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Divide and Learn: Early Contracting with Endogenous Threat*

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Abstract

An economic agent may engage into an early negotiation with the sole purpose of gathering information to improve his bargaining position. We analyze this issue in the context of a buyer/seller relationship, where the seller has private information on the future gains from trade, and the buyer can bypass at some preliminary stage. While both players can wait until uncertainty is resolved and trade efficiently ex-post, we show that the buyer may be better off by proposing an early contract. This early contract uses the sellers' private information to divide types in a way that makes costly bypass a credible threat. While some types of seller accept the contract because they gain more than in the status quo situation, other types only accept for fear that rejection would reveal too much information. We derive the whole set of equilibrium payoffs of this game, and study extensions to fit various economic situations.

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1 Introduction

In most contracting situations, the parties can choose to set the deal either before the uncertainty is realized - ex-ante or early contracting - or after - ex-post of late contracting. Both for efficiency and insurance reasons, many papers starting from Borch (1962), Arrow (1963) or Pauly (1968) have advocated the use of early contracting.[1] With this paper, we show that the information gathering aspect of early contracting may be a significant strategic dimension and help the uninformed party in reaping most of the surplus.

As an illustration, consider the situation where a trade between two parties B and S can generate a surplus of 1 with probability θ and 0 otherwise, with θ uniformly distributed on $[0, 1]$. If B and S wait for the observation of the surplus before negotiating trade, we assume that whole surplus will be gained by S. Therefore, in the absence of early contract, B obtains a zero payoff. But suppose that, at a cost $k = 2/3$ paid before the trade surplus is known, B can bypass S and reap the whole surplus.

If neither S nor B know the value of θ, they do not sign an early contract and B obtains a zero payoff. Indeed, S can ensure a payoff of $E(\theta) = 1/2$ by refusing to contract knowing that B has no reason to bypass as the cost k is greater than the total expected surplus. If instead S privately knows the value θ, our analysis implies that B can secure an expected payoff $v_B \approx 1/6$. Indeed, suppose that B proposes a early payment $T \gtrsim 1/3$ for the right to obtain the full surplus ex-post. We show that the unique equilibrium is for S to accept for all values of θ. Intuitively, it is only when θ is greater than T that S may have an incentive to refuse the contract. Refusal cannot occur in equilibrium because, with the belief that $\theta > T$ and no contract signed, $B's$ expected gain from bypassing is $E(\theta|\theta > T) > 2/3 = k$, implying that B would bypass. With T close to $1/3$, B can therefore secure a payoff close to $E(\theta) - 1/3 = 1/6$.

In the above example, the existence of private information hurts the informed party S by enabling the uninformed party B to appropriate a share of the surplus without effectively paying the cost of bypass. Indeed, the strategic design of the contract exploiting implicit revelation of information allows B to raise the credibility of the threat of bypass. To increase this credibility, B designs the contract so as to partition the information set the informed agents S in two subsets: the pieces of information leading S to accept irrespective of the credibility of the threat and the others. Then any rejection would reveal that the agent's information is in the latter

[1] See Laffont-Martimort (2002) for a modern treatment of the ex-ante complete contract approach.

sub-set. The sorting of the informed agent S is thus done so that the information conveyed by rejection makes the threat of bypass credible. Therefore, S has no other choice than to accept the offer at any level of private information. The informed agent is caught into an information trap where the acceptance by some types forces acceptance by all types. We refer to this strategy as *divide and learn*.

Situations as the one described above are quite common. Examples include:

- A buyer considers a make-or-buy decision faced to a better informed supplier. Bypass consists in investing to produce in-house. In this case, the information of the supplier could be about the match value of its product with the buyer's needs, but it could also be about the likelihood that entry of competing sellers improve the buyer's ex-post bargaining (hence about the future market structure).[2]

- A subcontractor produces a good customized to the need of a buyer. The buyer is better informed about future demand. Bypass consists in making the product suitable for other potential clients. Even if it is inefficient, it may be carried on if it allows the producer to obtain better terms ex-post when negotiating with the buyer.

- An innovator must choose between contracting with an existing producer better informed about market demand and setting his own company. In the latter case, there would be competition in differentiated products ex-post, which constitutes the threat point in the former case. The credibility of this threat depends on the cost of setting the company, the type of competition and the belief about market demand.

More generally our analysis applies when a party to future trade may undertake some action that raises his future bargaining position but lacks information on the opportunity of doing so. Depending on the context, we may interpret this action as a protection against a risk of hold-up, or as a threat used to expropriate the other party. Whenever the other party holds information relevant for the protection/expropriation

[2] An initial motivation for the paper was the situation faced by a local government relying on a utility for the provision of a local public good. For instance municipalities that wish to upgrade the quality of their telecommunication infrastructure may contract with the historical operator or build their own infrastructure. The latter option is inefficient as it implies duplication. In these situations, superior information by telecommunication operators may backfire into low profit under the threat of duplication.

decision, early contracting will help the uninformed party to raise his share of the surplus but to some limited extent. The limits of the divide and learn strategy is the cost of inducing types for which the threat is not credible to accept the offer, which generates a lower bound on the payoff the strategy can generate. To understand when the strategy is effective we focus on a very simple set-up.

We consider the relationship between a (zero cost) seller and a potential buyer, who will have a unit demand with some probability θ that is privately known by the seller. To highlight the rent shifting motive for early contracting, we focus in the main part of the paper on a situation where efficiency considerations are absent. For this purpose we assume that, conditional on the demand being positive, the reservation utility of the buyer is fixed at a known value. In this case, gains from trade are always realized ex-post and the only question is the division of the surplus between the buyer and the seller.

The buyer can wait until the demand is realized in which case a positive price will prevail, or he can invest in production facility and produce the good himself. Bypass is inefficient due to duplication of infrastructures. The buyer may also propose an advance purchase contract to the seller, under the threat of bypass. We assume that based on prior beliefs bypass is not a credible threat, but it would be if the buyer knew that θ is high. As discussed above, when a contract is proposed but refused by the seller, the buyer decides to bypass if his beliefs on demand are high enough. This interaction between the contract and post-rejection beliefs may be the source of a multiplicity of equilibria. We show that for each contract offered by the buyer, there will be at most two (pure strategy) continuation equilibria, one where rejection triggers bypass and one where it doesn't.

The ability of the buyer to reduce the rent of the seller depends on how this multiplicity is resolved. Before proceeding to the full characterization of the equilibrium set, we characterize extreme equilibrium payoffs, that obtain for particular selection rules among the multiple equilibria. In the *Maximal Scenario*, any rejection induces high enough beliefs to trigger bypass. Therefore, all the surplus in shifted to the buyer. In the *Minimal Scenario*, the belief induced by a rejection minimizes the probability of bypass, which also minimizes extraction of the surplus by the buyer. Under this scenario and absent ex-post inefficiencies, we show that the buyer and the seller either contract at the early stage for a fixed price or do not contract at all, depending on the credibility of the threat.

We then conclude that any allocation that yields payoffs between the above two scenario's payoffs can be supported in equilibrium and that an early contract is always offered.

In the second part of the paper, we propose several extensions to highlight various aspects of our model. First we introduce the possibility that the seller may not have any information. It illustrates the role of having heterogenous and private seller's information for the buyer to benefit from early contracting. Then we endogenize the seller's decision to enter the market initially by adding an ex-ante stage of investment. We derive conditions for such an investment to occur. At last, we extend the model to the case of random valuation instead of the binary value assumed above. Then early contracting entails efficiency gains due to the distortions associated with ex-post market power.

The idea that contract negotiation conveys information that may shape anticipated future rents and bargaining positions is reminiscent of the work on the dynamic ratchet effect (Weitzman (1980), Freixas, Guesnerie and Tirole (1985), Laffont and Tirole (1988)). In this literature, the information revealed during the course of the contractual relationship may be exploited latter on. The issue is then to design the contract so as to reduce inefficiencies arising due to the lack of commitment.[3] In our model, the information is conveyed by the refusal decision, as opposed to during the relationship, so that the prevailing ratchet effect may or may not raise efficiency, as illustrated by our extensions. The uninformed principal designs the contract so as to gain commitment and obtains a higher payoff than when the other party is also uninformed.

The general insight that rejection of a contractual offer reveals information about the agent goes back at least to Coase (1972) classical analysis of a durable good monopoly (see Hart and Tirole (1988)) and can also be found in the litigation literature (see Bebchuk (1984)). In the above cases, not participating reveals information favorable to the agent (low valuation). Our set-up differs as the contract is designed such that the information revealed by not participating is unfavorable to the agent, which allows to raise the participation level.

Recently, Philippon and Skreta (2012), as well as Tirole (2012), study optimal design of government programs aimed at improving market performance under adverse selection by allowing traders to opt out of the market and to engage in a public program. The participation decision then reveals information about agents, thereby changing beliefs about the value of trading on the market. Their models differ from ours in several respects. First the participation decision reveals information to third parties, i.e. the market participants. Second they focus on contractual externalities

[3] Along this line, Calzolari and Pavan (2006) study, in a Principal-Agent setting, the optimal contractual disclosure rule when the information revealed during the contractual relationship can be transmitted to another future Principal of the agent.

between multiple agents and on the efficiency of trade on the market. [4]

The most closely related works are Cramton and Palfrey (1995) and Celik and Peters (2011), who analyze from a mechanism design perspective situations where failure to contract triggers further interactions between the parties. Cramton and Palfrey (1995) introduce the notion of ratifiability that selects among implementable allocations using forward induction arguments, along the line of Grossman and Perry (1986). We do not apply this selection criterion in the main analysis as we aim at characterizing the whole set of equilibria, and the scenarios we consider emerge naturally in our set-up. Nevertheless, we characterize in Section 3.4 the set of ratifiable contracts and discuss the consequences of using this notion to select among equilibria. In particular we show that the equilibrium contract of the minimal scenario would emerge if the buyer were free to choose any contract verifying the strong version of ratifiability. In a related model, Celik and Peters (2011) show that a Principal may relax the participation constraint of one agent by proposing a mechanism that is refused by another agent with positive probability. Such a mechanism allows to alter the beliefs of the first agent on the value of interacting with the second agent. As Celik and Peters (2011) prove in their analysis, this possibility does not arise in our model.

Our analysis casts some light on the value of information in principal-agent models, by exhibiting a situation where the principal may benefit from negotiating with an superiorly informed agent. Kessler (1998) also found that, in a mixed model of regulation with both moral hazard and adverse selection, an agent may benefit from the possibility of remaining ignorant. In this approach, this is due to a change in likelihood of low-cost projects while in our article, information influences the perceived outside options of both parties, hence the way surplus is shared. Sobel (1993) studied a moral hazard setting where, despite participation being less costly when the agent is not informed, it may still be worth for the principal to face an informed agent. Indeed, the principal prefers to face an informed agent as the latter will choose the adequate actions. In our paper, information has no direct effect on efficiency, but only indirectly through the change in bargaining power driven by the impact of information on the outside options.[5]

Note at last that in our paper, the outside option of the informed agent depends on the belief hold on his own type, in the tradition of the type-dependent reserva-

[4]Contractual externalities arise for instance in Segal (1999), as well in the literature on auctions (see Jehiel and Moldovanu (1996)) and the literature on two-sided markets (see Jullien (2011)). In those papers as in our, the value of outside opportunities is endogenous.

[5]Note, contrarily to the Crémer, Khalil and Rochet (1998) for example, we do not discuss the incentives to acquire earlier some information that will be revealed ex-post.

tion utility models analyzed by Jullien (2000). Following this tradition, Rasul and Sonderegger (2010) analyzed how the difference between ex-ante and ex-post outside options can help the principal. In our paper, the difference between ex-ante and ex-post outside options has mainly an impact through the investment choice of the principal, and not of the agent.

The paper is organized as follows. Section 2 presents the base model. Section 3 derives the equilibrium payoffs and discusses ratifiability in our set-up. Section 4 proposes some extensions while Section 5 concludes.

2 Basic model

A buyer (she) may purchase one unit of a good from a seller (he). Both agents are risk-neutral. The seller's production cost is normalized to zero. The buyer's utility derived from consumption is either 0 or $U > 0$, where U is referred to as the buyer's valuation. The probability of her having a strictly positive utility for the good, denoted θ, is private information the seller and is referred to as the seller's type. The buyer holds prior beliefs on θ represented by a (strictly positive) density $f(.)$ and a cumulative distribution $F(.)$ defined over $[\underline{\theta}, \overline{\theta}] \subseteq [0, 1]$. We assume that the hazard rate $F(\theta)/f(\theta)$ is strictly increasing in θ.

There are two periods, sometimes called interim and ex-post. The first period is devoted to the contracting process and buyer's investment, while production and consumption take place during the second period. Periods also differ in the information available to the buyer. More precisely, during the first period, the buyer's valuation is unknown and the buyer may approach the seller to offer a contract that stipulates a transfer in exchange of the good. If the negotiation fails, the buyer has two options: she can either invest a fixed cost k which allows her to produce the good latter at a zero marginal cost. Or, she can wait until the second period. We refer to the first possibility as bypass.

At the beginning of the second period, the buyer privately discovers her valuation for the good. She is then able to buy ex-post one unit of the good from the seller, at an exogenous price $\alpha \leq U$. This ex-post price may have several interpretations, depending on the context. It may be the monopoly price set ex-post, $\alpha = U$, or it may also correspond to a 'catalogue' price posted by the seller that applies for all buyers.[6] As another example, in a regulatory context, the price α can be interpreted

[6]For instance, microprocessor suppliers propose a catalogue price for each processor that applies to all manufacturers, but negotiate specific rebates with large manufacturers.

as a price-cap imposed on a seller of an essential input.[7]

The negotiation between the buyer and the seller is summarized as follows:

Period 1:

1. The value of θ is realized and observed by the seller only.
2. The buyer makes a contractual offer $\mathcal{C}$ to the seller. The nature of $\mathcal{C}$ is detailed later on.
3. The seller accepts or rejects that offer.
   (a) If the offer is accepted, no bypass occurs;
   (b) If the offer is rejected, either the buyer bypasses the seller or she waits until the second period.

Period 2:

1. The buyer privately learns her valuation for the good.
2. If the seller and the buyer have previously agreed on an offer, it is implemented; otherwise a transaction may occur ex-post if the buyer has a positive demand and has not bypassed the seller.

The equilibrium concept is Perfect Bayesian Equilibrium. Note that the buyer does not learn θ but only her realized valuation.

We impose without loss of generality that early contracts preclude bypass. The reason is that, since bypass is inefficient, a contract inducing some bypass would be dominated by a contract replacing bypass with an option to sell at zero price. We also assume that the buyer does not offer a contract that would yield an equilibrium outcome with the same payoff as the absence of contract.

At stage 2 of the first period, the buyer and the seller negotiate the future terms of trade. To capture this situation in a simple way, assume that the buyer's offer $\mathcal{C}$ consists in a menu of two-part tariffs $(T(.), p(.))$ where T is a fixed payment and p is an option to buy at p, with $0 \leq p \leq U$.[8] If the parties fail to agree on the terms

[7] In network industries, such as telecommunication, energy or rail, access regulation often constrains the price charged by the incumbent operator for accessing its local network infrastructure.

[8] This condition entails no loss of generality. Indeed, as the realization of the demand is privately observed, the contract cannot be made contingent on this final demand. In this case of a unit demand, any (non-stochastic) contract can be summarized by the transfer if the buyer does not consume and the transfer if she consumes, hence by a two-part tariff.

of trade, the buyer is free to either bypass or wait the realization of the demand and buy ex-post.

As a benchmark, assume complete information on θ at the beginning of the first period. The buyer and the seller then agree on trade with no bypass, as this maximizes the social surplus. The sharing of this surplus between the parties depends on whether bypass is a relevant threat for the buyer. Bypass is a credible alternative when it yields an expected gain larger than the gain associated with waiting and buying the good at the ex-post price, or:

$$\theta U - k \geq \theta(U - \alpha) \Leftrightarrow \theta \geq \frac{k}{\alpha} \equiv \theta^B. \tag{1}$$

Contracting under complete information about θ would then result in a tariff $(T, p) = (0, 0)$ when $\theta \geq \theta^B$ as bypass is a credible threat. There would be no early contract when $\theta < \theta^B$, as the buyer would need to compensate the seller for the ex-post profit $\theta\alpha$. In both cases, the outcome is efficient. The same conclusion would hold, replacing θ by its expected value $\theta^e \equiv \mathbb{E}\{\theta\}$, if none of the parties were informed about the true value of θ.

As another benchmark, if the buyer could commit to always bypass when the seller rejects his offer, then she would be able to impose a zero price. In our model, there is no commitment on the decision wether to bypass if negotiation fails. As in Cramton and Palfrey (1995), this decision is taken considering the utility obtained in the continuation game. Since the seller has private information, rejection of the contractual offer may convey some information so that the buyer's bypass decision depends on the revised beliefs after rejection. The contract may therefore be designed so as to generate beliefs in case of rejection that help the buyer to restore her credibility. The objective of this paper is to analyze what the buyer can reach with this strategy.

3 Contracting under the threat of bypass

The objective of the section is to derive the contracts the buyer can propose at equilibrium. As we show below, multiple equilibria can emerge depending on the beliefs held by the parties at the different stages of the game. The roadmap of this section is therefore the following. Focusing on pure-strategy equilibria, for any contract offered by the buyer, we analyze the continuation equilibria. Then, selecting two particular, but well-chosen, continuation scenarios, we derive the corresponding optimal contracts. Using these contracts, we derive the whole set of payoffs that can be sustained in equilibrium. At last we discuss the link with the concept of ratifiability introduced by Cramton and Palfrey (1995).

3.1 Beliefs and equilibrium selection rules

As a first step of the analysis, let us derive the behaviors that a contract offered by the buyer may induce.

Consider the stage 3 of the game where a contract has been proposed. The seller's decision to accept or refuse the offer depends on his belief on the buyer's reaction if he refuses: the seller obtains 0 if the buyer bypasses after rejection, while he may obtain $\pi^R(\theta) \equiv \theta\alpha$ if there is no bypass. The buyer herself is influenced by her anticipation on the set of accepting sellers since her decision to bypass depends on her beliefs on the type of the rejecting seller. This circularity implies that there may exist multiple equilibria.

In order to focus on genuine multiplicity, we assume from now:

Assumption 1 *(i) If rejection does not trigger bypass and the seller is indifferent between accepting and rejecting, then the offer is rejected. (ii) If the buyer is indifferent between bypassing or not, then the buyer bypasses.*

When $\pi = \pi^R$ on some range of types and rejection does not trigger bypass, the same equilibrium payoffs can be reached with different seller's acceptance sets. Part (i) of Assumption 1 allows to get rid of this innocuous multiplicity. Part (ii) avoids some technicalities and is without consequences because in our model, the buyer can always break indifference by proposing a slightly modified contract that restoreS credibility of bypass. Note moreover that there cannot exist an equilibrium such that a contract offered is rejected with positive probability and there is no bypass. To see that, suppose instead that such an equilibrium exists. The seller must obtain at least $\pi^R(\theta)$ because he can reject the offer, implying that the buyer cannot gain by proposing the contract and thus would prefer not to propose anything.[9]

For a given contract $\mathcal{C}$, let $\pi(\theta)$ be the equilibrium profit of a θ-type seller can obtain by accepting the contract $\mathcal{C}$. Then, define the "rejection set" $\mathcal{R}$ as the set of seller's types:

$$\mathcal{R} = \left\{\theta \mid \pi(\theta) \leq \pi^R(\theta)\right\}.$$

A seller with type $\theta \in \mathcal{R}$ rejects the offer if he anticipates no bypass and accepts it otherwise as long as the profit in the contract offer is non-negative, which is a

[9]Recall that at indifference, the buyer offers no contract.

necessary condition for participation. A seller with type $\theta \notin \mathcal{R}$ accepts the offer in any continuation equilibrium.

Following a contract offer $\mathcal{C}$ inducing non-negative profit, two continuation equilibria may arise at most, one with bypass and one without bypass. Formally (remember that θ^B is defined by Equation (1)):

Lemma 1 *Following a contract offer $\mathcal{C}$ with rejection set $\mathcal{R}$:*

i) If $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} \geq \theta^B$, there is a unique continuation equilibrium where $\mathcal{C}$ is accepted by all types of sellers.

ii) If $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} < \theta^B \leq \bar{\theta}$, there are two continuation equilibria:

- *either $\mathcal{C}$ is accepted by all types of sellers;*
- *or $\mathcal{C}$ is accepted by the seller with types $\theta \notin \mathcal{R}$ only and no bypass follows a rejection.*

iii) If $\bar{\theta} < \theta^B$, there is a unique continuation equilibrium where $\mathcal{C}$ is accepted by the seller with type $\theta \notin \mathcal{R}$ and no bypass follows a rejection.

Proof. See the Appendix. ■

When beliefs on the seller's type are either sufficiently high or sufficiently low, there is only one continuation equilibrium following a contract offer by the buyer. In the former case, the offer is accepted by all types of sellers as any rejection triggers bypass; in the latter case, bypass is not credible and the buyer gives the seller more than his reservation utility π^R in order to induce acceptance.

The most interesting case lies in the intermediate situation where, following a contract offer, two continuation equilibria exist. Multiplicity emerges due to the self-enforcing nature of the seller's beliefs on bypass. Beliefs that there will be no bypass induce a large set of types to reject the offer (the set $\mathcal{R}$), making the buyer pessimistic enough about the chance to trade that she abstains from bypassing. On the contrary, in the first continuation equilibrium, the contract offer is accepted by the seller irrespective of his type, which is consistent with out-of-equilibrium buyer's beliefs that θ is large if rejection were to occur.

The outcome of the whole game between the seller and the buyer depends on the type of continuation equilibrium that prevails. Let us then define two possible selection rules of particular interest.

Definition 1 *Following a contract offer $\mathcal{C}$ such that $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} < \theta^B \leq \bar{\theta}$, we define:*

- *the **maximal scenario** as the selection rule where any contract $\mathcal{C}$ is accepted;*
- *the **minimal scenario** as the selection rule where, if $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} < \theta^B$, then the contract is accepted by types $\theta \notin \mathcal{R}$ only and there is no bypass following rejection.*

These selection rules may arguably appear as specific as they apply for any contract $\mathcal{C}$, i.e. on the equilibrium path but also out of the equilibrium path. There are, however, instrumental to characterize the set of equilibrium payoffs, as we show later on.

Simple cases. As the contract is proposed by the buyer, it is natural to see how far she may use her first-mover advantage to shift the gains from trade to her benefit. In this respect, one may consider the possibility that the buyer offers the simple contract $\mathcal{C}_0 \equiv (T = 0, p = 0)$. In light of Lemma 1, $\mathcal{C}_0$ is accepted in two simple cases.

Assume first that $\mathbb{E}\{\theta\} \geq \theta^B$. The buyer offering $\mathcal{C}_0$ implies that $\mathcal{R} = \left[\underline{\theta}, \bar{\theta}\right]$. Given that $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} = \mathbb{E}\{\theta\} \geq \theta^B$, all types of sellers accept in the unique continuation equilibrium.

Contract $\mathcal{C}_0$ could also be accepted when $\mathbb{E}\{\theta\} < \theta^B \leq \bar{\theta}$. Suppose, indeed, that the maximal scenario prevails. The buyer then anticipates that all contract offers should be accepted and would interpret contract rejection as a signal that θ is high. Contract rejection would then induce bypass because $\bar{\theta} \geq \theta^B$. With the maximal scenario, $\mathcal{C}_0$ is thus accepted by all types of sellers.

The possibility of proposing an early contract may sometimes have no value for the buyer. This arises in particular when bypass is never a credible threat, i.e. when $\theta^B > \bar{\theta}$. There exists in this case no values of θ which triggers bypass. The buyer has then no incentives to offer a contract and trade occurs ex-post at the price α.

3.2 Equilibrium contracts under the minimal scenario

We now focus on the case $\mathbb{E}\{\theta\} < \theta^B \leq \bar{\theta}$ and assume that the minimal scenario is the continuation rule for offers $\mathcal{C}$ for which there is a multiplicity of continuation equilibria[10]. In this scenario, contract $\mathcal{C}_0$ will never be proposed, for it would be

[10]This corresponds to the intermediate case stated in Lemma 1.

refused by all types of sellers and no bypass would follow. The seller would thus have strictly positive rents at equilibrium, either by accepting the contract if it generates high rents or by trading ex-post at price α.

Under the restrictions implied by the minimal scenario, the buyer has two options: either she offers a contract which induces full participation; or she offers a contract which induces partial participation, where the seller accepts the contract only if he obtains more than his reservation utility, and rejection does not trigger bypass. In fact, the buyer should never offer in equilibrium a contract with partial participation, for the seller's payoff would be at least $\pi^R(\theta)$. With partial participation the buyer's payoff would then be at most $\mathbb{E}\{\theta\}U - \mathbb{E}\{\pi^R(\theta)\} = \mathbb{E}\{\theta\}(U-\alpha)$, which is the minimal payoff that she obtains without offering any contract. Thus, either the buyer makes no offer or her offer is accepted by all types of sellers.

Let us now derive the optimal contract. For a given contract $\mathcal{C}$, let $(T(\theta), p(\theta))$ be the seller's preferred option when his information is θ, where $\pi(\theta) = T(\theta) + \theta p(\theta)$ is the corresponding profit. The buyer's expected utility when type θ accepts the contract is then $\theta U - \pi(\theta)$. We consider only offers that induce non-negative profit for the seller, i.e. $\pi(\theta) \geq 0$.[11]

For any equilibrium contract, we must have:

$$\min_{\theta} \left\{ \pi(\theta) - \pi^R(\theta) \right\} \leq 0. \tag{2}$$

Indeed a contract with $\min_{\theta} \left\{ \pi(\theta) - \pi^R(\theta) \right\} > 0$ would be always accepted by the seller and would leave the buyer with a payoff below her no-contract payoff.

Incentive compatibility requires that the profit obtained by a seller of type θ is maximal among the possible tariffs. The allocation is incentive compatible if only if:

$$\dot{\pi}(\theta) = p(\theta) \text{ for all } \theta,$$
$$p(.) \text{ is non-decreasing.}$$

The threat of bypass is credible when $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} \geq \theta^B$. In any equilibrium with full participation of the seller, the buyer makes sure that this constraint binds.[12]

Using the above incentive compatibility conditions, we show in the Appendix that $\mathcal{R}$ is an interval of the form $[\theta^R, \bar{\theta}]$. Intuitively, adding higher types to the rejection set, by reducing their promised expected profit, enhances the credibility of the threat

[11] In full generality, one could consider offers with negative profit for some types (who will then refuse the contract) and positive profit for other types. Such a possibility would always be dominated by offering the null contract $\mathcal{C}_0$ to those types who obtain a negative profit in the original contract.

[12] Otherwise, the buyer could increase her payoff by reducing the rent left to all types by $\varepsilon > 0$ without affecting the sellers' participation decision.

of bypass. The contract is thus designed to induce rejection on the upper end of the interval of types. One can then usefully define θ^D as:

Definition 2 *For* $\mathbb{E}\{\theta\} < \theta^B \leq \bar{\theta}$*,* θ^D *is the solution of* $\mathbb{E}\{\theta \mid \theta \geq \theta^D\} = \theta^B$.

By construction, θ^D is smaller than θ^B and from the above remarks, is such that if an equilibrium exists with a contract accepted by all types of sellers, all types below θ^D, and only these types, receive a profit larger than their reservation gain $\pi^R(.)$:

$$\mathcal{R} = [\theta^D, \overline{\theta}] \text{ and } \mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} = \theta^B.$$

The problem (P1) faced by the buyer when offering a contract inducing a credible bypass threat to the seller is then:

$$\max_{\{p(.),\pi(.)\}} \mathbb{E}\left\{\theta U - \pi(\theta)\right\}$$

$$\text{subject to } \forall\theta : \dot{\pi}(\theta) = p(\theta) \text{ and } p(.) \text{ non-decreasing,} \tag{3}$$

$$\forall\theta > \theta^D : \pi(\theta) \leq \pi^R(\theta), \tag{4}$$

$$\forall\theta < \theta^D : \pi(\theta) > \pi^R(\theta), \tag{5}$$

$$\pi(\theta^D) = \pi^R(\theta^D). \tag{6}$$

In equilibrium the buyer proposes a contract if the value of this program is larger than $\mathbb{E}\{\theta \times (U - \alpha)\}$ since the buyer would not bypass under her prior beliefs.

The following reasoning then simplifies the analysis. Given that types below θ^D and only those types must obtain a positive rent $\pi(\theta) > \pi^D(\theta)$, it must be the case that the price is strictly below α for all types $\theta \leq \theta^D$. Indeed, as the price is non-decreasing, if a price $p(\theta) > \alpha$ is accepted by $\theta < \theta^D$, then all types above θ trade at a price above α with a positive rent. We would then have $\theta^R < \theta^D$ and the bypass threat would not be credible. Given that $p(\theta) \leq \alpha$ for θ below θ^D, the buyer can always extend the contract above θ^D with prices below α while respecting the condition $\mathcal{R} = [\theta^D, \overline{\theta}]$. Thus we may restrict to prices below α. But with such prices, given the profit constraint at θ^D, the condition (4) holds while the condition (5) holds provided that $p(\theta) < \alpha$ for $\theta < \theta^D$. We may thus replace conditions (4) and (5) with

$$\forall\theta : p(\theta) \leq \alpha \text{ with strict inequality if } \theta < \theta^D.$$

In the proof of the next proposition we show that, if it exists, the solution of (P1) coincides with a contract offering to all types the same price $p < \alpha$. The fact that p is constant results from the tension between two effects. On the one hand, incentive compatibility requires that $p(.)$ be non-decreasing. On the other hand, p should be high enough for low θ to ensure a profit above π^R, and low enough for high θ to ensure a profit below π^R. This leads to a constant value for $p(.)$ as an optimal solution. The buyer's expected utility then equals $\mathbb{E}\{\theta\} U - T - p\mathbb{E}\{\theta\}$ where $T = \pi^R(\theta^D) - \theta^D p$, or $\mathbb{E}\{\theta(U-\alpha)\} + (p-\alpha)(\theta^D - \mathbb{E}\{\theta\})$. Given that $p - \alpha < 0$, the conclusion is that a contract is proposed if θ^D is small enough.

Proposition 1 *Assume that $\bar{\theta} \geq \theta^B > \mathbb{E}\{\theta\}$. In the minimal scenario, the buyer's expected payoff is $v^m = \mathbb{E}\{\theta\}U - \min\{\mathbb{E}\{\theta\}, \theta^D\}\alpha$. It is obtained by offering $\mathcal{C}^* = (T^* = \theta^D\alpha, p^* = 0)$ if $\theta^D < \mathbb{E}\{\theta\}$ (accepted by all θ) and no contract otherwise.*

Proof. See the Appendix. ■

In the minimal scenario, the buyer either offers a contract accepted by all types of sellers or has no incentive to propose any contract. In the first case, the buyer proposes an advanced payment with a rebate on the price α that would prevail ex-post if she does not propose anything. The limit on the buyer's ability to shift the surplus in her favor lies in the credibility of bypass. If she asks for too large a rebate, too many types of sellers may refuse and bypass after a rejection is no longer in her interest.

Early contracting allows the buyer to exploit information asymmetries so as to raise the credibility of the bypass threat, thereby increasing her bargaining position. While it is a dominant strategy for lower types of sellers to accept, higher types are trapped in a situation where refusing the offer would signal their type. This situation is represented in Figure 1.

Whether the buyer is able to exploit this mechanism or not (i.e., whether θ^B is larger than $\mathbb{E}\{\theta\}$ or not) depends on the incentives to bypass, which in turn depend on k and on the ex-ante price α. If α is too low or if k is too high, the threat of bypass is not credible; the buyer must then increase the profit left to the seller in order to shrink the set $\mathcal{R}$ so as to restore the credibility of bypass.

In case of contract $\mathcal{C}^*$, the buyer's expected utility depends non-monotonically on α. Raising α increases the credibility of bypass, which tends to decrease the seller's profit (in particular θ^D decreases). But it also raises the profit that has to be given up to lower types of sellers so as to maintain his unconditional participation to the contract (it raises $\pi^R(\theta^D)$ for a given value of θ^D). Thus a change in the ex-post bargaining power may benefit either the seller or the buyer.

3.3 Equilibrium set

We have highlighted so far two particular contracts, $\mathcal{C}_0$ and $\mathcal{C}^*$, that are equilibria in the two particular continuation rules defined above, the maximal scenario and

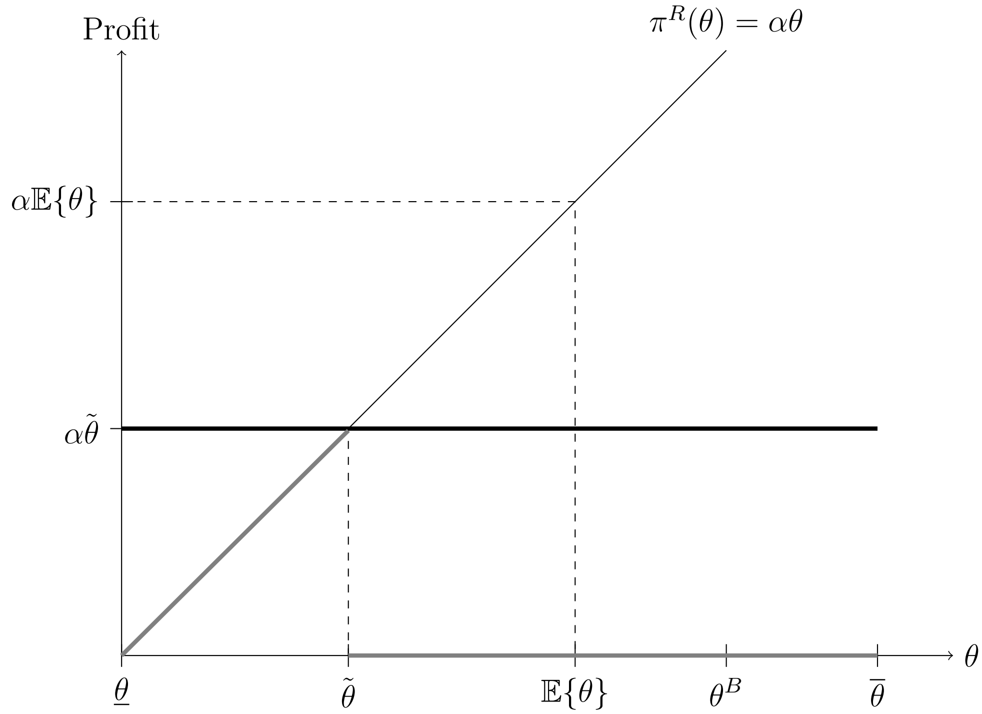

Figure 1: *Equilibrium allocation in the minimal scenario: all types in* $[\underline{\theta}, \tilde{\theta}]$ *accept and no bypass follows rejection; all types in* $[\tilde{\theta}, \overline{\theta}]$ *accept because bypass follows rejection.*

the minimal scenario. Other contracts can be offered at equilibrium, in particular in cases where the out-of-equilibrium continuation rules are different from those adopted on the equilibrium path. As an example, suppose that following an offer $\mathcal{C}_\epsilon \equiv (T = \epsilon, p = 0)$, with ϵ positive but small, the continuation equilibrium is the same as in the maximal scenario, whereas for any other offers it is given by the minimal scenario. Then, $\mathcal{C}_\epsilon$ is an equilibrium offer while $\mathcal{C}_0$ is not.

Let us now derive whole set of equilibrium payoffs. An equilibrium of the game can be described as, first, a contract offered by the buyer and, second, a mapping from contracts to continuation equilibrium allocations of the last stages. To check that a contract offer is indeed part of an equilibrium, and therefore to construct the set of equilibrium payoffs, one has to specify a continuation equilibrium for all possible contracts, possibly different on- and out-of-the equilibrium path, and check that this continuation equilibrium deters the buyer from deviating from the contract offer.

The usual way to check that an allocation can be sustained as an equilibrium outcome is to choose out-of-equilibrium continuation rules that are the least favorable to the deviating player, i.e. the minimal scenario. Hence, each party must obtain at least as much as the minimal payoff in any equilibrium: zero for the seller, v^m for the buyer (see Proposition 1). The next proposition shows that these conditions of minimal payoffs characterize the whole set of equilibrium allocation.

Proposition 2 *Suppose that $\bar{\theta} \geq \theta^B \geq \mathbb{E}\{\theta\}$. An allocation is an equilibrium if and only if it is incentive compatible and it generates interim payoffs $v \geq v^m$ for the buyer and $\pi(\theta) \geq 0$ $\forall \theta$ for the seller.*

Proof. See the Appendix. ■

The above proposition characterizes the set of equilibrium payoffs. It also implies that the equilibrium may not be efficient. In particular, the equilibrium set includes some equilibria with bypass when the contract is accepted and other equilibria with bypass with a probability smaller than one when the contract is refused and the buyer is indifferent.

To illustrate this last point, consider the contract where the buyer offers a fixed payment $T = (1-x)\alpha\theta^D$ where x is the probability of bypass when she is indifferent. With such an offer, the seller will accept for sure if and only if $\theta < \theta^D$. In this case, the buyer's expected payoff is given by:

$$v_B = \mathbb{E}\{\theta\} U - F(\theta^D)(1-x)\alpha\theta^D - x(1-F(\theta^D))k - (1-x)\int_{\theta^D}^{\bar{\theta}} \alpha\theta f(\theta)d\theta.$$

If the minimal scenario prevails for any other contract offer, this is an equilibrium if $v_B \geq v^m$, i.e. if x is sufficiently large.

3.4 Ratifiable allocations

In their original work, Cramton and Palfrey (1995) develop an approach of situations similar to this model based on an implementation perspective. They consider a mechanism design problem where agents can exercise a veto that induces a predefined default game. In our set-up, the default game is the continuation game of stage 3 (b) where the buyer decides to bypass with no contract prevailing. Defining a decision as a tariff (T, p), a decision rule associated to the mechanism design problem specifies a tariff $(T(\theta), p(\theta))$ as a function of an announcement of his type by the seller. Any agent may veto the decision (proposed by an hypothetical principal), in which case the default game is played. Cramton and Palfrey define an incentive compatible and individually rational (thereafter IC-IR) allocation as one that is accepted by the buyer and all types of sellers in a sequential equilibrium and that induces truthful announcement by the seller. Clearly any equilibrium allocation of our game is IC-IR. Cramton and Palfrey then propose further restrictions on allocations that reflect credibility considerations.[13]

The first concept, referred to as ratifiability, imposes that given a decision rule proposed, either i) there is no continuation equilibrium where some types of seller veto the allocation or ii) if there is one, there exists one continuation equilibrium where those types of sellers who veto are indifferent between vetoing or not. If we apply this definition to our model we find (see Appendix) that an IC-IR decision rule is ratifiable if and only if one out of the two following conditions holds (where $\mathcal{R}$ is defined as in Section 3.1):

R1. $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} \geq \theta^B$;

R2. There exists $\theta \geq \theta^B$ such that $\pi(\theta) = 0$.

The condition R1 is similar to the condition obtained for the contract proposed by the buyer in the minimal scenario, and, with the addition of positive profit, corresponds to case i) in the definition of ratifiability. Indeed under the minimal scenario we have found that the offer is credible and thus accepted only if this is the unique continuation equilibrium in stage 3. The condition R1 ensures rejection cannot occur in equilibrium, except by types with zero profit.

Ratifiability differs however from the minimal scenario due to the second condition. The reason is that it imposes only that the beliefs of the buyer following a veto be consistent with sellers' equilibrium behavior whenever possible. Under condition R2, after observing a veto, the buyer can put all the weight on types with zero profits and still bypass, making his beliefs on the seller's behavior self-consistent. Notice that, due to incentive compatibility conditions, it is equivalent to $\pi(\theta^B) = 0$. In particular, it implies that the zero price contract $\mathcal{C}_0$ is ratifiable if $\bar{\theta} \geq \theta^B$.

Cramton and Palfrey then strengthen the requirements on admissible decision rules to strong ratifiability. A decision rule is strongly ratifiable if either i) there is no continuation equilibrium where some types of sellers veto the allocation or ii) in any such continuation equilibrium, all types of sellers who veto are indifferent between vetoing or not. In our model, we show that an IC-IR decision rule is strongly ratifiable if and only if:

[13] Celik and Peters (2011) show that unanimous acceptation by all types of agents may restrict inefficiently the set of implementable allocations. However their proposition 2 shows that this is not the case in our set-up.

SR. $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} \geq \theta^B$.

As we can see strong ratifiability excludes the possibility that the beliefs concentrate on zero profit types, if condition R1 is violated.

There is a strong connection between ratifiability and our choice of scenarios, despite different approaches. To make the connection transparent, we may contrast our results with the equilibrium analysis using ratifiability as a selection criterion in our game. Suppose first that we assume that any contract inducing a ratifiable decision rule is accepted with probability 1. Then it is immediate from above that the buyer proposes the zero price contract $\mathcal{C}_0$. The equilibrium thus coincides with the equilibrium under the maximal scenario.

By contrast, assume that any contract inducing a strongly ratifiable decision rule is accepted with probability 1 while any other is rejected with positive probability. From the same argument as in Section 3.2, the buyer proposes a strongly ratifiable decision rule. As a consequence, whenever $\bar{\theta} \geq \theta^B > \mathbb{E}\{\theta\}$, the equilibrium outcome with this selection rule coincides with the equilibrium outcome under the minimal scenario.[14]

Thus strong ratifiability considerations provide further reasons to focus on the minimal scenario.

4 Extensions

4.1 Early contracting with uncertainty on the seller's information

The presence of seller's private information is crucial for the buyer to benefit from early contracting. But not all sellers have privileged information on the buyer's future demand, and the buyer probably does not know what a given seller's information set is. We take a closer look at this situation by assuming now that a share λ of sellers are informed about θ while a share $1-\lambda$ of sellers are not informed and hold the same ex-ante belief as the buyer.

Consider first the maximal scenario, assuming that $\mathbb{E}\{\theta\} < \theta^B$.[15] Take the extreme case where no seller has any information on θ, i.e. $\lambda = 0$. The buyer cannot learn anything from rejection so her ex-post beliefs are identical to her ex-ante ones.

[14] If $\theta^B \leq \mathbb{E}\{\theta\}$, the buyer will imposes the contract $\mathcal{C}_0$.

[15] When $\mathbb{E}\{\theta\} \geq \theta^B$ the contract $\mathcal{C}_0$ is the equilibrium contract for any scenario, as in the main section.

Early contracting is then of no use and the buyer will simply wait ex-post for the realization of her valuation. Suppose instead that some sellers have ex-ante private information on θ even if others do not (i.e. $\lambda \in (0,1)$). The same reasoning as in the baseline model applies then. The buyer will propose $\mathcal{C}_0$ and hold the belief that the seller knows that θ is large (above θ^B) if he rejects the offer. In the maximal scenario, the presence of uncertainty on the information set of the seller does not alter the buyer's ability to get all the surplus, except when there is no private information. Early contracting benefits the buyer as soon as some sellers have heterogenous and private information.

Let us now consider the minimal scenario assuming again $\theta^B > \mathbb{E}\{\theta\}$. As before, for any offer $\mathcal{C}$, we can define the set $\mathcal{R}$ of sellers that refuse this offer if they anticipate no bypass. Building on our previous work, we focus on the case of contract with a fixed payment T associated with an option to buy at price zero. As the buyer will always propose a fixed payment smaller than $\alpha\mathbb{E}\{\theta\}$, the set $\mathcal{R}$ can be further decomposed in two subsets. The first subset includes all the non-informed sellers, whose ex-ante beliefs on the gains from trade are given by $\mathbb{E}\{\theta\}$. The other subset is made of informed sellers with types greater than T/α. Then

$$\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} = \frac{(1-\lambda)\mathbb{E}\{\theta\} + \lambda \Pr(\theta \geq T/\alpha)\, \mathbb{E}\{\theta \mid \theta \geq T/\alpha\}}{1-\lambda+\lambda \Pr(\theta \geq T/\alpha)}$$

Suppose first that the buyer offers contract $\mathcal{C}^*$. With such a contract

$$\begin{aligned}\mathbb{E}\{\theta \mid \theta \in \mathcal{R}, \mathcal{C} = \mathcal{C}^*\} &= \frac{(1-\lambda)\mathbb{E}\{\theta\} + \lambda \Pr\left(\theta \geq \theta^D\right) \mathbb{E}\left\{\theta \mid \theta \geq \theta^D\right\}}{1-\lambda+\lambda \Pr\left(\theta \geq \theta^D\right)} \\ &= \frac{(1-\lambda)\mathbb{E}\{\theta\} + \lambda \Pr\left(\theta \geq \theta^D\right) \theta^B}{1-\lambda+\lambda \Pr\left(\theta \geq \theta^D\right)}\end{aligned}$$

For $\theta^B > \mathbb{E}\{\theta\}$, the RHS is less than θ^B as soon as some sellers are uninformed. The threat of bypass if the seller refuses the offer is then not credible. The buyer is forced now to increase the transfer proposed to the seller to make her threat credible again. To go further, let us define implicitly $\underline{\lambda}$ as the solution of the equation

$$\frac{(1-\underline{\lambda})\mathbb{E}\{\theta\} + \underline{\lambda}\mathbb{E}\{\theta \mid \theta \geq \mathbb{E}\{\theta\}\}}{1-\underline{\lambda}+\underline{\lambda}\Pr(\theta \geq \mathbb{E}\{\theta\})} = \theta^B, \tag{7}$$

and $T(\lambda)$ as the solution for $\lambda < \underline{\lambda}$ of[16]

$$\frac{(1-\lambda)\mathbb{E}\{\theta\} + \lambda \Pr(\theta \geq T(\lambda)/\alpha)\, \mathbb{E}\{\theta \mid \theta \geq T(\lambda)/\alpha\}}{1-\lambda+\lambda \Pr(\theta \geq T(\lambda)/\alpha)} = \theta^B. \tag{8}$$

[16]The left-hand side is increasing with T on the range $T \leq \mathbb{E}\{\theta\}$so that $T(\lambda)$ is uniquely defined for $\lambda < \underline{\lambda}$.

Proposition 3 *Assume that* $\bar{\theta} \geq \theta^B > \mathbb{E}\{\theta\}$ *and that only a share* $\lambda \in (0,1)$ *of sellers have ex-ante private information.*

1. *Under the maximal scenario,* $\mathcal{C}_0$ *is the equilibrium contract and all sellers accept this offer.*

2. *Under the minimal scenario,*

   i) for any $\lambda \leq \underline{\lambda}$*, the buyer will not propose any early contract and trade occurs at the ex-post price if the valuation is positive,*

   ii) for any $\lambda > \underline{\lambda}$*, the buyer will propose a contract with fixed payment* $T(\lambda)$ *decreasing with* λ*, from* $\alpha\mathbb{E}\{\theta\}$ *to* T^**.*

Proof. See the Appendix. ■

When the share of informed sellers is low, the buyer does not get much information when her contract is refused. The only way to get more information, and to make her threat credible, is then by increasing her transfer but it may reach the upper bound $\alpha\mathbb{E}\{\theta\}$ which occurs for $\lambda = \underline{\lambda}$. The seller will therefore propose no contract if the price to make this contract - and the associated threat - credible is above the expected payment of an ex-post deal. When the share of informed sellers is not too low, the buyer can propose a transfer lower than $\alpha\mathbb{E}\{\theta\}$ that screens sufficiently sellers to bring some information if the contract is refused. This transfer is increasing in θ^B and decreasing in λ. As the set of informed sellers increases, the information precision the buyer obtains following contract rejection also increases and, for a given offer, the expected gains from trade increase. Offering high payments to make her threat credible is then less useful and the buyer can keep a larger share of the surplus. This extension thus shows that private information and money are substitutable. When the proportion of informed sellers increases, the transfer necessary to make bypass a credible threat decreases.

4.2 Ex-ante investment in the minimal scenario

The situation considered so far concentrated on the buyer's incentives to invest in a bypass technology. We now investigate a polar case in which the seller has to decide whether to invest in the first place. To this end, let us focus on the minimal scenario and add an initial stage in the game where S must decide whether to invest or not. This investment has a cost $\kappa < k$ and is a necessary condition for ex-post production. The new period of investment is such that:

- Period 1:

    1. The seller learns the value of θ.
    2. The seller privately decides to invest or not.
    3. The buyer can make an early offer as previously.

The investment decision is now a signal of the information owned by the seller. Consider first the case without early contracting but where bypass is possible. There is always an equilibrium, mirroring the maximal scenario, where no investment takes place because the buyer would react by bypassing, with beliefs that θ is high if investment occurs. If $\kappa > \theta^D\alpha$, this is the only equilibrium because the seller would invest only if $\theta\alpha \geq \kappa$, which would reveal that $\theta > \theta^D$ and thus is large enough for bypass to be profitable for the buyer. On the other hand if $\kappa \leq \theta^D\alpha$, there is another equilibrium where the seller invests whenever $\theta \geq \kappa/\alpha$ and the buyer does not bypass.[17] Notice that total welfare maximization would require that the seller invests if $\theta \geq \kappa$. Thus there is insufficient investment when $\alpha < 1$ due insufficient congruence between private and social incentives.

Overall, due to the threat of bypass, there is insufficient investment in the absence of early contracting. One may conjecture that early contracting would exacerbate this insufficiency by raising the ability of the buyer to capture a substantive share of the value created by investment. We show below that this is not the case due to the nature of the equilibrium.

Let us denote by $\hat{\theta}$ the equilibrium cut-off type such that the seller with type $\theta \geq \hat{\theta}$ invests. Using the fact that previous results do not rely on the shape of the distribution below θ^B, and focusing on the minimal scenario, we can say that:

- If $\hat{\theta} < \theta^D$, then $\mathbb{E}\{\theta \mid \theta \geq \hat{\theta}\} < \theta^B$ and the buyer will offer $\mathcal{C}^*$ defined in Proposition 1, inducing a strictly positive profit for the seller.
- If $\hat{\theta} \geq \theta^D$, then $\mathbb{E}\{\theta \mid \theta \geq \hat{\theta}\} \geq \theta^B$ and the buyer will offer $\mathcal{C}_0$ leaving no profit to the seller.

Assume that $\hat{\theta} < \theta^D$ at equilibrium. Then, a θ-type seller anticipates an offer $T^* = \theta^D\alpha$ and thus invests if and only if $\theta^D\alpha \geq \kappa$. It follows that under the minimal scenario, there is an equilibrium where the seller invests irrespective of his information provided that κ is small.

[17] In this equilibrium, the buyer may or may not bypass when the seller does not invest, depending on whether k is larger or smaller that $\mathbb{E}\left\{\theta \mid \theta \leq \kappa/\alpha\right\}$.

Of course, there is still the possibility of a coordination failure where the seller does not invest, because the buyer would then hold very optimistic beliefs about θ and make an offer $\mathcal{C}_0$ accepted by the seller.

One can therefore summarize the investment stage of the whole game as follows.

Proposition 4 *Assume that after having learned the value of θ, the seller must choose to invest or not at a cost κ to produce the good ex-post. Then in the minimal scenario, there is an equilibrium where all types invest if $\kappa \leq \theta^D \alpha$; otherwise the seller does not invest.*

When the investment decision conveys information that makes the threat of bypass credible, there will be no investment by the seller both with and without early contracting. Early contracting is useless in this case as what would be needed to restore efficiency is ex-ante contracting, i.e. contracting before the seller decides to invest.

In constract, when κ is small, the investment decision conveys little information and leaves the buyer in weak position with a non-credible bypass threat. To restore credibility, the buyer uses early contracting with a Divide-and-Learn strategy. Thus the seller's unconditional expected payoff is lower with early contracting. But the reduction of the expected rent occurs through a reduction of the payoff of the highest types along with an increase of the rent of the lowest types of seller. As the buyer proposes a fixed advanced payment, accepting the offer becomes profitable even for types that would not invest based on their information about prospective sales.[18]

The critical level of cost for investment to occur depends on α, θ^B and the distribution of θ. In particular, increasing the seller's ex-post bargaining power, i.e. increasing α, has a ambiguous impact on investment. As increasing α makes the buyer's threat to bypass is more credible (θ^D decreases) but increases the seller's share when there is no bypass, it may either foster or deter investment.

4.3 Random valuation

We now extend our model by assuming that when demand is positive, the buyer's valuation U is a continuous random variable taking values in $(0, \bar{U}]$ and is known by the buyer only. As previously, the probability that demand is positive is θ and is known by the seller only. In the event that demand is positive, $D(p)$ denotes then the probability that U is larger than p, $W(p) = \int_p^{\bar{U}} D(t)dt$ is the buyer's expected surplus when the unit price is p, and $pD(p)$ is the seller's expected profit. conditional on

[18]The mechanism is reminiscent to adverse selection in insurance markets.

demand being positive. We assume that the price-elasticity $\varepsilon(p) = -pD'(p)/D(p)$ is increasing in p from $\varepsilon(0) = 0$ to a value larger than 1. We maintain the assumption that the ex-post price is $\alpha \leq p^M \in \arg\max pD(p)$.

The interim probability of sale for a given price p is the $\theta D(p)$, with the buyer's interim expected surplus and the seller's interim profit being respectively $\theta W(p)$ and $\theta pD(p)$. For this part we assume hat $F(\theta)/\theta f(\theta)$ is increasing in θ, which ensures that the equilibrium will be separating on the range of types accepting the contract.

As the buyer obtains $\theta W(\alpha)$ without bypass and $\theta W(0) - k$ with bypass, we can define the critical buyer's beliefs θ^B that makes bypass a credible threat as

$$\theta^B \equiv \frac{k}{W(0) - W(\alpha)}.$$

The seller's participation constraint now writes as:

$$\forall\theta : \pi(\theta) \geq \pi^R(\theta) = \theta\alpha D(\alpha).$$

Without loss of generality due to the assumption of unit demand, a contract $\mathcal{C}$ is again a menu of two-part tariffs $\{(T(.), p(.))\}$. The first-order incentive compatibility condition is now given by:[19]

$$\forall\theta : \dot{\pi}(\theta) = p(\theta)D(p(\theta)). \tag{9}$$

Consider first the interim negotiation in case where bypass is not possible. As the marginal cost of production is null, efficiency would dictate to set p equal to zero but such a price would only be compatible with the seller's full participation if it were compensated by high transfer $\bar{\theta}\alpha D(\alpha)$. The buyer will then trade off efficiency with the level of transfers. In our model, the seller has incentives to claim that θ is high so as to obtain higher profits. To reduce the seller's incentive to overstate the value of θ, the buyer will lower efficiency and rents for high types. The buyer then proposes a contract that involves both an increasing price $p(\theta)$ and a decreasing (net) rent $\pi(\theta) - \pi^R(\theta)$. As the price $p(\theta)$ will never exceed the ex-post price α, we may now have two subsets of types of seller. A seller with a lower type accepts the contract and sells the good if the valuation is positive and above the price $p(\theta)$. A seller with a higher type refuses the contract and sells the good ex-post at price α if the valuation is positive and above α. Formally, let us define $\phi(\theta)$ as the solution of

$$\varepsilon(\phi(\theta)) = \frac{1}{1 + \theta f(\theta)/F(\theta)}.$$

[19] We show in the Appendix that the second-order condition, $p(.)$ non-decreasing, is implied by modified monotone hazard rate property assumed at the beginning of this section and the assumption on the elasticity of demand.

Lemma 2 *Assume that bypass is not possible. Then, the allocation implemented by the buyer is such that* $\pi\left(\overline{\theta}\right) = \pi^R\left(\overline{\theta}\right)$ *and for all* $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, $p(\theta) = \min\{\phi\left(\theta\right), \alpha\}$*; in particular when* $\phi^{-1}\left(\alpha\right) < \theta < \bar{\theta}$*, the seller does not sign an interim contract (and obtains* $\pi\left(\theta\right) = \pi^R\left(\theta\right)$*)*

Proof. See the Appendix. ■

In contrast with the simple binary model, an early contract is proposed provided that $\phi^{-1}\left(\alpha\right) > \underline{\theta}$. While in the benchmark model with constant valuation, there was no reason to sign a contract, in this extension with random valuation, there is an efficiency motive as the contract, by using non-linear tariffs, allows to limit the negative impact of above-cost pricing ex-post. Therefore, the buyer has some incentives to propose a price in-between the efficient price and the status quo one. This reflects potential efficiency gains due to the fact that the utility is unknown. 'Partial participation' emerges however when α is small (and thus $\phi^{-1}\left(\alpha\right)$), as a results of the buyer's attempt to appropriate the sellers rent.

We now consider the possibility of bypass. The main methodological results derived in the binary case, in particular Lemma 1, still apply. Consequently, the multiplicity issue and the ways to deal with that multiplicity are unchanged. Under the maximal scenario, the buyer offers the contract $\mathcal{C}_0$, which is then accepted by all sellers. To highlight the difference with the benchmark model, we focus attention on the minimal scenario.

As in the case with no bypass, the minimal scenario involves a trade-off between rent extraction and efficiency. Moreover, the rents given up to the sellers should be such that the threat of duplication is credible, i.e. such that only the lower types sellers get more than their reservation profit. In the Appendix, we show that the buyer's problem can be formulated as follows:

$$\begin{aligned}
&\max_{\{p(.),\pi(.)\}} \mathbb{E}_\theta\left\{\theta W(p(\theta)) + \theta p(\theta) D(p(\theta)) - \pi(\theta)\right\}, \\
\text{subject to } \forall\theta : \;& \dot{\pi}(\theta) = p(\theta) D(p(\theta)), \\
& p(\theta) \in [0, \alpha] \text{ and is non-decreasing}, \\
& \pi(\theta^D) \geq \pi^R(\theta^D).
\end{aligned}$$

The program is similar to the binary case, adjusting for the new expected demand. The solution of the problem stated above has the following features:

Proposition 5 *Assume that* $\mathbb{E}\left\{\theta\right\} < \theta^B \leq \bar{\theta}$*. In the minimal scenario, the solution of the buyer's problem is unique and such that* $\pi^*(\theta^D) = \pi^R(\theta^D)$*. Moreover:*

- *If $\theta^D \leq \mathbb{E}\{\theta\}$, then a unique contract is proposed with $p(\theta) = 0$ and $T(\theta) = \pi^R(\theta^D)$ for all θ.*

- *If $\theta^D > \mathbb{E}\{\theta\}$, then there exists a unique $\theta^P < \theta^D$ characterized by*

$$\left(1 - \varepsilon\left(\phi\left(\theta^P\right)\right)\right)\left[\int_{\theta^P}^{\theta^D} F(\theta)\, d\theta - \int_{\theta^D}^{\bar{\theta}} \left(1 - F(\theta)\right) d\theta\right] = \varepsilon(\phi\left(\theta^P\right)) \int_{\theta^P}^{\bar{\theta}} \theta f(\theta)\, d\theta. \tag{10}$$

such that:

  - *for $\theta \leq \theta^* = \min\{\theta^P, \phi^{-1}(\alpha)\}$, the contract implements a price $p^*(\theta) = \phi(\theta)$.*
  - *for $\theta > \theta^* = \min\{\theta^P, \phi^{-1}(\alpha)\}$, the contract implements a price $p^*(\theta) = \phi(\theta^*)$.*

Proof. See the Appendix. ■

For $\mathbb{E}\{\theta\} \leq \theta^B < \overline{\theta}$, the solution can take three different forms. Two cases are qualitatively similar to the binary case. First, if $\theta^D \leq \mathbb{E}\{\theta\}$, all types of sellers are proposed the (efficient) null price and get the same profit equal to $\pi^R(\theta^D)$. Second, if θ^D is high enough so that $\theta^P > \phi^{-1}(\alpha)$, the solution with bypass corresponds to the solution without bypass. In this case, the threat of bypass is not credible enough for the buyer to exploit it in the negotiation.

The conclusions differ from the binary case in the intermediate case where $\theta^P < \phi^{-1}(\alpha)$. The prices are uniformly below the level α, increasing from zero up to a constant level below the status quo price α. The buyer is then able to design offers that induce full participation and reduces the price for higher types, thereby rasing efficiency when it is most valuable interim. This efficiency gain is achieved by shifting the rent away from the high types of seller toward the low types of sellers, with a divide-and-learn strategy. As long as θ^D is not too large, the buyer needs not leave too much rent to low types as their mass is relatively small. Whether it is worth doing it depends on the efficiency gains (with increases with α) and the cost in terms of rents, and our result states that it resumes to the comparison between θ^P and the participation threshold $\phi^{-1}(\alpha)$ without bypass.

Note that choosing the level of θ^P is equivalent to choosing the maximal price $p^* = p^*\left(\theta^P\right)$. The choice of θ^P reflects a standard trade-off between rent extraction and efficiency. Increasing the price p^* has two effects. First, the total surplus decreases by an amount that corresponds to the right-hand side of Equation (10). Second, the expected rent of the seller is affected positively for high types and negatively for low types. The left-hand side of Equation (10) corresponds to the expected decrease in the seller's profit. Whenever the latter is larger than the former, it is in the best interest of the buyer to increase the price targeted to seller informed that the expected demand is high.

5 Concluding remarks

Most of the principal-agent literature assumes that if an agent refuses the contract, no game is played. In many cases, for example in contracting between equal agents or sovereign States,[20] this assumption does not hold and there is no way to commit or control the outside opportunities. It is therefore important to have a better understanding of the game played when the contract may be refused. Our analysis shows that one party may engage into an advance negotiation with the sole purpose of gathering information to improve his bargaining position. Indeed when an agent has some superior information on a common value parameter, a principal can play some types against other and reduce the rents that must be left to some agents. More surprisingly, as this method can only be implemented when the agents have private information, the principal benefits from this asymmetry of information.

Our article brings new insights on ex-ante contracting but much work remains to fully understand this practice. For one thing we have assumed that only one party is informed. Strategic behaviors in early contract negotiation by multiple players remains to be understood. Implications for ex-ante information gathering should also be investigated. At last, although in our model the buyer cannot commit to a behavior after rejection of a contract offer, we assumed that she can commit not to renegotiate by offering a new contract. Allowing for renegotiations possibilities will be one of the challenge for future research.

References

Arrow K.J. (1963). "The Role of Securities in the Optimal Allocation of Risk-Bearing', *Review of Economic Studies*, Vol. 31, 91-96.

Bebchuk L.A. (1984). "Litigation and Settlement under Imperfect Information", *RAND Journal of Economics*, Vol. 15, 404-415.

[20]The negotiations over climate change are another good example of such a situation where the failure of a contract does not lead to no action.

Borch K. (1962). "Equilibrium in Reinsurance Markets", *Econometrica*, Vol. 30, 424-44.

Calzolari G. and A. Pavan (2006). "On the optimality of privacy in sequential contracting," *Journal of Economic Theory*, vol. 130(1), 168-204.

Celik G. and M. Peters (2011). "Equilibrium Rejection of a Mechanism", *Games and Economic Behavior*, Vol. 73(2), 375-387.

Coase R. (1972). "Durability and Monopoly", *Journal of Law and Economics*, 15(1), 143-149.

Cramton P. and T. Palfrey (1995). "Ratifiable Mechanisms: learning from disagreement", *Games and Economic Behavior*, Vol. 10(2), 255-283.

Crémer J., Khalil F. and J.C.Rochet (1998). "Strategic Information Gathering before a Contract if Offered", *Journal of Economic Theory*, Vol. 81, 163-200.

Grossman S. and M. Perry (1986). "Perfect sequential equilibrium," *Journal of Economic Theory*, Vol. 39(1), 97-119.

Freixas X., Guesnerie X. and J. Tirole (1985). "Planning under Incomplete Information: the Ratchet Effect", *Review of Economic Studies*, Vol. 52(2), l73-l9l.

Hart O. and J. Tirole (1988). "Contract Renegotiation and Coasian Dynalics", *Review of Economic Studies*, 55(4), 509-540.

Jehiel P. and B. Moldovanu (1996). "Strategic Nonparticipation", *Rand Journal of Economics*, Vol. 27(1), 84-98

Jullien B. (2000). "Participation Constraints in Adverse Selection Models", *Journal of Economic Theory*, Vol. 93(1), 1-47.

Jullien (2011). "Competition in Multi-Sided Markets: Divide-and-Conquer", *American Economic Journal: Microeconomics*, Vol. 3(4), 1-35.

Kessler A.S. (1998). "The value of Ignorance", *Rand Journal of Economics*, Vol. 29(2), 339-354.

Laffont J.J. and J. Tirole (1988). "The Dynamics of Incentives Contracts ", *Econometrica*, Vol. 56(5), 1153-1175.

Laffont J.J. and D Martimort (2002). *The Theory of Incentives*, Princeton University Press.

Pauly M. (1968). "The Economics of Moral Hazard: comments", *American Economic Review*, Vol. 58, 531-537.

Philippon T. and V. Skreta (2012). "Optimal Interventions in Markets with Adverse Selection", *American Economic Review*, Vol. 102(1), 1-28.

Rasul I. and S. Sonderreger (2010). "The role of the agent's outside options in principal-agent relationships", *Games and Economic Behavior*, Vol. 68, 781-788.

Segal I. (1999). "Contracting with Externalities", *Quaterly Journal of Economics*, 114(2), 337-338

Seierstad A. and K. Sydsaeter (1977). "Sufficient conditions in optimal control theory", *Internation Economic Review*, 18, 367-391.

Sobel J. (1993). "Information Control in the Principal-Agent Problem", *International Economic Review*, Vol. 34(2), 259-269.

Tirole (2012). "Overcoming Adverse Selection: How Public Intervention Can Restore Market Functioning", *American Economic Review*, Vol. 102(1), 29-59.

Weitzman (1980). "The 'Rachet Principle' and Performance Incentives", *Bell Journal of Economics*, Vol. 11(1), 302-308.

Appendix

Proof of Lemma 1. To begin with, consider the case where $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} \geq \theta^B$. There exists an equilibrium such that all types of sellers accept; in case of (out-of equilibrium) contract rejection, the buyer holds beliefs $\theta = \overline{\theta}$ and bypasses after after rejection.

Let us show now that there is no other type of equilibrium. Suppose on the contrary that some types θ reject the contract and that rejection is not followed by bypass. First, a seller with type $\theta \notin \mathcal{R}$ has no reason to refuse since that seller's profit is greater with the contract than without, even if the buyer bypasses. On the contrary sellers with type $\theta \in \mathcal{R}$ would then have an incentive to refuse the contract since they would then gain more, and would do so by assumption A.i. But with beliefs after rejection $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} \geq \theta^B$, the buyer would bypass. Therefore, when $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} \geq \theta^B$, the only possible equilibrium is such that all sellers accept and there is bypass in case of out-of-equilibrium rejection.

Consider the case $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} < \theta^B \leq \bar{\theta}$. There still exists a continuation equilibrium in which all sellers accept $\mathcal{C}$ and in the (out-of-equilibrium) event of contract rejection, the buyer holds beliefs $\theta = \bar{\theta} \geq \theta^B$ and bypasses. But there exists a second continuation equilibrium in which $\mathcal{C}$ is rejected by types $\theta \in \mathcal{R}$ and accepted by types $\theta \notin \mathcal{R}$. In the event of contract rejection, the buyer's beliefs about θ are pinned down by Bayes' law and must be equal to: $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\}$. Since $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} < \theta^B$, the buyer does not bypass in the event of rejection, thereby implying that types $\theta \in \mathcal{R}$ reject $\mathcal{C}$ and types $\theta \notin \mathcal{R}$ accept $\mathcal{C}$.

At last, if $\overline{\theta} < \theta^B$, there are no beliefs that can sustain bypass by the buyer in case of contract rejection. Therefore, only sellers with type $\theta \notin \mathcal{R}$ accept the contract at the equilibrium. ■

Proof of Proposition 1. As a preliminary stage, let us show that in any equilibrium where rejection triggers bypass,

1. $\pi(\underline{\theta}) \geq \pi^R(\underline{\theta})$,

2. $\pi(\overline{\theta}) \leq \pi^R(\overline{\theta})$,

3. the set of types obtaining no more than their reservation utility is an interval $\mathcal{R} = [\theta^R, \overline{\theta}]$.

Indeed, notice first that from the first-order incentive constraint, $\pi(\theta) - \pi^R(\theta)$ is a convex function.

1. To show the first result, suppose instead that $\pi(\underline{\theta}) < \pi^R(\underline{\theta})$. Because $\pi(\theta) - \pi^R(\theta)$ is convex, then $\mathcal{R}$ is an interval $[\underline{\theta}, \theta^R]$, with $0 < \theta^R \leq \bar{\theta}$. This implies that $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} \leq \mathbb{E}\{\theta\}$, which contradicts $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} \geq \theta^B > \mathbb{E}\{\theta\}$.

2. To show the second result, suppose this is not the case. Since $\pi(\theta) - \pi^R(\theta)$ is convex and $\dot{\pi}(\theta) - \dot{\pi}^R(\theta) = p(\theta) - \alpha$, it must be the case that $p(\overline{\theta}) > \alpha$. Therefore, there exists a maximal subset of types $]\theta', \overline{\theta}]$ such that $p(\theta) > \alpha$. Moreover $\theta' > \underline{\theta}$ because $\theta' = \underline{\theta}$ and (2) would imply that $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} = \underline{\theta} < \theta^B$ which contradicts our initial condition on the contract. At this point it must also be the case that $\pi(\theta') \leq \pi^R(\theta')$ because of claim 1. For those types in $[\theta', \overline{\theta}]$, let us change the contract as follows: $p(\theta) = \alpha$ and $\pi(\theta) = \pi(\theta') + \pi^R(\theta) - \pi^R(\theta') \leq \pi^R(\theta)$. This change of contract strictly benefits the buyer. Moreover, bypass in case of rejection remains a credible threat as $\mathbb{E}\{\theta \mid \theta \in \mathcal{R} \cup [\theta', \overline{\theta}]\} > \mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} \geq \theta^B$. This contradicts the optimality of the contract under the minimal scenario.

3. The last result follows directly from Claims 1 and 2 and the fact that $\pi(\theta) - \pi^R(\theta)$ is convex.

Consider now program $P1$. Notice that incentive compatibility along with the last constraint and $\pi(\theta^D) = \pi^R(\theta^D)$ imply that $p(\theta) \leq \alpha$ for θ below θ^D. Let us solve the relaxed problem obtained by replacing the last two constraints by $p(\theta) \leq \alpha$ for $\theta \leq \theta^D$. Using $\pi(\theta) = \pi^R(\theta^D) + \int_{\theta^D}^{\theta} p(\theta)\, d\theta$ and Fubini theorem, standard computations lead to the following relaxed program:

$$\max_{p(.)} \int_{\underline{\theta}}^{\theta^D} p(\theta)F(\theta)d\theta - \int_{\theta^D}^{\overline{\theta}} p(\theta)(1-F(\theta))d\theta + \pi^R(\theta^D)$$

$$\text{subject to } \forall\theta : p(.) \text{ non-decreasing.}$$

$$\forall\theta \leq \theta^D : p(\theta) \leq \alpha.$$

It is straightforward to show that $p(.)$ must be constant. Hence, the buyer's problem can be further simplified as $\max p(\theta^D - \mathbb{E}\{\theta\})$ s. t. $p \leq \alpha$. If $\theta^D > \mathbb{E}\{\theta\}$, then the buyer cannot do better than with no contract at all so that no contract is an equilibrium. If $\theta^D < E\{\theta\}$, then the buyer chooses $p = 0$ and obtains more than with no contract. The equilibrium offer is then:

$$T(\theta) = \theta^D\alpha \text{ and } p(\theta) = 0$$

which is accepted by all θ. ■

Ratifiable Mechanims. To study the ratifiability of a decision rule, Cramton and Palfrey (1995) define the credible veto belief as a probability distribution μ on type and a continuation equilibrium $\sigma(\mu)$ of the default game such that, following a refusal,

1. some types may veto the decision rule with positive probability
2. the types that strictly benefit from the decision rule (resp. the default game) compared to the default game (resp. decision rule) equilibrium never (resp. always) veto
3. the distribution μ is derived using Bayes' rule.

In our paper, the decision rule is given by the offer $(T(\theta), p(\theta))$ and the continuation equilibrium (bypass or wait) depends on the expected type $\mathbb{E}_\mu\{\theta\}$ following rejection with

$$\sigma(\mu) = \begin{cases} \text{bypass} & \text{if } \mathbb{E}_\mu\{\theta\} > \theta^B \\ \text{wait} & \text{if } \mathbb{E}_\mu\{\theta\} < \theta^B \\ \in \{\text{bypass, wait}\} & \text{if } \mathbb{E}_\mu\{\theta\} = \theta^B \end{cases}.$$

We can also write the payoff of the seller in this continuation game

$$\pi(\sigma(\mu)) = \begin{cases} 0 & \text{if } \mathbb{E}_\mu\{\theta\} > \theta^B \\ \theta\alpha & \text{if } \mathbb{E}_\mu\{\theta\} < \theta^B \\ \in \{0,\ \theta\alpha\} & \text{if } \mathbb{E}_\mu\{\theta\} = \theta^B \end{cases}$$

and the payoff of the buyer by

$$U(\sigma(\mu)) = \max\{\mathbb{E}_\mu\{\theta\}U - \theta^B\alpha, \mathbb{E}_\mu\{\theta\}(U-\alpha)\}.$$

The credible veto belief associated to the contract offer can be of two types (assuming again the the buyer bypass when indifferent) :

1. $\mathbb{E}_\mu\{\theta\} \geq \theta^B$ and $\sigma =$ bypass. As all the types in the support of μ have zero profit in the default game, they must also have zero profit in the contract offer.

2. $\mathbb{E}_\mu\{\theta\} < \theta^B$ and $\sigma =$ wait. Any seller in the support of μ must get less than $\theta\alpha$ in the contract offer.

The first case is easy to analyze since it only requires rejection induces bypass and that the rejecting types of seller get $\pi(\theta) = 0$. There exists such credible veto belief is $\pi(\theta) = 0$ for some $\theta \geq \theta^B$.

Let us look more closely at the second case. By the same argument as the one used in the proof of proposition 1, for any decision rule that is IR for the buyer and does not coincide with the no contract allocation, $\{\theta|\pi(\theta) < \theta\alpha\}$ is a non-empty interval. Its closure coincides with the set $\mathcal{R}$ defined in section 3.1. As a consequence, for any veto belief associated with no bypass, we must have $\mathbb{E}_\mu\{\theta\} = \mathbb{E}\{\theta|\theta \in \mathcal{R}\}$. It follows that there exists a credible veto belief associated with no bypass if and only if $\mathbb{E}\{\theta|\theta \in \mathcal{R}\} < \theta^B$.

To sum up, for an IC-IR decision rule, there exists a credible veto belief if either $pi(\theta) = 0$ for some $\theta \geq \theta^B$ with $\pi(\theta) = 0$ or $\mathbb{E}\{\theta|\theta \in \mathcal{R}\} < \theta^B$.

In Cramton and Palfrey (1995), a decision rule is said to be ratifiable either if there is no credible veto belief or if there exists one such that the system of belief μ put positive weight only on types who gain the same payoff by vetoing or by accepting. In our model, the two cases of ratifiability can be expressed as

1. $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} \geq \theta^B$ and $\pi(\theta) > 0$ for $\theta \geq \theta^B$

2.1 either $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} \geq \theta^B$ and for all $\theta \in \mathcal{R}$, $\pi(\theta) = \theta\alpha$ (this corresponds to the allocation with no bypass)

2.2 or there exists a system of belief μ such that $\mathbb{E}_\mu\{\theta\} \geq \theta^B$ and $\pi(\theta) = 0$ on the support of μ.

Lemma 3 *An IC-IR decision rule is ratifiable either if* $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} \geq \theta^B$ *or if* $\pi(\theta) = 0$ *for at least one type* $\theta \geq \theta^B$.

Proof. Suppose that $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} \geq \theta^B$. Then, either $\pi(\theta) > 0$ for all types above θ^B and condition 1 applies or $\pi(\theta) = 0$ for some type above θ^B and 2.2 applies. Suppose instead that $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} < \theta^B$. Then, condition 2.2 must hold. This is only possible if there is some $\theta \geq \theta^B$ that belongs to the support of μ and thus that obtain zero profit. Conversely, suppose that $\pi(\theta') = 0$ for θ'^B. Then we can assume that μ puts all the weight on θ'. .

Strong ratifiability holds if either there is no credible veto belief or for any credible veto belief, the seller with θ in the support of μ gets zero profit. Following the same logic as above, we obtain.

Lemma 4 *An IC-IR decision rule is strongly ratifiable if and only if* $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} \geq \theta^B$

Proof. Suppose first that $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} < \theta^B$. Then there exists a credible belief such that the buyer does not bypass. As the decision rule does not coincide with the no contract allocation, there exists some types of seller who strictly prefers to veto under this veto belief. Thus the decision rule is not strongly ratifiable. Suppose now that $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} \geq \theta^B$. If $\pi(theta)^B > 0$, this is true for all $\theta > \theta^B$ and there is no credible veto belief.

If $\pi(\theta)^B = 0$, consider any credible belief μ. By definition of a credible belief, only type θ such that $\pi(\theta) = 0$ can belong to the support of μ which is consistent of strong ratifiability. Notice that this latter case is only possible if $\mathbb{E}\{\theta\} \geq \theta^B$. ■

Proof of Proposition 2. Note first that any equilibrium allocation must be incentive compatible and individually rational for the seller. Consider now the condition on v.

The necessity is immediate because the offer $\mathcal{C} = \{\theta^D\alpha + \varepsilon, 0\}$ is accepted by all types of sellers and the buyer can also make no offer. Thus the buyer can ensure an expected payoff at least equal to $\mathbb{E}\{\theta\} U - \min\{\theta^D, \mathbb{E}\{\theta\}\}\alpha$.

For sufficiency consider the initial stage. Assume that $v \geq v^m$. Define the contract $\bar{\mathcal{C}}$ as a contract implementing the candidate allocation (it exists since the allocation is incentive compatible). We build the equilibrium by choosing the last stage selection mapping as follows:

i) if $\mathcal{C} = \bar{\mathcal{C}}$, the seller accepts for all type and rejection triggers bypass;

ii) If $\mathcal{C} \neq \bar{\mathcal{C}}$ and $\mathbb{E}\{\theta \mid \theta \in \mathcal{R}\} \geq \theta^B$, then the seller accepts for all type and rejection triggers bypass;

iii) Any other offer $\mathcal{C}$ is rejected by the seller with types $\theta \in \mathcal{R}$ and no bypass follows a rejection.

Thus the last stage continuation equilibrium coincides with the minimal scenario for all contracts except $\bar{\mathcal{C}}$ which coincide with the maximal scenario. This implies that the maximal surplus that the buyer can expect by offering $\mathcal{C} \neq \bar{\mathcal{C}}$ is $\mathbb{E}\{\theta\} U - \min\{\theta^D, \mathbb{E}\{\theta\}\}\alpha$. Thus it is optimal to offer $\bar{\mathcal{C}}$. Then condition i) ensures that the seller accepts for all θ. Finally $\theta^B \leq \bar{\theta}$ implies that bypass is credible.

■

Proof of Proposition 3.

As the first part of the proposition has been shown in the main text, let us focus on the second part. Note first, considering equation 8, that $\hat{T}$ is increasing in λ. Indeed, as $\mathbb{E}\{\theta\} < \theta^B$, the transfer should be greater than $\alpha\underline{\theta}$ to make the threat credible. This implies that $\mathbb{E}\{\theta\} < \mathbb{E}\{\theta \mid \theta \geq \hat{T}\}$ so the LHS of equation 8 increases with λ. For given value of θ^B, any increase in the value of λ should be compensated by $\hat{T}(\lambda)$.

Note also that the buyer will never propose a transfer T larger than $\alpha\mathbb{E}\{\theta\}$ as this represents her expected payment if she waits ex-post until the uncertainty is realized. For any transfer $T < \alpha\mathbb{E}\{\theta\}$, the ex-post belief following a rejection is smaller than the belief is the offer were $\mathbb{E}\{\theta\}$. Suppose that for an offer $\mathbb{E}\{\theta\}$ and the associated rejection set $\mathcal{R}$, we have $E\{\theta \mid \theta \in \mathcal{R}\} = \theta^B$. This is the case for $\lambda = \underline{\lambda}$ (see equation 7) so there is no valuable offer for the buyer that could make her threat to bypass credible in case of rejection. As we show above that the transfer was decreasing in λ, there will be no offer for $\lambda \leq \underline{\lambda}$. For $\lambda \leq \underline{\lambda}$, the offer will be given by the minimal value necessary to make the threat of bypass credible, i.e. $\hat{T}$. ■

Proof of Lemma 2. As said in the text, there will be two subsets of type, one with an allocation different from the status quo allocation and the other with allocation equivalent to the status quo. Let us denote t the cut-off type between those two subsets. The problem faced by the buyer is then to choose t and the allocation $(p(.), \pi(.))$ for θ below t, which can be stated as follows:

$$\max_{\{p(.),\pi(.),t\}} \int_{\underline{\theta}}^{\theta^p} [\theta W(p(\theta)) + \theta p(\theta) D(p(\theta)) - \pi(\theta)] f(\theta)\, d\theta + \int_t^{\bar{\theta}} \theta W(\alpha) f(\theta)\, d\theta$$
$$\text{subject to: } \forall \theta \in [\underline{\theta}, t] : \dot{\pi}(\theta) = p(\theta) D(p(\theta)),$$
$$\pi(t) = \pi^R(t).$$

Notice that we do not include the constraints $\pi(\theta) \geq \pi^R(\theta)$ and $0 < p(\theta) < \alpha$ since they are implied by the last two constraints.

We solve this problem in two steps, first by looking at the interval $[\theta, t]$ and then optimizing with respect to t.

The first part is solved using Pontryagin Principle. We define the Hamiltonian of the problem, with μ the co-state variable, as:

$$H = [\theta W(p(\theta)) + \theta p(\theta) D(p(\theta)) - \pi(\theta)] f(\theta) + \mu(\theta)[p(\theta) D(p(\theta))],$$

Using the sufficient theorems for concave objectives derived by Seierstad and Sydsaeter (1977), the following conditions must hold:

- $p(.)$ should maximize H so the first-order condition is:

$$p(\theta) \in \arg\max_p \theta W(p) + \left(\theta + \frac{\mu(\theta)}{f(\theta)}\right) pD(p).$$

- $\dot{\mu} = -\frac{\partial L}{\partial \pi} = f(\theta)$.
- $\dot{\pi} = p(\theta) D(p(\theta))$.
- $\mu(\underline{\theta}) = 0$.

The conditions stated above imply $\mu(\theta) = F(\theta)$. The first condition then implies that $p(\theta) = \phi(\theta)$ where $\phi(\theta)$ is implicitly defined by:

$$\theta\phi(\theta) D'(\phi(\theta)) + \frac{F(\theta)}{f(\theta)}[D(\phi(\theta)) + \phi(\theta) D'(\phi(\theta))] = 0 \Leftrightarrow \varepsilon(\phi(\theta)) = \frac{1}{1 + \frac{\theta f(\theta)}{F(\theta)}}.$$

Since $\varepsilon(p)$ and $\frac{1}{1+\frac{\theta f(\theta)}{F(\theta)}}$ are non-decreasing, $\phi(\theta)$ is non-decreasing.

The second part of the proof consists in optimizing with respect to the cut-off type t. Using classical results in dynamic control (see Seierstad-Sydsaeter (1987, chapter 5, Theorem 17)), this cut-off is such that

$$\max_t \int_{\underline{\theta}}^{t} \left[\theta W(\phi(\theta)) + \theta\phi(\theta)D(\phi(\theta)) - \pi(\theta)\right] f(\theta)\, d\theta + \int_t^{\bar{\theta}} \theta W(\alpha) f(\theta)\, d\theta$$

The first derivative is given by:

$$tW(\phi(t) + t\phi(t)D(p(\theta^p)) - \pi(t) - tW(\alpha)$$

Since by continuity, $\pi(t) = t\alpha D(\alpha)$, it can be written as

$$t\left[W(\phi(t) + \phi(t)D(\phi(\theta^p)) - W(\alpha) - \alpha D(\alpha)\right]$$

Using the fact that $W(p) + pD(p)$ and $\phi(\theta)$ are monotonic, the objective is quasi-concave in t. Canceling the derivative leads to $\phi(t) = \alpha$ for an interior solution. The optimal solution, denoted θ^{**} is then $\theta^{**} = \phi^{-1}(\alpha)$ if it is less than $\overline{\theta}$, $\theta^{**} = \overline{\theta}$ otherwise (notice that $\phi(\underline{\theta}) = 0$ hence $\underline{\theta}$ cannot be solution for $\alpha > 0$).

The price $p(\theta)$ is continuous and monotonic, equal to $\min\{\phi(\theta), \alpha\}$. Thus, the incentive compatibility conditions are globally satisfied. Moreover, the profit $\pi(\theta)$ is larger than $\pi^R(\theta)$ which ensures that the participation constraints are also satisfied. Hence, the solution to the relaxed problem is the optimal allocation proposed by a buyer. ■

Proof of Proposition 5. The first step of the proof consists in characterizing the set of equilibrium allocation under the minimal scenario while the second step maximizing the buyer's payoff within this set.

Note first that, both with full and partial participation contract, the condition $\pi(\theta^D) > \pi^R(\theta^D)$ must hold. Indeed, in contract with partial participation, there is bypass when the contract is refused so any contract should given the sellers at least as much as the reservation utility. Consider now a contract with full participation. As the lower bound of Θ^R, denoted here θ^p, should be greater that θ^D, this implies that $\pi(\theta^D) \geq \pi^R(\theta^D)$. Consider the following lemma.

Lemma 5 *Consider the minimal scenario and take any allocation that is feasible and incentive compatible. Then there exists a contract offer by the buyer and a continuation equilibrium that implements this allocation if and only if* $\pi(\theta^D) \geq \pi^R(\theta^D)$*;*

Proof of Lemma 5. We have seen above that any contract under the minimal scenario corresponds to an allocation that is feasible, incentive compatible and $\pi(\theta^D) \geq \pi^R(\theta^D)$. Let us then show that the condition is sufficient.

Let us assume first that $\pi(\theta^D) > \pi^R(\theta^D)$. Then, consider the contract $\mathcal{C} = (p(\theta), \pi(\theta))$ with full participation and bypass out-of-equilibrium. This contract with full participation implements the allocation. Indeed, since $\pi(\theta^D) > \pi^R(\theta^D)$, for all $\theta \leq \theta^D$, $\pi(\theta) > \pi^R(\theta)$ so $\theta^P > \theta^D$. It is then direct to see that the contract is accepted by all and so implements the initial allocation.

Let us now assume that $\pi(\theta^D) = \pi^R(\theta^D)$ in the allocation. We define the cut-off τ by $\tau = \max\{\theta \mid p(\theta) < \alpha\}$.

Consider the case where $\tau > \theta^D$ then the arguments are similar as above. Indeed, since $p(\theta)$ is increasing by incentive compatibility, then for all $\theta < \theta^D < \tau$, we have $p(\theta) < \alpha$. Since the slope of $\pi(\theta)$ is increasing with $p(\theta)$, for $\theta < \theta^D$, $\pi(\theta) > \pi^R(\theta)$ so $\theta^D = \theta^P$ and the contract with full participation implements the allocation.

Now, consider the case where $\tau < \theta^D$. Notice that in this case, we have $p(\theta) = \alpha$ for all $\theta > \tau$ which implies that $\pi(\theta) = \pi^R(\theta)$ for all these types. Also, for the same reason as above, we have for $\theta < \tau$, $\pi(\theta) > \pi^R(\theta)$. As before, any contract that replicates the allocation for $\theta < \tau$ implements the allocation. For example, consider the contract $\mathcal{C}$ with profit schedule $\pi^C(\theta) = \pi(\theta)$ for all types. Then, $\theta^P = \tau$, $\mathcal{C}$ is accepted by types less then τ and the allocation is implemented.

Finally, consider the case where $\tau = \theta^D$. Then, combining the above reasoning, when the contract $\pi^C(\theta) = \pi(\theta)$ is offered for all types, then there are two continuation equilibria: one where all types accept and rejection triggers bypass and another one where only types below θ^D accept and there is no bypass in case of rejection. Thus the first equilibrium implements $\pi(\theta)$. ■

The previous lemma provides a set of allocations that may be obtained in equilibrium. We can thus search for the preferred allocation of the buyer within the set of allocations that satisfies the properties of the lemma. It amounts to finding the solution to the $(\boldsymbol{P})$. Notice that the constraints imply that $\pi(\theta) \geq 0$ for all θ (because the slope is less than α) and $\pi(\theta) \geq \pi^R(\theta^D)$ for θ above θ^D. Notice also that $\pi(\theta^D) = \pi^R\left(\theta^D\right)$ is the only binding constraint.

From Jullien (2000), Theorems 3 and 4 (adapting the constraint $q \geq 0$ to $p \leq \alpha$), the solution is characterized by

- $p\left(\theta\right)$ is continuous;
- $\gamma^*\left(\theta\right) = 0$ for $\theta < \theta^D$ and $\gamma^*\left(\theta\right) = 1$ for $\theta \geq \theta^D$
- There exists θ^P such that:

- $p(\theta) = \phi(\theta) < \alpha$ if $\theta < \theta^P$, $p(\theta) = p^* \leq \alpha$ if $\theta \geq \theta^P$
- When evaluated at p^*, if $p^* > 0$ for all $\tau \in \left[\theta^P, \bar{\theta}\right]$

$$\int_{\theta^P}^{\tau} \frac{\partial}{\partial p}\left(\frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} W(p) + (\theta + \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)}) p D(p)\right)_{p=p^*} f(\theta)\, d\theta \geq 0 \tag{A1}$$

- and if $p^* < \alpha$, for all $\tau \in \left[\theta^P, \bar{\theta}\right]$

$$\int_{\tau}^{\bar{\theta}} \frac{\partial}{\partial p}\left(\frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} W(p) + (\theta + \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)}) p D(p)\right)_{p=p^*} f(\theta)\, d\theta \leq 0. \tag{A2}$$

We write

$$\begin{aligned} & \frac{\partial}{\partial p}\left(\frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} W(p) + (\theta + \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)}) p D(p)\right) \\ = & \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} D(p) + (\theta + \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)}) p D'(p) \\ = & D(p)\left(\frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} - (\theta + \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)}) \varepsilon(p)\right). \end{aligned}$$

Consider the case where $\underline{\theta} < \theta^P < \bar{\theta}$. Continuity implies that $p^* = \phi(\theta^P) > 0$. Then

$$\begin{aligned} \frac{F(\theta)}{f(\theta)} - (\theta + \frac{F(\theta)}{f(\theta)}) \varepsilon(p^*) &> 0 \text{ for } \theta > \theta^P \text{ as } p^* < \phi(\theta) \\ \frac{F(\theta) - 1}{f(\theta)} - (\theta + \frac{F(\theta) - 1}{f(\theta)}) \varepsilon(p^*) &< 0 \text{ for } \theta > \theta^P \text{ as } \varepsilon(p^*) < 1 \end{aligned}$$

Thus the LHS of $(A1)$ is quasi concave in τ, while the LHS of $(A2)$ is quasi convex. The condition $(A1)$ then holds for all τ if it holds for $\tau = \bar{\theta}$:

$$\int_{\theta^P}^{\bar{\theta}} \left(\frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} - (\theta + \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)}) \varepsilon(p^*)\right) f(\theta)\, d\theta \geq 0;$$

while the condition $(A2)$ holds for all τ if

$$\int_{\theta^P}^{\bar{\theta}} \left(\frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} - (\theta + \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)}) \varepsilon(p^*)\right) f(\theta)\, d\theta \leq 0.$$

Thus we have condition (10). Notice that this implies $\theta^P < \theta^D$, since otherwise the integral is negative.

Similarly if $p^* = \alpha$ we have

$$\int_{\theta^{**}}^{\bar{\theta}} \left(\frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} - (\theta + \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)})\varepsilon(r) \right) f(\theta)\, d\theta \geq 0;$$

and for $p^* = 0$ we have

$$\int_{\underline{\theta}}^{\bar{\theta}} \left(\frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} \right) f(\theta)\, d\theta = \theta^D - \mathbb{E}(\theta) \leq 0.$$

■