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“What Determines Market Structure? An Explanation from Cooperative Investment with Non-Exclusive Contracts”

Guillem Roig

Toulouse School of Economics

What Determines Market Structure? An Explanation from Cooperative Investment with Non-Exclusive Contracts*

Guillem Roig†

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Abstract

In a common agency setting, where the common buyer undertakes cooperative investment with her suppliers, we obtain a direct link between the level of ex-post competition and investment which affects the market structure of the supply side of the market. We show that more competitive equilibria are associated with a larger and more homogeneous distribution of investment among active suppliers, and an equilibrium with no investment might occur when competition is mild. In our model, buyer's investment works as a mechanism to incentivize competition, and its effectiveness is positively related to the level of competition ex-post. In general, the equilibrium investment profile is lower than efficiency, and we surprisingly find that higher competitive markets may sustain a larger number of suppliers.

Keywords: cooperative investment; investment distribution; competition.

JEL Classification Numbers: C72; D43; D44

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†Toulouse School of Economics. GREMAQ. Contact information: guillemroig182@gmail.com. I am grateful to Emanuele Bachiega, Jacques Crémer, Olga Gorelkina, Sjaak Hurkens, Edgardo Lara, Inés Macho-Standler, Antonio Miralles and Tommasso Valletti. I am also indebted to all the participants in the CREIP seminar, the seminar on Game Theory and Social Choice at UAB and the XXVIII Jornadas de Economía Industrial for all their suggestions and useful comments.

1 Introduction

In many situations, a party establishes relation specific investment not only to increase potential gains from a relationship but also to enhance his bargaining position over other agents. This is the case when the agent bearing the cost of investment receives the direct benefits, which allows him to demand more favorable trade conditions. However, there are situations when investment occurs even if the direct benefits are appropriated by the non-investing parties. This type of relation specific investment is called cooperative. An example is the case of the OEM (original equipment manufacturers) arrangements. Ernst (2000) states that: *"OEM refers to a transactional arrangement between a brand name company (OEM buyer) and the contractor (the supplier) where the buyer provides detailed technical blueprints and most of the components to allow the contractor to produce according to specification [...] as the OEM buyers are responsible for final product quality, typically those buyers must transfer key technology and timely information to their suppliers".*

An important characteristic of this type of agreements is that the common buyer avoids contracting with a single supplier. Instead, we observe that a buyer has a group of first-tier suppliers, complemented by the production undertaken by second-tier.[1] A first-tier supplier has a closer relationship with the buyer and in general the latter has undertaken some sort of relation-specific investment. Moreover, there is evidence on how first and second-tier suppliers are structured in different industries. For instance, in the cycling industry, most of the suppliers, providing raw materials and elaborated components to major OEM buyers, are rather homogeneous and the buyer has established some sort of specific relation with each one of them. Conversely, in other industries, the suppliers of intermediate products are rather heterogenous. The buyer establishes a specific relation with only a small group of them, which creates a well defined first and second-tier structure. This is the case in the IT industry where Kang et al (2007) state that: *"Dell deals with many different suppliers and among them, HIPRO has a dominant position"*. The picture below represents the market structure of these two industries, where the size of the boxes of the different suppliers illustrate their

[1] Under some circumstances, relation-specific investment with a specific agent is done strategically to foreclose possible competitors. With this spirit, we find the theoretical works of Farrell & Shapiro (1989), Valletti (2000), and Rey & Tirole (2007). Those papers, try to solve the opportunism that appears when one of the parties is locked into the relationship. Farrell & Shapiro (1989) consider the introduction of long-term contracts.

relative importance in the market.

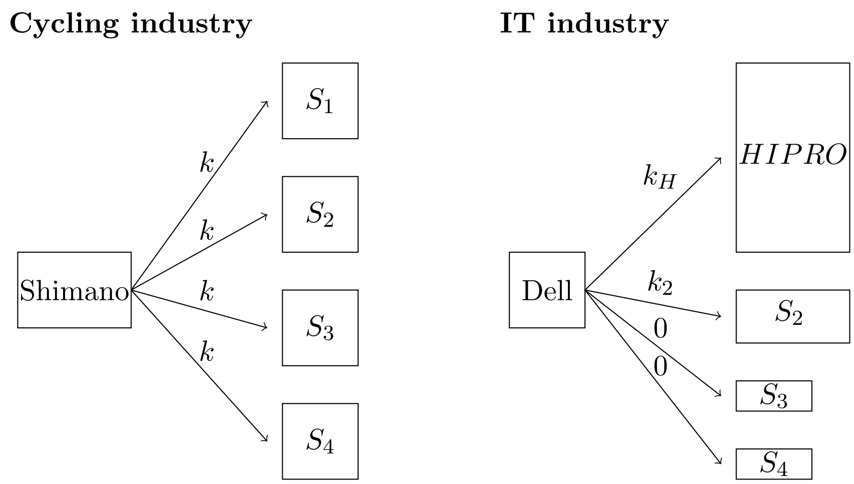


Figure 1: Market structure in two industries where the buyer undertakes cooperative investment with her suppliers. On the left, the cycling industry represents an homogeneous market structure where each supplier has a similar trading weight with respect to the others. On the right, the IT industry is characterized by a rather heterogenous structure, where one of the suppliers is larger than the rest. In this industry, we can easily differentiate between first-tier suppliers, those with a stronger relationship with the buyer such as HIPRO, and second-tier suppliers, those with a rather humble interaction with the former.

The aim of this paper is to study what are the incentives for the common buyer to establish specific relation investment with her suppliers and how she decides to distribute it. We want to understand what are the factors explaining the coexistence of first with second-tier suppliers and how investment shapes the market structure in the industry.

By using a common agency framework, where trading contracts are not exclusive, we show that the existence of cooperative investment and the way the buyer decides to distribute it among active suppliers depends on the degree of competition in the trading game. Using recent results on the markets and contracts literature, and assuming that suppliers are able to coordinate their out of equilibrium offers, the upper bound on the transfer that each supplier obtains depends on the "loss" of the trading surplus originated from his exclusion.[2] This is directly associated to the gains from trade that the rest of suppliers are able to attain. The higher the number of suppliers coordinating their out of equilibrium offers, the larger the gains from trade that can be generated and the lower are the equilibrium transfers of the latter. Hence, competition in our trading game is more aggressive the higher the number of

[2]Those refer to the works of Chiesa & Denicolò (2009,2012) and Martimort & Stole (2011).

active suppliers coordinating their offers.

In our model, investment exists as an instrument to enhance competition. The investment that a buyer undertakes to a given supplier, increases his relative importance with respect to the rest, and this boost his ability to generate trading surpluses. The "loss" from exclusion of any other supplier is consequently reduced and so is his equilibrium transfer. Whenever the competition in the trading game is aggressive, such that all the suppliers coordinate their out of equilibrium offers aimed at excluding any supplier i, the common buyer is interested in making all suppliers as homogeneous as possible, and she undertakes the same level of investment to all of them. Conversely, in situations where only a small group of suppliers coordinate their out of equilibrium offers, ex-post competition is reduced, and the interest of the buyer is to concentrate investment to a smaller group. Furthermore, the buyer might decide not to invest if the effect that investment has on the final trading allocation is so large that it generates a substantial market power of the former.

In general, since all direct gains from investment are appropriated by the non-investing party, we find that the equilibrium level of investment is lower than the efficient level. Moreover, when higher market competition emerges due to an increase on the number of active suppliers, the aggregate level of investment decreases. In this case, the effect that investment has on increasing the competitiveness of the market is reduced. Finally, the level of ex-post competition has an effect on the number of suppliers who decide to become active in the industry. In this regard, we find situations where anticipating a higher degree of competition gives higher incentives to potential suppliers to become active. In our model, a high intensity of competition attracts a larger number of suppliers, which in turn generates more competition. Thus, due to the linkages between competition, the number of active suppliers and investment, a competition authority should take into consideration not only how competition between the existing suppliers materialize, but also their actual number.

The remaining of the paper is organized as follows. In section (2), we give a summary on the existing literature. In section (3), we present the set-up of the model and establish the timing of the game. We proceed to solve the model backwards. Therefore, in section (4.1) we study the properties of the equilibrium trading allocation and the equilibrium transfers are obtained in section (4.2). We continue in section (4.3.1) by analyzing the link between investment and competition, and section (4.4) establishes the relation between the number of active suppliers and the level of ex-post competition. In section (5) we study the robustness of our finding, and we finally conclude in section (6). All proofs are in the appendix.

2 Literature

Most of the literature on relation-specific investment has centered in the case where investment directly benefits the party bearing the cost. The literature on investment generating a direct benefit to the non-investing party, cooperative investment, is much smaller and the main works are Che & Chung (1999), Che & Hausch (1999) and Hori (2006). All those models consider a bilateral trading game where one of the parties decides to undertake cooperative investment. This literature focuses on contract design and shows that with cooperative investment, the implementation of simple contracts cannot restore the efficient level of investment.[3] As a result, the introduction of more complex contracts such as option contracts are needed to restore efficiency. In this regard, Chih-Chi (2005) finds that, in a setting with asymmetric information on the bargaining game, the equilibrium investment profile is characterized by over-investment and an option contract is needed to restore the efficient investment profile. In the present paper, we do not restrict to the case of a bilateral monopoly, and in order to accommodate our theory to the case of OEM trading arrangements, we consider that a common buyer is able to trade with many suppliers at the same time. Because the existing literature has shown the zero value of contracting in settings where investment is purely cooperative, we work in a model where the signing of a contract before our trading game is not feasible.

Closer to our work is the literature considering that an economic agent undertakes cooperative investment with many others without creating de-facto foreclosure. In this literature, cooperative investment is modeled as learning by doing. Therefore, McLeod & Malcomson (1993), Bergemann & Välimaki (1996) and Burguet (1996), consider strategic learning in a multi-period setting. In those models, the supplier has private information which is revealed through experience with the buyer, and a more efficient relationship arises from experimentation. In more recent papers, Lewis & Yildirim (2002, 2005), consider a dynamic procurement

[3]This result contrast with the case of selfish investment. Here, a simple contract allows to restore full efficiency as in Edlin & Reichelstein (1996), Konakayama et al. (1986), Nöldeke & Schmidt (1995), Rogerson (1992) and Schmitz (2002b).

game where the cost of a supplier decreases when it has contracted with the buyer.[4] In their model, a part of the cost of the supplier is private information and the target of the buyer is to design a procurement mechanism to induce truth-telling. A duopoly market structure is assumed and the objective of the buyer is to exploit the learning economies and reduce information rents. The buyer then rotates her purchases with both suppliers to increase competition. However, when learning economies are important, the market tips into a monopolistic structure.[5]

Despite its appealing dynamic features, trade is exclusive with a single supplier in each period, and future gains from trade come from previous interactions, as more efficiency arises from learning. Yet, it is interesting to see that the equilibrium payment that a supplier obtains is related to the threat of being excluded from the current trading opportunity. Therefore, the more the supplier has experienced with a buyer, the higher is the likelihood that he trades the good in the current period, and the larger is his bargaining power. In our model, we also find that the more the buyer have invested with a given supplier, the higher is his "loss" of exclusion and the bigger the equilibrium transfers.

Since we consider that a buyer trades with different suppliers at the same time, the literature on common agency with non-exclusive contracts establishes a natural framework for our work. At this regard, we borrow from the theoretical works of Bernheim & Whinston (1986), Segal (1999) and Chiesa & Denicolò (2009, 2012) where different suppliers of an homogenous good contract with a single buyer. The former two works, establish what are the general conditions for an efficient allocation, and the latter two, prove the multiplicity of equilibrium transfers. The closest paper to ours is Chiesa & Denicolò (2012), where the authors focus attention on the analysis of the two extreme equilibria. We depart from their work in three important aspects. First, by imposing structure on the out of equilibrium offers - we assume that suppliers are able to coordinate their out of equilibrium offers - we give a characteriza-

[4]Lewis & Yildirim (2005) is a generalization of their previous work in which they assume that the learning acquired might disappear with a certain probability if the buyer decides to deal with another supplier. At this regard this model is related to the literature on switching costs. When switching costs are high, a buyer might induce suppliers to price more aggressively by credibly threading to replace the incumbent supplier with his rival. Once the incumbent is replaced, the likelihood to be hired again is reduced if switching costs are large and the buyer can exhort better terms of trade from the input supplier (switching costs increases the bargaining position of the buyer). In their setting, the buyer is sophisticated and he manages its procurements to control for switching costs.

[5]A similar result is found in Cabral & Riordan (1994).

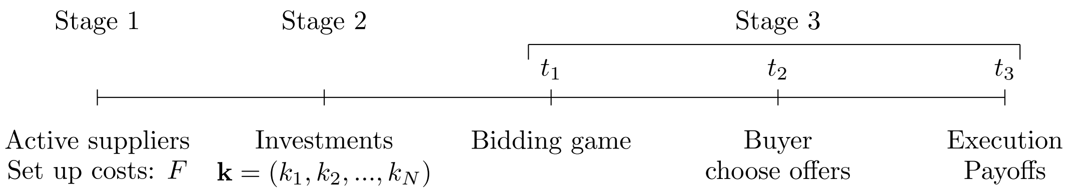
tion for a subset of equilibrium transfers. Those are equal to the “loss” of the trading surplus due to exclusion, which depends on the number of active suppliers coordinating their out of equilibrium offers. Second, we extend the strategy set by allowing the buyer to undertake cooperative investment. In this sense, the cost of production of each active supplier is endogenized, as well as the resulting market structure. Finally, since we are interested on how the level of competition and the distribution of investment affects the number of active suppliers, our setting endogenies the number of active principals in a common agency game.

Last but not least, our work relates to the literature on the “hold-up” problem where an early formulation is due to Klein, Crawford, & Alchian (1978) and Williamson (1979, 1983). The “hold-up” problem arises due to the fact that parties are unable to bargain over specific investment due to its unverifyiability. Here, the “hold-up” problem arises because investments are not contractable. Moreover, since investment is purely cooperative, the problem of being “held-up” is more severe since all direct gains from investment are appropriated by the non-investing party. We also find that the “hold-up” problem is related to the number of active suppliers. Since cooperative investment works as a mechanism to incentivize competition among suppliers, the investment decreases with the number of active suppliers as competition arises by other means.

3 Model

We consider a non-exclusive trading game where a monopolistic buyer is able to undertake cooperative investment with suppliers who produce an homogeneous input. We have a one shot game played in three stages. At the first stage, potential suppliers decide whether to become active in the market, and they pay a set-up cost $F > 0$. At stage two, the buyer makes cooperative investment with the suppliers in the industry. In particular, with N active suppliers, the buyer decides how much to invest with each one of them $k_i \geq 0$. The allocation of investment is represented by the vector $\mathbf{k} = (k_1, k_2, ..., k_N)$ and the total amount invested is the sum of individual investments $K = \sum_{i=1}^{N} k_i$. We denote by $\mathbf{k}_{-i}$ the vector of investment where the component i is not included.

At stage three, given the allocation of investment, trade takes place. Following the existing literature, we consider a bidding game in which trade is modeled as a first-price auction where suppliers simultaneously submit a menu of contracts and the buyer chooses the quantity she will purchase from each.[6] A typical contract is represented by $m_i = (x_i, T_i)$, where $x_i \geq 0$ is the quantity supplier i is willing to sell and $T_i \geq 0$ is the corresponding total payment or transfer from the buyer to supplier i. Because trade in our model is voluntary, we require that the null contract must be offered in equilibrium i.e. $m_i^0 = (0, 0)$, and this way the buyer is free not to trade with a given supplier. Hence, suppliers submit a collection of trading contracts, where the null contract is included. We summarize the moves of the game in the following timeline.



Our model is one of complete information, and due to its sequentiality, we obtain the sub-game perfect Nash equilibrium (SPNE).

3.1 Payoffs and Surplus

The payoff of the buyer and the suppliers are quasi-linear in transfers.[7] The buyer obtains

$$\Pi = U(X) - \sum_{i=1}^{N} T_i - \phi(K), \tag{3.1}$$

where $X = \sum_{i=1}^{N} x_i$ is the total quantity traded. The payoffs of each supplier gross of set-up costs F is

$$\pi_i(k_i) = T_i - C(x_i \mid k_i), \quad \forall i \in N, \tag{3.2}$$

where the amount of investment k_i directly affects the cost of production of supplier i.

Given the number of active suppliers N and the vector of investment $\mathbf{k}$, the maximum

[6] We are not considering the situation where the buyer is the one offering the trading contracts. If this was the case and because the set-up costs are strictly positive, any supplier will anticipate to be "held-up" and none of them will decide to become active in the industry. We need that the suppliers obtain a positive expected payoff, otherwise, the result will be a complete market shot-down with no trade in the market.

[7] This assumption means that all parties have a constant marginal utility for money and this is done for tractability since our analysis does not depend on welfare effects.

gains from trade are

$$TS^*(\mathbf{k}) = \max_{x_1,\ldots,x_n} \left[U\left(x_1 + \cdots + x_N\right) - \sum_N C(x_i \mid k_i) \right], \tag{3.3}$$

where $\mathbf{x}^* = (x_1^*, \ldots, x_N^*)$ is the vector of the efficient quantities and are the ones that solves the problem above. For later use, we denote by $X^*_{\{-H\}} = \sum_{j \neq H} x_j^*$ for $H \subset N$ the sum of the efficient individual quantities without taking the quantities of the subset of suppliers in H. We finish by stating the assumptions regarding the utility and cost functions.

1. $U'_x(\cdot) > 0,$ and $U''_{xx}(\cdot) < 0.$

2. $C'_x(\cdot) > 0,\ C''_{xx}(\cdot) > 0,\ C'_k(\cdot) < 0,\ C''_{xk}(\cdot) < 0$ and $C'''_{xkk}(\cdot) > 0.$

3. $\lim_{X \to 0} U'_x(\cdot) = +\infty$, $\lim_{X \to \infty} U'_x(\cdot) = 0$, $\lim_{x_i \to 0} C'_x(\cdot) = 0$ and $\lim_{x_i \to \infty} C'_x(\cdot) = +\infty.$

From these assumptions, we can establish the efficient investment profile. For a given number of active suppliers, the efficient investment maximizes welfare, trading surplus minus investment costs, and this is characterized in the following proposition.

Proposition 1. *In the efficient investment profile, the common buyer chooses the same investment for supplier, and this is defined by*

$$\phi'_K(K^*) = -C'_k\left(x_i^*(\mathbf{k}^*) \mid k\right), \quad \textit{for all} i \in N.$$

The efficient investment profile is such that marginal cost of investment equals the marginal benefit, and the formal proof is in page 34 in the appendix. The convexity of the cost function and the fact that the marginal reduction of production costs is also decreasing with investment, makes it optimal for each supplier to produce the same amount of the good. The least costly scheme is to make each supplier equally efficient and this is equivalent to choose the same level of investment to each one of them.

4 Analysis

We solve the model backwards. After presenting the equilibrium allocation and transfers, we proceed by characterizing the vector of cooperative investment. We analyze how the common buyer decides to allocate investment and to what extend the equilibrium investment profile differs from the efficient one. Later, we solve the first stage of the game and we give an intuition on how potential suppliers decide to become active in the industry.

4.1 Allocation in the trading game

We characterize the equilibrium allocation for a given number of active suppliers N and a vector of investment $\mathbf{k}$. Because the cost of production of each supplier depends only on his amount traded and there are no investment spillovers, the individual payoff of each supplier is not directly affected by the trading contracts submitted by the other suppliers. Therefore, given the trading contracts of all other suppliers, a supplier effectively plays a bilateral trading game with the buyer where he has all the bargaining power. Hence, he offers a trading contract that maximizes the potential gains from trade generated between him and the common buyer.[8] As a result, in a situation when all the other suppliers are offering the efficient allocation, it is optimal for supplier i to offer the efficient allocation as well, i.e.

$$U\left(\bar{X}^*_{-i}+x^*_i\right)-\sum_{j\neq i}T_j-C(x^*_i\mid k_i)>U\left(\bar{X}^*_{-i}+\hat{x}_i\right)-\sum_{j\neq i}T_j-C(\hat{x}_i\mid k_i)\text{ for any }\hat{x}_i\geq 0.$$

Consequently, there exists an equilibrium where each supplier offers the efficient allocation, and it is defined by the following system of equations:

$$U'_x\left(\sum_N x^*_i(\mathbf{k},N)\right)=C'_x(x^*_i(\mathbf{k},N)\mid k_i)\quad\text{for all }\ i\in N, \tag{4.1}$$

where the marginal cost of production of each supplier equals the marginal utility of the buyer.

The following two lemmas introduce some properties of the efficient allocation that will be useful in the remaining of the paper. We introduce the following definition.

Definition 1. ***(Allocative sensitivity)*** *The object dx^*_j/dk_i for $j\neq i$ is the "allocative sensitivity", and corresponds to the change on the efficient allocation due to an increase of investment undertaken to a competing supplier.*

[8] The first and the second conditions are called "individual excludability" and "bilateral efficiency" in the literature of market and contracts Bernheim & Whinston (1996) and Segal (1999).

Lemma 1. *In an equilibrium with the efficient allocation, for a given number of active suppliers, an increase on the level of investment to any supplier i, increases the amount of trade between the buyer and this supplier, but decreases the amount traded with all other suppliers. The total amount of trade increases:*

$$i)\ \frac{dx_i^*}{dk_i} > 0, \quad ii)\ \frac{dx_j^*}{dk_i} < 0 \ \text{ for all } \ j \neq i \quad \text{and} \quad iii)\ \frac{\partial}{\partial k_i} X^* > 0.$$

Hence, the higher the investment undertaken to supplier i the more efficient he becomes with respect to other suppliers. This entails that he trades more with the buyer and the other suppliers trade less. The magnitude of the "allocative sensitivity" depends on the primitives of the economy. Its intensity is implicitly defined in the appendix together with the formal proof of the lemma, page 32. Finally, because the economy is more efficient, the total amount traded is also higher.

The following lemma states how the individual and aggregate efficient amount of trade changes with the number of active suppliers.

Lemma 2. *In an equilibrium with the efficient allocation, for a given investment vector, the amount that each supplier trades with the buyer decreases with the number of active suppliers, but the aggregate amount of trade is larger.*

$$x_i^*(\mathbf{k}, N+1) < x_i^*(\mathbf{k}, N) \ \text{ for all } \ i \in N \quad \text{and} \quad X^*(\mathbf{k}, N+1) > X^*(\mathbf{k}, N).$$

The greater the number of active suppliers in the industry, the lower the amount of trade that a given supplier undertakes with the buyer. An increase on the number of suppliers creates a negative externality because each one trades less in equilibrium. However, the higher the number of active suppliers the larger is the total amount traded. Again, the formal proof is in the appendix, page 32, where the convexity of the cost of production is crucial for the result.[9]

[9]Because the equilibrium investment profile changes with the number of active suppliers, the negative externality can be even larger due to the the substitution effect among investments.

4.2 Equilibrium Transfers

By restricting attention to the efficient allocation, we proceed to characterize the equilibrium transfers. The literature on markets and contracts has shown that by no putting any restriction on the trading offers, there is multiplicity in the equilibrium transfers. In this work, we focus on a subset of equilibrium transfers which are obtained by assuming that suppliers are able to coordinate their out of equilibrium offers aimed at excluding any other supplier i from trade. The following two definitions are crucial to characterize the equilibrium transfers.

Definition 2. ***(Coordination)*** *A set H of suppliers coordinate their offers if the gains from trade that can be generated between them and the buyer is the largest.*

$$V_H\left(X^*_{-\{H\}} \mid b\right) = \max_{\{x_j\}_{j\in H}} \left[U\left(X^*_{-\{H\}} + \sum_{j\in H} x_j \mid \mathbf{k} \right) - \sum_{j\in H} C(x_j \mid k_j) \right]. \tag{4.2}$$

We denote by $\tilde{x}_j(\cdot \mid H)$ the quantity that solves the problem in expression (4.2). By assuming that suppliers are able to coordinate their offers we define the loss of exclusion.

Definition 3. ***(Exclusion loss)*** *Those are the trading gains that cannot be realized due to the exclusion from trade of a given supplier.*

The loss of exclusion for a given supplier is directly related to the gains than can be attained by the rest of the suppliers, and those gains depend on the number of suppliers coordinating their out of equilibrium offers.[10] Regarding the suppliers coordinating their offers, we assume that:

Assumption 1. *For a given investment profile -* $\mathbf{k}$ *- the set of suppliers who coordinate their out of equilibrium contracts to exclude supplier i are the most efficient ones*

$$J_i^{\mathbf{k}} = \{ j \in N \setminus \{i\} \;\; s.t\, k_j \geq k_h \;\; for\ all\, h \neq i \}\,.$$

Observe that we are not assuming that the investment of the buyer determines the cardinality of the set, we just state that, given a cardinality of the set $J_i^{\mathbf{k}}$, those suppliers belonging to this set will be the most efficient ones.

[10] In the literature of markets and contracts, this out of equilibrium contracts are called "latent" contracts and those are the offers or trading contracts that are never accepted by the buyer, but effectuate a constraint on the equilibrium transfer of suppliers.

Following Chiesa & Denicolò (2009), we know that the difference of the gains from trade that can be generated between the subset of suppliers in $J_i^{\mathbf{k}}$ and the common buyer with and without supplier i are equal to

$$V_{J_i^{\mathbf{k}}}\left(X^*_{-\{J_i^{\mathbf{k}}\}} \mid \mathbf{k}\right) - V_{J_i^{\mathbf{k}}}\left(X^*_{-\{J_i^{\mathbf{k}},i\}} \mid \mathbf{k}\right).$$

Then, with simple algebra and taking that the suppliers in $J_i^{\mathbf{k}}$ coordinate their out of equilibrium offers, we obtain that the exclusion loss of supplier i is

$$L_i\left(J_i^{\mathbf{k}}\right) = \left(U\left(X^*\right) - \sum_{j \in J_i^{\mathbf{k}}} C(x_j^* \mid k_j)\right) - V_{J_i^{\mathbf{k}}}\left(X^*_{-\{J_i^{\mathbf{k}},i\}} \mid \mathbf{k}\right), \tag{4.3}$$

where

$$V_{J_i^{\mathbf{k}}}\left(X^*_{-\{J_i^{\mathbf{k}},i\}} \mid \mathbf{k}\right) = \max_{\{x_j\}_{j \in J_i^{\mathbf{k}}}} \left[U\left(X^*_{-\{J_i^{\mathbf{k}},i\}} + \sum_{j \in J_i^{\mathbf{k}}} x_j \mid \tilde{x}_i = 0, \mathbf{k}\right) - \sum_{j \in J_i^{\mathbf{k}}} C(x_j \mid k_j)\right], \tag{4.4}$$

is the value function representing the maximum gains that can be obtained between the common buyer and the subset of suppliers in $J_i^{\mathbf{k}}$, by putting equal to zero the trading quantity of supplier i, keeping constant the production of the supplier not in $J_i^{\mathbf{k}}$, and choosing optimally the quantities of the suppliers belonging to $J_i^{\mathbf{k}}$.[11] The amount $\tilde{x}_j(\cdot \mid J_i^{\mathbf{k}})$ is the quantity traded that solves the problem in expression (4.4). The convexity of the cost function makes it straightforward to see that the "loss" of exclusion $L_i(J_i^{\mathbf{k}})$ is weakly decreasing in $J_i^{\mathbf{k}} : J_i^{\mathbf{k}} \supset J_i'^{\mathbf{k}} \Longrightarrow L_i(J_i'^{\mathbf{k}}) \leq L_i(J_i^{\mathbf{k}})$.[12] That is, the more the number of suppliers coordinating their out of equilibrium offers, the larger is the trading surplus that they can generate and the lower is the "loss" of exclusion.

Regarding the quantities that are submitted in the out of equilibrium offers, we introduce the following lemma that will be useful for the rest of the paper.

Lemma 3. *For any investment profile -* $\mathbf{k}$ *- and for any subset of supplier* $J_i^{\mathbf{k}} \subset N$*. The total*

[11] Chiesa & Denicolò (2009) consider the case where the set of suppliers is a singleton, and we extend their finding for any number of suppliers.

[12] In general this will be strictly increasing.

amount traded is higher when all active suppliers undertake trade with the buyer:

$$X^*(\mathbf{k}, N) > X^*_{-\{J_i^{\mathbf{k}}, i\}}(\mathbf{k}, N) + \sum_{j \in J_i^{\mathbf{k}}} \tilde{x}_j(\mathbf{k} \mid J_i^{\mathbf{k}}),$$

and

$$\tilde{x}_j(\mathbf{k} \mid J_i'^{\mathbf{k}}) > \tilde{x}_j(\mathbf{k} \mid J_i^{\mathbf{k}}) > x_j^*(\mathbf{k}); \quad \forall J_i'^{\mathbf{k}}, J_i^{\mathbf{k}} \in N \text{ and } J_i'^{\mathbf{k}} \subset J_i^{\mathbf{k}}.$$

The formal proof is relegated to the appendix in page 33. Due to the convexity of the cost function, the increase of the total amount traded due to an extra supplier always dominates the increase on the amount traded coming from the subset of suppliers in $J_i^{\mathbf{k}}$ that is needed to exclude the former, and this means that supplier "i" is not indispensable. It is immediate to see that the individual amount that any supplier $j \in J_i^{\mathbf{k}}$ submits in his out of equilibrium contract to exclude any supplier i is larger than his efficient amount i.e. $\tilde{x}_j(\mathbf{k} \mid J_i^{\mathbf{k}}) > x_j^*(\mathbf{k})$. Since they aim at excluding one supplier, they have to offer a larger amount to the buyer.

Because in our model trade is voluntary and the suppliers have the whole bargaining power, the equilibrium transfer to any supplier i is the maximal monetary amount such that the common buyer is indifferent between trading or excluding him from trade.[13] Hence, the equilibrium transfer to any supplier i cannot be greater than the exclusion loss $L_i\left(J_i^{\mathbf{k}}\right)$ for any $J_i^{\mathbf{k}}$ as the buyer will decide not to trade with him. This cannot be lower, as supplier i has an incentive to increase it. Hence, we center our attention to the case where the equilibrium transfer is equal to this cost of exclusion $T_i^e(J_i^{\mathbf{k}}) = L_i\left(J_i^{\mathbf{k}}\right)$. With these equilibrium transfers, we can easily obtain the equilibrium payoffs of the trading game. Those are stated in the following proposition.

Proposition 2. *For a given investment profile* $\mathbf{k}$ *and a subset of coordinating suppliers* $J_i^{\mathbf{k}}$*, the equilibrium payoffs are given by*[14]

$$\pi_i(\mathbf{k} \mid J_i^{\mathbf{k}}) = TS^*(\mathbf{k}) - \tilde{TS}_{-i}(\mathbf{k} \mid J_i^{\mathbf{k}}); \quad \forall i \in N, \tag{4.5}$$

[13]Chiesa and Denicolò (2009) talk about the threat that a given supplier is replaced from trade. They state that the upper-bound of the transfer that each supplier can ask for supplying the efficient amount x^* depends on the threat of being excluded from trade, and this is related on how aggressively any other supplier bids for quantities that are larger than the efficient ones.

[14]Remember that the trading surplus is given by $TS^*(\mathbf{k}) = \max_{x_1,\ldots,x_n}\left[U\left(x_1+\cdots+x_N\right) - \sum_N C(x_i \mid k_i)\right]$.

$$\Pi(\mathbf{k} \mid J^{\mathbf{k}}) = TS^*(\mathbf{k}) - \sum_i \Big(TS^*(\mathbf{k}) - \tilde{TS}_{-i}(\mathbf{k} \mid J_i^{\mathbf{k}})\Big) - \phi(K), \tag{4.6}$$

for each active supplier and the common buyer respectively, and $\tilde{TS}_{-i}(\mathbf{k} \mid J_i^{\mathbf{k}})$ *is the maximal trading surplus that can be generated with* $J_i^{\mathbf{k}}$.

The proof is relegated to the appendix, page 33. Because for a given investment profile, the payoff of each supplier i decreases with the number of suppliers in $J_i^{\mathbf{k}}$ a more competitive equilibrium is associated with a larger set $J_i^{\mathbf{k}}$. Hence, we introduce the following definition regarding the level of competition.

Definition 4. ***(Competition)*** *An equilibrium outcome is more competitive the lower the partition of the trading surplus that a supplier can appropriate. Hence, for a given number of active suppliers* N*, the most competitive equilibria is when* $|J_i^{\mathbf{k}}| = N - 1 = \bar{J}_i^{\mathbf{k}}$*. The least competitive is when* $|J_i^{\mathbf{k}}| = 1 = \underline{J}_i^{\mathbf{k}}$.

Having identified the equilibrium payoffs and defined the notion of competition of stage three, we can proceed to study stage two of the game when the buyer decides to effectuate cooperative investment to the active suppliers.

4.3 Investment profile

In this section, we provide a characterization of the equilibrium investment profile which depends on the level of competition in the trading game. We first analyze the way in which the buyer allocates investment among active suppliers and then we identify the equilibrium amount of investment. In equilibrium, investment suffers from the "hold-up" problem because all direct gains from investment are appropriated by suppliers. Yet, in general, the buyer sets a positive level of investment. Here, investment works as a mechanism to enhance competition, and its effectiveness depends on the degree of competition in the trading game. Accordingly, while we always find a positive level of investment when competition is the most aggressive, the buyer might decide not to invest in situations when competition is softer.

We start the analysis by introducing the following lemma, which states the way the common buyer decides to distribute investment.

Lemma 4. *With a set of suppliers* $J^{\mathbf{k}}$ *coordinating their out of equilibrium offers, the allocation of investment is:* $k_{j'} = k_j > k_\ell > k_m = 0$ *for any* $j', j \in J^{\mathbf{k}}$*;* $\ell \notin J^{\mathbf{k}}$ *and* $|\ell| = 1$ *and*

$\forall\, m \notin J^{\mathbf{k}}, \ell.$

The formal proof is relegated to the appendix, page 35, but the intuition is straightforward. The objective of the investment by the buyer is not to increase the potential gains from trade, but to reduce the equilibrium transfers to the active suppliers. Because investment to a given supplier increases the gains from trade that can be generated, since he becomes more efficient, it reduces the "loss" arising from the exclusion of any other supplier. As a result, the equilibrium transfers decrease with the investment undertaken by the buyer, and investment works as a mechanism to increase competition.

Hence, with the objective to constraint the equilibrium transfers of suppliers, the common buyer only invests towards those suppliers who coordinate their out of equilibrium offers, and in order to generate the maximum trading surplus of these coordinating suppliers, the buyer undertakes the same level of investment to each one of them. Additionally, in order to reduce the equilibrium transfer of the coordinating suppliers, the buyer sets a positive level of investment to another active supplier $\ell \in N$. But since this supplier will coordinates his out of equilibrium offers only with the set $\left(J^{\mathbf{k}} \setminus j\right)$ to exclude supplier $j \in J^{\mathbf{k}}$, and not to all suppliers, the investment undertaken to him is lower. Finally, the buyer does not invest towards those suppliers who do not effectuate any constraint to the equilibrium transfers of others, since any gains coming from investment will be totally appropriated by them. From the previous lemma, we directly obtain the following corollary.

Corollary 1. *In the most competitive equilibrium i.e.* $J_i^{\mathbf{k}} = N \setminus \{i\}$*, the distribution of investment coincides with the efficient distribution.*

So far, we have established how the buyer decides to allocate investment depending on the level of competition in stage three. The amount of investment undertaken is introduced in the following proposition, where we also compare to what extend the individual equilibrium amount of investment differs from the efficient one.

Proposition 3. *For a given number of active suppliers* $N \geq 2$*:*

i) In the most competitive equilibrium, the individual level of investment is always positive, *but lower than efficiency, and it is given by*

$$\phi_K'(K) = -\sum_{N-1}\left[\int_{x^*(\mathbf{k})}^{\tilde{x}(\mathbf{k}_{-i}|\bar{J}_i^{\mathbf{k}})} C''_{xk}(\tau)d\tau\right].$$

ii) In any other equilibria of stage three, the equilibrium investment depends on the "allocative sensitivity". Hence:

A) the equilibrium investment is zero if the "allocative sensitivity" for all $J^{\mathbf{k}} \subset N$ is such that

$$\left|\sum_{m\neq J^{\mathbf{k}},\ell}\frac{dx_m^*}{dk_j}\right| \geq \frac{-\sum_{i\in N\backslash\{j\}}\left[\int_{x_i^*(\mathbf{k})}^{\tilde{x}_i(\mathbf{k}|J^{\mathbf{k}})} C''_{xk}(\tau)d\tau\right]}{\sum_{i\in N}\left[\int_{X^*}^{X^*_{-\{J^{\mathbf{k}},\ell,i\}}+\sum_{j\in J}\tilde{x}_j(\mathbf{k}|J^{\mathbf{k}})+\tilde{x}_\ell(\mathbf{k}|J^{\mathbf{k}})} U''_{xx}(\tau)d\tau\right]} = \lambda(J^{\mathbf{k}}). \tag{4.7}$$

B) If (4.7) does not hold, then the equilibrium investment is positive and distributed as in lemma 4. The amount of investment is

$$\phi_K'(K) = -\sum_{i\in N\backslash\{j\}}\left[\int_{x_i^*(\mathbf{k})}^{\tilde{x}_i(\mathbf{k}|J^{\mathbf{k}})} C''_{xk}(\tau)d\tau\right] + \sum_i\sum_{m\neq J^{\mathbf{k}},\ell}\left[\int_{X^*}^{X^*_{-\{J^{\mathbf{k}},l,i\}}+\sum_{j\in J^{\mathbf{k}}}\tilde{x}_j(\mathbf{k}|J^{\mathbf{k}})+\tilde{x}_l(\mathbf{k}|J^{\mathbf{k}})} U''_{xx}(\tau)d\tau\right]\times\frac{dx_m^*}{dk_j},\ \ \forall j\in J^{\mathbf{k}}$$

and

$$\phi_K'(K) = -\int_{x_\ell^*(\mathbf{k})}^{\tilde{x}_\ell(\mathbf{k}|J^{\mathbf{k}})} C''_{xk}(\tau)d\tau + \sum_i\sum_{m\neq J^{\mathbf{k}},j}\left[\int_{X^*}^{X^*_{-\{J^{\mathbf{k}}\backslash\{j\},\ell\}}+\sum_{i\in J^{\mathbf{k}}\backslash\{j\}}\tilde{x}_i(\mathbf{k}|J^{\mathbf{k}})+\tilde{x}_\ell(\mathbf{k}|J^{\mathbf{k}})} U''_{xx}(\tau)d\tau\right]\times\frac{dx_m^*}{dk_\ell},\ \ \textit{for } i=\ell$$

where $K = |J^{\mathbf{k}}| \times k_j + k_\ell$.

The formal proof is in page 37 in the appendix. From this proposition and the previous lemma, we see that the buyer sets a positive level of investment due to strategic considerations. Investment is undertaken to constraint the equilibrium transfers of suppliers, but this enhancing competition mechanism is not sufficient to restore efficiency level of investment. Due to the nature of investment, the direct benefits are appropriated by the party that does not bear the costs of investment, and the effect that the investment has on constraining the equilibrium transfers of suppliers is of second order compared to the direct gains from trade.

Moreover, the constraining effect that investment has on the equilibrium transfers, when the equilibrium outcome is not the most competitive, is only beneficial for the buyer as long as the "allocative sensitivity" created by investment is not too large. Now, the investment undertaken to a given supplier not only makes him more efficient, but it also constraints the degree of how investment is able to enhance the competitive pressure among suppliers. Whether investment has a positive or negative net effect on increasing the competitiveness of the equilibrium outcome depends on how sensitive is the equilibrium allocation with respect to investment. When the effect is important, the investment of the buyer sensitively affects the equilibrium allocation of suppliers and this constraints the effectiveness of the out of equilibrium offers to constraint the equilibrium transfers. The result is that the suppliers whom the buyer decides to invest enjoy a market power position and the net effect on increasing the competitive pressure is negative. In this case, the buyer is better-off with a zero level of investment. When the effect is small, the investment still plays a positive net effect on increasing the competitive pressure and the buyer sets a positive level of investment.

Now, we turn on making a closer link between investment and competition. In the next section we provide a more detailed discussion on the results which we have already obtained, and we show how the investment per supplier evolves with the number of active suppliers in the industry.

4.3.1 Investment and competition

We have shown that the equilibrium investment profile depends on the competitiveness of the equilibrium outcome. In the most competitive equilibrium, the investment of the common buyer is perfectly symmetric and active suppliers are homogeneous is stage three. In this situation, the level of investment per supplier is also big and the objective of the buyer is to push competition. In any other less competitive equilibrium, the investment profile is asymmetric and suppliers are heterogeneous ex-post. Investment has a lower stake to increase competition and its level is generally low.

To establish a closer link between investment and competition, we study how the individual equilibrium investment evolves with respect to the number of suppliers. We show that the larger the number of active suppliers, the lower is the investment per supplier. This indicates that whenever competition is enhanced due to more suppliers, the effect that investment has on fostering competition is reduced. Accordingly, the buyer sets a lower level of investment. The following lemma states the evolution of the level of investment per supplier.

Lemma 5. *When the number of active suppliers is $N \geq 2$, the investment per supplier decreases with the number of active suppliers, regardless of the level of ex-post competition.*

$$k_j(N) > k_j(N+1); \quad \forall j \in J^{\mathbf{k}}.$$

The main factor driving the result comes from the fact that, the higher is the number of suppliers, the lower is the amount that the buyer trades with each one. Hence, since the buyer is effectuating less trade with each supplier, the equilibrium transfer is smaller and the constraining effect of investment on the transfers is reduced. The formal proof is presented in the appendix, page 40.

We illustrate the result in figure 2.[15] The thick solid line represents the efficient investment per supplier, and without loss of generality, we draw the two extreme equilibria. Accordingly, the solid line depicts the most competitive outcome and the dashed line stands for the least competitive. With less than two suppliers, the equilibrium investment is zero, as the buyer does not appropriate any direct gains coming from investment. With no competition the "hold-up" problem is maximal. Whenever there are two active suppliers in the industry, we start to have a positive level of investment. Whether the investment of the supplier in the least competitive equilibrium is above or below the one in the most competitive depends on the degree of the "allocative sensitivity". In this regard, the picture on the left depicts a situation where the investment has a small effect on the allocation of the non-investing suppliers, while the one on the right represents the opposite case.

We proceed to show how the aggregate investment is affected by the number of active suppliers. The result is introduced in the following proposition.

Proposition 4. *Regardless of the level of competition in the trading game, the aggregate level of investment decreases with the number of active suppliers in the industry when $N > 2$. The aggregate investment tends to zero when the number of active suppliers tends to ∞.*

[15]This is just an illustration not computed.

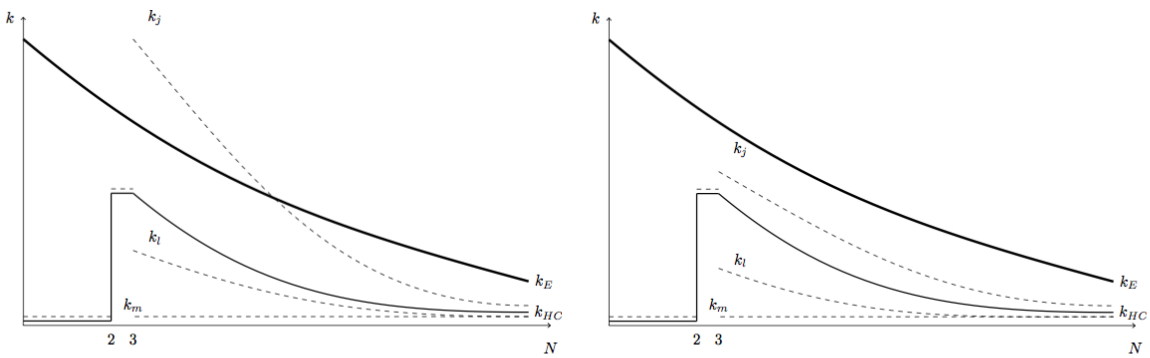


Figure 2: Investment per supplier as a function of the number of active suppliers. The thick black line stands for the efficient investment. The black thin line corresponds to the highest level of competition and the dashed lines the least severe competitive equilibrium. The figure on the left stands for the case when the "allocative sensitivity" is small and the one in the right when it is big.

We have already seen that the investment per supplier decreases with the number of active suppliers. However, how the aggregate investment evolves depends on whether the individual reduction of investment dominates the effect of having an extra supplier. In the appendix, page 41, we show that the first effect dominates and the results are the ones represented in figure 3. As above, the thick black line stands for the aggregate efficient investment. The grey

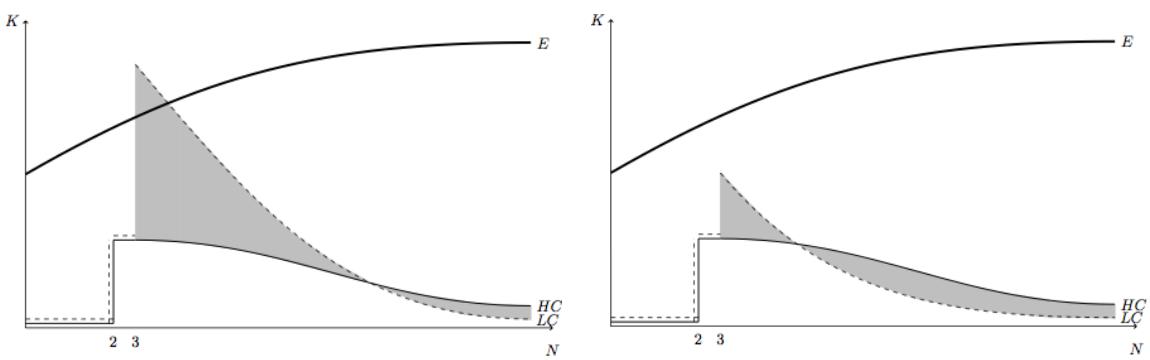


Figure 3: Aggregate investment as a function of the number of active suppliers. The thick black line stands for the efficient investment. The black thin line is the one corresponding to the situation where the equilibrium outcome is the most competitive, and the dashed line is the one where competition is the least severe. The grey area are all the feasible aggregate investments arising from the different types of equilibrium. The figure in the left stands for the case where the "allocative sensitivity" is small and the one in the right when this is big.

area is the feasible aggregate investment that can be achieved depending on how competitive is the equilibrium outcome.

4.4 Active suppliers

In this section, we study how many suppliers becomes active. A given supplier becomes active as long as the expected payoffs are above the set-up costs F. Therefore, for a given number of active suppliers N, and any a set of suppliers $J^{\mathbf{k}}$ coordinating their out of equilibrium offers, the expected payoff of any active supplier is equal to

$$\begin{aligned}\mathbb{E}\left[\pi_i\left(\mathbf{k} \mid N, J^{\mathbf{k}}\right)\right] &= \Pr\left(i = j \in J^{\mathbf{k}}\right) \times \pi_j\left(k_j \mid \mathbf{k}, N, J^{\mathbf{k}}\right) + \Pr\left(i = l\right) \times \pi_l\left(k_l \mid \mathbf{k}, N, J^{\mathbf{k}}\right) \\ &\quad + \Pr\left(i = m \notin \{J^{\mathbf{k}}, l\}\right) \times \pi_m\left(0 \mid \mathbf{k}, N, J^{\mathbf{k}}\right) \\ &= \frac{\left|J^{\mathbf{k}}\right|}{N} \times \pi_j\left(k_j \mid \mathbf{k}, N, J^{\mathbf{k}}\right) + \frac{1}{N} \times \pi_l\left(k_l \mid \mathbf{k}, N, J^{\mathbf{k}}\right) \\ &\quad + \frac{N - \left(\left|J^{\mathbf{k}}\right| + 1\right)}{N} \times \pi_m\left(0 \mid \mathbf{k}, N, J^{\mathbf{k}}\right),\end{aligned}$$

and by substituting this by the equilibrium payoffs that we have obtained in proposition 2 we get

$$\begin{aligned}\mathbb{E}\left[\pi_i\left(\mathbf{k} \mid N, J^{\mathbf{k}}\right)\right] &= \frac{\left|J^{\mathbf{k}}\right|}{N} \times \left(TS^*(\mathbf{k}) - \tilde{TS}_{-j}(\mathbf{k} \mid J^{\mathbf{k}})\right) + \frac{1}{N} \times \left(TS^*(\mathbf{k} \mid J^{\mathbf{k}}) - \tilde{TS}_{-l}(\mathbf{k} \mid J^{\mathbf{k}})\right) \\ &\quad + \frac{N - \left(\left|J^{\mathbf{k}}\right| + 1\right)}{N} \times \left(TS^*(\mathbf{k} \mid J^{\mathbf{k}}) - \tilde{TS}_{-m}(\mathbf{k} \mid J^{\mathbf{k}})\right) \\ &= TS^*(\mathbf{k}) - \frac{\left|J^{\mathbf{k}}\right|}{N} \times \tilde{TS}_{-j}(\mathbf{k} \mid J^{\mathbf{k}}) - \frac{1}{N} \times \tilde{TS}_{-l}(\mathbf{k} \mid J^{\mathbf{k}}) \\ &\quad - \frac{N - \left(\left|J^{\mathbf{k}}\right| + 1\right)}{N} \times \tilde{TS}_{-m}(\mathbf{k} \mid J^{\mathbf{k}}),\end{aligned}$$

where the equilibrium vector of investment $\mathbf{k}$ as in proposition 3.

Then, a supplier decides to become active in the industry if the expected payoffs are above the set-up costs, i.e. $\mathbb{E}\left[\pi_i\left(\mathbf{k} \mid N, J^{\mathbf{k}}\right)\right] \geq F$. In our model, potential suppliers do not only consider how the gains from trade are distributed but also they take into consideration the amount of investment of the buyer and its allocation. The following proposition establishes how suppliers' expected payoffs changes with both the number of active suppliers and the number of suppliers belonging to the set $J^{\mathbf{k}}$.

Proposition 5. *The expected payoff of any supplier is decreasing with the number of active suppliers N and increasing with the number of suppliers who coordinate their out of equilibrium* *offers* $J^{\mathbf{k}}$.

$$\frac{\partial \mathbb{E}\left[\pi_i\left(\mathbf{k} \mid N, J^{\mathbf{k}}\right)\right]}{\partial N} < 0, \quad \frac{\partial \mathbb{E}\left[\pi_i\left(\mathbf{k} \mid N, J^{\mathbf{k}}\right)\right]}{\partial J^{\mathbf{k}}} > 0.$$

The first result is necessary to obtain a unique equilibrium in the number of active suppliers in the industry, and the second tells us how expected payoffs evolve with the degree of competition. With regards to the number of active suppliers, we see that there are two effects that go into the same direction. The likelihood of being a supplier whom the buyer decides to invest decreases with the number of suppliers for a given set $J^{\mathbf{k}}$. There is also an indirect effect, that is also negative, and comes from the changes in the investment profile and its effect to the trading surplus that is generated. In lemma 5, we showed that the per supplier level of investment decreases with the number of active suppliers in the industry.

Considering the change in the expected payoffs with respect to the number of suppliers belonging to $J^{\mathbf{k}}$, we see that the overall effect is positive. There is a positive direct effect that comes from the increased probability of being the supplier to whom the buyer invests and a positive indirect effect that originates from the change created to the investment profile. The negative direct effect comes from a higher intensity of competition which generates a smaller partition of the trading gains.

Competition brings about a larger and more evenly distribution of investment, and this translates to a larger expected payoffs for suppliers. As a result, a larger number of active suppliers can be sustained in equilibrium. This result is stated in the following corollary.

Corollary 2. *Whenever the buyer undertakes a positive level of investment, a higher competitive equilibrium generates higher expected payoffs for the suppliers. The contrary occurs when investment does not materialize.*

$$\begin{aligned}
&\mathbb{E}\left[\pi_i\left(\mathbf{k} \mid J^{\mathbf{k}}\right) \mid \mathbf{k} \neq \mathbf{0}\right] \geq \mathbb{E}\left[\pi_i\left(\mathbf{k} \mid J'^{\mathbf{k}}\right) \mid \mathbf{k} \neq \mathbf{0}\right], \\
&\mathbb{E}\left[\pi_i\left(\mathbf{k} \mid J^{\mathbf{k}}\right) \mid \mathbf{k} = \mathbf{0}\right] < \mathbb{E}\left[\pi_i\left(\mathbf{k} \mid J'^{\mathbf{k}}\right) \mid \mathbf{k} = \mathbf{0}\right] \quad \textit{for} \quad J'^{\mathbf{k}} \subset J^{\mathbf{k}}.
\end{aligned}$$

We leave the formal proof in the appendix and we provide a short explanation here. In the case where there is no investment, either because this does not arise in equilibrium, or the buyer does not have the technology, lower levels of competition entail a larger number of active suppliers. Because entry in the industry occurs until the rents are dissipated, a larger number of suppliers can coexist when they appropriate a larger proportion of the gains from trade. This is the case when competition is milder as each supplier obtains more than his marginal contribution to the surplus. With positive investment, the analysis is richer, now the amount and the distribution of investment depends on the level of competition ex-post. From the previous analysis we have established that the total amount invested is in general larger whenever competition is more intense. This, together with a more homogeneous distribution of investment makes suppliers to expect larger payoffs even if the partition of the trading surplus is less favorable. A smaller partition of a larger surplus dominates a more favorable partition of a smaller one.

In the figure below, we provide a graphical interpretation of the results introduced in the previous discussion. We illustrate the number of active suppliers as a function of the level

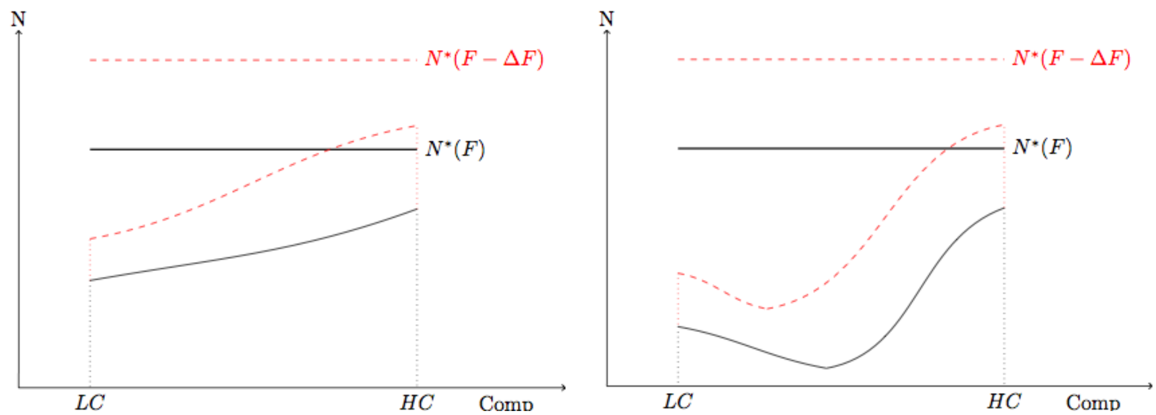


Figure 4: Number of active suppliers as a function of competition for a given level of set-up costs. The dashed line in red, represents a situation with a decrease of ΔF in the set-up costs. The figure on left depicts a situation where the "allocative senistivity" is small and the one on the right is when this is large.

of competition and the magnitude of the set-up costs F. The picture on the left, depicts a situation where the "allocative sensitivity" is moderate and the buyer always sets a positive level of investment regardless of the competitiveness of the equilibrium outcome. In this case, a larger number of active suppliers is achieved with a more competitive outcome. Conversely, the figure on the right stands for a situation where the "allocative sensitivity" is sufficiently big such that the buyer decides not to invest with low competitive outcomes. Hence, in the set of equilibria where no investment takes place, the number of active suppliers that can be sustained in equilibrium increases with lower competition. Finally, a reduction of the entry costs will translate into a larger number of active suppliers but, in general, the equilibrium is characterized by too little entry.

5 Discussion

In this paper, we have considered the case where investment by the buyer is purely specific. As a result, the default payoff of any supplier with not trade is equal to zero, regardless of the level of investment undertaken by the buyer. How our results change when suppliers' default payoff depends on investment? What would happen if the outside option is endogenized by the existence of another buyer whom suppliers might trade with? In such a situation, cooperative investment would increases the outside option of suppliers as they get better trading terms with other potential buyers. The IT industry provides some evidence where Kang et al. (2007) state that: *"after winning an order from Dell, HIPRO found it easier to approach other OEM buyers (i.e., Cisco) and its importance with dealing with Dell increased with respect to other suppliers"*.

Hence, investment may not only help a given supplier to establish a closer relationship with the investing buyer but it can also encourage to initiate trade with other buyers. This is the case when investment is aimed at reducing the production costs, because having more efficient suppliers is in the interest of the industry as a whole. Consequently, investment not only works as a mechanism to enhance competition among suppliers, but it may also create competition downstream because some buyers, by free-riding from the investment undertaken in the industry, may decide to become active.

Therefore, what is the equilibrium investment profile if buyers can free-ride on the investments undertaken by other competitive buyers? We believe that the problem of being "held-up" is even more severe. Again this would depend on the degree of investment spillovers and whether suppliers can also sign non-exclusive contracts with the existing buyers in the economy. At this regard Felli and Roberts (2013) consider a matching model with many buyers where parties can undertake specific investment.[16] Our approach is to study a similar problem but without assuming de-facto foreclosure. Nonetheless, we leave it as topic for future research.

In what follows, we analyze the robustness of our findings by considering the case of selfish

[16] Another paper studying a similar issue is Samuelson (2013).

investment. Now, specific investment does not reduce the cost of production but has a direct impact on the buyer's valuation for the traded good. Hence, the utility of the buyer directly depends on the total amount invested S, i.e. $U\left(X(S,N) \mid S\right)$, and it has increasing returns on investment $U'_S(\cdot) > 0$ and $U''_{xS}(\cdot) > 0$. The production cost of each supplier does not directly depend on investment $C\left(x_i(S,N)\right),\ \forall i \in N$, and the cost of investment $\psi(S)$ is increasing and convex.

With selfish investment, the investing party appropriates all direct benefits from investment, whose efficiency level is implicitly given by

$$\psi'_S(S) = U'_S(X^*(S) \mid S).$$

The equilibrium investment for the most and the least competitive equilibrium are respectively given by

$$\psi'_S(S) = U'_S\left(X^*(S) \mid S\right) - \sum_{i=1}^{N} \int_{\tilde{X}_{-i}(S|\bar{J})}^{X^*(S)} U''_{xS}(\tau) d\tau,$$

$$\begin{aligned} \psi'_S(S) = U'_S\left(X^*(S) \mid S\right) - \sum_{i=1}^{N} \int_{X^*_{-\{i,h\}}(S)+\tilde{x}_h(S,\underline{J})}^{X^*(S)} U''_{xS}(\tau) d\tau \\ + \sum_{i}^{N} \sum_{j \neq i,h} \left(\int_{X^*(S)}^{X^*_{-\{i,h\}}(S)+\tilde{x}_h(S,\underline{J})} U''_{xx}(\tau) d\tau \right) \times \frac{dx^*_j}{dS}, \quad h \in \underline{J}. \end{aligned}$$

Now, the equilibrium investment is always positive because the buyer appropriates all direct benefits coming from investment. The direct benefit is represented by the first part of each expression above. Yet, the "hold-up" problem exists as some part of the benefits from investment are appropriated by the suppliers, which is represented by the rest of the terms. The way investment affects the efficient allocation has an effect on the equilibrium investment profile. However, contrary to the case with cooperative investment, this externality has a positive effect, and lower levels of competition are associated to larger amounts of aggregate investment when the "allocative sensitivity" is large.

Additionally, with selfish investment, a larger number of active suppliers entails higher investment. With more suppliers the part of the trading surplus appropriated by the buyer is larger. In the figure, we observe that whenever the number of suppliers is arbitrarily large the equilibrium level of investment coincides with efficiency.

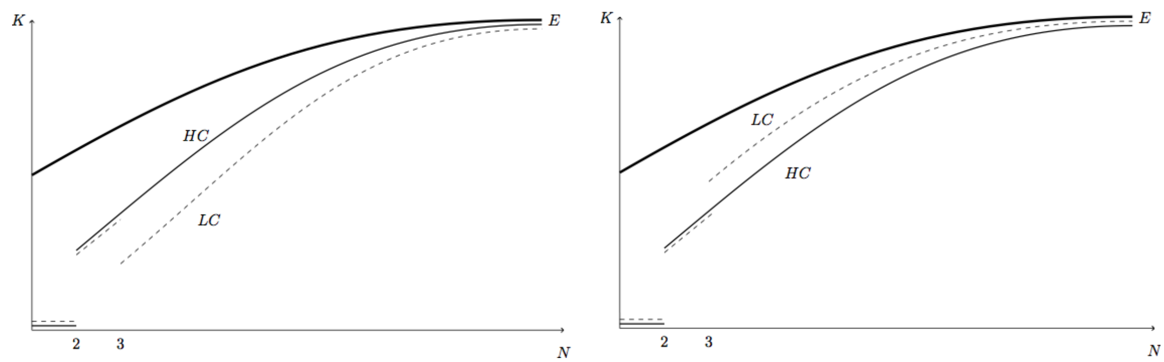


Figure 5: Aggregate investment as a function of the number of active suppliers. The thick black line represents the efficiency level and the thin and dashed line stand for the equilibrium level profile in the two extreme equilibrium of the trading game. The left hand side is a situation where the "allocative sensitivity" dx_j^*/dS is low. The one on the right represents the case where this change is large.

6 Conclusion

The economics of specialization makes the relation-specific investment a current growing phenomenon, whose analysis is essential to understand the well functioning of market transactions. In the present paper, we have considered the case where a monopolistic buyer decides on specific relation investment. Departing from most of the existing literature, we analyzed cooperative rather than selfish investment, and we obtained that both the distribution and the aggregate level of investment depends on the level of competition of the equilibrium outcome.

When investment is cooperative, there exists an endemic "hold-up" problem, since the investing party does not appropriate the direct gains coming from investment. Yet, the buyer sets a positive level of investment because, investment to a given supplier, constraints the equilibrium transfers of the remaining ones. Hence, in equilibrium, the buyer invests to foster competition among suppliers. We find that, a more competitive equilibrium is characterized by higher levels of investment and investment pushes competition even further. When the number of suppliers increases, competition is intensified, and the incentives to invest are reduced. The model then establishes a positive relation between the level of set-up costs and investment, and in order to incentivize investment, entry costs should be "sufficiently" large. The intuition is similar to the models of patent protection, where a concentrated market is more suitable for undertaking investment. However, the mechanism is very different from this literature. In our model investment works as a instrument to increase competition and this increases the rents that can be appropriated by the investing party. Hence, a competition authority should only be concerned on establishing the conditions to ensure effective competition in the market place and not on increasing the number of competitors.

Furthermore, the present paper provides a theory explaining the industry configuration of the supply side of the market. In situations when the equilibrium outcome is competitive, the buyer distributes his investment homogeneously among suppliers, creating an ex-post homogenous market structure. Conversely, when the equilibrium outcome is less competitive, the investment of the buyer is asymmetric and it is concentrated to a small group of suppliers. This explains the coexistence of first with second tier suppliers. With regards to the number of active suppliers, we also find that it depends on the level of competition. Because a higher level of competition entails larger investment, a bigger number of suppliers have incentives to become active. Hence, we find a reverse effect on entry and competition. In general the equilibrium is characterized by too little entry. Our model then illustrate two different sources of inefficiency and restoring full efficiency seems a daunting task.

Finally, the theory provided in this paper can be empirically tasted. The first thing is to see whether cooperative investment is more likely to appear in more competitive industries and the second is to observe whether cooperative investment is larger in industries characterized by large set-up or entry costs.

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Appendix

A Appendix

Lemma 6. *The loss of the trading surplus of any supplier i is bigger with larger investments of the buyer. Then, for $\mathbf{k}'$ and $\mathbf{k}$ such that we have $k_i' \geq k_i$, $\forall i \in N$.*

$$L_i\left(\mathbf{k}' \mid \bar{J}_i\right) = TS^*(\mathbf{k}') - TS^*_{-i}(\mathbf{k}'_{-\mathbf{i}}) > TS^*(\mathbf{k}) - TS^*_{-i}(\mathbf{k}_{-\mathbf{i}}) = L_i\left(\mathbf{k} \mid \bar{J}_i\right).$$

Proof. The previous claim is equivalent to

$$TS^*(\mathbf{k}') - TS^*(\mathbf{k}) > TS^*_{-i}(\mathbf{k}'_{-i}) - TS^*_{-i}(\mathbf{k}_{-i}).$$

We show that for any number of active suppliers N, the potential gains from trade are bigger when the buyer invests more $TS^*(\mathbf{k}') > TS^*(\mathbf{k})$

$$\begin{aligned}
TS^*(\mathbf{k}) &= U\left(X^*(K)\right) - \sum_i C(x_i^*(\mathbf{k}) \mid k_i) \\
&= U\left(X^*(\mathbf{k})\right) + U\left(X^*(\mathbf{k}')\right) - U\left(X^*(\mathbf{k}')\right) - \sum_i C(x_i^*(\mathbf{k}) \mid k_i) \\
&\leq U\left(X^*(\mathbf{k})\right) - U\left(X^*(\mathbf{k}')\right) + TS^*(\mathbf{k}') \\
\Longrightarrow & TS^*(\mathbf{k}') - TS^*(\mathbf{k}) \geq U\left(X^*(\mathbf{k}')\right) - U\left(X^*(\mathbf{k})\right) = \int_{X^*(\mathbf{k})}^{X^*(\mathbf{k}')} U_x(\tau) d\tau > 0,
\end{aligned}$$

and the last inequality comes from the optimal allocation represented in (4.1) that dictates $X^*(\mathbf{k}') > X^*(\mathbf{k})$.

From the previous, we know that $TS^*(\mathbf{k}') - TS^*(\mathbf{k}) > U\left(X^*(\mathbf{k}')\right) - U\left(X^*(\mathbf{k})\right) = \underline{D}$. Furthermore, we can show that:

$$\begin{aligned}
TS^*_{-i}(\mathbf{k}'_{-i} \mid \bar{J}_i) &= U\left(\tilde{X}_{-i}(\mathbf{k}' \mid \bar{J}_i)\right) - \sum_{j \neq i} C(\tilde{x}_j(\mathbf{k}'_{-i} \mid \bar{J}_i) \mid k_j') \\
&< U\left(\tilde{X}_{-i}(\mathbf{k}' \mid \bar{J}_i)\right) - \sum_{j \neq i} C(\tilde{x}_j(\mathbf{k}'_{-i} \mid \bar{J}_i) \mid k_j) \\
&= U\left(\tilde{X}_{-i}(\mathbf{k}' \mid \bar{J}_i)\right) + U\left(\tilde{X}_{-i}, (\mathbf{k} \mid \bar{J}_i)\right) - U\left(\tilde{X}_{-i}(\mathbf{k} \mid \bar{J}_i)\right) - \sum_{j \neq i} C(\tilde{x}_j(\mathbf{k}'_{-i}, \mid \bar{J}_i) \mid k_j) \\
&< U\left(\tilde{X}_{-i}(\mathbf{k}' \mid \bar{J}_i)\right) - U\left(\tilde{X}_{-i}(\mathbf{k} \mid \bar{J}_i)\right) + TS^*_{-i}(\mathbf{k}_{-i} \mid \bar{J}_i) \\
\Longrightarrow & TS^*_{-i}(\mathbf{k}' \mid \bar{J}_i) - TS^*_{-i}(\mathbf{k} \mid \bar{J}_i) < U\left(\tilde{X}_{-i}(\mathbf{k}' \mid \bar{J}_i)\right) - U\left(\tilde{X}_{-i}(\mathbf{k} \mid \bar{J}_i)\right) = \overline{D},
\end{aligned}$$

and it is easy to see that

$$\underline{D} > \overline{D} \to U\left(X^*(\mathbf{k}')\right) - \left(X^*(\mathbf{k})\right) > U\left(\tilde{X}_{-i}(\mathbf{k}' \mid \bar{J}_i)\right) - U\left(\tilde{X}_{-i}(\mathbf{k} \mid \bar{J}_i)\right)$$
$$= \int_{X^*(\mathbf{k})+\tilde{X}_{-i}(\mathbf{k}|\bar{J}_i)}^{X^*(\mathbf{k}')+\tilde{X}_{-i}(\mathbf{k}'|\bar{J}_i)} U_x(\tau)d\tau > 0.$$

And this is the case as the increase in the total mount traded from an extra supplier is higher when the buyer increases his total amount of investment $X^*(\mathbf{k}') - \tilde{X}_{-i}(\mathbf{k}' \mid \bar{J}_i) > X^*(\mathbf{k}) - \tilde{X}_{-i}(\mathbf{k} \mid \bar{J}_i)$. □

B Appendix

Proof of lemma 1: differentiating the first-order conditions for x_j^* given in (4.1) with respect to k_i we obtain:

$$U''_{xx}(X^*) \times \sum_{h=1}^{N} \frac{dx_h^*}{dk_i} = C''_{xx}(x_j^* \mid k_j) \times \frac{dx_j^*}{dk_i}. \tag{B.1}$$

Since the left hand side is independent of j we find that all dx_j^*/dk_i have the same sign. Now suppose also dx_i^*/dk_i has that same sign. Then also the sum has that same sign and since $U''_{xx}(\cdot) < 0$, and $C''_{xx}(\cdot) > 0$ this leads to a contradiction because the right and the left hand side have different signs. Now suppose $dx_i^*/dk_i < 0$. The other signs therefore have to be positive. By (B.1) we find $\sum_{h=1}^{N}(dx_h^*/dk_i) < 0$, but the first-order condition for x_i^* differentiated with respect to k_i is

$$U''_{xx}(X^*) \times \sum_{h=1}^{N} \frac{dx_h^*}{dk_i} = C''_{xx}(x_i^* \mid k_i) \times \frac{dx_i^*}{dk_i} + C''_{xk}(x_i^* \mid k_i),$$

which would then have a positive left hand side and a negative right hand side due to $C''_{xk}(\cdot) < 0$ and this leads again to a contradiction. We thus have shown point (i) and (ii). Again by (B.1) point (iii) follows from $\partial X^*/\partial k_i = \sum_{h=1}^{N}(dx_h^*/dk_i)$.

Proof of lemma 2: The results comes directly from the concavity of the utility function and the convexity of the cost function. Without loss of generality, we assume that the equilibrium allocation is symmetric.[17] The amount traded in equilibrium needs to satisfy

$$U'_x\left(N + 1 \times x^*(\mathbf{k}, N+1)\right) = C'_x\left(x^*(\mathbf{k}, N+1) \mid k\right).$$

We proof the claim by contradiction, assume that $x^*(\mathbf{k}, N+1) \geq x^*(\mathbf{k}, N)$, then we have that $N + 1 \times x^*(\mathbf{k}, N+1) > N \times x^*(\mathbf{k}, N)$. By the concavity of $U'_x(\cdot)$ and optimality it has to be the case

[17] Our results do not change for an asymmetric equilibrium allocation where we have to substitute $N + 1 \times x^*(\mathbf{k}, N+1)$ by $\sum_{i=1}^{N+1} x_i^*(\mathbf{k}, N+1)$.

that

$$C_x'\left(x^*(\mathbf{k},N+1)\mid k\right)=U_x'\left(N+1\times x^*(\mathbf{k},N+1)\right)<U_x'\left(N\times x^*(\mathbf{k},N)\right)=C_x'\left(x^*(\mathbf{k},N)\mid k\right),$$

but by the convexity of $C_x'(\cdot)$ this implies that $x^*(\mathbf{k},N+1)<x^*(\mathbf{k},N)$, which leads to a contradiction. From the previous, we see that that $X^*(\mathbf{k},N+1)>X^*(\mathbf{k},N)$ comes directly.

Proof of lemma 3: We have to show that

$$X^*(\mathbf{k},N)>X^*_{-\{J_i^{\mathbf{k}},i\}}(\mathbf{k},N)+\sum_{j\in J_i^{\mathbf{k}}}\tilde{x}_j^J(\mathbf{k}\mid J_i^{\mathbf{k}}).$$

For any investment profile $\mathbf{k}$ and a set $J_i^{\mathbf{k}}$, we know that $\sum_{h\neq\{J_i^{\mathbf{k}},i\}}x_h^*=X^*_{-\{J_i^{\mathbf{k}},i\}}$. Hence, the expression above is equivalent to $\sum_{j\in J_i^{\mathbf{k}}}x_j^*+x_i^*>\sum_{j\in J_i^{\mathbf{k}}}\tilde{x}_j(\mathbf{k}\mid J_i^{\mathbf{k}})$. Therefore since $x_i^*>0$ if $\sum_{j\in J_i^{\mathbf{k}}}\left(x_j^*-\tilde{x}_j(\mathbf{k}\mid J_i^{\mathbf{k}})\right)>0$ we are done. Observe that for a given investment profile, if the above is true it also has to be true for any $j\in J_i^{\mathbf{k}}$, hence $x_j^*>\tilde{x}_j(\mathbf{k}\mid J_i^{\mathbf{k}})$. If the contrary occurs, $x_j^*<\tilde{x}_j(\mathbf{k}\mid J_i^{\mathbf{k}})$, then from the equilibrium allocation we have

$$U_x'\left(X^*_{-\{J_i^{\mathbf{k}},i\}}+\sum_{j\in J_i'}\tilde{x}_j(\mathbf{k}\mid J_i^{\mathbf{k}})\right)=C_x'(\tilde{x}_j(\mathbf{k}\mid J_i^{\mathbf{k}}))>C_x'(x_j^*)=U_x'(X^*),$$

and by the concavity of U we prove the claim. The previous also implies that for any $j\in J_i^{\mathbf{k}}$ we have $\tilde{x}_j(\mathbf{k}\mid J_i^{\mathbf{k}})>x_j^*$. Using the same procedure we can easily prove that for any $J_i^{\prime,\mathbf{k}}\subseteq J_i^{\mathbf{k}}$ we have

$$X^*_{-\{J_i^{\mathbf{k}},i\}}+\sum_{j\in J_i^{\mathbf{k}}}\tilde{x}_j(\mathbf{k},\mid J_i^{\mathbf{k}})\geq X^*_{-\{J_i^{\prime\mathbf{k}},i\}}+\sum_{j\in J_i^{\prime\mathbf{k}}}\tilde{x}_j(\mathbf{k}'\mid J_i^{\prime\mathbf{k}}),$$

and we also obtain that $\tilde{x}_j(\mathbf{k}'\mid J_i^{\prime\mathbf{k}})\geq\tilde{x}_j(\mathbf{k}\mid J_i^{\mathbf{k}})$.

Proof of proposition 2: Here, we consider the equilibrium where the suppliers submit the efficient allocation. The equilibrium transfer for supplier i depend on the set $J_i^{\mathbf{k}}$ and for a given investment profile $\mathbf{k}$ this is equal to

$$T_i(J_i^{\mathbf{k}})=U\left(X^*\right)-\left(\max_{\{x_j\}_{j\in J_i^{\mathbf{k}}}}\left[U\left(X^*_{-\{J_i^{\mathbf{k}},i\}}+\sum_{j\in J_i^{\mathbf{k}}}x_j\mid\tilde{x}_i=0,\mathbf{k}\right)-\sum_{j\in J_i^{\mathbf{k}}}C(x_j\mid k_j)\right]+\sum_{j\in J_i^{\mathbf{k}}}C(x_j^*\mid k_j)\right).$$

Operating we obtain:

$$
\begin{aligned}
T_i(J_i^{\mathbf{k}}) &= U\left(X^*\right) - \left(\max_{\{x_j\}_{j\in J_i^{\mathbf{k}}}} \left[U\left(X^*_{-\{J_i^{\mathbf{k}},i\}} + \sum_{j\in J_1} x_j \mid \tilde{x}_i = 0, \mathbf{k}\right) - \sum_{j\in J_i^{\mathbf{k}}} C(x_j)\right] + \sum_{j\in J_i^{\mathbf{k}}} C(x_j^* \mid k_j)\right) \\
&= U(X^*) - \sum_{j\in J_i^{\mathbf{k}}} C(x_j^* \mid k_j) - \left[U\left(X^*_{-\{J_i^{\mathbf{k}},i\}} + \sum_{j\in J_i^{\mathbf{k}}} \tilde{x}_j(\mathbf{k}, J_i^{\mathbf{k}}) \mid \tilde{x}_i = 0, \mathbf{k}\right) - \sum_{j\in J_i^{\mathbf{k}}} C(\tilde{x}_j(\mathbf{k}, J_i^{\mathbf{k}}))\right] \\
&= U(X^*) - \sum_{j\in J_i^{\mathbf{k}}} C(x_j^* \mid k_j) - \left[U\left(X^*_{-\{J_i^{\mathbf{k}},i\}} + \sum_{j\in J_i^{\mathbf{k}}} \tilde{x}_j(\mathbf{k}, J_i^{\mathbf{k}}) \mid \tilde{x}_i = 0, \mathbf{k}\right) - \sum_{j\in J_i^{\mathbf{k}}} C(\tilde{x}_j(\mathbf{k}, J_i^{\mathbf{k}}))\right] \\
&+ \left[\sum_{j\notin J_i^{\mathbf{k}},i} \left(C(x_j^* \mid k_j) - C(x_j^* \mid k_j)\right)\right] + \left[C(x_i^* \mid k_i) - C(x_i^* \mid k_i)\right] \\
&= TS^*(\mathbf{k}) - \left[U\left(X^*_{-\{J_i^{\mathbf{k}},i\}} + \sum_{j\in J_i^{\mathbf{k}}} \tilde{x}_j(\mathbf{k}, J_i^{\mathbf{k}}) \mid \tilde{x}_i = 0, \mathbf{k}\right) - \sum_{j\in J_i^{\mathbf{k}}} C(\tilde{x}_j(\mathbf{k}, J_i^{\mathbf{k}}) \mid k_j) - \sum_{j\neq J_i^{\mathbf{k}},i} C(x_j^* \mid k_j)\right] \\
&+ C(x_i^* \mid k_i) \\
&= TS^*(\mathbf{k}) - \tilde{TS}_{-i}(\mathbf{k} \mid J_i^{\mathbf{k}}) + C(x_i^* \mid k_i).
\end{aligned}
$$

By putting this to the payoff functions in (3.1) and (3.2), we obtain the result.

Proof of proposition 1: We start the proof by calculating the change on the trading surplus by an increase of investment. Later we show that the trading surplus is maximized by setting the same level of investment to all active suppliers.

By taking the first order condition of the trading surplus with respect to investment

$$
\begin{aligned}
\frac{\partial TS(\mathbf{k})}{\partial k_i} &= U_x'\left(\sum_N x_i^*(\mathbf{k})\right) \times \sum_{j=1}^N \frac{\partial x_j^*(\mathbf{k})}{\partial k_i} - C_x'\left(x_i^*(\mathbf{k}) \mid k_i\right) \times \sum_{j=1}^N \frac{\partial x_j^*(\mathbf{k})}{\partial k_i} - C_k'\left(x_i^*(\mathbf{k}) \mid k_i\right) \\
&- \phi_K'(K) \times \frac{\partial K}{\partial k_i} = 0 \\
&\Longrightarrow \left[U_x'\left(\sum_N x_i^*(\mathbf{k})\right) - C_x'\left(x_i^*(\mathbf{k}) \mid k_i\right)\right] \times \sum_{j=1}^N \frac{\partial x_j^*(\mathbf{k})}{\partial k_i} - C_k'\left(x_i^*(\mathbf{k}) \mid k_i\right) - \phi_K'(K) = 0 \\
&\Longrightarrow \phi_K'(K^*) = -C_k'\left(x_i^*(\mathbf{k}^*) \mid k_i\right), \quad \forall i \in N,
\end{aligned}
$$

and the last line comes from the application of the envelope theorem.

We proceed to show that the distribution of investment that maximizes the trading surplus is symmetric. At this purpose, we compare the benefits of an increase of investment by the buyer in a situation where this increase is distributed symmetrically, to an extreme case where it is allocated to only one of the suppliers. Later, we see that this result can be easily extended to any asymmetric distribution.

Consider a symmetric distribution of an aggregate investment K, and each suppliers trades an amount $x_i^*(\mathbf{k}, N)$. The buyer then distributes an arbitrarily small increase of investment ΔK. A symmetric distribution gives a new new vector of investment $\mathbf{k}^s$ where $k^s = k + \frac{\Delta K}{N}$. An a asymmetric distribution implies that for a given supplier $j \in N$, $k_j^a = k + \Delta K$ and for all $i \neq j$ the investment stays the same $k_i^a = k$. The extra gains of both strategies are $\Delta^s = TS^*(\mathbf{k}^s) - TS^*(\mathbf{k})$ and $\Delta^a = TS^*(\mathbf{k}^a) - TS^*(\mathbf{k})$, which is approximately equal to $\Delta^s \approx -\sum_{i=1}^{N} C_k'\left(x_i^*(\mathbf{k}^s) \mid k_i^s\right)$ and $\Delta^a \approx -C_k'\left(x_j^*(\mathbf{k}^a) \mid k_j^a\right)$.[18] Hence, by comparing both gains we obtain

$$\Delta^s - \Delta^a > 0 \Longrightarrow -\sum_{i=1}^{N} C_k'\left(x_i^*(\mathbf{k}^s) \mid k_i^s\right) > -C_k'\left(x_j^*(\mathbf{k}^a) \mid k_j^a\right)$$

$$\Longrightarrow -(N-1)C_k'\left(x_i^*(\mathbf{k}^s) \mid k_i^s\right) > -C_k'\left(x_j^*(\mathbf{k}^a) \mid k_j^a\right) + C_k'\left(x_j^*(\mathbf{k}^s) \mid k_j^s\right) = -\int_{x_j^*(\mathbf{k}^s)}^{x_j^*(\mathbf{k}^a)} C_{xkk}'''(\tau)d\tau,$$

where the last equality comes from the application of the fundamental theorem of calculus.

The previous condition is always fulfilled because the left hand side is positive and the right hand side is negative. This is the case due to assumption $C_{xkk}'''(\cdot) > 0$, and by lemma 1 where we showed that for any $k_i' > k_i$ then $x_j^*(\mathbf{k}', N) > x_j^*(\mathbf{k})$.

We can easy extend this reasoning with any other asymmetric distribution. Consider, the buyer distributes the increase of investment to any subset $J^{\mathbf{k}}$ of suppliers. Then for any $j \in J^{\mathbf{k}}$ we have $k_j^J = K + \frac{\Delta K}{|J^{\mathbf{k}}|}$ where the denominator is the cardinality of the set. Therefore for any $J^{\mathbf{k}} \subset N$ we obtain

$$-(N - \left|J^{\mathbf{k}}\right|) \times C_k'\left(x_i^*(\mathbf{k}^s) \mid k_i^s\right) > -\sum_{j \in J^{\mathbf{k}}} \int_{x_j^*(\mathbf{k}^s)}^{x_j^*(\mathbf{k}^J)} C_{xkk}'''(\tau)d\tau,$$

and this is always the case by the same argumentation as before.

The argument here is local, but it can be easily extended globally. To illustrate this case, consider a situation where the buyer allocates her investment to only one supplier. Then, by following the same argument as before, there exist local deviations where the buyer redistributes the investment to other active suppliers. Indeed, there will always be local deviations until the investment is symmetrically distributed.

Proof of lemma 4: We start from considering the case where the equilibrium outcome is the most competitive i.e. $J_i^{\mathbf{k}} = N \setminus \{i\} = \bar{J}_i^{\mathbf{k}}$ and we show that the buyer will set the same level of investment to

[18] Observe that those gains are approximated because we only take into consideration direct gains. It is true that with an asymmetric distribution of investment there are always reduction of production costs for the non investing suppliers via the reduction of the equilibrium allocation. However this effect is of second order and we neglect it here.

all suppliers. We proceed by construction. First, we show that an increase of the level of investment of supplier $j \neq i$ decreases the payoff of any supplier i. Second, we see that those payoffs are minimized if the buyer sets the same level of investment to all suppliers.

The equilibrium payoff for every supplier i in the most competitive equilibrium is equals to $\pi_i(\mathbf{k} \mid \bar{J}_i^{\mathbf{k}}) = TS^*(\mathbf{k}) - TS^*_{-i}(\mathbf{k}_{-i} \mid \bar{J}_i^{\mathbf{k}})$. And the second part is the maximization of the trading surplus with all the rest of the suppliers. The payoff of any supplier i decreases with the investment undertaken to any other supplier by the amount

$$\begin{aligned}\frac{\partial \pi_i(\mathbf{k} \mid \bar{J}_i^{\mathbf{k}})}{\partial k_j} &= \left[\frac{\partial TS^*(\mathbf{k})}{\partial k_j} - \frac{\partial TS^*_{-i}(\mathbf{k}_{-i} \mid \bar{J}_i^{\mathbf{k}})}{\partial k_j}\right] = -C'_k(x_j^*(\mathbf{k}) \mid k_j) + C'_k(\tilde{x}_j(\mathbf{k}_{-i} \mid \bar{J}_i^{\mathbf{k}}) \mid k_j) \\ &= \int_{x_j^*(\mathbf{k})}^{\tilde{x}_j(\mathbf{k}_{-i} \mid \bar{J}_i^{\mathbf{k}})} C''_{xk}(\tau) d\tau < 0.\end{aligned}$$

The first equality is obtained by applying the envelope theorem, and the last strict inequality comes from assumption 2 and lemma 3. Setting an equal level of investment comes directly from an observation of the second part of the payoffs. Because $TS^*_{-i}(\mathbf{k}_{-i} \mid \bar{J}_i^{\mathbf{k}})$ does not depend on the investment undertaken to supplier i, minimizing his payoffs is equivalent to set the vector of investments $\mathbf{k}_{-i}$ such that expression $TS^*_{-i}(\mathbf{k}_{-i} \mid \bar{J}_i^{\mathbf{k}})$ is maximized. By the same argument as in proposition 1 we obtain that $k_j = k$ for all $j \in N$.

We use the same procedure to show the distribution of investment by any other level of competition i.e $J^{\mathbf{k}} < N \setminus \{i\}$. By calculating the first order condition and applying the envelope theorem, we get how the payoffs of any supplier i changes with the investment done to any other supplier j

$$\begin{aligned}\frac{\partial\left(\left[TS^*(\mathbf{k}) - \tilde{TS}_{-i}(\mathbf{k} \mid J_i^{\mathbf{k}})\right]\right)}{\partial k_j} &= -C'_k(x_j^*) + C'_k(\tilde{x}_j^J(J_i^{\mathbf{k}})) \\ &\quad - \left(\sum_{m \neq J_i^{\mathbf{k}}, i}\left[U'_x\left(X^*_{-\{J_i^{\mathbf{k}}, i\}} + \sum_{j \in J_i^{\mathbf{k}}} \tilde{x}_j\right) - C'_x(x_m^*)\right] \times \frac{dx_m^*}{dk_j}\right) \\ &= \int_{x_j^*(\mathbf{k})}^{\tilde{x}_j(\mathbf{k} J_i^{\mathbf{k}})} C''_{xk}(\tau) d\tau - \sum_{m \neq J_i^{\mathbf{k}}, i}\left(\int_{X^*}^{X^*_{-\{J_i^{\mathbf{k}}, i\}} + \sum_{j \in J} \tilde{x}_j} U''_{xx}(\tau) d\tau\right) \frac{dx_m^*}{dk_j}.\end{aligned}$$

The first part is similar to the one with the most competitive outcome and the part comes from the fact that when constraining the transfers of all suppliers, the allocation of the suppliers that do not coordinate their equilibrium offers remains unchanged. As before, the first part is negative by assumption $C'''_{xk}(\cdot) < 0$ and lemma 3. The second is positive by assumption $U''_{xx}(\cdot) < 0$ and by lemmas 1 and 3. Moreover, the magnitude of this second part depends on the degree of the "allocative sensitivity" dx_m^*/dk_j. From the same argument as before, the buyer sets the same level of investment to the suppliers in $J_i^{\mathbf{k}}$.

The second part of the constructive argument comes directly from the fact that an increase of the investment to any supplier $i = l$ constraints the equilibrium transfer of the set of suppliers in $J_i^{\mathbf{k}}$. Since the group of suppliers $j \in J_i^{\mathbf{k}}$ constraint all active suppliers, their level of investment is larger than for the supplier that only puts a constraint on the set of suppliers $J_i^{\mathbf{k}}$. Finally, it is optimal for the buyer to set a level of zero investment to all the other suppliers $k_m = 0$ for $m \neq J, l$, because those suppliers do not constraint the payment of any other supplier.

Proof of proposition 3: That investment is symmetric when the equilibrium outcome is the most competitive is proven in lemma 4. Each supplier then trades the same amount with the buyer and the former does not depend on the identity of supplier i. Hence, the individual amount of investment is obtained by the first order condition.

$$\begin{aligned}\frac{\partial \Pi(\mathbf{k} \mid \bar{J}^{\mathbf{k}})}{\partial k} &= -C_k'\left(x^*(\mathbf{k}) \mid k\right) - \left[-C_k'\left(x^*(\mathbf{k}) \mid k\right)\right] - \sum_{N-1}\left[-C_k'\left(x^*(\mathbf{k}) \mid k\right) + C_k'\left(\tilde{x}(\mathbf{k}_{-1} \mid \bar{J}^{\mathbf{k}}) \mid k\right)\right] \\ &\quad - \phi_K'(K) \times \frac{\partial K}{\partial k} = 0 \\ &\Longrightarrow \phi_K'(K) = -\sum_{N-1}\left[\int_{x^*(\mathbf{k})}^{\tilde{x}(\mathbf{k}_{-1}|\bar{J}^{\mathbf{k}})} C_{xk}''(\tau)d\tau\right] \equiv \zeta.\end{aligned}$$

In the first line of the equation we see that the direct benefit from investment is fully appropriated by the supplier. This result comes directly from the fact that investment in this case is cooperative and the party that does not invest appropriates all the rents.[19] However, an increase on the amount invested to a given supplier creates a negative indirect externality to all other suppliers as their "loss" of exclusion is reduced. To establish that for a given number of suppliers N, the buyer under-invest in equilibrium, we compare the right hand side of the efficient investment profile with the one that we have just obtained. We get that the former is always larger than the latter by assumption $C_{xk}''(\cdot) < 0$ and lemma 3.

$$\begin{aligned}&- C_k'\left(x^*(\mathbf{k}) \mid k\right) > \zeta \rightarrow -C_k'\left(x^*(\mathbf{k}) \mid k\right) > -\sum_{N-1}\left[\int_{x^*(\mathbf{k})}^{\tilde{x}(\mathbf{k}_{-i}|\bar{J}_i^{\mathbf{k}})} C_{xk}''(\tau)d\tau\right] \\ &\rightarrow \sum_{N-1}\left[C_k'\left(\tilde{x}(\mathbf{k_{-i}} \mid \bar{J}_i^{\mathbf{k}}) \mid k\right) - C_k'\left(x^*(\mathbf{k}) \mid k\right)\right] - C_k'\left(x^*(\mathbf{k}) \mid k\right) > 0 \rightarrow \int_{X^*(\mathbf{k})}^{\tilde{X}^*_{-i}(\mathbf{k}_{-i})} C_{xk}''(\tau)d\tau > 0.\end{aligned}$$

To show point ii) we calculate the first order condition of the buyer's payoff with respect to individual level of investment k_j whenever $j \in J_i^{\mathbf{k}}$. The payoff of the common buyer and this is $\Pi(\mathbf{k} \mid J^{\mathbf{k}}) = TS^*(\mathbf{k}) - \sum_i \left[TS^*(\mathbf{k}) - \tilde{TS}_{-i}(\mathbf{k} \mid J_i^{\mathbf{k}})\right]$. What differs from the most competitive equilibria is the term $\tilde{TS}^J_{-i}(\mathbf{k} \mid J_i^{\mathbf{k}})$, which depends on the vector of all investments. Because only a set of suppliers coordinate their out of equilibrium offers, there is only re-optimization of the amount traded by only those set of suppliers. Hence, the unchanged equilibrium quantities, $X^*_{-\{J_i^{\mathbf{k}},i\}}(\mathbf{k})$, depend on

[19] This is also the case because we have considered a bidding game where the suppliers submit the contracts to the buyer which gives all the bargaining power to the suppliers.

the overall investment profile $\mathbf{k}$, and when deriving the equilibrium investment profile, there will be an extra effect coming from this unchanged efficient equilibrium, which stays absent in the most aggressive equilibrium due to the envelope condition.

Therefore, by calculating the first order condition with respect to k_j, and by applying the envelope theorem we obtain

$$\begin{aligned}\frac{\partial\Pi(\mathbf{k}\mid J^{\mathbf{k}})}{\partial k_j} &= -C_k'\left(x_j^*(\mathbf{k})\mid k_j\right)+C_k'\left(x_j^*(\mathbf{k})\mid k_j\right)-\sum_{i\in N\setminus\{j\}}\left[-C_k'\left(x_i^*(\mathbf{k})\mid k_i\right)+C_k'\left(\tilde{x}_i(\mathbf{k}\mid J^{\mathbf{k}})\mid k_i\right)\right]\\
&\quad+\sum_{i\in N}\sum_{m\neq J^{\mathbf{k}},l}\left(U_x'\left(X^*_{-\{J^{\mathbf{k}},l\}}+\sum_{j\in J^{\mathbf{k}}}\tilde{x}_j(\mathbf{k}\mid J^{\mathbf{k}})+\tilde{x}_l(\mathbf{k}\mid J^{\mathbf{k}})\right)-C_x'(x_j^*\mid k_j)\right)\times\frac{dx_m^*}{dk_j}\\
&\quad-\phi_K'(K)\times\frac{\partial K}{\partial k_j}=0\\
\implies \phi_K'(K) &= -\sum_{i\in N\setminus\{j\}}\left[\int_{x_i^*(\mathbf{k})}^{\tilde{x}_i(\mathbf{k}|J^{\mathbf{k}})}C_{xk}''(\tau)d\tau\right]\\
&\quad+\sum_i\sum_{m\neq J^{\mathbf{k}},l}\left[\int_{X^*}^{X^*_{-\{J^{\mathbf{k}},l\}}+\sum_{j\in J^{\mathbf{k}}}\tilde{x}_j(\mathbf{k}|J^{\mathbf{k}})+\tilde{x}_l(\mathbf{k}|J^{\mathbf{k}})}U_{xx}''(\tau)d\tau\right]\times\frac{dx_m^*}{dk_j}\equiv\aleph(J^{\mathbf{k}})\ \ \forall j\in J^{\mathbf{k}}.\end{aligned}$$

Again, the first line of of the equation is the direct benefit from investment, and it is fully appropriated by the supplier. The other extra parts are the once referring to an increase of the competitive pressure. To determine whether the investment is positive, we need to study the sign of $\aleph^J(J^{\mathbf{k}})$. The first sum is positive and the second negative for the same argument as provided in lemma 4. Later, we provide a formal analysis establishing the conditions where the investment is positive, and this would depend on the degree of the "allocative sensitivity" i.e. dx_m^*/dk_j. If this is too big, then the second part of $\aleph(J^{\mathbf{k}})$ dominates and the buyers sets a zero level of investment. We proceed by calculating the first order condition with respect to the supplier that restricts the payoff of the most efficient suppliers $J^{\mathbf{k}}$ who, in lemma 4, we have denoted by l. Hence, by applying the same reasoning as before, we obtain

$$\phi_K'(K)=-\int_{x_l^*(\mathbf{k})}^{\tilde{x}_l(\mathbf{k})}C_{xk}''(\tau)d\tau+\sum_i\sum_{m\neq J^{\mathbf{k}},l}\left[\int_{X^*}^{X^*_{-\{J\setminus\{i\},l\}}+\sum_{j\in J\setminus\{i\}}\tilde{x}_j(\mathbf{k}|J^{\mathbf{k}})+\tilde{x}_l(\mathbf{k}|J^{\mathbf{k}})}U_{xx}''(\tau)d\tau\right]\times\frac{dx_m^*}{dk_j}\equiv\eta(J^{\mathbf{k}}),\tag{B.2}$$

where the quantities in the previous expressions are given by

$$\begin{aligned}\{\tilde{x}_j\}_{j\in J^{\mathbf{k}}}&=\arg\max\left[U\left(X^*_{-\{J^{\mathbf{k}},l\}}+\tilde{x}_l(\mathbf{k}\mid J^{\mathbf{k}})+\sum_{j\in J_i^{\mathbf{k}}}x_j(\mathbf{k}\mid J^{\mathbf{k}})\right)-\sum_{j\in J^{\mathbf{k}}}C(x_j\mid k_j)\right],\\
\tilde{x}_l&=\arg\max\left[U\left(X^*_{-\{J_i^{\mathbf{k}},l\}}+\sum_{j\in J^{\mathbf{k}}}\tilde{x}_j(\mathbf{k}\mid J^{\mathbf{k}})+x_l(\mathbf{k}\mid J^{\mathbf{k}})\right)-C(x_l\mid k_l)\right].\end{aligned}$$

We proceed to study under which conditions the buyer decides to set a positive level of investment, i.e. $\aleph(J^{\mathbf{k}}) > 0$.

$$\left| \sum_{m \neq J^{\mathbf{k}}, l} \frac{dx_m^*}{dk_j} \right| < \frac{-\sum_{i \in N \setminus \{j\}} \left[\int_{x_i^*(\mathbf{k})}^{\tilde{x}_i(\mathbf{k}|J^{\mathbf{k}})} C''_{xk}(\tau) d\tau \right]}{\sum_{i \in N} \left[\int_{X^*}^{X^*_{-\{J^{\mathbf{k}},l\}} + \sum_{j \in J} \tilde{x}_j(\mathbf{k}|J^{\mathbf{k}}) + \tilde{x}_l(\mathbf{k}|J^{\mathbf{k}})} U''_{xx}(\tau) d\tau \right]} = \lambda(J^{\mathbf{k}}).$$

In order to prove that the previous exist we introduce the following result, whose derivation comes directly from Chiesa and Denicolò (2009). The function $V\left(X^*_{-\{J^{\mathbf{k}},l\}}\right)$ is well defined due to Inada conditions. By the envelop theorem we have $V'\left(X^*_{-\{J^{\mathbf{k}},l\}}\right) > 0$ and $V''\left(X^*_{-\{J^{\mathbf{k}},l\}}\right) < 0$, which implies that the function is strictly increasing and strictly concave. By the implicit function theorem, we find that

$$\frac{\partial \tilde{x}_j^J\left(X^*_{-\{J^{\mathbf{k}},l\}}\right)}{\partial X^*_{-\{J^{\mathbf{k}},l\}}} = \frac{U''(\cdot)}{C''(\cdot) - U''(\cdot)} < 0.$$

And the reduction is of lower magnitude because

$$\frac{\partial \tilde{x}_j^J\left(X^*_{-\{J^{\mathbf{k}},l\}}\right)}{\partial X^*_{-\{J^{\mathbf{k}},l\}}} = \frac{U''(\cdot)}{C''(\cdot) - U''(\cdot)} < -1 \to 0 < -C''(\cdot).$$

We proceed to show existence.

$$\aleph(J^{\mathbf{k}}) = -\sum_{i \in N \setminus \{j\}} [C'_k(\tilde{x}_i \mid k_i) - C'_k(x_i^* \mid k_i)] + \sum_{i \in N} \sum_{m \neq J^{\mathbf{k}}, l} \left[U'_x \left(X^*_{-\{J^{\mathbf{k}},l\}} + \sum_{j \in J^{\mathbf{k}}} \tilde{x}_j + \tilde{x}_l) - U'_x(X^*) \right) \right] \times \frac{dx_m^*}{dk_j}. \tag{B.3}$$

A Taylor approximation for the utility and production costs are:

$$C'_k(\tilde{x}_i \mid k_i) \approx C'_k(x_i^* \mid k_i) + \sum_{t=1}^{n} \frac{1}{t!} \frac{\partial^t}{\partial x^t \partial k} C\big(\tilde{x}_i \mid k_i\big) \times (\tilde{x}_i - x_i^*)^k,$$

$$U'_x\left(X^*_{-\{J^{\mathbf{k}},i\}} + \sum_{j \in J^{\mathbf{k}}} \tilde{x}_j + \tilde{x}_l\right) \approx U'_x(X^*) + \sum_{t=1}^{n} \frac{1}{t!} \frac{\partial^t}{\partial x^t} U\big(X^*_{-\{J^{\mathbf{k}},i\}} + \sum_{j \in J^{\mathbf{k}}} \tilde{x}_j + \tilde{x}_l\big) \times \left(\left(X^*_{-\{J^{\mathbf{k}},i\}} + \sum_{j \in J^{\mathbf{k}}} \tilde{x}_j + \tilde{x}_l \right) - X^* \right)^k,$$

and by taking the first order approximation, we have:

$$C'_k(\tilde{x}_i \mid k_i) - C'_k(x_i^* \mid k_i) \approx C''_{xk}\big(\tilde{x}_i \mid k_i\big) \times (\tilde{x}_i - x_i^*),$$

$$U'_x\left(X^*_{-\{J^{\mathbf{k}},i\}} + \sum_{j \in J^{\mathbf{k}}} \tilde{x}_j + \tilde{x}_l\right) - U'_x(X^*) \approx U''_{xx}\big(X^*_{-\{J^{\mathbf{k}},i\}} + \sum_{j \in J^{\mathbf{k}}} \tilde{x}_j + \tilde{x}_l\big) \times \left((X^*_{-\{J^{\mathbf{k}},i\}} + \sum_{j \in J^{\mathbf{k}}} \tilde{x}_j + \tilde{x}_l) - X^* \right).$$

If we introduce this to expression (B.3) we obtain:

$$\begin{aligned}\aleph(J^{\mathbf{k}}) \approx &- \sum_{i\in N\setminus\{j\}} \left[C'_{xk}\big(\tilde{x}_i \mid k_i\big) \times (\tilde{x}_i - x_i^*)\right] \\ &+ \sum_{i\in N} \sum_{m\neq J^{\mathbf{k}},i} \left[U''_{xx}\big(X^*_{-\{J^{\mathbf{k}},i\}} + \sum_{j\in J^{\mathbf{k}}} \tilde{x}_j + \tilde{x}_l\big) \times \left(\sum_{j\in J^{\mathbf{k}}} \tilde{x}_j - \sum_{j\in J^{\mathbf{k}}} x_j^* - x_i^*\right)\right] \times \frac{dx_m^*}{dk_j}.\end{aligned} \tag{B.4}$$

We also know that the efficient allocation for the group of suppliers in $J^{\mathbf{k}}$ most efficient supplier dictates that $U'_x\left(X^*_{-\{J^{\mathbf{k}},i\}} + \sum_{j\in J^{\mathbf{k}}} \tilde{x}_j + \tilde{x}_l\right) = C'_x(\tilde{x}_j \mid k_j)$, and by differentiating by k_j we get:

$$\begin{aligned}&U''_{xx}(X^*_{-\{J^{\mathbf{k}},i\}} + \sum_{j\in J^{\mathbf{k}}} \tilde{x}_j + \tilde{x}_l) \times \left[\sum_{m\neq J^{\mathbf{k}},i} \frac{\partial x_m^*}{\partial k_j} + \frac{\partial \tilde{x}_j}{\partial k_j}\right] = C''_{xk}(\tilde{x}_i) + C''_{xx}(\tilde{x}_i) \times \frac{\partial \tilde{x}_i}{\partial k_j} \\ &\to\ U''_{xx}(X^*_{-\{J^{\mathbf{k}},i\}} + \sum_{j\in J^{\mathbf{k}}} \tilde{x}_j + \tilde{x}_l) = \frac{C''_{xk}(\tilde{x}_i) + C''_{xx}(\tilde{x}_i) \times \frac{\partial \tilde{x}_i}{\partial k_j}}{\left[\sum_{m\neq J^{\mathbf{k}},i} \frac{\partial x_m^*}{\partial k_j} + \frac{\partial \hat{x}_j}{\partial k_j}\right]}.\end{aligned} \tag{B.5}$$

Finally, by introducing this to expression (B.3) we obtain

$$\begin{aligned}\aleph(J^{\mathbf{k}}) \approx &- \sum_{i\in N\setminus\{j\}} \left[C''_{xk}\big(\tilde{x}_i \mid k_i\big) \times (\tilde{x}_i - x_i^*)\right] \\ &+ \sum_{i} \left[\left(C''_{xk}(\tilde{x}_i) + C''_{xx}(\tilde{x}_i) \times \frac{\partial \tilde{x}_i}{\partial k_j}\right) \times \left(\sum_{j\in J^{\mathbf{k}}} \tilde{x}_j - \sum_{j\in J^{\mathbf{k}}} x_j^* - x_i^*\right) \times \left(\frac{\sum_{m\neq J^{\mathbf{k}},i} \frac{\partial x_m^*}{\partial k_j}}{\sum_{m\neq J^{\mathbf{k}},i} \frac{\partial x_m^*}{\partial k_j} + \frac{\partial \hat{x}_j}{\partial k_j}}\right)\right],\end{aligned} \tag{B.6}$$

and we see that the first part is positive and the second is negative. However, the magnitude of the second part depends crucially on the term $\left(\sum_{m\neq J^{\mathbf{k}},i} \frac{\partial x_m^*}{\partial k_j} / \sum_{m\neq J^{\mathbf{k}},i} \frac{\partial x_m^*}{\partial k_j} + \frac{\partial \hat{x}_j}{\partial k_j}\right)$ and this term has little effect, if the term $\left|\sum_{m\neq J^{\mathbf{k}},i} \frac{\partial x_m^*}{\partial k_j}\right|$ is not too large. Moreover, it is also the case that the maximizer $\tilde{x}_j$ is negatively affected by a change of $X^*_{-\{J^{\mathbf{k}},i\}}$, and we have previously seen that this effect is of lower magnitude. In words, if the investment that the buyer undertakes to supplier $j \in J^{\mathbf{k}}$ does not have much effect on the efficient allocation for the non investing suppliers, the buyer decides to set a positive level of investment and the reasoning is the one that we have exposed in the main text. By using a similar procedure we obtain the conditions for the investment of the second most efficient supplier to be positive.

Proof of lemma 5: We show that the higher the number of suppliers, the lower is the individual investment undertaken with any supplier. We start with the case when competition is the most aggressive and we then proceed to analyze the case when suppliers tacitly collude to reduce the level of competition, and without loss of generality, we will pay attention to the situation where competition is the least severe.

Define $D^{HC}(N) = \zeta(\Delta N) - \zeta(N)$ the difference in the left hand side whenever competition is severe

$$D(N) = \zeta(\Delta N) - \zeta(N) = -\int_{N\times x_j^*(\Delta N)}^{N\times\tilde{x}_j(N|\bar{J}^{\mathbf{k}})} C''_{xk}(\tau)d\tau + \int_{(N-1)\times x_j^*(N)}^{(N-1)\times\tilde{x}_j(N-1|\bar{J}^{\mathbf{k}})} C''_{xk}(\tau)d\tau$$
$$= \int_{N\times\tilde{x}_j(N|\bar{J}^{\mathbf{k}})+(N-1)\times x_j^*(N)}^{N\times x_j^*(\Delta N)+(N-1)\times\tilde{x}(N-1|\bar{J}^{\mathbf{k}})} C''_{xk}(\tau)d\tau < 0,$$

and it is negative due to assumption $C'''_{xk}(\cdot) < 0$ and by the concavity of the problem we know that the upper part of the integral is bigger than the lower part that is $(N-1)\times\left(\tilde{x}_j(N-1 \mid \bar{J}^{\mathbf{k}}) - x_j^*(N)\right) > N\times\left(\tilde{x}_j(N \mid \bar{J}^{\mathbf{k}}) - x_j^*(\Delta N)\right)$.

We proceed to show that the higher the number of supplier the lower is the investment undertaken to each supplier $j \in J^{\mathbf{k}}$. Without loss of generality, we consider the case of the least competitive equilibrium and we make use of the following difference $D^{LC}(N) = \aleph^1(\Delta N) - \aleph^1(N) < 0$.

$$\aleph(\Delta N \mid \underline{J}) - \aleph(N \mid \underline{J}) = \int_{(N-1)\times x_j^*(N)}^{(N-1)\times\tilde{x}_j(N-1|\underline{J})} C''_{xk}(\tau)d\tau - \int_{N\times x_j^*(\mathbf{k}_{+i},\Delta N)}^{N\times\tilde{x}_j(N|\underline{J})} C''_{xk}(\tau)d\tau$$
$$+\sum_{i\neq\underline{J}}\sum_{m\neq\underline{J},i}\left[\int_{X^*}^{X^*_{-\{\underline{J},i\}}+\tilde{x}_j(\underline{J})} U''_{xx}(\tau)d\tau\right]\times\frac{dx_m^*(N)}{dk_j}$$
$$-\sum_{i+1\neq\underline{J}}\sum_{m\neq\underline{J},i}\left[\int_{X^*}^{X^*_{-\{\underline{J},i\}}+\tilde{x}_j(\underline{J})} U''_{xx}(\tau)d\tau\right]\times\frac{dx_m^*(\Delta N)}{dk_j}$$
$$= \int_{N\times\tilde{x}_j(N|\underline{J})+(N-1)\times x_j^*(\mathbf{k},N)}^{N\times x_j^*(\mathbf{k}_{+i},\Delta N)+(N-1)\times\tilde{x}_j(N-1|\underline{J})} C''_{xk}(\tau)d\tau + \kappa(N) < 0,$$

where

$$\kappa(N) \equiv \sum_{i\neq\underline{J}}\sum_{m\neq i,\underline{J}}\left[\int_{X^*}^{X^*_{-\{\underline{J},i\}}+\tilde{x}_j(\underline{J})} U''_{xx}(\tau)d\tau\right]\times\frac{dx_m^*(N)}{dk_j} - \sum_{i+1\neq\underline{J}}\sum_{m\neq\underline{J},i}\left[\int_{X^*}^{X^*_{-\{\underline{J},i\}}+\tilde{x}_j(\underline{J})} U''_{xx}(\tau)d\tau\right]\times\frac{dx_m^*(\Delta N)}{dk_j},$$

and it is negative due to assumption $C''_{xk}(\cdot) < 0$ and by the concavity of the problem we know that the upper part of the integral is bigger than the lower part which is equivalent to $(N-1)\times\left(\tilde{x}_j(N-1 \mid \underline{J}) - x_j^*(N)\right) > N\times\left(\tilde{x}_j(N \mid \underline{J}) - x_j^*(\mathbf{k}_{+i},\Delta N)\right)$. It is also the case that $\kappa(N) < 0$ as it is easy to show that $|dx^*(N)/dk_j| > |dx^*(\Delta N)/dk_j|$ and this is of first order effect.

Proof of proposition 4: For the case of efficiency, it is easy to see that as the objective is to maximize welfare, the higher the total amount transacted the higher will be the aggregate level of investment. Then, we see that the aggregate level of investment will be increasing with the number of active suppliers. Nevertheless, as the cost of investment is convex the level of aggregate investment is concave with the number of active suppliers. Regarding the equilibrium aggregate investment, the result when replacement is undertaken by only one supplier is immediate. From lemma 4, we know that the buyer only sets a positive level of investment to two active suppliers and from lemma 5 we know that the per-supplier level of investment decreases with the amount of active suppliers. Hence, the aggregate level of investment decreases with the number of active suppliers.

For the most competitive equilibrium the analysis is less direct and we see that the aggregate level of investment will be decreasing if the following holds

$$N \times \zeta(\Delta N) < (N-1) \times \zeta(N) \to \zeta(\Delta N) - \zeta(N) < -\frac{\zeta(N)}{N},$$

and both the left hand side and the right hand side are negative. The above condition is equivalent to $\frac{\zeta(N)}{N} < \zeta(N) - \zeta(\Delta N)$ and we know that this is always true by applying the Jensen's inequality to the concave function ζ.

In the limit, we have that the competition arising between suppliers is large and the effect that investment have on increasing further the level of ex-post competition is small. As a result, cooperative investment decreases and it tends to zero with an arbitrary large number of active suppliers. Technically, the term dictating the level of individual investment converges to 0, i.e. $\lim_{N\to+\infty} \zeta(N) = \lim_{N\to+\infty} \aleph(N, J^{\mathbf{k}}) = \lim_{N\to+\infty} \eta(N, J^{\mathbf{k}}) = 0$ for all $J^{\mathbf{k}} \subset N$.

Proof of proposition 5: For a given number of active suppliers N and a set of suppliers J. The expected payoff for each supplier is

$$\mathbb{E}\left[\pi_i\left(\mathbf{k} \mid N, J^{\mathbf{k}}\right)\right] = TS^*(\mathbf{k}) - \frac{1}{N}\left[\left|J^{\mathbf{k}}\right| \times \tilde{TS}_{-j}(\mathbf{k} \mid J^{\mathbf{k}}) + \tilde{TS}_{-l}(\mathbf{k} \mid J^{\mathbf{k}}) + \left(N - 1 - \left|J^{\mathbf{k}}\right|\right) \times \tilde{TS}_{-m}(\mathbf{k} \mid J^{\mathbf{k}})\right].$$

We start by considering how the expected payoff changes with an increase on the number of active suppliers and this is approximately equal to:

$$\begin{aligned}
&= \frac{-N^2 \times TS^*(\mathbf{k}) + \left|J^{\mathbf{k}}\right| \times \tilde{TS}_{-j}(\mathbf{k}) + \tilde{TS}_{-l}(\mathbf{k}) + \left(\left|J^{\mathbf{k}}\right| + 1\right) \times \tilde{TS}^{J}_{-m}(\mathbf{k}^J)}{N^2} \\
&+ \frac{\partial TS^*(\mathbf{k})}{\partial N} - \frac{\left|J^{\mathbf{k}}\right|}{N} \times \frac{\partial \tilde{TS}_{-j}(\mathbf{k})}{\partial N} - \frac{1}{N} \times \frac{\partial \tilde{TS}_{-l}(\mathbf{k})}{\partial N} - \frac{N - \left(\left|J^{\mathbf{k}}\right| + 1\right)}{N} \times \frac{\partial \tilde{TS}_{-m}(\mathbf{k})}{\partial N} \\
&+ \left(\frac{\partial TS^*(\mathbf{k})}{\partial \mathbf{k}} - \frac{\left|J^{\mathbf{k}}\right|}{N} \times \frac{\partial \tilde{TS}_{-j}(\mathbf{k})}{\partial \mathbf{k}} - \frac{1}{N} \times \frac{\partial \tilde{TS}^{J}_{-l}(\mathbf{k}^J)}{\partial \mathbf{k}^J} - \frac{N - \left(\left|J^{\mathbf{k}}\right| + 1\right)}{N} \times \frac{\partial \tilde{TS}_{-m}(\mathbf{k})}{\partial \mathbf{k}}\right) \times \frac{\nabla \mathbf{k}}{\partial N},
\end{aligned}$$

where the first line represents the direct effect, and the next two are the indirect effects. It is easy to see that the whole effect is negative i.e. $\partial\left(\mathbb{E}\left[\pi_i\left(\mathbf{k} \mid N, J^{\mathbf{k}}\right)\right]\right)/\partial N < 0$. The direct effect is negative, since $TS^*(\mathbf{k}) > \tilde{TS}_{-h}(\mathbf{k} \mid J^{\mathbf{k}})$ for any $h \in N$, and this tells us that the likelihood of being the supplier to which the buyer decides to invest is smaller. The second line, in the previous equation is also negative as the effect that an extra supplier has in the gains from trade deceases with N due to its concavity. Finally, we also obtain a negative impact from the last line. By lemma 6, in the first part of the appendix, we know that $\partial TS^*(\mathbf{k}^J)/\partial \mathbf{k} > \partial \tilde{TS}_{-h}(\mathbf{k} \mid J^{\mathbf{k}})/\partial \mathbf{k}$ for any $h \in N$ but this effects is negative by the fact that $\nabla \mathbf{k}^J/\partial N < 0$ by lemma 5.

By fixing the number of active suppliers, we consider how the expected payoff changes with an increase in the number of suppliers belonging to $J^{\mathbf{k}}$. This effect is approximately given by:

$$\begin{aligned}
&\frac{\partial \left(\mathbb{E}\left[\pi_i \left(\mathbf{k} \mid N, J^{\mathbf{k}}\right)\right]\right)}{\partial J^{\mathbf{k}}} = \\
&\underbrace{-\frac{1}{N}\left(\tilde{TS}_{-j}(\mathbf{k}) - \tilde{TS}_{-m}(\mathbf{k})\right)}_{(+)} \\
&+ \underbrace{-\frac{1}{N} \times \left(\left|J^{\mathbf{k}}\right| \times \frac{\partial \tilde{TS}_{-j}(\mathbf{k})}{\partial J^{\mathbf{k}}} + \frac{\partial \tilde{TS}_{-l}(\mathbf{k})}{\partial J^{\mathbf{k}}} + \left(N - 1 - \left|J^{\mathbf{k}}\right|\right) \times \frac{\partial \tilde{TS}_{-m}(\mathbf{k})}{\partial J^{\mathbf{k}}}\right)}_{(-)} \\
&+ \underbrace{\left[\frac{\partial TS^*(\mathbf{k})}{\partial \mathbf{k}} - \frac{1}{N} \times \left(\left|J^{\mathbf{k}}\right| \times \frac{\partial \tilde{TS}_{-j}(\mathbf{k})}{\partial \mathbf{k}} + \frac{\partial \tilde{TS}_{-l}(\mathbf{k})}{\partial \mathbf{k}} + \left(N - 1 - \left|J^{\mathbf{k}}\right|\right) \times \frac{\partial \tilde{TS}_{-m}(\mathbf{k})}{\partial \mathbf{k}}\right)\right] \times \frac{\nabla \mathbf{k}}{\partial J^{\mathbf{k}}}}_{(+)},
\end{aligned}$$

where the first and the second line represents the direct effect and the third line is for the indirect effect. While the first line of the direct effect is positive due to $\tilde{TS}_{-j}(\mathbf{k} \mid J^{\mathbf{k}}) < \tilde{TS}_{-m}(\mathbf{k}^J \mid J^{\mathbf{k}})$, we have that the second line is negative due to a lower partition of the trading surplus due to larger competition. The indirect effect is also negative as $\partial TS^*(\mathbf{k})/\partial \mathbf{k} > \partial \tilde{TS}_{-h}(\mathbf{k})/\partial \mathbf{k}$ for any $h \in N$ due to lemma 6 and $\nabla \mathbf{k}/\partial J^{\mathbf{k}} > 0$. The whole effect is in general positive and we have that $\partial \left(\mathbb{E}\left[\pi_i \left(\mathbf{k} \mid N, J^{\mathbf{k}}\right)\right]\right)/\partial J^{\mathbf{k}} > 0$, and the payoffs per supplier increase whenever competition is more aggressive.

Proof of corollary 2: This is straightforward to proof. We make explicit use of lemma 5. Whenever the "allocative sensitivity" is to large such that the equilibrium investment is zero, i.e. $\mathbf{k}^J = 0$ then we know that $\tilde{TS}_{-j}(\mathbf{k}^J \mid J^{\mathbf{k}}) = \tilde{TS}_{-m}(\mathbf{k}^J \mid J^{\mathbf{k}})$ and $\nabla \mathbf{k}^J/\partial J^{\mathbf{k}} > 0$. Then we obtain

$$\frac{\partial \left(\mathbb{E}\left[\pi_i \left(\mathbf{k} \mid N, J^{\mathbf{k}}\right)\right]\right)}{\partial J^{\mathbf{k}}} = -\frac{1}{N} \times \left(\left|J^{\mathbf{k}}\right| \times \frac{\partial \tilde{TS}_{-j}(\mathbf{k})}{\partial J^{\mathbf{k}}} + \frac{\partial \tilde{TS}_{-l}(\mathbf{k})}{\partial J^{\mathbf{k}}} + \left(N - 1 - \left|J^{\mathbf{k}}\right|\right) \times \frac{\partial \tilde{TS}_{-m}(\mathbf{k})}{\partial J^{\mathbf{k}}}\right) < 0.$$

Whenever the "allocative sensitivity" is not to large such that the equilibrium investment is positive then the result follows directly from lemma 5.