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“Competition in the Market for Flexible Resources: an application to cloud computing”

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Competition in the Market for Flexible Resources: an application to cloud computing*

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Abstract

This paper considers firms' incentives to invest in local and flexible resources when demand is uncertain and correlated. Before demand is realized, two firms decide to invest in their local capacity. Provider(s) of flexible resource observe these decisions and invest in their capacity. After demand is realized, firms buy flexible resource if demand exceeds their local capacity. I find that market power of the monopolist providing flexible resources distorts investment incentives, while competition mitigates them. The extent of improvement depends critically on demand correlation and the cost of capacity: under social optimum and monopoly, if the flexible resource is cheap, the relationship between investment and correlation is positive, and if it is costly, the relationship becomes negative; under duopoly, the relationship is positive. The analysis also sheds light on some policy discussions in markets such as cloud computing.

Keywords: capacity investment, cloud computing, competition, demand correlation

JEL Classification: D4, L8

1 Introduction

For firms in various industries, capacity investment decision involves investing early in their own capacity before demand for their products is realized, and such investment is difficult to reverse. After the demand is realized, firms have the option to undertake a second investment in a flexible resource to accommodate the excess demand, for instance by outsourcing. In the IT sector, cloud computing provides such an opportunity for outsourcing. Cloud computing is fundamentally the leasing of computer services, including computing power and storage, but

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on an unprecedented scale. While local computing capacity can support the average demand of the firm, cloud computing is able to scale services on demand and accommodate the workload that exceeds what the local capacity can handle.¹ Accordingly, firms can use cloud computing as a flexible resource for business continuity and disaster recovery plans.²

Moreover, in the cloud computing market, computing demand is uncertain as demand varies daily; and correlated at a global level. For example, a U.S. cloud provider such as Amazon, Google and Microsoft could have customers from Europe as well as Australia. Correlation is therefore driven to some extent by geography: computing demands from countries that are close to each other are positively correlated; demands from countries that are located in different time zones are negatively correlated. Moreover, as argued by Harms and Yamartino (2010), even the largest cloud provider will not be able to eliminate uncertainty and correlation.³

This paper focuses on the problem of capacity investment in two resources when demand is uncertain and correlated. In the cloud computing example, capacity is a key part of competition in this industry. In the introductory phase, it is common that cloud providers build far more capacity than needed, and one does not expect capacity to be an issue in this growing phase. However, as cloud computing enters a more mature phase, capacity may become constrained as demand grows quickly.⁴⁵ For example, on August 25, 2013, Amazon seems to struggle to keep up with the growing computing demand, and an IT problem at one of its datacenters has caused many users of major web services such as Instagram, Netflix, Vine and Airbnb to experience lengthy delays and reduced data transfer speeds for several hours.⁶ Amazon's web stores, Microsoft's outlook.com, Google's Gmail email service and the YouTube video site have also faced similar glitches from time to time. This raises a number of interesting questions: what is the profit-maximizing investment strategy in flexible resource such that the problem of quality degradation can be avoided? How should we promote efficient investment from a public policy perspective?

The contribution of this paper is twofold: first, I consider investment in two resources: firms first invest in their local capacity, and later can use flexible resources as an alternative sourcing option to cover temporary shortage of local resources; Second, I focus on uncertain and correlated demand; whereas the existing literature either assumes one type of resources

¹The U.S. National Institute of Standards and Technology provides five defining characteristics of cloud computing: on-demand service, broad network access, resource pooling, rapid elasticity and measured service. This paper focuses on the definition of on-demand service and rapid elasticity.

²Business continuity and disaster recovery plans minimize any disruption of business operation due to insufficient local capacity or failure of critical systems.

³In the cloud computing market, retailers increase computing demand during the holiday season; and businesses need more computing power during the tax season. However, this type of correlation is not correlation across firms, and is therefore not the focus of this paper.

⁴International Data Corporation (IDC) estimates that worldwide spending on public cloud services is expected to reach \$47.4 billion in 2013 and \$107 billion in 2017, which represents a growth rate five times that of the IT industry as a whole.

⁵Capacity can be interpreted in two ways: number of physical servers or service quality. In the former case, there is a maximum traffic that each server can handle. In the latter case, even if the capacity does not hit the limit, high demands can put a costly strain on servers, which results in poor quality of service.

⁶BBC news, "Instagram, Vine and Netflix hit by Amazon glitch," available at <http://www.bbc.co.uk/news/technology-23839901>, August 26, 2013 (accessed on August 27, 2013).

or ignores demand correlation. An interesting finding is that investment can increase with correlation, which is in contrast to the common belief that only negative correlations are valuable because the provider can aggregate demand and reduces the risk.⁷ The reason why providers invest more as correlation increases is that when capacity is cheap, providers can benefit more from high demand realizations without worrying about the risk of low demand realizations.

Two firms, whose demand is uncertain and correlated, make their investment decision in local resource under demand uncertainty. Observing firms' local investment, providers of flexible resource (e.g. Amazon, Google and Microsoft) decide how much to invest in capacity, and set the price for their flexible resource (e.g. Amazon Web Services (AWS), Google Compute Engine, Microsoft Azure). After demand is realized, firms can buy flexible resources if demand exceeds their local capacity.

I consider both cases of monopoly and duopoly in providing the flexible resource. As should be expected, investment is suboptimal in the monopoly market. Particularly, the provider of the flexible resource tends to underinvest in its capacity with respect to the socially optimal level, whereas firms tend to overinvest in their local capacity. Such inefficiency comes from market power of the monopolist. Firms invest in local capacity to avoid being exploited by the monopolist, which in turn reduces investment incentive of the monopolist.

Competition always mitigates the underinvestment problem, but more interestingly, the extent of improvement depends crucially on demand correlation and the cost of capacity. Both socially optimal and monopoly investment in flexible resource increases with correlation if the investment cost of flexible resource is small enough, and decreases with correlation if the flexible resource is costly. The reason is that as correlation increases, firms either "win big" when demand realization is high for both firms or "lose big" when demand realizations is low for both firms. If the flexible resource is cheap, the planner or the provider need not worry about "losing". Rather, they will focus on reaping benefits from the "winning" outcome, and therefore they invest more as correlation increases. On the contrary, if the flexible resource is expensive, then "losing" is costly, and thus they invest less as correlation increases.

Under duopoly, I show that investment in flexible resource is increasing in correlation for high or low correlations with a numerical example. The reason for not observing the negative relationship between investment and correlation in this case, as opposed to the social optimum and the monopoly case, is that firms rely more on the flexible resource as competition between providers lowers the price of flexible resources. Firms' incentive to capture the windfall from the "winning" high demand realizations increasingly outweighs their incentive to avoid the risk of "losing" as correlation increases. Knowing this, each provider is willing to build a bigger capacity of flexible resources. These results suggest that information on the cost condition and the degree of demand correlation have important consequences for investment. They also explain the need for far more data on costs and demand in order to underpin the appropriate degree of competition in an industry. I will discuss in more detail the implications of competition on investment in the cloud computing industry in the penultimate section.

⁷See, for instance, p. 218 of Bayrak et al. (2011).

1.1 Literature

This paper is closely related to the literature on capacity and resource flexibility in operational management. However, unlike this paper, this literature either studies monopolistic models that cannot explain the effect of competition or studies a competitive setting without demand correlation. For example, Lee (2009) studies the optimal capacity investment of a computing service provider in a single resource in the absence of correlated demands. Niyato, Chaisiri and Lee (2009) study the optimal choice of private and public computing service in the monopoly and oligopoly market, but again in a context without correlated demands. Both Van Miegham (1998), and Bish and Wang (2004) study the optimal investment strategy in flexible resources when a monopolist faces uncertain demands for its two products, which corresponds to the social optimum in this model. However, they did not identify the problem of suboptimal investment, and more importantly, how to correct the problem. There are few papers that study firms' choice of technology in a competitive setting. See, for instance, Goyal and Netessine (2007) and Anupindi and Jiang (2008). However, these papers focus on the production stage, without taking into account the incentives to provide flexible resource.

This paper is also related to the literature on Real Options (RO) in finance, which focuses on the role of RO in providing flexibility to management decisions. However, unlike financial assets, IT investments are not tradable, and therefore cannot be priced at the value of risk; rather they are priced by a third party, which is the service provider in this case. See, for instance, Angelou and Economides (2005), Benaroch and Kauffman (1999) and Kauffman et. al. (2002) for details on the limitation of RO's applicability in IT investments. Moreover, the RO literature usually assumes that the value of investment projects is uncorrelated, whereas demand correlation plays an important role here.

2 The Model

Consider two firms, 1 and 2, that need to build capacity in order to serve their customers. To do this, they can either invest in their own local resource L or they can buy flexible resources K from the market. The difference lies in that investments in local resources are irreversible and these resources are for the exclusive use of the investing firm, while flexible resources can be bought from the market instantly when needed and released when not needed. An example of flexible resources is cloud computing as cloud computing power is provisioned as an on-demand service. The firm gets a profit π for each consumer served.

Investment technology. The unit cost of local resource and flexible resource are denoted by c_L and c_K respectively. I assume that local resource is supplied competitively, so that firms can buy L at a price c_L . The flexible resource market can be either a monopoly or a duopoly.

Demand. The demand for the final services of the two firms is uncertain and correlated. More specifically, demands for firm 1 and 2, denoted by x and y respectively, are drawn from a joint distribution $h(x, y)$, with support $[0, \infty) \times [0, \infty)$. The demand of firm 1, x , is given by the marginal distribution $f(x) = \int_0^\infty h(x, y) dy$. Similarly, the demand of firm 2, y , is given by $g(y)$. In the following analysis, I focus on the case where demands (x, y) follow an exponential

distribution with $\lambda = 1$,⁸ but in Appendix E I show that the main results carry through in the linear case. More particularly, the exponential distribution can be described as follows. The marginal distributions $F(x)$ and $G(y)$ and marginal densities $f(x)$ and $g(y)$ are respectively

$$\begin{aligned} F(x) &= 1 - e^{-x}, \\ G(y) &= 1 - e^{-y}, \\ f(x) &= e^{-x}, \\ g(y) &= e^{-y}. \end{aligned}$$

The joint distribution function $H(x, y)$ and joint density function $h(x, y)$ follow Gumbel (1960):

$$\begin{aligned} H(x, y) &= (1 - e^{-x})(1 - e^{-y})(1 + \alpha e^{-x-y}), \\ h(x, y) &= e^{-x-y}[1 + \alpha(2e^{-x} - 1)(2e^{-y} - 1)], \end{aligned}$$

where $-1 < \alpha < 1$ is a measure of correlation.⁹

We consider the following game:^{10,11}

- Stage 1: firm 1 and 2 invest in their own local capacity L_1 and L_2 simultaneously;
- Stage 2: the provider(s) invest(s) in capacity of flexible resources K ;
- Stage 3: the provider(s) set(s) a per unit price of flexible resource p ;
- Stage 4: demands (x, y) are realized and firms decides whether and how much to buy the flexible resource.

It is clear that in Stage 4, if a firm's demand spikes above its local capacity, it will purchase flexible resources as long as the price is less than π . In other words, a firm's demand for flexible resources is price-inelastic.¹²

For simplicity, I make the following assumptions. First, $\pi > c_L$, so there is incentive to purchase local resources. Second, I focus on the more interesting case where $c_K < c_L$. For example, it is common in practice that cloud computing exhibits significant economies of scale. To facilitate our analysis, I focus on the specification with $\pi = 1$, $c_L = 0.5$ and $c_K \in [0, 0.5]$.¹³

⁸A distribution is exponential when $F(\lambda, x) = 1 - \lambda e^{-\lambda x}$ is satisfied.

⁹Strictly speaking, $\rho = \frac{\text{cov}(x, y)}{\sqrt{\text{var}(x)\text{var}(y)}}$ is the coefficient of correlation, but since α and ρ move in the same direction (more precisely, $\rho = \frac{\alpha}{4}$, see Equation (3.10) on p. 706 of Gumbel (1960)), there is no loss of generality in saying that α is a measure of correlation.

¹⁰I do not model entry here, but I expect the same qualitative result with entry. Although entry will lower the price, the underinvestment problem still exists as long as $p > c_K$.

¹¹Section 6.1 considers alternative timing.

¹²Qualitative results for the monopoly case would be similar if we consider elastic demand. As for the duopoly case, however, if we consider elastic demand, we can no longer follow the approach of de Frutos and Fabra (2011), who study a sequential capacity-price game under demand uncertainty with price-inelastic demands. Interested reader can see Reynolds and Wilson (2000) for a discussion of the two-stage game under the assumption of downward-sloping and uncertain demand.

¹³These assumptions are innocuous for two reasons. First, setting $\pi = 1$ is only a normalization, and it will not affect the qualitative conclusion. Second, the main results hold more generally as long as the flexible resource is more efficient, i.e. $c_K < c_L$.

Third, when users are indifferent between buying and not buying the flexible resource, it will always buy for some exogenous reasons such as reputation: if its customer's demand is not served, the customer will never purchase from that firm again. The solution concept adopted here is subgame perfect equilibrium (SPE).

3 Social Optimum

The benevolent planner chooses L_1, L_2, K so as to maximize social welfare. Fig. 1 illustrates the basic structure of the demand for flexible resources.

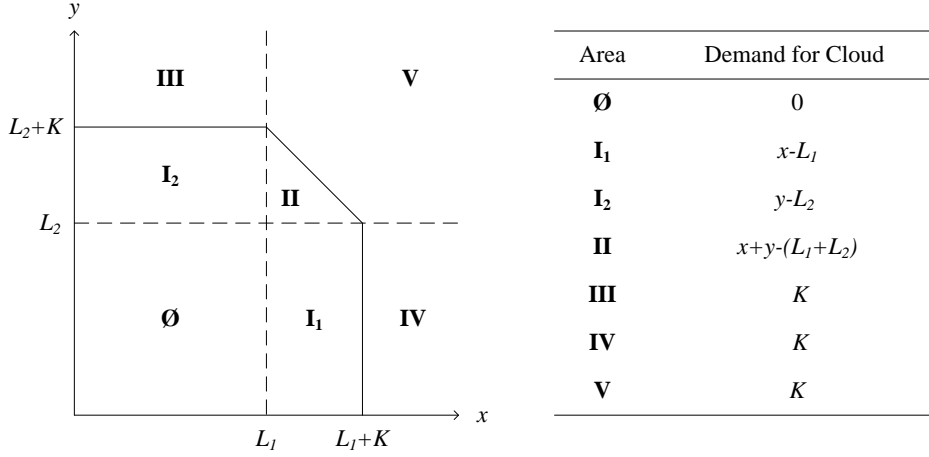


Figure 1: Demand for Flexible Resources.

In *Area* \emptyset , both firms have sufficient local capacity to serve their customers, and therefore there is no demand for cloud. *Area* I_1 captures the situation where firm 2's local capacity is enough to cover its demand, but firm 1's demand exceeds its local capacity and will therefore purchase flexible resources. *Area* I_2 illustrates the reverse situation where only firm 2 buys cloud. In *Area* II , both firms buy cloud. In all the cases above, all demands are served. *Area* III represents the situation where firm 1 has enough local capacity, while firm 2 has too much demand such that the flexible resource provider is capacity constrained. *Area* IV shows the reverse situation: firm 1 has too much demand, while firm 2's local capacity is sufficient. *Area* V captures the situation where the demands of both firms are extremely high such that it exhausts the capacity of the flexible resource provider. Thus the social welfare is given by

$$\begin{aligned}
\max_{L_1, L_2, K} S &= \int_{\emptyset+I_1+I_2+II} (x+y)h(x,y)dydx + \int_{III} (x+L_2+K)h(x,y)dydx \\
&+ \int_{IV} (L_1+K+y)h(x,y)dydx + (L_1+L_2+K) \int_V h(x,y)dydx \\
&- c_K K - c_L(L_1+L_2).
\end{aligned} \tag{1}$$

Let $\Omega(L_1, L_2, K)$ denote the probability of (x, y) falling in areas $\{III\} + \{IV\} + \{V\}$. The social planner only invests in flexible resources, and the socially optimal investment is given by

$$\Omega(0, 0, K) = 1 - \int_0^K \int_0^{K-x} h(x, y)dydx = c_K.$$

The optimal capacity is such that the social marginal benefit equals the marginal cost.

Proposition 1. *The social planner only invests in the flexible resource, and the socially optimal investment in flexible resource increases with demand correlation if c_K is small, but decreases with demand correlation if c_K approaches c_L .*

Proof. See Appendix A. □

The intuition behind Proposition 1 runs as follows. The social planner only invests in flexible resources because $c_K < c_L$. As the demand correlation increases, so does the probability of getting either high demand realizations or low demand realizations from both firms: the firms either “win big” or “lose big.” The impact of an increase in demand correlation therefore depends on the cost of the flexible resource. If the investment cost is sufficiently low, then “losing” is cheap and the planner would focus on reaping the benefits of high demand realizations. Therefore, investment increases with correlation for low cost. On the contrary, if investment cost is large enough, the planner aims at minimizing the risk of “losing,” so investment decreases with correlation.

4 Monopoly

Suppose now that there is a monopoly provider for the flexible resource that chooses p and K to maximize its expected profit. Proceeding by backward induction, given L_1 , L_2 , K and monopoly price p^m , the demand for cloud is the same as in Fig. 1 as long as $p^m \leq \pi$. As the monopolist can extract all the value of its cloud service, it is obvious that

$$p^m = \pi \tag{2}$$

in Stage 3.

The investment of the provider is determined by

$$\Omega(L_1, L_2, K) = 1 - \int_0^L \int_0^{L+K} h(x, y) dy dx + \int_L^{L+K} \int_0^{2L+K-x} h(x, y) dy dx = c_K, \tag{3}$$

In Stage 1, expecting that $p^m = \pi$, firm 1 chooses its local capacity L_1 so as to maximize its profit:

$$\max_{L_1} \int_0^{L_1} x f(x) dx + \int_{L_1}^{\infty} L_1 f(x) dx - c_L L_1.^{14}$$

The first two terms show that the whole demand is served when demand is below local capacity, whereas capacity is saturated when demand exceeds local capacity. The last term represents the total spending in local capacity.

Then, the first-order condition determines the equilibrium investment of L_1 :

$$1 - F(L_1) \leq c_L. \tag{4}$$

¹⁴The firm only gets positive profit from its local capacity because the surplus of the consumers, who are served by utilizing the flexible resource, are extracted entirely.

The second-order condition is also satisfied.

Analogously, for firm 2, the equilibrium investment of L_2 is determined by

$$1 - G(L_2) \leq c_L. \quad (5)$$

The market equilibrium is characterized by Equations (2), (3), (4) and (5). It is clear that, unlike the social optimum, firms invest in a positive amount of local capacities; and unlike the duopoly case, firms' investments are independent of the provider's investment strategy.

Proposition 2. *In the market with a monopolistic flexible resource provider, the provider underinvests in the flexible resource relative to the social optimum, while the firms overinvest in their local capacity.*

Proof. See Appendix B. □

The intuition behind Proposition 2 is as follows. The monopolist sells the flexible resource at a monopoly price, which extracts all consumer surplus. Anticipating this, the firm will invest in L , even if L is a less efficient technology compared with K , in order to gain part of the consumer surplus. As a consequence, the benefit of investing in the flexible resource is lower for the monopolist than for the social planner, and hence the monopolist underinvests.¹⁵

To solve the problem, the regulator may ban local investments of the firms. However, this is a rather heavy-handed approach. Firms may prefer local computing for a variety of legitimate reasons. For instance, flexible resources are valuable for the firm as they offer the flexibility to modify a prior investment strategy as more information becomes available over time. More particularly, in case of “good news” the firm can scale up their services, and in case of “bad news” it can scale down. Therefore, firms are willing to pay extra to buy the flexible resource even though it is more expensive ($p^m > c_L$). Indeed, statistics shows that cloud computing is appealing to industries that have high variability in data traffic such as medical research and drug discovery in the healthcare sector.¹⁶

Therefore, I consider a lighter form of intervention. Since surplus appropriation originates from market power, it seems reasonable to investigate whether introducing more competition in the market—thereby forcing down the price—would incentivize the provider and the firms to behave optimally. As we will see later, the extent to which competition improves investment incentives is subtler than it appears as it varies with demand correlation and investment cost.

Let us now turn to the impact of correlation.

Proposition 3. *In the decentralized case with a single provider, there is positive local investment; and the monopolist's investment in flexible resource increases with demand correlation if c_K is small, but decreases with demand correlation if c_K approaches c_L .*

Proof. See Appendix C. □

¹⁵Notice that Proposition 2 holds more generally for any rationing rule. The reason is that users pay the monopoly price, and hence the rationing rule will not affect local investment.

¹⁶World Economic Forum (2010) identifies the healthcare industry as one of the major sectors which can benefit from cloud computing.

The impact of an increase in demand correlation on both socially optimal and equilibrium investment depends on whether the flexible resource is significantly more efficient than the local resource. The intuition of Proposition 3 is in the same spirit as Proposition 1. However, the monopolist's investment is more likely to be decreasing in demand correlation as shown in the following corollary:

Corollary 1. *The smallest c_K under which investment in flexible resource decreases with demand correlation is larger at the social optimum than under monopoly.*

Proof. See Appendix D. □

The intuition behind Corollary 1 is that local investment is zero at the social optimum and positive in the monopoly case. Thus, the planner will not run into the risk of not being able to sell the flexible resource to firms that receive low demand and buy local resources only. As a consequence, the planner can better enjoy the possible windfall from high demand realizations than the monopolist.

5 Duopoly

Let us now consider the case of competing providers. They play the game as before.¹⁷ I solve the problem proceeding backwards. In the capacity-price stage, I apply some results in de Frutos and Fabra (2011), henceforth FF, which can be summarized as follows. In their paper, two firms make sequential capacity-price decision under demand uncertainty in markets with price-inelastic demands. They show that

- Proposition 7 of FF. The only equilibrium in the pricing stage is a mixed-strategy equilibrium.
- Proposition 8 of FF. Capacity choices are asymmetric.
- Proposition 9 of FF. If the density function of demand is non-decreasing, then the equilibrium is unique.

For a given L_1 and L_2 , there is a stochastic demand function for the flexible resource that is price-inelastic. Thus, we can apply FF's results in the continuation game, where the aggregate capacity is defined by $K(L_1, L_2)$, the capacity chosen by the smaller provider $k^-(L_1, L_2)$, the capacity chosen by the larger provider $k^+(L_1, L_2)$, and the equilibrium expected profits of the two providers $\pi^-(L_1, L_2)$ and $\pi^+(L_1, L_2)$.

The main difference between this paper and FF is that the first stage in this paper is absent in FF. FF assume that demand is exogenously given, while here the demand for the

¹⁷Since there is demand uncertainty, this exercise requires more than just applying the classical result of Kreps and Scheinkman (1983), which proves outcome equivalence between the capacity-price game and the Cournot game. As pointed out by de Frutos and Fabra (2011), the introduction of demand uncertainty rules out the existence of symmetric equilibria due to a difference in marginal revenue between the large firm and the small firm even if the two firms are symmetric ex ante.

flexible resource is endogenously determined by investments in local capacity and the strength of demand correlation. Therefore, unlike the monopoly case, firms' investments are no longer independent of the provider's strategy. This poses several difficulties in the analysis.

First, the endogenously determined demand function for the flexible resource is not necessarily non-decreasing, which means that the equilibrium in the continuation game may not be unique. If this is the case, we focus on the most symmetric case, where the difference between the big firm and the smaller firm is minimized, meaning that the degree of competitiveness is maximized.

Second, this introduces strategic interaction between the two firms: each firm's investment changes the demand for the flexible resource, which affects providers' investments and in turn affects the rival firm's investment. To simplify the analysis, I assume that L_1 and L_2 are chosen cooperatively such that $L_1 = L_2 = L$. The two firms maximize the following joint profit:¹⁸¹⁹

$$\max_L [S(L) - \pi^+(L) - \pi^-(L)] - 2c_L L,$$

where $S(L)$ is the social surplus given by Equation (1). The surplus is shared between the firms and the providers (but not the consumers). This is because demand is inelastic, so firms can extract all consumer surplus.

Solving the above problem yields the equilibrium investment in local capacity L_1^d and L_2^d , where d denotes duopoly. Then, we can also determine the equilibrium investment in flexible resource $K^d(L_1^d, L_2^d)$.

As should be expected, competition always increases social welfare as compared to the monopoly case because it mitigates the underinvestment problem in flexible resources and the overinvestment problem in local resources. A formal proof is provided in Appendix F. More interesting is that the extent of improvement depends crucially on the cost of capacity and the degree of correlation, which is shown in the following numerical example.²⁰

Fig. 2 plots, for a given c_K , flexible resource investment against demand correlation. Social optimum is shown with a solid line, the duopoly case is drawn as a dotted line, and the monopoly case is illustrated by a long-dashed line.

The main observations in Fig. 2 are summarized in the following remark.

Remark 1. *Comparing the socially optimal, monopoly and duopoly solutions,*

- (i) *When c_K is sufficiently small, both the planner and the monopolist's investments in flexible resources increase with correlation. As c_K approaches c_L , both of these investments decrease with correlation. The threshold level such that the impact of correlation changes is larger at the social optimum than it is under monopoly.*

¹⁸Under this assumption, rationing rule does not affect investments in local and flexible resources.

¹⁹Even though I assume cooperative investment, the two firms act differently from the case with a single firm because the two firms cannot share their local capacity.

²⁰The main difficulty in solving for an explicit solution in the duopoly case stems from the fact that the demand for the flexible resource is endogenously determined by L and α , and this, in turn, affects the mixed strategy in prices of the provider. Consequently, it is difficult to characterize the profit function of the firm without using a numerical method.

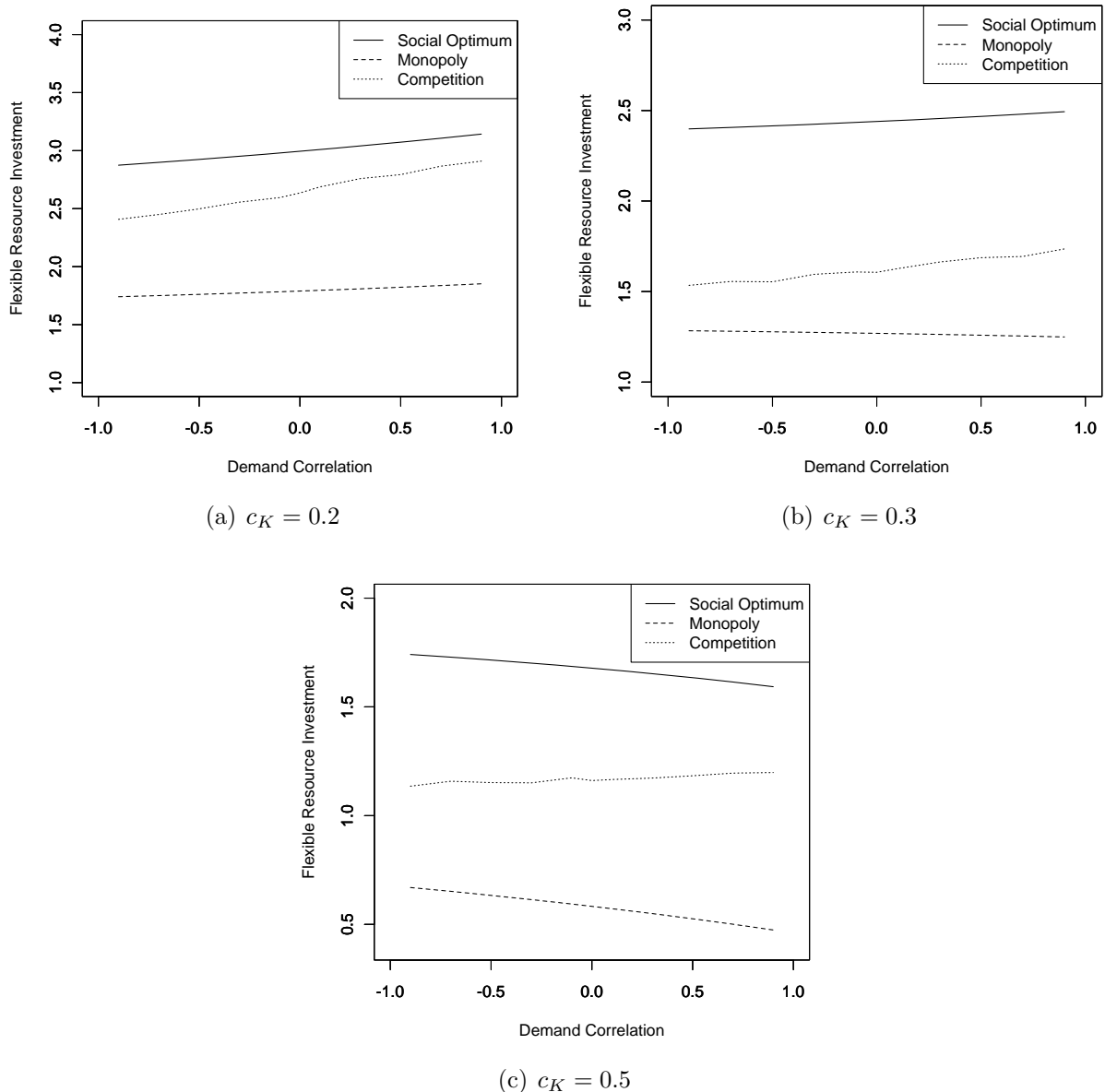


Figure 2: Flexible Resource Investment and Demand Correlation for different values of c_K .

(ii) Under duopoly, it can be shown that for high or low correlations, the investment in flexible resource is increasing in correlation.

Part (i) is already shown in Propositions 1 and 3, and Corollary 1. As for part (ii), the intuitive reason for not observing a negative relationship between investment and correlation under duopoly, unlike the socially optimal and monopoly regimes, is as follows. Under the socially optimal and monopoly regimes, local investment does not vary with correlation: at the social optimum local investment is zero; in the monopoly case firms pay the monopoly price, and thus their local investment is not affected by correlation. Unlike these regimes, in the duopoly case firms pay less than the monopoly price and are therefore more willing to switch to buying the flexible resource in order to capture the possible windfall of high demand realizations. As a consequence, firms invest less in local capacities, and hence providers invest more in flexible resources as correlation increases.

6 Discussion

6.1 Alternative Timing

My analysis focuses on the timing where firms invest first. It fits the scenario where some flexible resources such as cloud computing offers more flexibility in managing demand uncertainty than local resources. However, one could alternatively consider the case where firms observe the provider's investment in flexible resources before deciding their own local investment. In this setting, firms still overinvest in L , and providers still underinvest in K , provided price is chosen after the capacity decision because the monopoly price will emerge as long as demand is inelastic. Another alternative is to consider the case where p is chosen prior to L , but the underinvestment problem will still occur because the provider will never charge $p = c_K$ as its profit will become zero and it will not have any incentive to invest. Moreover, it is difficult to think of a situation in practice that fits the scenario of choosing price prior to capacity.

6.2 Remedies

Although it is always more efficient for firms to use the flexible resource, there are two reasons that prevent everyone from using the flexible resource only: first, the stochastic nature of demand prevents the provider from contracting over the amount of investment ex ante; second, the provider of the flexible resource cannot commit to marginal-cost pricing. As a consequence, firms rely more on local capacity and the provider underinvests.

Throughout the paper, I focus on non-contingent and linear pricing.²¹ One can think of other pricing structures such as non-linear tariffs and contingent pricing. First, considering non-linear tariffs, it is common for cloud providers such as Amazon, Dropbox and Google to use non-linear pricing for their storage service: they provide basic service for free, and then offer additional storage capacity for a fee. However, we can easily see that non-linear pricing does not solve the underinvestment problem because the provider will underinvest as long as $p > c_K$.

Second, considering contingent pricing, such practice is not very popular in the market for cloud computing: with the exception of AWS, which uses both contingent and non-contingent pricing, other large cloud providers such as Azure, Google and IBM rarely use spot pricing. On the contrary, in the electricity wholesale market, electricity is bought and sold at spot prices.²² Yet, there is only one kind of capacity: firms typically buy energy from electricity companies, but do not generate their own electricity (although some firms may have their own emergency electricity generator, they are not for regular use). As argued by Carr (2005) and Jeff Bezos in Stone (2013), they both envisioned today's IT supply would transform from

²¹Non-contingent pricing means that prices are determined before demand is realized, whereas contingent pricing are state-dependent.

²²The electricity literature (see, for instance, Borenstein and Holland (2005), Murphy and Smeers (2005), Joskow and Tirole (2007), and Léautier (2011)) mostly considers a two-stage game, in which firms choose their capacity first, and then they bid prices for each state of the world in a spot market. See also Crew, Fernando and Kleindorfer (1995) for a survey of the literature on peak-load pricing.

companies' private capacity into a centralized utility service, just like how electricity became a utility a century ago. It is therefore interesting to think about how spot pricing can change investment incentives in an environment with both flexible and local resources, as in the case of cloud computing, where firms buy flexible resource for its instant scalability and own local resource for data security and privacy reasons. A formal model of contingent pricing would entail a trade-off as follows: the provider tends to price high during peak periods, which induces firms to invest more in local capacity; but it tends to price low during off-peak periods, which induces firms to rely more on flexible resource. Consequently, the extent to which investment is distorted depends on the relative strength of these two effects. If, for instance, the second effect dominates, then contingent pricing can potentially remedy the problem of underinvestment in flexible resources. Despite this additional trade-off created by contingent pricing, investment decision still depends fundamentally on the degree of correlation and the cost of capacity, and therefore all the main qualitative results of this paper should remain valid.

Finally, it may be worthwhile to consider a subsidy. Suppose the regulator introduce a subsidy s for investment in flexible resource. The cost of flexible resource becomes $c_K - s$, so the provider will be more willing to offer a lower price. At the same time, it also has more incentives to undertake investment in flexible resource, which could potentially mitigate the underinvestment problem.

6.3 Policy Implications

Cloud computing has emerged as a new business model for computing and storage resource management for firms, and a new source of entertainment and communication services for consumers. As the cloud market is still in its infancy, many classic economic issues such as pricing, investment strategies, the appropriate market structure, competition policy, privacy and security concerns are still unclear.²³ We take the first step to understand the impact of competition on investment in this industry.

Although there are a number of competitors in the cloud computing market such as AWS,

²³Recently, there has been a flurry of research on the opportunities and obstacles for the adoption of cloud services; see, for example, Armbrust et al. (2009), Harms and Yamartino (2010), and Marston et al. (2011). They mainly focus on three layers of the cloud architecture: infrastructure, platform, and application. However, as argued in Bayrak et al. (2011), such categorization are useful only in defining technological differences, but not so much in analyzing their economic impact. Indeed the existing literature on cloud computing are mostly descriptive, and only rarely is the problem approached from a theoretical perspective. Fershtman and Gandal (2012) raise important economic issues of cloud computing such as changes in the strength of network effects, compatibility among software applications, the development of standards, and the market structure that should emerge. However, most of these topics have already been well-documented in a separate literature; in order to work on theoretical advancement, one needs to clearly delineate the unique features of the cloud computing market.

Recent efforts to expand the theoretical study of cloud computing include Wang (2014), who studies the adoption of cloud services within a moral hazard framework, and this paper. However, they differ in two respects. First, this paper is about capacity investment, while Wang focuses on the problem of migration, which means that there is no investment on the provider's side. Second, this paper studies the effect of competition, but such effect is absent in Wang.

Azure, Google and IBM/SoftLayer, market power exists. For instance, large cloud providers build hyperscale datacenters that exhibit significant increasing returns to scale, which could come from the centralization of computing resources or from volume discount on the components that providers use to build their datacenter.²⁴ As a result, smaller firms may not be able to compete with these incumbents. Moreover, many consumers prefer to buy service from well-known brands because they expect higher quality. This raises concerns about the degree of competitiveness of this market.

This model predicts that the impact of competition on investment depends crucially on the investment cost. It is often argued that cloud computing reduces the cost of investing in computing power significantly. While the marginal cost of producing an extra unit of storage or computing power is close to zero, the costs of electricity for powering up the machines, cooling the systems, as well as management, maintenance and implementation of the software and hardware in a large server farm is far from negligible.²⁵ Therefore, information on the cost structure in the cloud computing industry should have been gathered and analyzed as it has important consequences for investment.

7 Conclusion

This paper has analyzed firms' incentives to invest in local and flexible resources. I find that market power of the monopolist providing flexible resources distorts investment, and competition always improves social welfare. The extent of improvement depends on demand correlation and investment cost. If investment cost is small, investment under social optimum, monopoly and competition is increasing in correlation; if cost is large, investment under competition is still increasing in correlation, whereas that under social optimum and monopoly goes in opposite direction. I have also examined the potential merits of policies such as spot pricing and a subsidy for investment in flexible resource to remedy the underinvestment problem.

These results have implications for investment decision in outsourcing, particularly in the market for cloud computing. Admittedly, the cloud computing market is growing unpredictably, and there is no clear indication or consensus on how it will develop. For now, this paper shows that even if the cloud computing market follows the footsteps of the electricity market and providers eventually adopt spot pricing, a similar trade-off that we derived here will arise. Therefore, analyzing data on cost and demand represents a useful first step towards a fuller understanding of the nascent industry.

I list some important topics that lie beyond the scope of this paper, but would be appropriate for further work. The first is to consider product differentiation. For example, assuming that cloud computing services (such as Dropbox storage services) and local storage services are differentiated—how, then, would the investment strategy change? Second, it would be interesting to study the consequences of vertical integration. For instance, what will happen if upstream cloud computing firms such as Microsoft and Google also enter the downstream

²⁴See Harms and Yamartino (2010) for more examples of how firms benefit from economies of scale.

²⁵In September 2012, the New York Times reported that “the digital warehouses use about 30 billion watts of electricity, roughly equivalent to the output of 30 nuclear power plants.”

market of software applications?

Appendices

A Proof of Proposition 1

The social optimum is obtained by differentiating Equation (1) with respect to L_1 , L_2 and K .

The F.O.C. with respect to L_1 is given by

$$\{IV\} + \{V\} \leq c_L.$$

Similarly, the F.O.C. with respect to L_2 is

$$\{III\} + \{V\} \leq c_L.$$

Finally, the F.O.C. with respect to K is:

$$\{III\} + \{IV\} + \{V\} \leq c_K.$$

As $\{III\} + \{IV\} + \{V\} > \{IV\} + \{V\}$ or $\{III\} + \{V\}$, the marginal benefit of investing in the flexible resource is always higher than that of local capacity. Furthermore, the marginal cost of investing in the flexible resource is lower ($c_K < c_L$). Then we must have $L_1^* = L_2^* = 0$, where asterisk denotes the socially optimal level of investment. Since $c_K < c_L < \pi$, all F.O.C. are satisfied with equality.

The socially optimal investment in the flexible resource is determined by the F.O.C. with respect to K , which can be rewritten as

$$F(K, \alpha, c_K) = \int_0^K \int_0^{K-x} h(x, y) dy dx - 1 + c_K = 0.$$

By implicit function theorem,

$$\frac{\partial K}{\partial \alpha} = -\frac{\frac{\partial F}{\partial \alpha}}{\frac{\partial F}{\partial K}}.$$

We can show that

$$\frac{\partial F}{\partial K} = \int_0^K e^{-K} [1 + \alpha(2e^{-x} - 1)(2e^{x-K} - 1)] dx$$

is positive. Moreover, we have

$$\begin{aligned} \frac{\partial F}{\partial \alpha} &= \int_0^K \int_0^{K-x} e^{-x-y} (2e^{-x} - 1)(2e^{-y} - 1) dy dx \\ &= -e^{-K} [K + 3e^{-K} + 2Ke^{-K} - 3]. \end{aligned}$$

It can be shown that there exists a \bar{K}^* such that $\frac{\partial F}{\partial \alpha} < 0$ when $K > \bar{K}^*$, and $\frac{\partial F}{\partial \alpha} > 0$ when $K < \bar{K}^*$. In addition, it is obvious that K decreases with c_K . Therefore, if c_K is small such that $K > \bar{K}^*$, then $\frac{\partial K}{\partial \alpha} > 0$. On the contrary, if c_K is large, K is small such that $K < \bar{K}^*$, then $\frac{\partial K}{\partial \alpha} < 0$.

B Proof of Proposition 2

For firm 1, its equilibrium investment is determined by

$$1 - F(L_1) = c_L,$$

As $1 - F(L_1) > \{IV\} + \{V\}$, we must have $L_1^m > L_1^* = 0$, and hence there is overinvestment. The same happens for firm 2.

For the flexible resource provider, its equilibrium investment K^m is determined by

$$\begin{aligned} \max_K \Pi = & \int_0^{L_1} \int_{L_2}^{L_2+K} (y - L_2)h(x, y)dydx + \int_{L_1}^{L_1+K} \int_0^{L_2} (x - L_1)h(x, y)dydx \\ & + \int_{L_1}^{L_1+K} \int_{L_2}^{L_1+L_2+K-x} (x + y - L_1 - L_2)h(x, y)dydx \\ & + K \left[\int_0^{L_1} \int_{L_2+K}^{\infty} h(x, y)dydx + \int_{L_1+K}^{\infty} \int_0^{L_2} h(x, y)dydx \right. \\ & \left. + \int_{L_1}^{\infty} \int_{L_2}^{\infty} h(x, y)dydx - \int_{L_1}^{L_1+K} \int_{L_2}^{L_1+L_2+K-x} h(x, y)dydx \right] - c_K K. \end{aligned}$$

which gives us

$$\Omega(L_1^m, L_2^m, K^m) = c_K = \Omega(0, 0, K^*).$$

Suppose that the flexible resource provider invests K such that $L^m + K = K^*$. Since $L^m > 0$, it must be $\Omega(L_1^m, L_2^m, K) < \Omega(0, 0, K^*)$, which means such K cannot be the equilibrium. Therefore, the flexible resource provider must invest K^m such that $L^m + K^m < K^*$, which means that $K^m < K^*$ (underinvestment).

C Proof of Proposition 3

The monopolist's investment is determined by

$$F(K, \alpha, c_K) = \int_0^L \int_0^{L+K} h(x, y)dydx + \int_L^{L+K} \int_0^{2L+K-x} h(x, y)dydx - 1 + c_K = 0.$$

By implicit function theorem,

$$\frac{\partial K}{\partial \alpha} = -\frac{\frac{\partial F}{\partial \alpha}}{\frac{\partial F}{\partial K}}.$$

It is straightforward to show that

$$\begin{aligned} \frac{\partial F}{\partial K} = & \int_0^L e^{-x-L-K} [1 + \alpha(2e^{-x} - 1)(2e^{-L-K} - 1)]dx \\ & + \int_0^L e^{-y-L-K} [1 + \alpha(2e^{-y} - 1)(2e^{-L-K} - 1)]dy \\ & + \int_L^{L+K} e^{-2L-K} [1 + \alpha(2e^{-x} - 1)(2e^{-x-2L-K} - 1)]dy > 0, \\ \frac{\partial F}{\partial \alpha} = & \int_0^L \int_0^{L+K} e^{-x-y}(2e^{-x} - 1)(2e^{-y} - 1)dydx \\ & + \int_L^{L+K} \int_0^{2L+K-x} e^{-x-y}(2e^{-x} - 1)(2e^{-y} - 1)dydx. \end{aligned}$$

Similar to the proof in Appendix A, there exists \bar{K}^m such that $\frac{\partial F}{\partial \alpha} < 0$ when $K > \bar{K}^m$; and $\frac{\partial F}{\partial \alpha} > 0$ when $K < \bar{K}^m$. Moreover, as K^m is decreasing in c_K , then if c_K is small such that $K > \bar{K}^m$, then $\frac{\partial K}{\partial \alpha} > 0$. On the contrary, if c_K is such that $K < \bar{K}^m$, then $\frac{\partial K}{\partial \alpha} < 0$.

D Proof of Corollary 1

From the proof in Appendices A and C, it suffices to show $\frac{\partial F^*}{\partial \alpha}(K^*) < \frac{\partial F^m}{\partial \alpha}(L^m, K^m)$, where both terms integrate the same function over the respective area as shown in Fig. 3. The difference between $\frac{\partial F^*}{\partial \alpha}(K^*)$ and $\frac{\partial F^m}{\partial \alpha}(L^m, K^m)$ lies in the shaded area. Comparing integrations over the triangles and the trapezium, we can conclude that the above condition is satisfied because the triangles have higher values of x or y .

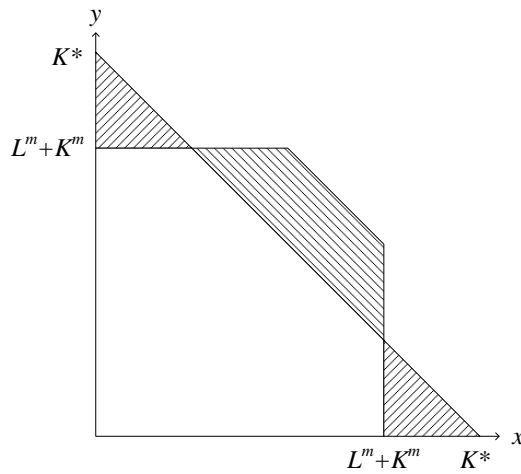


Figure 3: Investment under Social Optimum and Monopoly.

We therefore have

- If $\frac{\partial F^m}{\partial \alpha} < 0$, then $\frac{\partial F^*}{\partial \alpha} < 0$. Both $\frac{\partial K^*}{\partial \alpha}, \frac{\partial K^m}{\partial \alpha} > 0$, which is true for small c_K .
- If $\frac{\partial F^*}{\partial \alpha} > 0$, then $\frac{\partial F^m}{\partial \alpha} > 0$. Both $\frac{\partial K^*}{\partial \alpha}, \frac{\partial K^m}{\partial \alpha} < 0$, which is true for large c_K .
- For medium c_K , $\frac{\partial F^*}{\partial \alpha} < 0$ and $\frac{\partial F^m}{\partial \alpha} > 0$. Then, $\frac{\partial K^*}{\partial \alpha} > 0$ and $\frac{\partial K^m}{\partial \alpha} < 0$.

Thus, under social optimum there is a larger range of c_K under which investment increases with correlation as compared to the monopoly case.

E Linear Example

E.1 Social Optimum

The relationship between investment in flexible resource and demand correlation at the social optimum is slightly different when demands are uniformly distributed. To see this, consider a joint distribution $h(x, y)$ as follows:

- *Positive correlation.* With probability ρ , only pairs of demands on the $x = y$ line are possible (perfect positive correlation). With probability $1 - \rho$, demands are uniformly distributed on a unit square $[0, 1] \times [0, 1]$ (independent demands). We can use ρ as a measure of positive correlation.
- *Negative correlation.* With probability ρ , only pairs of demands on the $x + y = 1$ line are possible (perfect negative correlation). With probability $1 - \rho$, demands are uniformly spread over a unit square $[0, 1] \times [0, 1]$ (independent demands). We can use $-\rho$ as a measure of negative correlation.

Since $c_K < c_L < \pi$, all the F.O.C. are satisfied with equality. In the case of positive correlation, the optimal capacity is chosen such that the marginal benefit is equal to the marginal cost:

$$\rho\left(1 - \frac{K}{2}\right) + (1 - \rho)\frac{1}{2}(2 - K)^2 = c_K.$$

Note that $K \geq 1$ because $c_K \leq 0.5$. Differentiating K with respect to ρ , we find that K^* increases with ρ .

In the case of negative correlation, we have

$$K^* = \max \left\{ 1, 2 - \sqrt{\frac{2c_K}{1 - \rho}} \right\}.$$

Note that $K \geq 1$. The reason is that if demands are perfectly negatively correlated and investment is less than 1, then marginal benefit always exceeds cost. When $K > 1$, the optimal investment is determined by

$$(1 - \rho)\frac{1}{2}(2 - K)^2 = c_K.$$

It is easy to see that K^* increases with $-\rho$.

We therefore have

Result 1. *In the case of uniformly distributed demands, the social planner only invests in the flexible resource, and the socially optimal investment always increases with demand correlation.*

The reason is that, for uniformly distributed demands, the marginal benefit of expanding capacity always increases as correlation increases.

E.2 Monopoly Case

In the monopoly case, the result in the linear example is the same as Proposition 3 in the main text. To keep things simple, further assume that $c_K \in [0.25, 0.5]$ such that $L_1 + K$ and $L_2 + K$ are smaller than 1. In the case of positive correlation, the monopolist chooses K such that

$$\rho\left(\frac{1}{2} - \frac{K}{2}\right) + (1 - \rho)\left(\frac{3}{4} - K - \frac{1}{2}K^2\right) = c_K.$$

In the case of negative correlation, the monopolist choice of K solves

$$\rho(1 - 2K) + (1 - \rho)\left(\frac{3}{4} - K - \frac{1}{2}K^2\right) = c_K.$$

We therefore have

Result 2. *In the case of uniformly distributed demands, there is positive local investment; and the monopolist's investment in flexible resource increases with demand correlation if c_K is small, but decreases with demand correlation if c_K approaches c_L .*

F Competition Improves Social Welfare

Competition always increases social welfare because it mitigates the underinvestment and overinvestment problem.

- $K^d \geq K^m$: The F.O.C. of K in the monopoly case is

$$\{III\} + \{IV\} + \{V\} = c_K.$$

As for the duopoly case, we refer to Equation (12) in FF: the F.O.C. of K is

$$1 - D(K) = c_K,$$

where $D(K)$ is the demand for the flexible resource. Since firms only buy the flexible resource when demand is above their local capacity, this condition can be rewritten as

$$\frac{\{III\} + \{IV\} + \{V\}}{1 - \int_0^{L_1} \int_0^{L_2} h(x, y) dy dx} = c_K.$$

Therefore, $K^d \geq K^m$ because $1 - \int_0^{L_1} \int_0^{L_2} h(x, y) dy dx < 1$. Note that $K^d = K^m$ only when $L_1, L_2 = 0$.

- $p^d \leq p^m$: Under duopoly, providers of the flexible resource randomize over price with the upper bound of π (see Proposition 7 of FF).
- $L^d \leq L^m$: Firms invest less in local resource under duopoly because the price of it is lower.

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