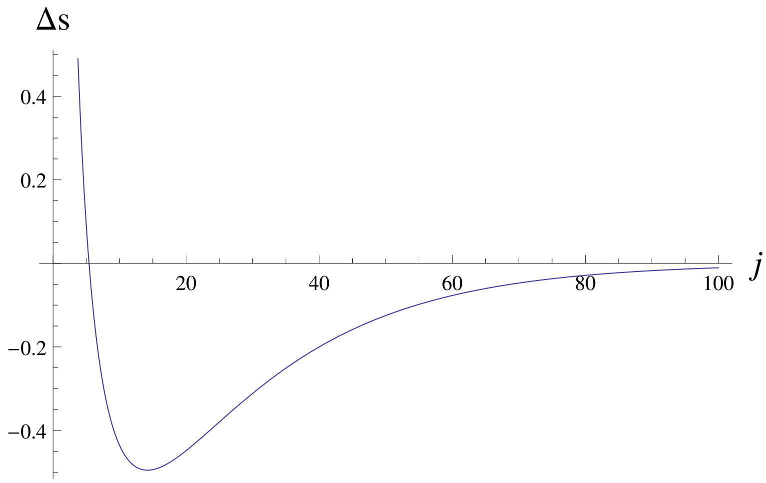
Proposition 5 *The change in the income share from the equilibrium without unbundling to the equilibrium with unbundling, $\Delta s(j)$, (i) is continuous in j, (ii) it is decreasing in j for $j<j_-$ and increasing thereafter, with $j_-=\lambda^{-1}\left(-3W(3)-\log\left(1-3W(3)\right)\right)$, (iii) it is convex for $j<j_c$ and concave thereafter, with $j_c<j_-$, (iv) $\Delta s(0)=\infty$, $\Delta s(\infty)=0$ and $\Delta s(\lambda^{-1}\left(-W(1)-\log\left(1-W(1)\right)\right))=0$.*

This proposition implies that *(i)* top-bottom inequality increases with unbundling and *(ii)* middle-productivity countries are the most likely to lose in absolute terms. Figure 4 illustrates

[25] To derive this expression note that $s^{\text{without}}(z)=\lambda e^{-\lambda\left(-\frac{1+\log\left(ze^{-z}\right)}{\lambda}\right)}=\lambda z e^{1-z}$. In addition, to express the income share with unbundling, note that $s^{\text{with}}(j)=-\frac{dz}{dj}\iff s^{\text{with}}(z)=-\frac{1}{\frac{dj}{dz}}$. Using that $\frac{dj}{dz}=-\frac{1-z}{\lambda z}$,we have that $s^{\text{with}}(z)=\frac{\lambda z}{1-z}$. The change in income share in terms of j is $\Delta s_j=\frac{\lambda W(-\exp(-1-\lambda j))}{1+W(-\exp(-1-\lambda j))}-\lambda\exp(-\lambda j)$.

[26] To see this, note that $\Delta s(z)$ is continuous, increasing for z for $z\in(1-3W(1/3),1]$ and decreasing otherwise. Moreover, $\Delta s(0)=0$, $\frac{d\Delta s}{dz}(0)<0$, $\frac{d\Delta s}{dz}(1)=\infty$ and the result follows. Also note that $\Delta s(z)$ is convex for all $z\in[0,1]$.

Figure 4: Change in World Income Shares



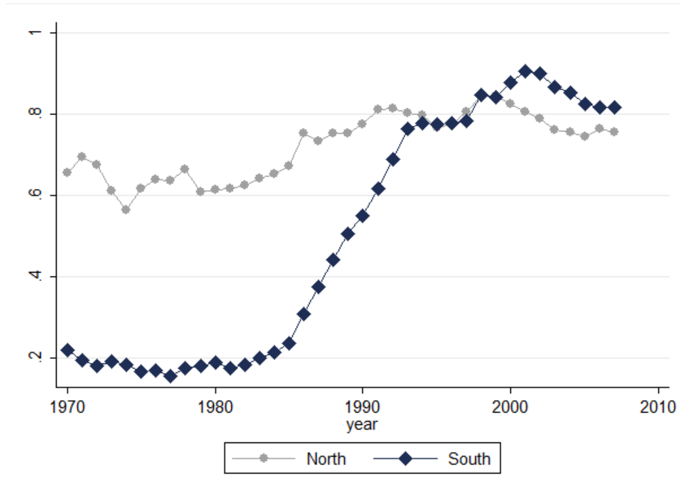
a generic case. Without unbundling of production, the demand of capital is determined by the number of varieties a country produces. In contrast, with unbundling, the demand of capital is determined by the intermediates in which the country specializes. The most productive country gains the most because it specializes in the most capital-intensive intermediates. Low productive countries specialize in low-capital-intensive intermediates but they do not lose much because they accumulated a small amount of capital in the equilibrium without unbundling. The main losers are middle-productivity countries. These countries accumulated a sizeable amount of capital in the equilibrium without unbundling. However, they now compete against more productive countries and end up specializing in relatively low-capital-intensive intermediates and, thus, with less capital and a lower income share. In other words, there is a large mismatch between the capital they accumulated in the equilibrium without unbundling and the needed to produce the equilibrium intermediates with unbundling. Since world output increases, a lower income share does not imply that a country loses in absolute terms. However, middle-productivity countries are the most likely to lose in absolute terms.

Some of these predictions are in line with the observed changes in the world income distribution between 1990 and 2008.[27] Consistent with our model, top-bottom income inequality increased during this period. For instance, the 90th percentile to 10th ratio of income per capita rose from 24 to 28 and the 95th-5th ratio went from 38 to 42. To test the prediction that the income shares of most productive countries increases while they declined for the rest, we have regressed income per capita growth between 1990-2008 on the country's TFP ranking in 1988 from Hall and Jones (1999). We find that the coefficient of this regression is negative, which is supportive of our prediction. However, the coefficient is not precisely estimated and it is not significant at conventional levels.[28] Indeed, many other factors have affected the world

[27] We choose to finish at 2008 to exclude the effects of the Great Recession.

[28] Figure 8 in the Appendix reports the distribution of the income per capita growth in this period over the

Figure 5: Ratio of Value of Exported Intermediates to Final Goods.



Source: Feenstra World Trade Database. To classify goods as intermediates, we use the end-use classification of Feenstra and Jensen (2012). Southern countries are defined as countries with GDP per capita (PPP) lower than 50 percent of the United States in 2000.

income distribution during this period and empirically disentangling the effects of unbundling on the world income distribution is beyond the scope of this paper.

4 Extensions

In this section we make three extensions to the baseline model and we provide an exact microfoundation to the production function of varieties we assumed in the baseline model. The first extension studies the effects of southern countries joining the global supply chain. The second extension analyzes the effects of a labor saving technology, computerization, on inequality. In the last extension, we study how the diffusion of technology changes the world income distribution.

4.1 South Joins the Global Supply Chain

One interpretation of the increasing importance of trade in intermediates is that southern countries have joined the global supply chain (e.g., Baldwin, 2012). Figure 5 reports evidence supporting this view. It decomposes the ratio of exported intermediates to final goods between northern and southern countries. Note that trade in intermediates increased in both northern and southern countries after late 1980s but the most dramatic increase was in southern countries. For southern countries, the ratio was roughly constant around .2 before the 1990s, when it sharply increased and it has converged to around .8 in the late 2000s.

TFP ranking of the countries.

Motivated by this evidence, we analyze the effect on the world income distribution of southern countries joining the global supply chain. We consider a world of J countries and define as South the set of countries with a productivity level θ below $\underline{\theta}$. We compare two equilibria. *Before* the South joins the global supply chain: all countries trade varieties but only countries with productivity θ above $\underline{\theta}$ can trade intermediates. *After* the South joins the global supply chain: all countries trade both varieties and intermediates.

The equilibrium *after* southern countries join the global supply chain is the same as in the baseline model (subsection 3.2.2). The income share of each country j is given by $s_j^{after} = -dz^{after}/dj$, where the assignment of intermediates to countries is given by equation (23).

The equilibrium income share *before* the South joins the global supply chain is a piecewise function that specifies the income share for northern and southern countries separately. Southern countries are those with low productivity levels, that is, countries $j > \underline{j}$, where $\underline{j} = \frac{1}{\lambda}\ln\left(\frac{\lambda}{\underline{\theta}}\right)$. As southern countries only trade varieties, their income shares, implied by the trade balance condition, are

$$s_j^{before} = \theta(j) = \lambda\exp(-\lambda j), \qquad \text{for } j > \underline{j}.$$

Northern countries trade both varieties and intermediates. The trade balance of each northern country $j < \underline{j}$ implies that[29]

$$s_j^{before} = -\frac{dz^{before}}{dj}\left(1 - \frac{\int_{\underline{j}}^{\infty}\mu_j dj}{\int_0^{\infty}\mu_j dj}\right), \qquad \text{for } j < \underline{j}, \tag{24}$$

where z^{before} is the equilibrium assignment of intermediates when only northern countries trade intermediates.

Therefore, we need to derive the equilibrium assignment of intermediates to compute the income share of northern countries before the South joins the global supply chain. To derive the assignment, we proceed in an analogous way as in Section 3.2.2 and solve equation (22) with the terminal condition $z(\underline{j}) = 0$. That is, the South (countries with $j > \underline{j}$) does not participate in intermediates trade. The equilibrium assignment is given by

$$j = -\frac{1 - z^{before}}{\lambda} - \frac{C_1^*(\underline{j})}{\lambda^2}\log\left(1 - \frac{\lambda(1 - z^{before})}{C_1^*(\underline{j})}\right),$$

[29] Denoting by $\xi_{\underline{j}}$ the amount of varieties produced by southern countries (i.e., $\xi_{\underline{j}} = \sum_{j=\underline{j}}^{J}\mu_j$), the trade balance of northern countries j becomes

$$\frac{\mu_j}{N}\left(\sum_{i=1}^{J}p_iY_i - p_jY_j\right) + Z_j\frac{N - \xi_{\underline{j}} - \mu_j}{N}\sum_{i=1}^{J}p_iY_i = \frac{N - \mu_j}{N}p_jY_j + (1 - Z_j)\frac{\mu_j}{N}\sum_{i=1}^{J}p_iY_i.$$

Rearranging, $s_j^{before} = Z_j^{before}\left(1 - \frac{\xi_{\underline{j}}}{N}\right)$ and taking the limit to a continuum of countries becomes (24).

where $C_1^*(\underline{j})$ is an integrating constant. We show in Appendix C.1 that $z^{before}(j;\underline{j})$ is decreasing in $\underline{j}$. This is illustrated in Figure 6a for two different $\underline{j}$. It means that if there are more countries participating in intermediates trade ($\underline{j}$ larger), each northern country specializes in more capital-intensive intermediates (higher z). Finally, note that, by definition, $z^{before}(j;\underline{j}=\infty) = z^{after}(j)$.

We can write the change in the world income distribution when the South joins the global supply chain as

$$\Delta s_j = \begin{cases} -z'(j) - \lambda e^{-\lambda j} & \text{if } j > \underline{j} \text{ (Southern country)}, \\ -z'(j) + z'(j;\underline{j})(1 - e^{-\lambda \underline{j}}) & \text{if } j < \underline{j} \text{ (Northern country)}. \end{cases}$$

Proposition 6 *When the South joins the global supply chain, all northern countries increase their income shares. If $\underline{j} < j^*$, southern countries with $j \in [\underline{j}, j^*]$ increase their income share and the rest decrease their share, where $j^* = -\lambda^{-1}(W(1) + \log(1 - W(1)))$. If $\underline{j} > j^*$, the income share of all southern countries declines.*

The reason for these results is as follows. For southern countries, we have the same comparison as in Section 3.2.2. Their income shares increase if they can produce more intermediates than their share of varieties. Therefore, if the country is productive enough, it produces enough intermediates and accumulates more capital participating in the global supply chain, thereby increasing its income share. For northern countries, there are two effects: *(i)* selection effect: they produce less intermediates but they are more capital-intensive and *(ii)* market size effect: northern countries sell intermediates to all the countries, not only in the North. The overall effect is positive because northern countries specialize in more capital-intensive intermediates and sell them to a bigger market. Figure 6b illustrates the change in the world income distribution.

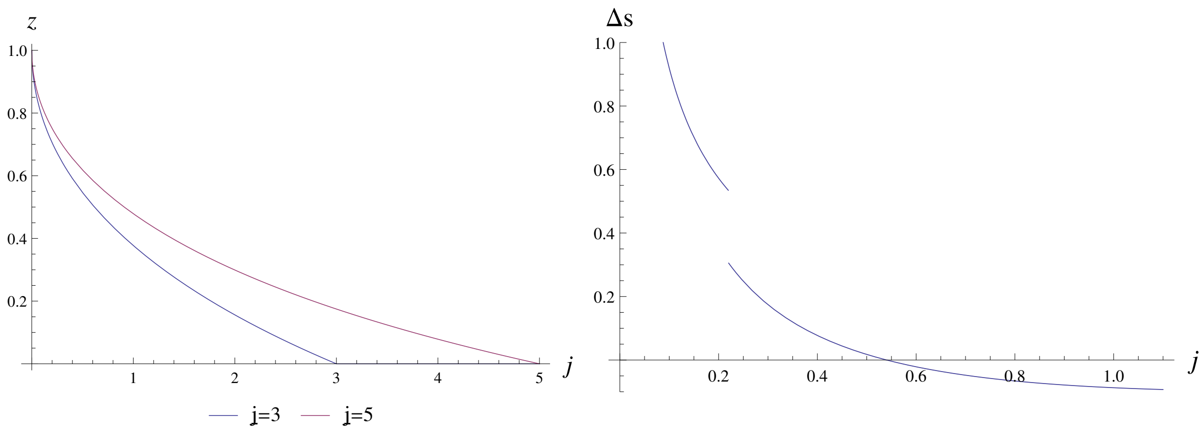
In this section, we have assumed, for simplicity, that southern countries either fully participated or did not participate in intermediates trade. In Section 2 of the Online Appendix we relax this assumption and we assume that a fraction $\alpha(j)$ of a country participates in intermediates trade, where $\alpha(j)$ is a decreasing function of j. We show numerically that the same qualitative results hold. As $\alpha(j)$ increases, the income share increases in countries with $j < j^*$ and it decreases in the rest.

4.2 Computerization

The adoption of Information Technologies has been pointed out as one important reason behind the unbundling of production (see, for example, Basco and Mestieri, 2013). Moreover, Autor et al. (2003) among others have argued that computerization, by eliminating labor-intensive tasks, has also changed the income distribution within countries. In this extension, we analyze how the effects of computerization on the income distribution depend on the trade regime.

Figure 6: South joins the Gobal Supply Chain

(a) Change in Equilibrium Assignment with $\underline{j}$ (b) Change in the World Income Distribution



As discussed in equation (5), a bundle of intermediates of different labor-intensity must be assembled to produce a variety v,

$$x_j(v) = \exp\left[\int_0^1 \beta(z)\ln a_j(z,v)dz\right],$$

where $\beta(z)$ is a weight on intermediate z, with $\int_0^1 \beta(z)dz = 1$. We model computerization as a shift in the weighting function $\beta(z)$ that reduces the weight to labor-intensive (low z) intermediates.

More precisely, we assume that the distribution $\beta(z)$ has a monotonically decreasing probability ratio (MPR), where the probability ratio is defined as

$$\mathcal{I}(z) = \frac{\beta(z)}{B(z)},$$

and $B(z)$ denotes the cumulative distribution of z.[30] We assume that computerization induces a shift in $\beta(z)$ that can be ranked in terms of the probability ratio. Supposing that γ indexes computerization, we assume that $\mathcal{I}(z;\gamma)$ is monotonically increasing in γ. Eeckhoudt and Gollier (1995) show that a monotone increase in the probability ratio implies a first-order stochastic dominant shift.[31] Accordingly, we define computerization as an increase in γ. That is, an increase in γ implies that, ceteris paribus, less relatively labor-intensive intermediates

[30] This condition has been applied in other economic contexts, see Hopkins and Kornienko (2004) and the references therein. The normal, uniform and exponential distribution among other distributions satisfy this condition.

[31] Moreover, they also show that a Monotone Likelihood Ratio (MLRP) order implies the Monotone Probability Ratio order. Thus, MPR is more stringent than first-order stochastic dominance but less stringent than MLRP.

are needed to produce each variety.

For example, one family of distributions satisfying the MPR ordering is given by

$$\beta(z) = \begin{cases} 0 & \text{if } z < \gamma, \\ \frac{1}{1-\gamma} & \text{if } z \in [\gamma, 1], \end{cases} \tag{25}$$

where γ is the index of computerization. When $\gamma = 0$, there is no computerization and $\beta(z) = 1$, as we assumed in the baseline model. For $\gamma > 0$, the most labor-intensive intermediates $z < \gamma$ are no longer required to produce varieties.

In the equilibrium without unbundling, the income share of each country depends only on the number of varieties and it is given by $s_j = \mu_j / \int_{j \in \mathcal{J}} \mu_j dj$, which is independent on the weighting function $\beta(z)$. However, computerization decreases the demand of labor, which in equilibrium increases the capital income share.

To analyze the equilibrium with unbundling, note that computerization changes the equilibrium assignment. Proceeding as in section 3.2.2, the assignment function is characterized by the following differential equation

$$(1 - z(j)) \left(\frac{z''(j)}{z'(j)} + z'(j) \frac{\beta'(z(j))}{\beta(z(j))} \right) - z'(j) = \frac{\theta'(j)}{\theta(j)} = -\lambda.$$

Note that $\beta(z)$ enters into the assignment function through its semi-elasticity, $z'(j)\beta'(z)/\beta(z)$.

The solution to this differential equation with boundary conditions $z(0) = 1$ and $z(\infty) = 0$ is given by[32]

$$j(z) = \frac{1}{\lambda} \int_z^1 \mathcal{I}(x, \gamma)(1 - x) dx.$$

The income share in terms of z is

$$s(z) = \frac{\lambda}{\mathcal{I}(z, \gamma)(1 - z)}. \tag{26}$$

With unbundling, computerization changes the world income distribution. From equation (26), we see that γ affects the income shares through the inverse probability ratio, $\mathcal{I}(z, \gamma)$, and the equilibrium assignment $z(j(\gamma))$. On the one hand, by assumption, $\mathcal{I}(z, \gamma)$ is increasing in γ, which reduces the income share. On the other hand, $z(j(\gamma))$ increases with γ, each country j is now assigned to a higher z intermediate, which raises the income share.[33] Therefore, the overall effect on the income share (26) is ambiguous. The next Proposition shows that it depends on the country ranking.

[32]Note that if $\beta(z) = 1$, we obtain that $j(z) = \lambda^{-1}(z - \log z - 1)$ as in the baseline model. Also, note that, for simplicity, we are reporting the case in which the support of intermediates remains $[0, 1]$. Appendix C.2 discusses the case when $\beta(z)$ takes the form of (25), in which the support changes with computerization.

[33]Note from equation (4.2) that $j(z, \gamma)$ increases monotonically with an increase in $\mathcal{I}(z, \gamma)$.

Proposition 7 *In the equilibrium without unbundling, computerization does not affect inequality between countries. In the equilibrium with unbundling, computerization increases the income share for countries with $j \in [0, j_1)$ and decreases it for countries with $j > j_2$. If $\beta(z)$ is given by equation (25), an increase in γ increases the income share of countries $j < j^*$, while it decreases in the rest. Computerization raises the capital income share in all countries and both trade regimes.*

Proposition 7 implies that top-bottom inequality unambiguously increases. The reason is that all countries specialize in more capital-intensive intermediates. However, this shift in the pattern of specializing disproportionately favors the most productive countries, which can now specialize in even more capital-intensive intermediates. This is the reason why the income share raises at the top. The least productive countries do not benefit from computerization because $\beta(z)$ does not change much at the extreme of the distribution.

To sum up, in this section we have shown that the effects of computerization on the world income distribution depend on the trade regime. Without unbundling, computerization does not change the relative income of countries. In contrast, with unbundling, computerization leads all countries to specialize in more capital-intensive intermediates, which exacerbates the income differences between countries. Moreover, computerization always raises the capital income share. This empirical prediction is consistent with the finding of Karabarbounis and Neiman (2013) that the labor-share has declined in most countries.

4.3 Diffusion of technology

The source of comparative advantage in our model is technology. In the baseline model we assumed that technology is exogenous and constant. However, technology diffuses over time and low-productivity countries eventually learn the innovations that the countries in the technological frontier make. In this section we analyze how the diffusion of technology changes the world income distribution with and without unbundling.

We assumed, consistent with the data, that productivity follows an exponential distribution

$$\theta(j) = \lambda \exp(-\lambda j).$$

We model technological catch-up of low-productivity countries as a decline in the parameter λ from λ_1 to $\lambda_2 < \lambda_1$. This implies a first-order stochastic shift in the distribution of productivities in the world.[34]

[34] Note that this formulation implies a counterfactual decline in the TFP level of the most productive countries. We choose this formulation for notational convenience. It can be verified that the same results on income shares apply if we define technological catch-up as a change only in the slope of the original exponential function, $\theta(j) = \lambda \exp(-(\lambda - \xi)j)$ with $\xi > 0$. This formulation would avoid reducing TFP in absolute levels for the most productive countries. However, the same results go through in terms of income shares because, in relative terms, we still have a decline in TFP for the most productive countries and the assignment function would

Proposition 8 *Diffusion of technology leads to convergence in income with and without unbundling. Moreover, the income share increases in more low-productivity countries when there is unbundling of production.*

These results are illustrated in Figure 7. Without unbundling of production, the income share of country j is $s_j^{without} = \theta(j)$. Note that changes in productivity directly affect the income share. It is then straightforward to see that the income share increases in low-productivity countries ($j > \bar{j}$) and declines in the rest ($j < \bar{j}$).[35] Therefore, diffusion of technology leads to convergence in income shares.

With unbundling of production, the income share of country j is $s_j^{with} = -z'(j)$. It means that productivity affects the income share through the endogenous assignment of intermediates. To understand the effect of technology diffusion on the income share, first notice how the assignment function changes,

$$\Delta j = \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)(z - \log z - 1) > 0.$$

This change in the assignment function implies that low-productivity countries are climbing up the ladder of global supply chains by producing higher z intermediates. This new selection of intermediates results in an increase in the income share of low-productivity countries ($j > j^\dagger$) and a decline in the rest ($j < j^\dagger$). The reason is that, due to the diffusion of technology, low-productivity countries can now produce more intermediates, thereby increasing their income share. This result implies that diffusion of technology leads to income convergence.

Finally, we compare the changes in the world income distribution under the two trade regimes. It is straightforward to check that

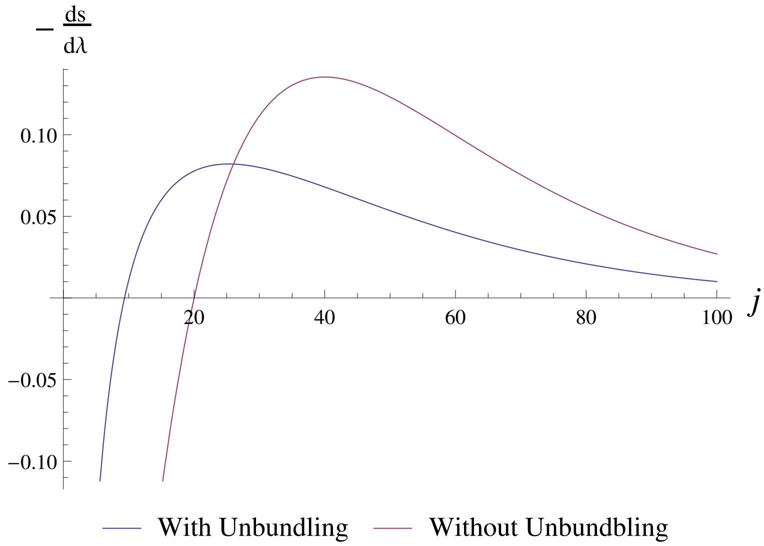
$$\frac{\partial s_j^{with}}{\partial \lambda}\Big|_{j=\bar{j}} < 0.$$

This inequality implies that $\bar{j} > j^\dagger$, which means that in the equilibrium with unbundling the income share increases for a larger mass of low-productivity countries. In particular, the income share of countries with $j \in \left(j^\dagger, \bar{j}\right)$ raises with unbundling but falls without unbundling. The intuition is that, in the equilibrium with unbundling, the relative productivity (not the absolute level) determines the assignment of intermediates. The slope of the distribution of productivities flattens with the diffusion of technology, which results in countries with productivity $j \in \left(j^\dagger, \bar{j}\right)$ gaining comparative advantage against nearby more-productive countries, which allows them to climb the supply chain ladder and produce relatively more capital-intensive intermediates.

remains unaltered as $\theta'(j)/\theta(j) = -\lambda + \xi$.

[35] Using that $\theta(j)$ follows an exponential distribution, the threshold $\bar{j}$ is $\bar{j} = \frac{\ln\left(\frac{\lambda_1}{\lambda_2}\right)}{\lambda_1 - \lambda_2}$.

Figure 7: Change in the World Income Distribution with a change in λ



To sum up, in this section we have shown that diffusion of technology leads to convergence in income under the two trade regimes. However, the mass of low-productivity countries benefiting from technological catch-up is larger in the trade equilibrium with unbundling of production.

4.4 Endogenous number of varieties

In this section we provide an exact microfoundation to the exogenous number of varieties that we postulated in the baseline model.

We assume that there exists an innovation sector that produces new varieties. The innovators sell the patents to the producer of varieties. Inventors extract all the surplus of the producer of varieties, who has monopoly rights on the production of the variety.

The final good is needed to produce innovation. In particular, we follow Jones (1995) and assume that the production function of ideas perceived by an innovator is

$$\mu_j = \phi_j i_j,$$

where $\phi_j = (\widetilde{\kappa}\theta_j)^{1-\lambda}(i_j)^{\lambda-1}$ and i_j is the amount of final good devoted to innovation. One can think of ϕ_j as the probability of finding a new idea, which is increasing with the productivity in the country and decreasing with the number of innovators looking for a new idea.

The innovator sells the blueprint to the producer of a variety who has monopoly rights on the production of the variety. However, we assume that there exists a competitive fringe that can copy the variety at a marginal cost $(1+\sigma)$ higher than the blueprint's marginal cost. This imposes a constraint on the price that the monopoly producer of any variety v can charge,

$$p_j^x(v) = (1+\sigma)MC_j(v),$$

where $MC_j(v)$ denotes the marginal cost of production using the blueprint. The demand of variety v at this price is

$$x_j(v) = \frac{1}{(1+\sigma)MC_j(v)} \frac{\sum_{i=1}^J p_i Y_i}{N}.$$

Thus, the profits of the producer of a variety are total revenues less variable costs and the price of purchasing the idea p_j^R,

$$\pi_j(v) = \frac{(1+\sigma)MC_j(v)}{(1+\sigma)MC_j(v)} \frac{\sum_{i=1}^J p_i Y_i}{N} - \frac{MC_j(v)}{(1+\sigma)MC_j(v)} \frac{\sum_{i=1}^J p_i Y_i}{N} - p_j^R,$$

which simplifies to

$$\pi_j(v) = \frac{\sigma}{1+\sigma} \frac{\sum_{i=1}^J p_i Y_i}{N} - p_j^R.$$

Profit maximization in the innovation sector implies that

$$p_j = \phi_j p_j^R.$$

Given that innovators extract all the rents of the producer of varieties, it follows that $p_j^R = \frac{\sigma}{1+\sigma} \frac{\sum_{i=1}^J p_i Y_i}{N}$.

Note that we can set $p_j = 1$ and find that the amount of final good used in the innovation sector in country j is

$$i_j = p^{R^{\frac{1}{1-\lambda}}} \widetilde{\kappa} \theta_j.$$

Finally, we can use this equation and the production function of varieties to find that the number of varieties in country j is

$$\mu_j = \kappa \theta_j,$$

with $\kappa = \widetilde{\kappa}^{1-\lambda} \left(\frac{\sigma}{1+\sigma} \frac{\sum_{i=1}^J Y_i}{\sum \theta_j} \right)^{\lambda}$.

To sum up, this section has provided an exact microfoundation to the exogenous production function of varieties assumed in the baseline model. Note that this solution is the same with and without unbundling. The only difference is that the level of world output and, thus, κ is higher in the equilibrium with unbundling. Moreover, the relative number of varieties, which determines the relative income share without unbundling, is independent of κ.

5 Concluding Remarks

In this paper we have developed a framework to study how the international unbundling of production changes the world income distribution. In our setup, countries only differ in their productivity. Each variety requires a bundle of intermediates, which use capital and labor in different proportions.

We showed that in the steady-state without unbundling (only varieties are costlessly traded), the world income share is determined by the fraction of varieties that each country produces. We also provided a microfoundation to show that the measure of varieties is proportional to the productivity of a country. We found that the world income distribution changes with unbundling (intermediates can also be traded). The world income share of a country depends on the intermediates that each country produces.

Our first main result is that unbundling of production brings about symmetry breaking. That is, countries with the same productivity have the same income in the equilibrium without unbundling. In contrast, unbundling of production leads to divergence in income levels. The intuition is that arbitrarily small differences in productivity translate into comparative advantage differences in capital-intensive intermediates. Specialization in capital-intensive intermediates, induces capital accumulation, thereby reinforcing the initial comparative advantage. As a result, specialization in capital-intensive intermediates increases the capital-labor ratio of a country, which translates into a higher income share.

Our second main result is to show that unbundling of production raises top-bottom inequality and it generates non-monotonic changes in the world income distribution. The largest fall in income shares is in middle-productivity countries. The reason is that the most productive countries specialize in capital-intensive intermediates and, thus, accumulate more capital and become relatively richer. Middle-productivity countries lose relatively more because they produce a sizeable amount of varieties and, thus, they accumulated a considerable amount of capital in the equilibrium without unbundling. However, with unbundling, the stock of capital only depends on the intermediates in which the country specializes. Since the country has intermediate productivity, it specializes in relatively low-capital-intensive intermediates. Thus, it accumulates relatively less capital and it ends with a lower income share.

We showed that when southern countries join the global supply chain (participate in trade in intermediates), the income shares of all northern and the most productive southern countries increase, while they decrease for the rest of southern countries. The reason is that northern countries specialize in more capital-intensive intermediates and sell them to a larger market. However, only productive enough southern countries are able to climb up the ladder of global supply chains to specialize in sufficiently capital-intensive intermediates.

We also analyze how the effect of a labor-saving technology, computerization, depends on the trade regime. Without unbundling, computerization has no effect on the world income distribution. In contrast, with unbundling, computerization exacerbates inequality between countries. The reason is that with computerization all countries specialize in more capital-intensive intermediates, which disproportionately favors the most productive countries.

Finally, our model predicts that diffusion of technology leads to income convergence. However, the mass of countries benefiting from technological catch-up is larger in the trade equilibrium with unbundling of production. The reason is that in these countries relative productivity declines, which reduces the income share without unbundling. However, relative productivity falls less than in their nearby more-productive countries, which allows them to produce more capital-intensive intermediates and increase their income share with unbundling.

The unbundling of production is exogenous in the model. Nonetheless, in practice, firms adopt technologies (for example, computers and the internet) to be able to offshore part of the production process. We plan on extending our framework to analyze the interdependence between technology adoption and trade. We have only considered two factors of production: capital and labor. Although we think of capital in broad terms, which could also include human capital (along the lines of Matsuyama, 2004), a more careful investigation of the distinctive effects of human capital accumulation would be another interesting extension of the model.

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A Tables and Figures

Table 1: TFP and Exports in Labor-Intensive Industries

$$X_{ict} = \alpha + \beta \cdot \text{TFP}_c \cdot \text{Labor Intensity}_{it} + \delta_i + \delta_c + \delta_t + \varepsilon_{ict}$$

	(1)	(2)	(3)	(4)
$\text{TFP}_c \cdot \text{Labor-Intensity}_{it}$	-1.20	-1.05	-1.44	-1.31
	(0.61)	(0.65)	(0.81)	(0.86)
Country Fixed Effects	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes
Intermediates Fixed Effects	Yes	Yes	Yes	Yes
Year*Country FE	No	Yes	No	Yes
Year*Intermediates FE	No	No	Yes	Yes
Observations	240,397	240,397	240,397	240,397

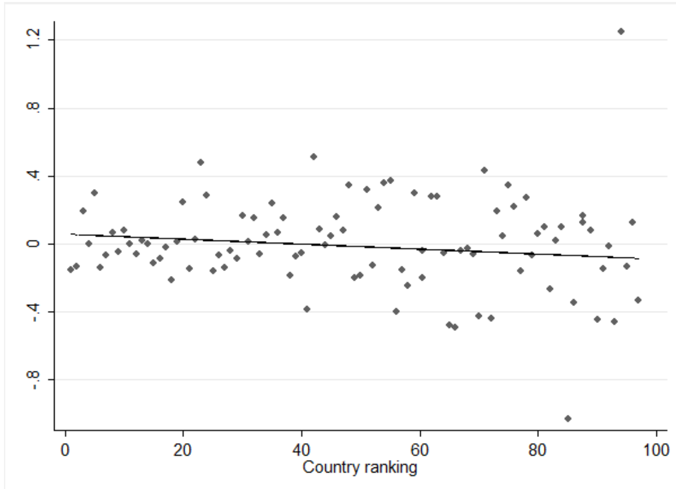
Notes: Standard errors are clustered at country level. Dependent variable X_{ict} is the log of world exports of intermediates i of country c in year t from 1994 to 2008. Our data is disaggregated at 6-digit NAICS. To classify intermediates, we use the definition in Feenstra and Jensen (2012). TFP is total factor productivity from Hall and Jones (1999) in 1988 and Labor-intensity is computed from NBER CES manufacturing industry database.

Table 2: Capital-Intensity at different levels of aggregation

	Capital-Intensity	
	Varieties (3-digit NAICS)	Intermediates (6-digit NAICS)
Unconditional Mean	1.13	1.13
Unconditional Std. Deviation	0.37	0.66
Within Std. Deviation	0.40	0.52
Within Range	1.64	2.02

Notes: Within Std. Dev. for varieties refers to the standard deviation of the average capital intensity at three digits of aggregation. Within Std. Dev. for intermediates refers to the average of the standard deviation computed at six-digit level conditional on belonging to a given 3-digit NAICS category (i.e., a variety). While the unconditional std. deviation assumes that all intermediates are used to produce a given variety, within std. deviation assumes that only the 6-digit intermediates that start with the same 3-digit are used to produce a given 3-digit NAICS (i.e, to produce, for example, the variety 311 only 311XXX intermediates are used). Analogous definitions are used to compute the range. Note that, in practice, intermediates with different 3-digits are used to produce a given variety, thus, the right measure lies between the two. Capital-intensity is computed as the ratio between total real capital stock and value added from NBER CES manufacturing industry database. We report the results for year 1990 (the year around which we assume the unbundling of production started). Tables 1 and 2 in the on-line appendix show that we obtain the same qualitative results if we use 1980 or 2000.

Figure 8: GDP per capita Growth (1990-2008): Difference from the mean



Source: GDP per capita (PPP) is obtained from World Development Indicators (World Bank). Country ranking is the TFP ranking from Hall and Jones (1999). The line represents the predicted values of a linear of regression of both variables, excluding China (the point in the upper-left-side). The negative coefficient is significant at 90%. Without excluding China, the coeficient remains negative but it is not significantly different from zero.

B Proofs of Propositions in Section 3

Proof of Proposition 2 Equation (20) can be derived as follows. Denoting by $x_j = z_j - \frac{z_j^2}{2}$, equation (20) implies that $x_j - x_{j+1} = d$ for some $d > 0$. Moreover, $\sum_{j=1}^{J} x_j = 1/2$. These two conditions imply that $d = 1/2J$. Thus, $x_j = x_{j+1} + d$ and $x_j = (J - j)/2J$. Solving for z_j, we find equation (20).That is, using the definition of x_j in terms of z_j we obtain $-\frac{z_j^2}{2} + z_j - \frac{J-j}{2J} = 0$. The unique solution of this second-order equation that is between zero and one is $z_j = 1 - \sqrt{j/J}$. Note that it satisfies the boundary conditions, $z_J = 0$ and $z_0 = 1$. Alternatively, the same thresholds can be derived by taking the limit for $J \to \infty$ and working with a differential equation, as in Section 3.2.2. In this case, the differential equation governing the assignment is $(1 - z(j))\frac{z''(j)}{z'(j)} - z'(j) = 0$,with terminal conditions $z(0) = 1$ and $z(J) = 0$. The solution to this differential equation is (20).

Proof of Proposition 3 we need to check that $\Delta s_2 = \frac{1}{2}(z^* - \theta_2) < 0$,where z^* is the solution to the threshold z^* that divides the intermediates produced by each country and given by the next expression, $A(\theta, z^*) = B(z^*)$,where $A(\theta, z) = \left(\frac{\theta_1}{\theta_2}\right)^{\frac{1}{1-z}}$ and $B(z) = \frac{\frac{1}{2} - \left(z - \frac{z^2}{2}\right)}{\left(z - \frac{z^2}{2}\right)}$. Note that, on the one hand, $A(\theta, z)$ is increasing in z with $A(\theta, z = 0) = \theta_1/\theta_2 > 1$ and $\lim_{z\to 1} A(\theta, z) = \infty$. On the other hand, $B(z)$ is decreasing in z with $\lim_{z\to 0} B(z) = \infty$ and $B(z = 1) = 0$. This implies that the solution to this equation is unique. Moreover, z^* is continuous and monotonically decreasing with θ_1/θ_2. Moreover, since $\theta_1 + \theta_2 = 1$, $\theta_2 < \theta_1$

implies that $\theta_2 \in (0, 1/2)$. We know that for $\theta_2 = 1/2$, $z^* = 1 - \sqrt{1/2} < 1/2$. Therefore, for $\theta_2 = 1/2$, it is verified that $\Delta s_2 < 0$. Thus, we only need to check that $\Delta s_2 < 0$ for $\theta_2 = \varepsilon$, where ε is a positive number. In other words, we need to show that $z^* < \varepsilon = \theta_2$. When $\theta_2 = \varepsilon$, the equilibrium threshold z is implicitly defined by $\varepsilon^{\frac{1}{1-z}} = \frac{z - \frac{z^2}{2}}{\frac{1}{2} - \left(z - \frac{z^2}{2}\right)}$. Note that $z = 0$ does not solve this equation. In particular, the left-hand side is larger than the right-hand side. Moreover, it is straightforward to check that $z = \varepsilon$ does not solve this equation either. In addition, the left-hand side is smaller than the right-hand side. Thus, Bolzano's Theorem guarantees that there exists $z^* \in (0, \varepsilon)$ that solves this equation.

Proof of Proposition 4 and 5 The change in the income share of country j is

$$\Delta s(z) = z\lambda \left(\frac{1}{1-z} - e^{1-z} \right). \tag{B.1}$$

Note that at $\Delta s(1) = \infty$ and that $\Delta s(0) = 0$. Also, note that $1/(1-z)$ is increasing in the relevant domain while e^{1-z} is decreasing. Moreover, at $z = 0$, $e > 1$ and at $z = 1$, $\infty > 1$. Thus, the two curves cross once (actually at a value $1 - W(1) \simeq .43$).

$$\frac{d\Delta s}{dz} = \lambda \left(\frac{1}{(1-z)^2} - e^{1-z}(1-z) \right). \tag{B.2}$$

It is readily verified that the derivative is positive for $z \in (1 - 3W(1/3) \simeq .23, 1]$ and negative otherwise. Moreover $\frac{d\Delta s}{dz}(0) < 0$ and $\frac{d\Delta s}{dz}(1) = \infty$.

The second derivative is

$$\frac{d^2\Delta s}{dz^2} = \lambda \left(\frac{2}{(1-z)^3} + e^{1-z}(2-z) \right), \tag{B.3}$$

which is positive for all $z \in [0, 1]$.

Finally we can analyze the shape of $\Delta s(j)$ given the previous results given that we can write $\Delta s(j(z))$. Recall that the mapping of z to j is continuously decreasing with $s(0) = \infty$ and $s(1) = 0$. This shows that $\Delta s_j(0) = \infty$, $\Delta s_j(j) > 0$ for $j < j(1 - W(1))$ and increasing for $j > j(1 - W(1))$, and $\Delta s_j(\infty) = 0$. Using the implicit function theorem we have that $d\Delta s_j/dj = d\Delta s_j(z(j))/dj$

$$\frac{d\Delta s_j}{dj} = \frac{d\Delta s_j(z(j))}{dz(j)} \frac{dz(j)}{dj} = \frac{d\Delta s_j(z(j))}{dz(j)} \frac{1}{\frac{dj}{dz}}. \tag{B.4}$$

As $\frac{dj}{dz} < 0$, we have that $\frac{d\Delta s_j}{dj}$ is decreasing for $j \in [0, j(1 - 3W(1/3))$ and increasing thereafter. Moreover, note that as $\frac{d\Delta s_j(z(j))}{dz(j)}\Big|_{z=0}$ is bounded and $\frac{dz(j)}{dj}\Big|_{z=0} = \infty$ we have that $\frac{d\Delta s_j}{dj}\Big|_{j=\infty} = 0$. Thus, as we have a function it cannot be convex throughout its support.

Finally, for the second derivative, using that

$$\frac{d^2 z(j)}{dj^2} = -\frac{d^2 j(z)}{dz^2}\left(\frac{1}{\frac{dj(z)}{dz}}\right)^3 \tag{B.5}$$

we have that

$$\frac{d^2\Delta s_j}{dj^2} = \frac{d}{dj}\left(\frac{d\Delta s_j(z(j))}{dz(j)}\frac{1}{\frac{dj}{dz}}\right) \tag{B.6}$$

$$= \frac{d^2\Delta s_j(z(j))}{dz(j)^2}\left(\frac{1}{\frac{dj}{dz}}\right)^2 - \frac{d\Delta s_j(z(j))}{dz(j)}\frac{d^2 j(z)}{dz^2}\left(\frac{1}{\frac{dj(z)}{dz}}\right)^3. \tag{B.7}$$

The first term is always positive. The second term has the first derivative of the share which is decreasing and then increasing in j, the derivative of $dj(z)/d(z)$ which is always negative and the term

$$\frac{d^2 j(z)}{dz^2} = \frac{1}{\lambda z^2}, \tag{B.8}$$

which is always positive. For $j < j(1-3W(1/3)$, we have that $z > 1-3W(1/3)$, which implies that the second derivative is unambiguously convex. For $j > j(1-3W(1/3))$ we have that $z < 1-3W(1/3)$ and the sign is ambiguous. We have that

$$\frac{d^2\Delta s_j(z(j))}{dz(j)^2} - \frac{d\Delta s_j(z(j))}{dz(j)}\frac{d^2 j(z)}{dz^2}\frac{1}{\frac{dj(z)}{dz}} = \lambda\frac{1-e^{1-z}(1-z)^5+2z}{z(1-z)^3} \tag{B.9}$$

As $1+2z$ is increasing in z and $e^{1-z}(1-z)^5$ is decreasing, and the numerator evaluated at 0 is negative $1-e<0$ and at $1/2$ is positive $2 > e^{1/2}/2$, we have a unique solution such that below a critical threshold the equation is concave. This threshold is given by the solution to $1-e^{1-z}(1-z)^5+2=0$ which is approximately $z = .123$ (and thus smaller than $1-3W(1/3)$).

C Proofs and detailed derivations in Section 4

C.1 Results in Section 4.1

Note that the general solution to the differential equation is

$$\frac{\lambda(1-z(j))}{C_1} = 1 + W\left(\frac{e^{-1-\frac{\lambda^2(j+C_2)}{C_1}}}{C_1}\right). \tag{C.1}$$

The first boundary condition is that $z(0) = 1$, which yields

$$W\left(\frac{e^{-1-\frac{\lambda^2 C_2}{C_1}}}{C_1}\right) = -1. \tag{C.2}$$

Thus,

$$\frac{e^{-1-\frac{\lambda^2 C_2}{C_1}}}{C_1} = -e^{-1} \tag{C.3}$$

and we can express C_2 as

$$C_2 = \frac{-C_1 \log(-C_1)}{\lambda^2}. \tag{C.4}$$

Substituting in the general solution and simplifying, we find that

$$j = -\frac{1-z}{\lambda} - \frac{C_1}{\lambda^2}\log\left(1 - \frac{\lambda(1-z)}{C_1}\right). \tag{C.5}$$

or alternatively,

$$\frac{\lambda(1 - z(j))}{C_1} = 1 + W\left(-e^{-1-\frac{\lambda^2 j}{C_1}}\right) \tag{C.6}$$

The second terminal condition is that $z(\underline{j}) = 0$. Thus, substituting in the previous equations we have that

$$\underline{j} = -\frac{1}{\lambda} - \frac{C_1}{\lambda^2}\log\left(1 - \frac{\lambda}{C_1}\right). \tag{C.7}$$

Note that for $\underline{j} > 0$, it has to be the case that $C_1 > \lambda$ or that $C_1 < 0$. Rearranging, we find that

$$-\lambda(1 + \lambda\underline{j}) = C_1 \log\left(1 - \frac{\lambda}{C_1}\right). \tag{C.8}$$

The left hand side of this expression is negative and decreasing in $\underline{j}$. Note for $\underline{j} \to \infty$ it becomes $-\infty$. This implies that $C_1 = \lambda$ or that $C_1 = -\infty$. The latter case would yield a constant function in the assignment function, which cannot be a solution. Thus, we select $C_1 = \lambda$. Moreover, as the domain of W is $[-1, \infty)$ the branch of C_1 that can solve (C.6) is $C_1 \geq \lambda$. Note that this implies that the solution $C_1(\underline{j})$ is continuous in the parameter $\underline{j}$. For $0 < \underline{j} < \infty$, as the left hand is negative and decreasing in $\underline{j}$. Moreover, the first derivative of the right hand of (C.8) is

$$\frac{\lambda}{C_1 - \lambda} + \log\left(\frac{C_1 - \lambda}{C_1}\right). \tag{C.9}$$

Note that this function asymptotes to $+\infty$ when $C_1 \to \lambda$ and to 0 when $C_1 \to \infty$. The second derivative is

$$\frac{-\lambda^2}{C_1(C_1 - \lambda)^2} < 0. \tag{C.10}$$

for all $C_1 \geq \lambda$. Thus, as the function is strictly convex over $[C_1, \infty)$, we have that it is monotonically decreasing. This implies, the first derivative (C.9) is always positive. We have shown that the right hand side of (C.8) is monotonically increasing and convex. Moreover, it is readily verified that the range of the right hand side is $(\infty, 0]$. Hence, the solution to (C.8) exists and is unique. Moreover, as the left hand side is decreasing in $\underline{j}$, we have that $C_1(\underline{j})$ is decreasing in $\underline{j}$.

The solution for $\underline{j} < \infty$ can be characterized implicitly proceeding in an analogous manner as in (C.3) to obtain

$$C_1^* = \frac{\lambda(1+\lambda \underline{j})}{(1+\lambda \underline{j}) + W\left(-e^{-(1+\lambda \underline{j})}(1+\lambda \underline{j})\right)}. \tag{C.11}$$

Note that for $\underline{j} = \infty$, $C_1 = \lambda$ and otherwise $C_1 > \lambda$. If $j \to 0$ we have that $C_1 = \infty$ and we obtain a flat assignment (the only country produces everything). Indeed, C_1 is monotonically declining in $\underline{j}$

Thus, the equilibrium assignment is characterized by

$$j = -\frac{1-z}{\lambda} - \frac{C_1^*(\underline{j})}{\lambda^2} \log\left(1 - \frac{\lambda(1-z)}{C_1^*(\underline{j})}\right). \tag{C.12}$$

The assignment $j(z, \underline{j})$ is decreasing in j. To see this, take the derivative of (C.12) with respect to $\underline{j}$ to find that

$$\frac{\partial j(z, \underline{j})}{\partial \underline{j}} = \frac{C^{*'}(\underline{j})}{\lambda^2}\left(\frac{(1-z)\lambda}{(1-z)\lambda - C^*(\underline{j})} - \log\left(1 - \frac{(1-z)\lambda}{C^*(\underline{j})}\right)\right). \tag{C.13}$$

The term inside brackets is always negative (it is minus (C.9)). Thus the partial derivative is negative. As a result, taking the derivative of the inverse function, we have that $z(j; \underline{j})$ is also decreasing in $\underline{j}$. This is illustrated in figure 6a.

Finally, we are interested in computing the cross-partial of $z(j; \underline{j})$. Note that

$$\frac{\partial j(z; \underline{j})}{\partial z} = -\frac{1-z}{C_1(\underline{j}) - \lambda(1-z)} \tag{C.14}$$

$$\frac{\partial z(j; \underline{j})}{\partial j} = -\frac{C_1(\underline{j}) - \lambda(1 - z(j; \underline{j}))}{1 - z(j; \underline{j})}. \tag{C.15}$$

$$\tag{C.16}$$

Taking the derivative of the second equation we find that

$$\frac{\partial^2 z(j; \underline{j})}{\partial z \partial \underline{j}} = \frac{-(1-z)C_1'(\underline{j}) - C_1 \partial z / \partial \underline{j}}{(1-z)^2} > 0 \tag{C.17}$$

as $1-z \geq 0$, $C_1' < 0$, $C_1 > 0$ and $\partial z/\partial \underline{j} < 0$, the previous equation is unambiguously positive.

For the southern countries, those countries with $j > \underline{j}$ where $\theta(\underline{j}) = \underline{\theta}$, the world income share is just the relative number of varieties

$$s_j^B = \frac{\mu_j}{\int \mu_j dj} = \lambda \exp(-\lambda j). \tag{C.18}$$

For the northern countries $j < \underline{j}$, the income share calculation differs from (16) because the demand of intermediates comes only for countries that are integrated in the global supply chain. Following the same steps as above, it is straightfoward to derive that the income share is

$$s_j^B = -\left(z_j^B\right)' \left(1 - \frac{\int_{\underline{j}}^{\infty} \mu_j dj}{\int_0^{\infty} \mu_j dj}\right), \tag{C.19}$$

where z_j^B is the equilibrium assignment of intermediates when only northern countries can trade intermediates.

Characterization of the changes in the world income distribution The change in the world income distribution is thus given by

$$\Delta s_j(z) = -z'(j) - \left((1-\mathbb{1}_{\underline{j}})z'(j;\underline{j})\left(1-e^{-\lambda \underline{j}}\right) + \mathbb{1}_{\underline{j}}\lambda e^{-\lambda j}\right) \tag{C.20}$$

where $\mathbb{1}_{\underline{j}}$ is an indicator function that takes value of 1 if $j > \underline{j}$ and zero otherwise. The first term $z'(j)$ refers to the derivative of (23), which is the particular case $z'(j;\underline{j} = \infty)$. From (C.17) we have that $z'(j) < z'(j;\underline{j})$. However, the presence of the extra term, implies that

$$\begin{aligned} \frac{d^2 z(j;\underline{j})\left(1-e^{-\lambda \underline{j}}\right)}{dj d\bar{j}} &= \frac{d}{d\underline{j}}\left(\lambda - \frac{C_1 \underline{j}}{1-z(j;\underline{j})}\right)(1-e^{-\lambda \underline{j}}) && \text{(C.21)} \\ &= \frac{-C_1'(1-z) - C_1 \partial z/\partial \underline{j}}{(1-z)^2} > 0 && \text{(C.22)} \end{aligned}$$

as $C_1 > \lambda$, $0 < z < 1$, $C_1' < 0$ and $\partial z \partial / \underline{j} < 0$.[36] This implies, that for $j < \underline{j}$ the equilibrium with the integrated world generates a higher share than the equilibrium in which the south does not participate in unbundling. Thus, this shows that the "North" always increases its share of world output with the south joining the global supply chain.

For the south there are two possible cases, either all countries lose or some lose and the southern countries with highest TFP win. To see this, we show that the income shares with complete unbundling is decreasing in j faster than without the south joining the global supply chain. And that depending on $\underline{j}$, the income share of the most productive southern country without unbundling can be either higher or lower than in the final equilibrium.

We analyze when the two curves cross. Note that the income share before we have the South joining

$$s(j) = \lambda e^{-\lambda j}$$

can be expressed in terms of the ex-post assignment $j(z)$, to obtain

$$s(z) = \lambda e^{1-z+\log z}.$$

The income share when all countries join is

$$s^{unbundling}(z) = \frac{\lambda z}{1-z}. \tag{C.28}$$

Equating the previous two equations, we find that the solution is

$$\tilde{z} = 1 - W(1) \tag{C.29}$$

Thus, this implies that in order to have a crossing $\underline{j}$ has to be less than

$$j(z = 1 - W(1)) = \frac{1 - W(1) - \log(1 - W(1)) - 1}{\lambda} = -\frac{W(1) + \log(1 - W(1))}{\lambda}. \tag{C.30}$$

[36] One can further characterize the function

$$\begin{aligned}
\frac{d^2 z(j;\underline{j})\left(1-e^{-\lambda \underline{j}}\right)}{dj^2} &= -\frac{j''(z)}{(j'^3} && \text{(C.23)}\\
&= C_1 \frac{C_1 - (1-z)\lambda}{(1-z)^3} > 0 && \text{(C.24)}\\
\frac{d^3 z(j;\underline{j})\left(1-e^{-\lambda \underline{j}}\right)}{dj^3} &= -j'''(z'^4 - 3j''z''(z'^2 && \text{(C.25)}\\
&= \frac{3C_1}{(1-z)(C_1-\lambda(1-z))^5} - 2\lambda C_1 \frac{(C_1 - \lambda(1-z))}{(1-z)^4} && \text{(C.26)}\\
\frac{d^2 z(j;\underline{j})\left(1-e^{-\lambda \underline{j}}\right)}{dj^2 d\underline{j}} &= \frac{-(2C_1 - \lambda(1-z))(1-z)C_1' + C_1 \partial z/\partial \underline{j}(-3C_1 + 2\lambda(1-z)}{(1-z)^4} > 0. && \text{(C.27)}
\end{aligned}$$

The result that these derivatives are unambiguously signed follows from the fact that $C_1 > \lambda$, $0 < z < 1$, $C_1' < 0$ and $\partial z \partial \underline{j} < 0$.

Otherwise there is not solution because the two lines do not cross. This completes the proof of the proposition

C.2 Proof of Propositions in Section 4.2

Proof of Proposition 7 Consider an increase in γ. From equation (26), the overall effect on (26) is ambiguous,

$$\frac{ds(j)}{d\gamma} = -\lambda \frac{\mathcal{I}_\gamma (1-z) - \mathcal{I} z_\gamma}{(\mathcal{I}(z)(1-z))^2}. \tag{C.31}$$

Note however that at the very top $z(j=0) = 1$, thus the top country increases its share unambiguously. Moreover, as at $j = \infty$, $z = 0$, we have that $z_\gamma(j = \infty) = 0$, thus the worst country unambiguously loses income share. As the $s(j)$ is continuous the first result follows.

For the particular case described in (25) we have that

$$j(z) = \int_z^1 \frac{(1-x)}{\lambda(x-\gamma)} dx = \frac{z - 1 - (1-\gamma)\log\left(\frac{z-\gamma}{1-\gamma}\right)}{\lambda}. \tag{C.32}$$

This equation defines implicitly $z(j, \gamma)$. We find that

$$\frac{dz}{d\gamma} = 1 + \log\left(\frac{z-\gamma}{1-\underline{z}}\right)\frac{z-\gamma}{1-z} > 0 \tag{C.33}$$

which is positive for all $z \in [\gamma, 1)$ and zero at $z = 1$. Moreover, this derivative is monotonically decreasing in z. In this case the index of the hazard rate is γ, and

$$\frac{d\mathcal{I}}{d\gamma} = \frac{1 - z_\gamma}{(z-\underline{z})^2}. \tag{C.34}$$

Finally note that the income share has to be normalized by the support of the distribution

$$s(z;\gamma) = \frac{\lambda}{\mathcal{I}(z,\gamma)(1-z(\gamma))(1-\gamma)}. \tag{C.35}$$

Thus, as

$$\frac{ds(j)}{d\gamma} > 0 \Longleftrightarrow \mathcal{I} z_\gamma (1-\gamma) + \mathcal{I}(1-z) - \mathcal{I}_\gamma (1-z)(1-\gamma) > 0. \tag{C.36}$$

Substituting, and arranging the terms, we find that the derivative is

$$\frac{(z-1)(z+\gamma-2) + (1-\gamma)^2 \log\left(\frac{z-\gamma}{1-\gamma}\right)}{(1-z)(z-\gamma)}$$

It is readily verified that this is a continuous function on its domain. Also, for $z = \gamma$, this function is $-\infty$, also for $z = 1$, the function is positive and equal to $(1+\gamma)/(1-\gamma)$. Taking the first order derivative of this expression and equating it to zero, it can be verified that it only has an interior extremum at

$$z^* = \frac{1+\gamma}{2} \tag{C.37}$$

and this is a maximum. This implies that this function only crosses once the zero in the relevant domain at a value $z < z^*$.

C.3 Derivation of Results in Section 4.3

The productivity distribution moves from being distributed exponential with parameter λ_1 to $\lambda_2 < \lambda_1$. The change in the assignment function is readily computed as

$$\Delta j = \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)(z - \log z - 1). \tag{C.38}$$

Thus, except for $z = 1$, for which $\Delta j = 0$ we have that the change is positive. Not only that, as $\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$ and

$$\frac{d}{dz}(z - \log z - 1) = 1 - \frac{1}{z} < 0 \tag{C.39}$$

Thus, the poorest countries are climbing up the ladder of global supply chains and producing higher z intermediates. The change in the income share is given by

$$\frac{ds(j)}{d\lambda} = -\frac{d}{d\lambda}\left(\lambda - \frac{\lambda}{1+W(-e^{-1-\lambda j})}\right) = -1 + \frac{\lambda j W(-e^{-1-\lambda j})}{(1+W(-e^{-1-\lambda j}))^3} + \frac{1}{1+W(-e^{-1-\lambda j})}. \tag{C.40}$$

Note that $\frac{ds(0)}{d\lambda} = \infty$ and $\frac{ds(\infty)}{d\lambda} = 0$. The second term is negative and increasing. It asymptotes towards $-\infty$ for $j = 0$ and towards zero as $j \to \infty$. The last term is positive and decreasing, asymptotically towards 1 as $j \to \infty$. The ratio of these last two terms is

$$\frac{\lambda j W(-e^{-1-\lambda j})}{(1+W(-e^{-1-\lambda j})^2} \tag{C.41}$$

It is negative, increasing and bounded between $[-.5, 0]$. Perhaps the simplest way is to analyze it is to realize that

$$\frac{ds(j)}{d\lambda} = \frac{s_j}{\lambda} + \frac{\lambda j W(-e^{-1-\lambda j})}{(1+W(-e^{-1-\lambda j}))^3}. \tag{C.42}$$

We have that the first term is positive and decreasing and dominates for $j \to 0^+$, meaning that the function is decreasing in 0^+. As s_j is decreasing and the second term is increasing but negative the overall behavior is ambiguous. However, it must exist a region in which $\frac{ds(j)}{d\lambda}$ as the overall integral of s_j is one, so if some countries increase their share some others have to lose it. Expressing s_j we find that the sign of the derivative coincides with the sign of

$$\frac{\lambda j}{(1+W(-e^{-1-\lambda j}))^2}-1, \tag{C.43}$$

or alternatively, whether

$$\lambda j-(1+W(-e^{-1-\lambda j}))^2 \lesseqgtr 0. \tag{C.44}$$

Note that both terms are equal to zero for $j=0$. The slope of the first term is λ, while

$$\frac{d}{dj}(1+W(-e^{-1-\lambda j})^2=-2\lambda W(-e^{1-j\lambda}). \tag{C.45}$$

It is readily verified that

$$-2\lambda W(-e^{1-j\lambda})>\lambda \tag{C.46}$$

if and only if $j\in[0,\frac{-1+2\log(2)}{2\lambda})$. Thus, we have that

$$\lambda j-(1+W(-e^{-1-\lambda j}))^2<0 \tag{C.47}$$

for $j\in[0,\frac{-1+2\log(2)}{2\lambda})$. Moreover, as λj grows at a slower speed than $(1+W(-e^{-1-\lambda j})^2$ for all $\frac{-1+2\log(2)}{2\lambda}$, it means that will exist a unique $j^\dagger>\frac{-1+2\log(2)}{2\lambda}$ such that for all $j<j^\dagger$ equation (C.44) is negative, and positive for $j>j^\dagger$. Moreover, this implies that

$$\frac{d^2 s(j^\dagger)}{d\lambda dj}<0 \tag{C.48}$$

as otherwise it would not be possible to reach zero as $j\to\infty$. This observation, joint with the fact that

$$\frac{d^2}{dj^2}\left(\frac{ds(j)}{dj}\right)\bigg|_{j=0}>0 \tag{C.49}$$

implies that the function ds/dj is convex for $j\in[0,j^{\dagger\dagger})$ with $j^{\dagger\dagger}>j^\dagger$ and concave thereafter.

We finally compare this trickle-down process of technology with what would happen in a world without unbundling. In this case, income distribution is given by $s(j)=\lambda e^{-\lambda j}$. So

$$\frac{d}{d\lambda}(\lambda e^{-\lambda j})=(1-\lambda j)e^{-\lambda j}. \tag{C.50}$$

Thus, we see that countries with $j<1/\lambda$ increase their and the rest decrease their share. For the shape of the change, we have that

$$\frac{d}{dj}\left(\frac{d}{d\lambda}(\lambda e^{-\lambda j})\right) = (-2+\lambda j)e^{-\lambda j}\lambda < 0 \iff j < 2/\lambda, \tag{C.51}$$

$$\frac{d^2}{dj^2}\left(\frac{d}{d\lambda}(\lambda e^{-\lambda j})\right) = (3-\lambda j)e^{-\lambda j}\lambda^2 < 0 \iff j > 3/\lambda. \tag{C.52}$$

Thus the function is decreasing for $j < 2/\lambda$ and then increasing, convex for $j < 3/\lambda$ and then concave. We compare the threshold for which countries increase their share in the equilibrium without unbundling with the one for the equilibrium with unbundling.

We evaluate equation (C.44) at $j = 1/\lambda$,

$$\lambda/\lambda - (1 - W(-e^{-1-\lambda/\lambda})^2 \simeq 1 - (1 + .1586)^2 = -.34 < 0 \tag{C.53}$$

this shows that the range of countries that increase their income share is larger in the equilibrium without unbundling than with unbundling. Moreover, as both changes in the share are convex in this regime and we have that the slope in the positive region of $ds/d\lambda$ is higher in the equilibrium with unbundling. Next we evaluate the minimum value of ds/dj without unbundling $j = 2/\lambda$ at $d^2s/djds$ with unbundling

$$\frac{d}{dj}\frac{ds^{unbundling}(j)}{d\lambda}\Bigg|_{j=2/\lambda} = 2\lambda W(-1/e^3)^2 \frac{4 + W(-1/e^3)}{(1 + W(-1/e^3))^5} \simeq .029\lambda > 0. \tag{C.54}$$