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Abstract

In this paper, we study how access pricing affects network competition when subscription demand is elastic and each network uses non-linear prices and can apply termination-based price discrimination. In the case of a fixed per minute termination charge, we find that a reduction of the termination charge below cost has two opposing effects: it softens competition but helps to internalize network externalities. The former reduces mobile penetration while the latter boosts it. We find that firms always prefer termination charge below cost for either motive while the regulator prefers termination below cost only when this boosts penetration.

Next, we consider the retail benchmarking approach (Jeon and Hurkens, 2008) that determines termination charges as a function of retail prices and show that this approach allows the regulator to increase penetration without distorting call volumes.

Keywords: Mobile Penetration, Termination Charge, Access Pricing, Networks, Interconnection, Regulation, Telecommunications.

JEL numbers: D4, K23, L51, L96

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1 Introduction

In most countries, there are competing wireless telecommunication networks. Even though different operators may adopt different standards,[1] there is ubiquitous interconnection across different networks in that a customer can communicate with any other customer regardless of whether the latter subscribes to the same or to a rival network. This interconnection requires operators' agreement over termination charges: How much should operator A pay to operator B in case a call originating from network A terminates on network B and vice versa? Since this termination charge enters as a cost (for off-net calls) and as revenue (from termination of incoming calls), it affects competition in the retail market, which in turn determines the total number of subscribers. In this paper, we study the socially and privately optimal termination charges when subscription demand is elastic.

Although inelastic subscription demand is a standard assumption in the literature on two-way access pricing,[2] this assumption is often not satisfied even in developed countries. For instance, according to the latest statistics from International Telecommunication Union (ITU),[3] the mobile penetration rate (defined as mobile cellular subscribers per 100 inhabitants) in 2007 is 83.88 for Japan, 90.20 for Korea and 86 for USA.[4] In developing countries, the mobile penetration rate is low but growing fast: for instance, in Africa, the average penetration rate is 28.49 in 2007 but the number of subscribers increased by a factor 5 between 2002 and 2007. What is more striking is the comparison with the fixed phone lines. For instance, in Africa, the fixed phone penetration rate is extremely low and changes little (it increased from 2.69 to 3.21 between 2002 and 2007).[5] The overall trend in developing countries shows that fast growing mobile communications have overtaken and replaced the stagnating fixed phone communications as the main means of telecommunications.

The huge impact of mobile telecommunications on economic development leads us to raise the following questions. First, what is the socially optimal regulation of termination charges when one accounts for the social welfare gains generated by a boost in mobile penetration? Second, in the absence of regulation, what is the termination charge that firms

[1] This is the case in USA and India where there is no mandatory standard. For instance, in India, Reliance Communications, the industry number two, uses CDMA standard while its two state-run rivals, Bharat Sanchar Nigam and Mahanagar Telephone Nigam, use GSM standard (Financial Times, "Indian operators battle for 3G share", June 8 2009).

[2] This literature is about how termination charges affect competition among interconnected networks and the pioneers are Armstrong (1998) and Laffont, Rey and Tirole (1998a,b).

[3] See http://www.itu.int/ITU-D/icteye/Indicators/Indicators.aspx (accessed on June 17, 2009).

[4] The average penetration rate in Europe is 111.14 but this high rate is due to subscribers having multiple phone numbers.

[5] In India, the fixed phone penetration rate even decreased from 3.93 to 3.37 for the same period while the mobile subscribers increased by a factor 18 during the same period to 19.98 in 2007.

would agree on? Does the private incentive coincide with the social incentive in terms of choosing termination charge or boosting penetration rates? We address these questions in a setting with Calling-Party-Pays (CPP) principle[6] in which consumers' subscription demand is elastic and network operators compete with non-linear prices and can apply termination-based price discrimination. We study two different approaches to determine termination charges.

First, we consider the standard approach based on a fixed (per-minute) termination charge and study how the termination charge affects profits, penetration, and social welfare. We find that both the firms and the regulator want to depart from cost-based termination charge (and hence want to distort call volumes) in order to affect consumer subscription. In particular, a reduction in termination charge creates two opposing effects: It softens competition but it also helps to internalize network externalities. The former reduces mobile penetration while the latter expands it. Depending on which of the two effects dominates, there can be conflicts or alignments of interests between the firms and the regulator regarding whether they prefer termination charge below or above cost.

Second, we study the retail benchmarking approach that determines termination charges as a function of retail prices. We extend the approach from the setting without termination-based price discrimination and with inelastic subscription demand (considered in Jeon and Hurkens, 2008) to the setting with termination-based price discrimination and with elastic subscription demand. We show that for a given fixed (reciprocal) termination charge, we can find a family of access pricing rules that induce firms to charge on-net price equal to on-net cost and off-net price equal to off-net cost but the equilibrium fixed fee decreases with the strength of the feedback from the retail prices to access payment. The result implies that the regulator can increase consumer subscription without creating any distortion in call volumes. Our access pricing rules intensify retail competition since a firm can reduce its access payment to rival firms by reducing its average retail prices.

In the case of the standard approach, we extend the models of Gans and King (2001) and Calzada and Valletti (2008) (who consider inelastic subscription demand) and Dessein (2003) (who does not allow for termination-based price discrimination). Our innovation is to identify the interplay between the two opposing effects associated with a change in termination charge. When total subscription demand is inelastic, firms suffer from the usual business stealing effect and prefer termination charge below cost to soften competition in

[6]CPP is prevalent while Receiving-Party-Pays (RPP) is used in some countries such as the U.S.A. and Hong Kong. See Jeon-Laffont-Tirole (2004) and Cambini-Valletti (2008) for the analysis of RPP with inelastic subscription demand.

the case of termination-based price discrimination (Gans and King, 2001): when termination charge is lower than termination cost, on-net price is higher than off-net price and therefore consumers prefer to belong to the smallest network all other things being equal, which reduces firms' incentive to steal customers. Therefore, firms prefer termination charge below cost whereas the regulator prefers termination charge equal to cost. Recently, Calzada and Valletti (2008)[7] and Armstrong and Wright (2009)[8] find a similar result.[9]

When total subscription demand is elastic, on top of the business stealing effect firms suffer from a network externality effect. In order to isolate the effect of internalizing network externalities from the competition-softening effect, we first study a benchmark of "two interconnected islands" in which each island is occupied by a monopolist facing an elastic subscription demand. There is no competition between the two monopolists since consumers cannot move from one island to the other. In this benchmark, when a monopolist attracts an additional customer, he creates positive externalities to the other monopolist since the latter's consumers can enjoy off-net calls to the additional customer. Since the two monopolists fail to fully internalize these externalities, the total number of subscribers is smaller than the number when both islands are occupied by one identical monopolist. We find that termination charge below cost, by increasing the degree of interconnection, helps them to internalize better the network externalities and thus expands market penetration.[10] When consumers enjoy off-net calls, firms realize that raising one's fixed fee reduces the size of the other network and thus hurts its own customers. This negative feedback on one's own customers increases with the degree of interconnection.

It is useful to note that competing firms would like to choose termination charge in order

[7] They consider a Logit model with inelastic subscription (i.e. full subscription) while Gans and King (2001) consider the Hotelling model with inelastic subscription.

[8] They consider an extension of the Hotelling model that allows for elastic demand. Although they focus on the case in which the fixed to mobile (FTM) termination charge and the mobile to mobile (MTM) termination charge are the same due to arbitrage, when they study the case in which the two can be separately chosen, they find that firms prefer the MTM charge below cost (due to competition-softening effect à la Gans-King) while the regulator prefers the charge above cost.

[9] Very recently, Jullien, Rey, and Sand-Zantman (2009) extend LRT (1998a, b)'s model of inelastic participation by adding "pure receivers" who get a constant utility from receiving calls (independently of the volume of calls) and have elastic participation. They find that in the absence of termination-based price discrimination, this induces firms to prefer termination charge above cost because of a competition-softening effect: losing a caller to a rival network increases the profit that the original network makes from terminating calls on its pure receivers. They find that this result extends to the case with termination-based price discrimination if there are many pure receivers with elastic subscription demand.

[10] To some extent, this effect is similar to the result of Katz and Shapiro (1985) that an increase in compatibility among competing networks increases the total number of subscribers. However, in their paper, firms are engaged in Cournot competition and the cost of achieving compatibility is a fixed cost and hence does not directly affect the retail competition (i.e. only demand increasing effect of compatibility remains). In our model, firms compete with non-linear tariffs and interconnection is mediated by the access charge that directly affects retail competition through off-net price: in particular, reducing access charge below cost distorts off-net call volume.

to make the outcome closer to the outcome that a monopolist owning both networks would choose. When termination charge is equal to termination cost and subscription demand is elastic, there are two possible cases: market penetration under duopoly can be either higher or lower than the one under monopoly. In the first case, firms want to decrease market penetration (and thus, consumer surplus) while, in the second case, they want to increase market penetration (or consumer surplus). A rather surprising result is that in both cases, firms prefer having termination charge below cost. The reason is that consumer surplus is larger under duopoly than under monopoly exactly when the business stealing effect dominates the network externality effect so that firms prefer to soften competition, which requires a low termination charge. Consumer surplus is lower under duopoly than under monopoly exactly when the network externality effect dominates the business stealing effect. Thus firms prefer to internalize the network effect better, which again requires termination charge below cost. The regulator always prefers larger consumer surplus so that in the first case it prefers to strengthen competition by means of a termination charge above cost, while in the second case it favors a termination charge below cost (in fact, in this case, the socially optimal access charge is lower than the one preferred by the firms).

Our result in the standard approach based on a fixed termination charge is reminiscent of Dessein (2003)'s finding. Dessein considers a setting without termination-based price discrimination and with elastic demand and shows that firms again prefer to have a termination charge below cost while the regulator prefers a termination charge above cost when the business stealing effect dominates. However, since without termination-based price discrimination, termination charges do not affect profits in the extreme case of inelastic subscription,[11] he does not clearly disentangle the two opposing effects as we do. In Dessein's model, both business stealing and network externality effects are present. However, without termination-based price discrimination it is not clear how a change in termination charge can soften competition or help to internalize the network externality. In contrast, with termination-based price discrimination, there is only a business stealing effect in the extreme case of competition with inelastic subscription (as in Gans and King (2001)) while there is only a network externality effect in the other extreme case of two interconnected islands (*i.e.*, no competition with elastic subscription).

Most of the literature on two-way access pricing considers the termination charge as a fixed (per minute) price.[12] In this paper we depart from this approach and are inter-

[11] This is the so-called profit neutrality result: Laffont-Rey-Tirole (1998a), Dessein (2003), Hahn(2004).

[12] See, for instances, Armstrong (1998), Armstrong and Wright (2009), Calzada and Valletti (2008), Cam-

ested in applying a retail benchmarking approach to determine termination charges. In Jeon and Hurkens (2008) we introduced this approach successfully in a framework without termination-based price discrimination and with inelastic subscription demand. We showed that when networks compete in linear prices, by choosing the benchmarking rule appropriately, a regulator can achieve the Ramsey outcome, without knowing demand function. We also showed that when networks compete with non-linear prices, our approach allows to achieve both static efficiency (marginal cost pricing) and dynamic efficiency (optimal investment). The approach also allows to increase consumer surplus by inducing lower fixed fees. However, if firms compete in non-linear prices and subscription demand is inelastic, lower fixed fees do not increase social welfare. To maximize social welfare it is enough to set the termination charge equal to cost. In the present paper, with elastic subscription demand, the retail benchmarking approach may be more useful since it induces lower fixed fees which increases penetration, which in turn affects social welfare.

We show that for a given fixed (reciprocal) termination charge, we can find a family of access pricing rules parameterized by $\kappa(\leq 1)$ such that all the access pricing rules in the family induce firms to charge on-net price equal to on-net cost and off-net price equal to off-net cost but the equilibrium fixed fee decreases with κ where $\kappa = 0$ corresponds to the standard approach based on the fixed termination charge. The result implies that the regulator can increase mobile penetration without creating any distortion in call volumes. The regulator may also use the rule to maintain the number of subscribers (and therefore consumer surplus) and to reduce the distortion in call volume. Such a rule would improve efficiency and increase profits. Furthermore, we show that when the regulator and the firms have the same information about demand and cost structure, there is a simple modification of our rules that achieves the Ramsey outcome (i.e. firms charge all prices including the fixed fee just at costs and consumer subscription is maximized).

The rest of the paper is organized as follows. In section 2, we introduce the Logit model formulation of network competition with elastic subscription demand,[13] explain how rational consumer expectations are formed and describe the Ramsey benchmark. Expectations about network size are important since consumers care about the size of each network. In section 3 we characterize the unique symmetric equilibrium in case of a fixed per minute termination

bini and Valletti (2008), Carter and Wright (1999, 2003), Dessein (2003, 2004), Gans and King (2000, 2001), Hahn (2004), Hermalin and Katz (2001, 2004), Laffont, Marcus, Rey and Tirole (2003), Laffont, Rey and Tirole (1998a,b), Jeon, Laffont and Tirole (2004), Valletti and Cambini (2005) and Wright (2002).

[13] For an introduction of logit models see Anderson and de Palma (1992) and Anderson, de Palma and Thisse (1992).

charge close to cost. In order to disentangle the business stealing effect from the network externality effect, we also analyze a model of two interconnected monopolistic networks. In section 4 we introduce the retail benchmarking approach and show that it outperforms any rule based on fixed (per minute) termination charges. In section 5 we show that, if the regulator has the same information as the networks, a minor modification of the benchmarking rule induces the Ramsey outcome. Section 6 concludes. The Appendix collects some proofs.

2 The model

Our model is standard and identical to that of Laffont-Rey-Tirole (1998b) except the elastic subscription demand for which we use the Logit specifications. After presenting the model, we describe rational expectations and the Ramsey outcome.

2.1 The Logit Model

We consider competition between two mobile operators.[14] Each firm i ($i = 1, 2$) charges a fixed fee F_i and may discriminate between on-net and off-net calls. Firm i's marginal on-net price is written p_i and its off-net price is written $\hat{p}_i$. The total mass of consumers is normalized to 1. Consumer's utility from making calls of length q is given by a concave and increasing utility function $u(q)$. Demand $q(p)$ is defined by $u'(q(p)) = p$. The marginal cost of a call equals c and the termination cost equals $c_0 (\leq c)$. The reciprocal access price (equivalently, termination charge) is denoted a. Therefore, the marginal cost of on-net calls is c while that of off-net calls is $\hat{c} = c + a - c_0$. Networks incur a fixed cost of f per subscriber. We define $v(p) = u(q(p)) - pq(p)$. Note that $v'(p) = -q(p)$. We also make the standard assumption of a balanced calling pattern, which means that the fraction of calls originating from a given subscriber of a given network and completed on another given (including the same) network is equal to the fraction of subscribers to the terminating network.

The timing of the game is as follows:

First, a reciprocal access price (= termination charge) a is chosen either by the firms or by the regulator. Second, each firm i (= 1, 2) chooses simultaneously retail tariffs $T_i = (F_i, p_i, \hat{p}_i)$. Third, consumers form expectations about the number of subscribers of each network i (β_i) with $\beta_1 \geq 0$, $\beta_2 \geq 0$ and $\beta_1 + \beta_2 \leq 1$ and make subscription decisions. We will write $\beta_0 = 1 - (\beta_1 + \beta_2)$ for the number of consumers expected to remain unsubscribed.

[14] We consider neither the fixed phone networks nor the calls between the fixed phone networks and the mobile phone networks.

We consider rational expectations equilibria (see the next subsection for details). This implies that all consumers will have the same expectations. Given such expectations, utility from subscribing to network 1 equals

$$V_1 = \beta_1 v(p_1) + \beta_2 v(\hat{p}_1) - F_1,$$

while subscribing to network 2 yields

$$V_2 = \beta_2 v(p_2) + \beta_1 v(\hat{p}_2) - F_2.$$

Finally, not subscribing at all yields utility V_0.

Define $U_1 = V_1 + \mu\varepsilon_1$, $U_2 = V_2 + \mu\varepsilon_2$, and $U_0 = V_0 + \mu\varepsilon_0$. The parameter $\mu > 0$ reflects the degree of product differentiation in a Logit model. A high value of μ implies that most of the value is determined by a random draw so that competition between the firms is rather weak. The noise terms ε_k are random variables of zero mean and unit variance, identically and independently double exponentially distributed. They reflect consumers' preference for one good over another. A consumer will subscribe to network 1 if and only if $U_1 > U_2$ and $U_1 > U_0$; he will subscribe to network 2 if and only if $U_2 > U_1$ and $U_2 > U_0$; he will not subscribe to any network otherwise. The probability of subscribing to network i is denoted by $\wp_i$ (where $\wp_0$ represents the probability to remain unsubscribed). This probability is given by

$$\wp_i = \frac{\exp[V_i/\mu]}{\sum_{k=0}^{2} \exp[V_k/\mu]}. \tag{1}$$

Expectations are rational if $\wp_i = \beta_i$ for all i. For given rational expectations, the profit of network i is equal to

$$\Pi_i = \wp_i \left[\wp_i (p_i - c) q(p_i) + \wp_j (\hat{p}_i - \hat{c}) q(\hat{p}_i) + F_i - f \right] + \wp_i \wp_j (a - c_0) q(\hat{p}_j).$$

The first term reflects the retail profit made on subscribers while the second term reflects the wholesale profit from termination of incoming calls.

Consumer surplus in the Logit model has been derived by Small and Rosen (1981) as (up to a constant)

$$CS = \mu \ln \left(\sum_{k=0}^{2} \exp(V_k/\mu) \right) = V_0 - \mu \ln(\wp_0), \tag{2}$$

where the right-hand side follows from (1). Clearly, consumer surplus is increasing in market penetration $1 - \wp_0$.

2.2 Rational Expectations

As stated before, for expectations to be rational, $\beta_i = \wp_i$ must hold for all i. For any price schedules (T_1, T_2), such self-fulfilling expectations exist as these are fixed points of the mapping $\wp : \Delta^2 \to \Delta^2$ where $\wp(\beta_1, \beta_2) = (\wp_1, \wp_2)$. We can show:

Lemma 1 *For any price schedules (T_1, T_2), rational expectations $(\wp_1, \wp_2)$ are uniquely defined as long as μ is sufficiently high.*

Proof. See Appendix. ■

If μ is relatively low (that is, operators are highly substitutable) and rational expectations are not uniquely defined, one can potentially construct many equilibria by having even the tiniest of deviations lead to expectations (and thus subscriptions) that jump discontinuously, in the direction that makes such deviations unprofitable. We find it reasonable to restrict attention to equilibria where rational expectations are continuous functions of the tariffs. Uniqueness of continuous rational expectations requires a milder condition on μ.

In particular, let expectation β be rational when tariffs (T_1, T_2) are chosen, where $T_i = (F_i, p_i, \hat{p}_i)$ for $i = 1, 2$. Let $T_1' = (F_1', p_1', \hat{p}_1')$ be an alternative tariff with small deviations in usage prices and fixed fee such that

$$\beta_1 v(p_1) + \beta_2 v(\hat{p}_1) - F_1 = \beta_1 v(p_1') + \beta_2 v(\hat{p}_1') - F_1'.$$

That is, the alternative tariff yields exactly the same utility if expectations are not changed. Then it is clear that expectation β is also rational when tariffs (T_1', T_2) are chosen. The restriction of continuous rational expectations then implies that expectations must remain fixed when tariffs are changed locally such that utility for its subscribers remains constant, given these expectations. By repeatedly applying the same argument one can see that continuous rational expectations will remain fixed when p_1 is changed into c and $\hat{p}_1$ is changed into $\hat{c}$, as long as F_1 is changed into $F_1' = \beta_1(v(c) - v(p_1)) + \beta_2(v(\hat{c}) - v(\hat{p}_1)) + F_1$. The same can be done with prices of network 2. In the next section we use this argument to establish the perceived marginal cost pricing principle, which says that it is optimal for each firm to set usage prices at perceived marginal cost. Now, given that firms use perceived marginal cost pricing, the uniqueness of rational expectations is guaranteed if termination charge is close to termination cost and $\mu > v(c)/4$, which we will assume.

Assumption 1

$$\mu > v(c)/4. \tag{3}$$

Lemma 2 *If firms set usage prices equal to perceived marginal cost, rational expectations are unique if $a \approx c_0$ and Assumption 1 holds.*

Proof. See Appendix.

It will be convenient for the subsequent analysis to establish the relation between fixed fee F and the number of subscribers per firm $\wp$ in a symmetric equilibrium candidate in which firms price usage at perceived marginal cost, $(F, c, \hat{c})$. From (1) and the assumption of rational expectations one obtains immediately:

$$F = \wp(v(c) + v(\hat{c})) - \mu \ln\left(\frac{\wp}{1 - 2\wp}\right) - V_0. \tag{4}$$

2.3 Ramsey outcome

Consider now the Ramsey outcome defined as the outcome that maximizes social welfare under the break-even constraint of each firm. First, because of symmetry, we can look for the outcome among symmetric tariffs $T = (F, p, \hat{p})$. Second, given $T = (F, p, \hat{p})$, maximizing social welfare requires each firm to realize zero profit. Otherwise, the social planner can further decrease F, which increases social welfare because it increases the number of subscribers and therefore creates positive network externalities on the existing subscribers. Third, given the binding zero profit constraint, maximizing consumer surplus (2) is equivalent to maximizing $V(= V_1 = V_2)$, which requires marginal cost pricing ($p = \hat{p} = c$) and $F = f$. Therefore, the Ramsey outcome is characterized by

$$p = \hat{p} = c,\ F = f.$$

Note that in the case of inelastic subscription, the Ramsey outcome is characterized by $p = \hat{p} = c$ and $F \geq f$ since the fixed fee does not affect social welfare. With elastic subscription, because of the positive network externalities, the social planner wants to increase the number of subscribers as long as the break-even constraint is satisfied: in fact, the first best outcome requires the social planner to subsidize each firm because of network externalities. Namely, the social planner would like to set usage prices equal to marginal cost and choose fixed fee F and per firm number of subscribers $\wp$ as to maximize the sum of consumer surplus and industry profit,

$$V_0 - \mu \ln(1 - 2\wp) + 2\wp(F - f),$$

subject to the rational expectations condition (4). It is easily verified that this requires $F < f$.

3 Competition with a fixed per minute termination charge

In this section, we analyze the case with a constant reciprocal termination charge a.

3.1 Preliminary results

3.1.1 Marginal cost pricing

Let $R(p) = (p-c)q(p)$. Although the number of subscribers $\wp_i$ and $\wp_j$ depend on V_0 and tariff schedule T_1, T_2. we will omit arguments for expositional simplicity. Profit can be written as follows

$$\Pi_i = \wp_i[\wp_i R(p_i) + \wp_j R(\hat{p}_i) + F_i - f] + \wp_i \wp_j (a - c_0)(q(\hat{p}_j) - q(\hat{p}_i)). \tag{5}$$

Firm i maximizes profits by setting T_i, holding T_j constant. Note that a change in marginal price p_i or $\hat{p}_i$ while holding F_i fixed will affect not only the number of i's subscribers but also that of j's subscribers. For example, a decrease in F_i will make network i more attractive and will thus attract some subscribers of j and will also attract some consumers who previously did not subscribe to any network. This then makes it also more attractive to subscribe to network j relative to staying unsubscribed, because of the network effect. It will be convenient to apply a change of variables and let network i maximize profits by choosing p_i, $\hat{p}_i$, and $\wp_i$, holding p_j, $\hat{p}_j$ and F_j fixed. This can be done because of the assumption of continuous rational expectations, as explained in the previous section.

Note that from (1) one immediately deduces that

$$\frac{\wp_i}{1 - \wp_i} = \frac{\exp[V_i/\mu]}{\exp[V_j/\mu] + \exp[V_0/\mu]}. \tag{6}$$

From (6), one has

$$F_i = \wp_i v(p_i) + \wp_j v(\hat{p}_i) - \mu \log \left[\frac{\wp_i}{1 - \wp_i} (\exp[V_j/\mu] + \exp[V_0/\mu]) \right].$$

Holding everything but p_i and F_i fixed, one obtains

$$\partial F_i / \partial p_i = \wp_i v'(p_i).$$

Similarly, holding everything but $\hat{p}_i$ and F_i fixed, one obtains

$$\partial F_i / \partial \hat{p}_i = \wp_j v'(\hat{p}_i).$$

Note that if p_i is changed while keeping $\wp_i$, $\hat{p}_i$, p_j, $\hat{p}_j$ and F_j fixed, then also $\wp_j$ will remain fixed. Maximizing Π_i with respect to on-net price p_i (keeping $\wp_i$ fixed) thus yields

$$0 = \frac{\partial \Pi_i}{\partial p_i} = \wp_i^2 (R'(p_i) + v'(p_i)) = \wp_i^2 (p_i - c) q'(p_i).$$

Hence, $p_i = c$. In words, on-net calls are priced at marginal cost.

Maximizing Π_i with respect to off-net price $\hat{p}_i$ (keeping $\wp_i$ fixed) yields

$$0 = \frac{\partial \Pi_i}{\partial \hat{p}_i} = \wp_i \wp_j (R'(\hat{p}_i) + v'(\hat{p}_i) - (a - c_0) q'(\hat{p}_i)) = \wp_i \wp_j (\hat{p}_i - c - a + c_0) q'(\hat{p}_i).$$

Hence, $\hat{p}_i = c + a - c_0 \equiv \hat{c}$. In words, off-net calls are priced at perceived marginal cost (i.e. the off-net marginal cost). We thus obtain the standard "perceived" marginal cost pricing result under non-linear pricing as in LRT (1998b).

Summarizing, we have:

Proposition 1 *Under the assumption of continuous rational expectations, it is optimal for each firm to price on-net call at the marginal cost* (c) *and off-net call at the perceived off-net marginal cost* ($c + a - c_0$).

In the sequel we will write $v = v(c)$ and $\hat{v} = v(\hat{c})$ to reduce notation.

3.1.2 Net business stealing vs. net network externality

For the equilibrium analysis and for comparative statics exercises we will perform later, it will be necessary to know how the number of subscribers to one of the networks changes when fixed fees are varied. Note that an increase in the fixed fee of network 1, *everything else equal*, will decrease the number of subscribers to network 1 and will increase the subscribers to network 2. However, a change in F_1 also affects rational expectations. In particular, consumers will realize that network 1 will become smaller and that may make network 2 also less attractive. So an increase in the fixed fee of network 1 could potentially reduce the number of subscribers to network 2. The following lemma describes exactly when this happens.

Lemma 3 *Suppose networks use tariffs $T_1 = (F_1, c, \hat{c})$ and $T_2 = (F_2, c, \hat{c})$. Then*

$$\frac{\partial \wp_1}{\partial F_1} = -\frac{\wp_1}{d\mu^2}((1-\wp_1)\mu - \wp_2(1-\wp_1-\wp_2)v) \tag{7}$$

and

$$\frac{\partial \wp_2}{\partial F_1} = -\frac{\wp_1\wp_2}{d\mu^2}(\hat{v}(1-\wp_1-\wp_2)-\mu) \tag{8}$$

where

$$d\mu^2 = \mu^2 + \mu[(\wp_1^2 - \wp_1)v + (\wp_2^2 - \wp_2)v + 2\wp_1\wp_2\hat{v}] + \wp_1\wp_2(1-\wp_1-\wp_2)(v^2-\hat{v}^2).$$

If a is close to c_0 and Assumption 1 holds, then $d > 0$ and

$$\frac{\partial \wp_1}{\partial F_1} < 0, \quad \frac{\partial \wp_1}{\partial F_1} + \frac{\partial \wp_2}{\partial F_1} < 0$$

and

$$\frac{\partial \wp_2}{\partial F_1} > 0 \quad \textit{if and only if} \quad \nabla = \mu - \hat{v}(1-\wp_1-\wp_2) > 0.$$

Proof. See Appendix for the proof.

When one firm increases its fixed fee, it loses subscribers such that total market penetration ($\wp_1 + \wp_2$) decreases. However, whether the rival firm loses or gains subscribers depends on the sign of $\nabla = \mu - v(\hat{c})\wp_0$. If $\nabla > 0$, the rival firm gets more subscribers while if $\nabla < 0$, it gets less subscribers. In what follows, we will say that there is *a net business stealing effect* if $\nabla > 0$ and there is *a net network externality effect* if $\nabla < 0$. For instance, in the extreme case of inelastic and full subscription (i.e. $\wp_1 + \wp_2 = 1$), we have $\nabla = \mu > 0$ since there is only a business stealing effect: all consumers who leave firm 1 subscribe to firm 2. In general, when subscription is elastic, there exists a network externality effect since an increase in the fixed fee of firm 1 reduces the total number of subscribers ($\wp_1 + \wp_2$), which reduces the utility from subscribing to any given network. If the network externality effect dominates the business stealing effect, the number of subscribers of firm 2 decreases as firm

1 increases its fixed fee.[15]

3.1.3 Unique symmetric equilibrium

Under the perceived marginal cost pricing ($p_1 = p_2 = c$ and $\hat{p}_1 = \hat{p}_2 = \hat{c}$), profits can be rewritten as

$$\Pi_i = \wp_i[\wp_j R(\hat{c}) + F_i - f].$$

Thus

$$\frac{\partial \Pi_i}{\partial F_i} = \frac{\partial \wp_i}{\partial F_i}[\wp_j R(\hat{c}) + F_i - f] + \wp_i[\frac{\partial \wp_j}{\partial F_i} R(\hat{c}) + 1].$$

So the first order condition reads

$$0 = \frac{\partial \wp_i}{\partial F_i}(F_i - f) + \wp_i + R(\hat{c})\left(\wp_j \frac{\partial \wp_i}{\partial F_i} + \wp_i \frac{\partial \wp_j}{\partial F_i}\right).$$

Solving for a symmetric solution, and using the marginal effects on the number of subscribers of networks 1 and 2 with respect to a change in the fixed fee of network 1 derived in Lemma 3, yields

$$F - f = \frac{-\wp - R(\hat{c})\frac{\wp^2(-1+2\wp)}{\mu - \wp(1-2\wp)(v+\hat{v})}}{\frac{\partial \wp_i}{\partial F_i}}.$$

This can be manipulated to yield $F = F^{equil}(\wp, a)$ where

$$F^{equil}(\wp, a) := f + \frac{\mu - (1-2\wp)\wp(v + \hat{v} + R(\hat{c}))}{(1-\wp)\mu - \wp(1-2\wp)v}(\mu - \wp(v - \hat{v})) \tag{9}$$

On the other hand, rational expectations, by means of equation (4), need to be satisfied. We define:

$$F^{RE}(\wp, a) := \wp(v + \hat{v}) - V_0 - \mu \ln\left[\frac{\wp}{1-2\wp}\right]. \tag{10}$$

[15] More precisely, an increase in F_1 induces some subscribers of 1 to switch to firm 2 and some others to become unsubscribed. If consumers do not immediately adjust their expectations, the proportion of all the consumers that leave firm 1, and then go to firm 2 (rather than becoming unsubscribed), is proportional to β_2/β_0. Once consumers realize that the value of subscription is reduced because there are more unsubscribed consumers, some of the subscribers of firm 2 will become unsubscribed. So firm 2 gains some subscribers due to the business stealing effect, but also loses some subscribers due to the network externality effect. Clearly, if β_0 is relatively large, a relatively large fraction of consumers leaving firm 1 will become unsubscribed, so that the network externality effect becomes large. In order to see whether the net effect is positive or negative, note that, in the Logit model $\log(\beta_2) - \log(\beta_0) = (V_2 - V_0)/\mu$. This implies that (when all usage prices are c)

$$\beta_2 = \beta_0 \exp[((1-\beta_0)v - F_2 - V_0)/\mu].$$

The derivative of the right-hand side with respect to β_0 equals $(1 + \beta_0(-v/\mu))\exp[((1-\beta_0)v - F_2 - V_0)/\mu]$, which is positive if and only if $\nabla = \mu - (1 - \beta_1 - \beta_2)v > 0$.

The equilibrium number of subscribers per firm is thus found by solving $F^{equil}(\wp, a) = F^{RE}(\wp, a)$. We will denote this solution by $\wp(a)$. In particular, for $a = c_0$ the solution is given by

$$\left[f + \mu \frac{\mu - 2\wp(1-2\wp)v}{(1-\wp)\mu - \wp(1-2\wp)v}\right] - \left[2\wp v - V_0 - \mu \ln\left(\frac{\wp}{1-2\wp}\right)\right] = 0.$$

It can be shown (using Assumption 1) that there is a unique solution $\wp^* = \wp(c_0)$ to this equation. There will then also be a unique solution for $a \neq c_0$ for small enough $|a - c_0|$. Moreover, $F^{RE}(\wp, c_0) > F^{equil}(\wp, c_0)$ if and only if $\wp < \wp^*$. That is, the rational expectations curve cuts the equilibrium curve from above. Fig. 1 illustrates these findings for a given access charge a.

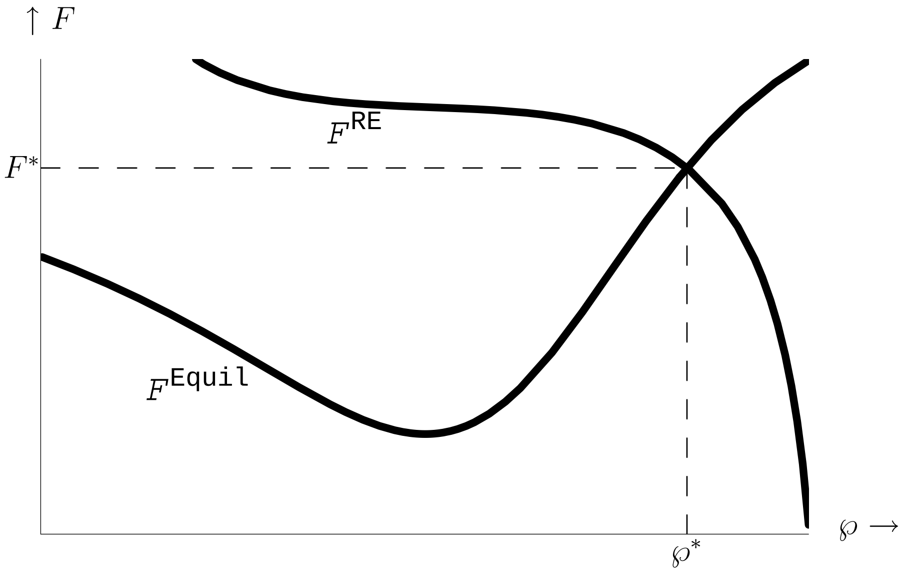

Figure 1: Symmetric equilibrium.

Proposition 2 *Under Assumption 1, for $|a - c_0|$ small enough, there exists a unique symmetric equilibrium $(F, p, \hat{p})$. This solution is given by $p = c$, $\hat{p} = \hat{c}$ and $F = F^{RE}(\wp(a), a)$.*

We will be particularly interested in how profits, consumer surplus and social welfare vary with a. It turns out that analyzing these effects is not straightforward since there are two opposing effects at work. First, firms may want to use the termination charge to soften price competition to raise fixed fees and profits. This is the only force at work in Gans and King (2001) where subscription demand is inelastic. However, in the Logit model with elastic subscription demand there is a second force at work, namely network externalities. Firms may have a common incentive to increase market penetration as this increases the value of subscription to each customer. Note that the second force works against the first one since softening competition would cause a reduction in the number of subscribers. It is not obvious which of the two effects dominates. Moreover, in case the network externality effect dominates, it is also not clear whether firms or the regulator would like to increase or decrease the termination charge. Therefore, in what follows, we first analyze an extreme case of two interconnected monopoly networks facing elastic subscription demand; this case allows us to isolate the network externality effect. After that we turn to the case of competing networks facing elastic subscription demand.

3.2 Benchmark: two interconnected monopolists

There are two islands and firm i $(= 1, 2)$ operates only in island i. Each island has a population normalized to 1/2. Inhabitants of an island cannot (or simply do not want to) subscribe to the operator of the other island. Hence, the two firms do not compete for the same customers. However, the inhabitants care indirectly about the pricing policy of the monopolist on the other island since it affects subscription rates and thus affects how many calls can be made to its subscribers.

As before, given retail tariffs, consumers form expectations over the number of subscribers of network 1 (β_1) and network 2 (β_2), with $\beta_i \geq 0$ and $\beta_1 + \beta_2 \leq 1$. Given such expectations, utility from subscribing to network i (for inhabitants of island i) equals V_i where V_1, V_2, V_0 are defined as before. Define $U_1 = V_1 + \mu\varepsilon_1$, $U_2 = V_2 + \mu\varepsilon_2$, and $U_0 = V_0 + \mu\varepsilon_0$. Consumers from island $i = 1, 2$ subscribe (to network i) when $U_i > U_0$ and remain unsubscribed otherwise. The number of subscribers on island i now equals

$$\tilde{\wp}_i = \frac{1}{2} \times \frac{\exp[V_i/\mu]}{\exp[V_i/\mu] + \exp[V_0/\mu]}, \tag{11}$$

since inhabitants of island i can only choose between subscribing (to network i) or remaining unsubscribed.[16] Rational expectations imply $\tilde{\wp}_i = \beta_i$. In the Appendix we show how rational expectations are affected by a marginal change in firm 1's fixed fee, at a symmetric equilibrium candidate $(\tilde{F}, c, \hat{c})$:

Lemma 4 *In the Logit model with two interconnected monopolists, at a symmetric equilibrium candidate $(\tilde{F}, c, \hat{c})$ with number of subscribers per firm equal to $\tilde{\wp}$, we have*

$$\frac{\partial \tilde{\wp}_1}{\partial F_1} = \frac{-1}{\tilde{d}\mu^2} \tilde{\wp}((1 - 2\tilde{\wp})(\mu - \tilde{\wp}(1 - 2\tilde{\wp})v) < 0 \tag{12}$$

[16] In this subsection on interconnected monopolists we use tildes to distinguish the symbols from the case of interconnected duopoly, whenever they are different.

and

$$\frac{\partial\tilde{\wp}_2}{\partial F_1} = \frac{-1}{\tilde{d}\mu^2}\tilde{\wp}^2(1-2\tilde{\wp})^2\hat{v} < 0. \tag{13}$$

where

$$\tilde{d} = [\mu - \tilde{\wp}(1-2\tilde{\wp})(v+\hat{v})][\mu - \tilde{\wp}(1-2\tilde{\wp})(v-\hat{v})]/\mu^2.$$

Proof. See Appendix for the proof. ■

Hence an increase of the fixed fee of firm 1 definitively results in a decrease of the number of subscribers of firm 2. This is the network externality effect.

Given $p_1 = p_2 = c$ and $\hat{p}_1 = \hat{p}_2 = \hat{c} := c + a - c_0$, profits can be rewritten as

$$\Pi_i = \tilde{\wp}_i[\tilde{\wp}_j R(\hat{c}) + \tilde{F}_i - f].$$

So the first order condition reads

$$0 = \frac{\partial\tilde{\wp}_i}{\partial\tilde{F}_i}(\tilde{F}_i - f) + \tilde{\wp}_i + R(\hat{c})\left(\tilde{\wp}_j\frac{\partial\tilde{\wp}_i}{\partial\tilde{F}_i} + \tilde{\wp}_i\frac{\partial\tilde{\wp}_j}{\partial\tilde{F}_i}\right).$$

Solving for a symmetric solution, using Lemma 4, yields

$$\tilde{F} - f = \frac{-\tilde{\wp} - R(\hat{c})\frac{-\tilde{\wp}^2(1-2\tilde{\wp})}{\mu-\tilde{\wp}(1-2\tilde{\wp})(v+\hat{v})}}{\frac{\partial\tilde{\wp}_i}{\partial\tilde{F}_i}}.$$

This can be manipulated to yield $\tilde{F} = \tilde{F}^{equil}(\tilde{\wp}, a)$ where

$$\tilde{F}^{equil}(\tilde{\wp}, a) = f + \frac{\mu - (1-2\tilde{\wp})\tilde{\wp}(v + \hat{v} + R(\hat{c}))}{(1-2\tilde{\wp})(\mu - \tilde{\wp}(1-2\tilde{\wp})v)}(\mu - \tilde{\wp}(1-2\tilde{\wp})(v-\hat{v})). \tag{14}$$

It is readily verified that the right-hand side of this equation is decreasing in a at $a = c_0$:

$$\frac{\partial\tilde{F}^{equil}(\tilde{\wp}, a)}{\partial a}|_{a=c_0} = -\tilde{\wp}q(c)\frac{\mu - 2\tilde{\wp}(1-2\tilde{\wp})v}{\mu - \tilde{\wp}(1-2\tilde{\wp})v}.$$

To have rational expectations fulfilled in this two island model, we obtain from (11)

$$\tilde{F}^{RE} = \tilde{\wp}(v+\hat{v}) - V_0 - \mu\ln\left[\frac{2\tilde{\wp}}{1-2\tilde{\wp}}\right]. \tag{15}$$

Note that, at $a = c_0$, the right-hand side of this equation is decreasing in a:

$$\frac{\partial\tilde{F}^{RE}(\tilde{\wp}, a)}{\partial a}|_{a=c_0} = -\tilde{\wp}q(c).$$

Hence, a marginal increase of a above c_0 makes the rational expectations curve drop by more than the equilibrium condition curve. This means that the number of subscribers goes down when a is increased above c_0. This is illustrated in Fig. 2 below.

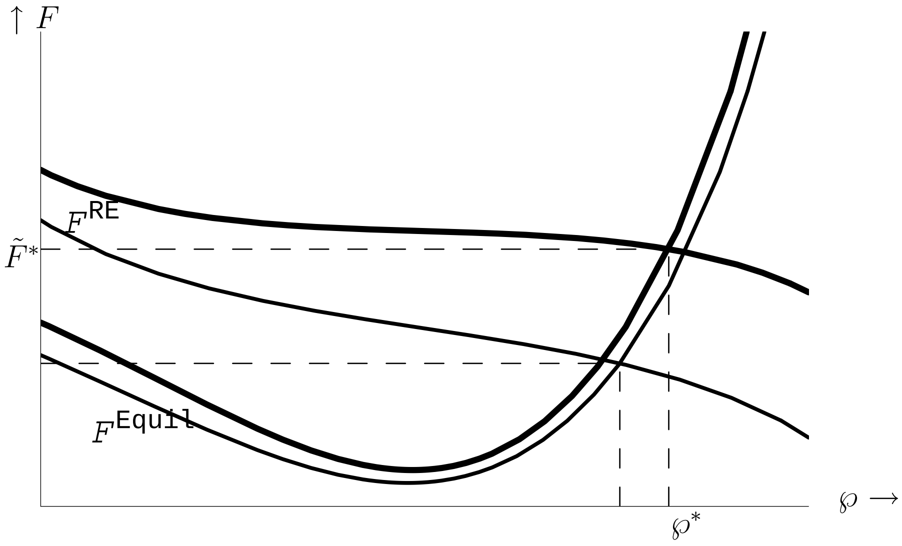

Figure 2: An increase in a above c_0 leads to lower market penetration.

Lemma 5 *In the Logit model with two interconnected monopolists, an increase in the termination charge above* c_0 *lowers overall market penetration and equilibrium fixed fees.*

So if firms want to increase market penetration, they want termination charge below cost. The intuition is that firms realize that raising one's fixed fee reduces the size of the other network and thus hurts its own customers. However, they fail to internalize the fact that this also hurts the other network, and therefore they set a too high fixed fee. By having $a < c_0$, the value of making off-net calls is higher. This means that subscribers of a given network care more about the size of the other network. An increase in the fixed fee of network 1 will now thus reduce the size of the other network more than when $a = c_0$, which in turn hurts 1's own consumers more than when $a = c_0$. Hence, letting $a < c_0$ exacerbates the negative effect of raising one's fee on its own subscribers and firms therefore lower the fixed fee and this increases market penetration. In other words, $a < c_0$ induces them to better internalize network externalities.

The above Lemma suggests that firms would prefer an access charge below termination as this increases market penetration and equilibrium fixed fees. The following proposition makes this formal.

Proposition 3 *In the Logit model with two interconnected monopolists, firms prefer access fee $a < c_0$. This also improves consumer surplus.*

Proof. See Appendix for the proof. ■

To provide the intuition for the result, it is useful to note that the two monopolists would like to have the number of subscribers closer to the one that would be chosen by an integrated monopolist operating in both islands. Given that the two monopolists do not fully internalize positive network externalities, they end up having a number of subscribers smaller than the one chosen by an integrated monopolist. Therefore, they want to increase the subscribers by choosing an access charge below the termination cost.

3.3 Interconnected duopoly

We now return to the case of competing interconnected duopolists. As explained before, the case of interconnected duopolists exhibits both network externalities and business stealing effects. We here analyze the effect of a change in termination charge around c_0 for profits, consumer surplus and social welfare.

We first analyze how an increase in a effects market penetration. Let us define $h(\wp, a) = F^{equil}(\wp, a) - F^{RE}(\wp, a)$.

$$h(\wp, a) = \frac{\mu - (1-2\wp)\wp(v + \hat{v} + R(\hat{c}))}{(1-\wp)\mu - \wp(1-2\wp)v}(\mu - \wp(v - \hat{v})) - \wp(v + \hat{v}) + V_0 + f + \mu \ln\left[\frac{\wp}{1-2\wp}\right].$$

We have already established that there is a unique solution $\wp(a)$ of $h(\wp, a) = 0$. Moreover, at the solution $h_\wp > 0$. Hence,

$$\wp'(a) = -\frac{h_a}{h_\wp};$$

$\wp'(a)$ and h_a have opposite signs. We have

$$\frac{\partial h(\wp, a)}{\partial a}|_{a=c_0}(\wp, c_0) = \frac{\wp^2 q(c)}{(1-\wp)\mu - \wp(1-2\wp)v}((1-2\wp)v - \mu).$$

We conclude that for $\nabla^* = \mu - (1-2\wp^*)v > 0$ an increase in a above c_0 will increase the equilibrium number of subscribers, while for $\nabla^* < 0$ such an increase in a results in a decrease in the equilibrium number of subscribers.

Lemma 6 *Let* $\nabla^* = \mu - (1-2\wp^*)v$.

$$\frac{d\wp^*(a)}{da}|_{a=c_0} > 0 \textit{ if and only if } \nabla^* > 0$$

and

$$\frac{d\wp^*(a)}{da}|_{a=c_0} < 0 \text{ if and only if } \nabla^* < 0.$$

If $\nabla^* < 0$, then $\partial\wp_2/\partial F_1 < 0$: the network externality effect dominates the business stealing effect. Therefore, as in the case of two interconnected monopolists facing elastic subscription demand, a decrease in a below c_0 increases the equilibrium number of subscribers by inducing the firms to internalize better the network externality. On the contrary, if $\nabla^* > 0$, then $\partial\wp_2/\partial F_1 > 0$: the business stealing effect dominates the network externality effect. Then, as in the case of competing duopoly facing inelastic subscription demand, a decrease in a below c_0 decreases the equilibrium number of subscribers by softening competition. Therefore, one would expect that firms would prefer termination charge below cost for opposite reasons: in order to boost market penetration for $\nabla^* < 0$ and in order to reduce market penetration for $\nabla^* > 0$. We now proceed to verify that indeed firms always prefer below cost termination charges.

Let $H(a,\wp)$ denote the profit a firm makes when it has $\wp$ subscribers, access charge is a and its fixed fee is $F^{RE}(\wp, c_0)$. That is

$$H(a,\wp) = \wp(\wp R(\hat{c}) + F - f) = \wp(\wp(R(\hat{c}) + v(c) + v(\hat{c})) - V_0 - \mu\log[\wp/(1-2\wp)] - f).$$

We will be interested in knowing what happens with this profit at $a = c_0$ when $\wp$ is moved away from the corresponding equilibrium value $\wp^*$. Note that we know that per consumer profit at the equilibrium equals $F - f$, which by (9) equals (at $a = c_0$)

$$\mu\frac{\mu - 2\wp^*(1-2\wp^*)v}{(1-\wp^*)\mu - \wp^*(1-2\wp^*)v}.$$

Hence,

$$\begin{aligned}
\frac{\partial H}{\partial\wp}(c_0,\wp^*) &= H(c_0,\wp^*)/\wp^* + \wp^*\left[2v(c) - \frac{\mu}{\wp^*(1-2\wp^*)}\right] \\
&= \mu\frac{\mu - 2\wp^*(1-2\wp^*)v}{(1-\wp^*)\mu - \wp^*(1-2\wp^*)v} + 2\wp^*v - \frac{\mu}{1-2\wp^*} \\
&= \frac{-\wp^*(\mu-(1-2\wp^*)v)(\mu - 2\wp^*(1-2\wp^*)v)}{(1-2\wp^*)((1-\wp^*)\mu - \wp^*(1-2\wp^*)v)}.
\end{aligned}$$

Therefore, if Assumption 1 is satisfied, the sign of this derivative is opposite to the sign of $\nabla^* = \mu - (1-2\wp^*)v$. Thus, if an increase of a above c_0 increases (decreases) market penetration, profits decrease (increase) with the number of subscribers *along* the rational expectations curve, in a neighborhood around c_0.

Next, we have to account for the fact that when a is varied, the rational expectations curve, and thus the equilibrium, will change. The partial effect on profits (keeping $\wp^*$ fixed) equals

$$\frac{\partial H}{\partial a} = (\wp^*)^2(a - c_0)q'(\hat{c}),$$

so that at $a = c_0$ a marginal change in a does not affect profits directly. The extra profit made through access revenues is just offset by the decrease in fixed fee. However, profits are affected indirectly by a change in market penetration.

$$\frac{dH}{da} = H_a + H_\wp \times \wp'(a).$$

Since the sign of $H_\wp$ is the opposite of the sign of $\wp'(a)$ at $a = c_0$, one observes that profits are always decreasing in a in a neighborhood around c_0. Firms thus always prefer a termination charge below cost.

Proposition 4 *Firms prefer a termination charge below cost $a < c_0$.*

How does social welfare change when the access charge is changed? Social welfare is the sum of consumer surplus and industry profit. The expression for consumer surplus in the Logit model has been derived by Small and Rosen (1981) as (up to a constant)

$$CS(a, \wp(a)) = \mu \ln\left(\sum_{j=0}^{2} \exp(V_j/\mu)\right) = V_0 - \mu \ln(1 - 2\wp(a)).$$

Hence, consumer surplus is not directly affected by the access charge, but only through the equilibrium number of subscribers. Clearly, consumer surplus is increasing in the number of subscribers:

$$\frac{\partial\, CS}{\partial \wp} = \frac{2\mu}{1 - 2\wp} > 0.$$

$$SW(a, \wp(a)) = CS(a, \wp(a)) + 2H(a, \wp(a)).$$

We thus obtain, at $a = c_0$,

$$\begin{aligned} \frac{d\ SW}{d\ a} &= \wp'(a)CS_\wp + 2H_a + 2H_\wp\wp'(a) \\ &= \wp'(c_0)\left(\frac{2\mu}{1 - 2\wp} + 2\frac{-\wp(\mu - (1 - 2\wp)v)(\mu - 2\wp(1 - 2\wp)v)}{(1 - 2\wp)((1 - \wp)\mu - \wp(1 - 2\wp)v)}\right) \end{aligned}$$

It can be established that the term in brackets is strictly positive when $\mu > v/4$. This means that social welfare increases as market penetration increases.

Proposition 5 *Let $\wp^*$ denote the number of subscribers per firm in the equilibrium when $a = c_0$. If $\mu > (1 - 2\wp^*)v$, the number of subscribers, and thus social welfare, increases as a is increased above c_0. If $\mu < (1 - 2\wp^*)v$, the number of subscribers, and thus social welfare, increases as a is decreased below c_0: in this case, the socially desirable access charge is lower than the one that maximizes industry profit.*

Both the firms and the regulator want to divert from the termination charge equal to termination cost in order to affect the number of subscribers. The firms want to make the number of subscribers closer to the number chosen by a monopolist owning both networks while the regulator always wants to increase the number. When there is a net business-stealing effect (i.e. $\nabla^* > 0$), there is a conflict of interest between the firms and the regulator since the firms want to decrease the number of subscribers, which requires them to choose a below c_0 in order to soften competition. When there is a net network externality effect (i.e. $\nabla^* < 0$), there is a congruence of interests between the firms and the regulator in the sense that firms want to increase the number of subscribers, which again requires them to choose a below c_0 in order to internalize network externalities. However, the firms do not internalize the positive effect that an increase in their network size has on consumer surplus and therefore the socially preferred access charge is lower than the one preferred by the firms.

Of course, in order to choose between $a > c_0$ and $a < c_0$, the regulator must possess some information about the demand side such that she is able to determine whether or not $\mu > (1 - 2\wp^*)v$ holds.

To conclude this section, we illustrate the results obtained by means of numerical examples. We assume the same demand functions and cost parameters (in cents) as de Bijl and Peitz (2004) and López and Rey (2009): $q(p) = (20 - p)^2/0.015$, $c_0 = 0.5$, $c = 2$, $f = 1000$, $\mu = 3000$. Note that $v(c) = 10800$ so that $v(c)/2 > \mu > v(c)/4$.

In the first case (left panel) we assume $V_0 = 5000$. This means the outside option is relatively attractive and network effects will be important. In this case, for $a = c_0$, the (symmetric) equilibrium has $F \approx 2372$ and total market penetration $2\wp^* \approx 0.58$. So $\nabla^* = \mu - (1 - 2\wp^*)v(c) \approx -1517 < 0$ and the network effect dominates.

The graphs illustrate that (i) market penetration decreases with access charge, (ii) industry profits are maximized at $a = 0$, (iii) social welfare is maximized at $a = 0$ (Bill and Keep).

In the second case (right panel) we assume $V_0 = 0$. In this case, for $a = c_0$, the (symmetric) equilibrium has $F \approx 5859$ and total market penetration $2\wp^* \approx 0.86$. So $\nabla^* = \mu - (1 - 2\wp^*)v(c) \approx 1535 > 0$ and the business stealing effect dominates.

The graphs illustrate that (i) market penetration increases with access charge, (ii) industry profits are maximized at $a = 0 < c_0$ (Bill and keep), (iii) social welfare is maximized at about $a = 5.4$.

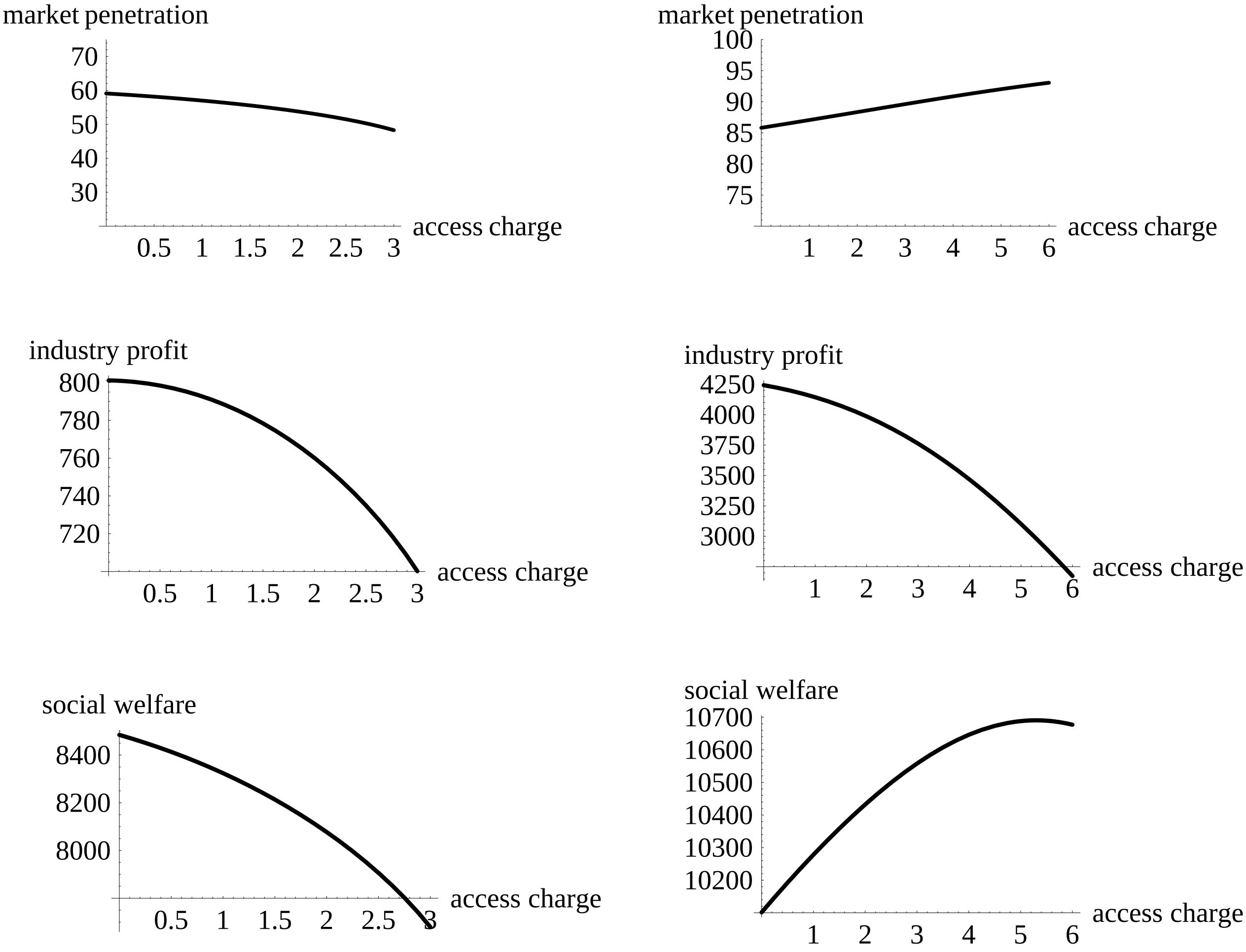

Figure 3: Illustration of the effect of access charges when network effects (left panel) or business stealing effect (right panel) are important.

4 Retail Benchmarking Approach

In this section, we generalize the retail benchmarking approach introduced in Jeon and Hurkens (2008). Jeon and Hurkens (2008) consider the case without termination-based price discrimination and with inelastic subscription of all consumers and find a class of access pricing rules parameterized by κ that achieves the marginal cost pricing (i.e. the call charge equal to c). We generalize the previous result in three dimensions. First, we allow for termination-based price discrimination. Second, we consider a (Logit) model with elastic subscription demand. Third, in this setting, for a given fixed access charge a, we find a class of access pricing rules parameterized by κ that induces each network to choose the on-net price equal to the on-net marginal cost and the off-net price equal to the off-net marginal cost.

Before generalizing the retail benchmarking approach, we remind the regulator's information constraint and the result from Jeon and Hurkens (2008).

4.1 Assumption and Result from Jeon and Hurkens (2008)

We maintain the same constraint on the regulator's information as in our previous paper:

• *The regulator's informational constraint*: On the one hand, we assume that the regulator has limited information about the market such that she is not informed about the individual demand function $q(p)$, each firm's subscription demand function and the value of the fixed cost f. On the other hand, she knows the marginal cost c and the termination cost c_0. Furthermore, she and consumers observe retail prices $[(p_1, \widehat{p}_1, F_1), (p_2, \widehat{p}_2, F_2)]$. Moreover, we need to assume that the regulator can observe average retail prices,[17] which means that she is able to observe realized demand.

The firms are assumed to know all the relevant information regarding both the demand and the cost sides as in Jeon and Hurkens (2008).

In a model without termination-based price discrimination and inelastic subscription of all consumers, we find that the following family of access pricing rules parameterized by $\kappa(< 1)$ induces each firm to adopt the marginal cost pricing (i.e. $p_i = c$):

$$a_i = c_0 + \kappa \left(\frac{F_i + p_i q(p_i)}{q(p_i)} - c \right), \tag{16}$$

where a_i represents the access charge that firm i pays to each rival firm. $\kappa = 0$ corresponds to the fixed access charge equal to the termination cost. According to the rule, the mark-up of the access charge that firm i pays to each rival firm is equal to the firm i's average price mark-up multiplied by κ. We find that the retail benchmarking rule intensifies retail competition such that higher values of κ translate into lower equilibrium fixed fee. However,

[17] For instance, the Spanish telecommunication agency (Comisión del Mercado de las Telecomunicaciones) publishes data on each network's average price.

when all consumers subscribe to one of the two networks, the level of fixed fee does not affect social welfare and choosing termination charge equal to termination cost is enough to achieve the Ramsey outcome.

4.2 Generalization

Consider a given fixed and reciprocal access charge a that can be different from c_0. Let $\pi_i(a)$ denote network i's retail profit per customer gross of the fixed cost;

$$\pi_i(a) \equiv \wp_i(p_i - c)q(p_i) + \wp_j(\widehat{p}_i - (c + a - c_0))q(\widehat{p}_i) + F_i$$

Therefore, given the fixed access charge a, network i's profit is given by

$$\Pi_i = \wp_i \left[\pi_i(a) + \wp_j(a - c_0)q(\widehat{p}_j) - f\right]$$

We remind from the previous sections that in this case, network i finds it optimal to choose $p_i = c$ and $\widehat{p}_i = c + a - c_0$.

We now propose the following generalization of our access pricing rule: *in addition to paying the fixed (per-minute) access charge a,* network i pays an access charge a_i determined by

$$a_i q(\widehat{p}_i) = \kappa \pi_i(a).$$

so that total termination charge that network i pays equals

$$\lambda(a, \kappa) := a + \kappa \frac{\pi_i(a)}{q(\widehat{p}_i)}. \tag{17}$$

Under our generalized access pricing rule, network i's profit is given by:

$$\Pi_i = \wp_i \left[\pi_i(a) + \wp_j(a - c_0)q(\widehat{p}_j) - f\right] + \wp_i\wp_j \left[-a_i q(\widehat{p}_i) + a_j q(\widehat{p}_j)\right].$$

which is equal to

$$\Pi_i = \wp_i \left[\pi_i(a) + \wp_j(a - c_0)q(\widehat{p}_j) - f\right] - \kappa\wp_i\wp_j \left[\pi_i(a) - \pi_j(a)\right]. \tag{18}$$

The second term of the R.H.S. of equation (18) shows well that our access pricing rules adds a sort of competition between the two firms in terms of the profit per customer $\pi_i(a)$ such that the firm which extracts more (less) surplus from consumers is punished (rewarded) with the additional net access payment (revenue). The intensity of this competition increases with κ. Rearranging (18) gives

$$\Pi_i = \wp_i \left[(1-\kappa\wp_j)\pi_i(a) - f\right] + \kappa\wp_i\wp_j \left[\pi_j(a) + (a-c_0)q(\widehat{p}_j)\right]. \tag{19}$$

As in the previous sections, we can apply a change of variables and let network i maximize profits by choosing p_i, $\widehat{p}_i$ and $\wp_i$, holding p_j, $\hat{p}_j$ and F_j fixed. Note that if p_i is changed while keeping $\wp_i$, $\hat{p}_i$, p_j, $\hat{p}_j$ and F_j fixed, then also $\wp_j$ will remain fixed. Note also that in a Logit model, we always have $\wp_i < 1$ and $\wp_j < 1$. Therefore, for $\kappa \leq 1$, maximizing Π_i with respect to on-net price p_i (keeping $\wp_i$ fixed) is equivalent to maximizing π_i. In other words, as long as $\kappa \leq 1$, κ does not affect the optimal choice of p_i. Since we know from the previous sections that network i chooses $p_i = c$ when $\kappa = 0$, network i chooses $p_i = c$ for $\kappa \leq 1$. For the same reason, network i chooses $\widehat{p}_i = c + a - c_0$ for $\kappa \leq 1$. Therefore, the class of access pricing rules induces networks to charge on-net price (off-net price) equal to on-net marginal cost (off-net marginal cost).

We now study the equilibrium fixed fee. From $p_i = c$ and $\widehat{p}_i = c + a - c_0$, we have $\pi_i(a) = F_i$. Then, (18) becomes

$$\Pi_i = \wp_i \left[F_i + \wp_j A(a) - f\right] - \kappa\wp_i\wp_j \left[F_i - F_j\right] \tag{20}$$

where $A(a) \equiv (a-c_0)q(c+a-c_0)$. The second term of the R.H.S. of equation (20) clearly shows that our access pricing rule creates extra competition in terms of fixed fee: the firm charging a higher (lower) fixed fee is punished (rewarded) with the additional net access payment (revenue). Rewriting equation (20) yields

$$\Pi_i = \wp_i \left[F_i - \wp_j \left[\kappa\left(F_i - F_j\right) - A(a)\right] - f\right].$$

The first order derivative with respect to F_i is given by:

$$\frac{\partial\wp_i}{\partial F_i}\left[F_i - \wp_j\left[\kappa\left(F_i - F_j\right) - A(a)\right] - f\right] + \wp_i\left[(1-\kappa\wp_j) - \left[\kappa\left(F_i - F_j\right) - A(a)\right]\frac{\partial\wp_j}{\partial F_i}\right].$$

Solving for a symmetric solution yields:

$$F = f - \wp A(a) - \frac{\wp}{\frac{\partial\wp_i}{\partial F_i}}\left[(1-\kappa\wp) + A(a)\frac{\partial\wp_j}{\partial F_i}\right]. \tag{21}$$

From $\frac{\partial \wp_i}{\partial F_i} < 0$, F decreases with κ when a is close to c_0. This is very intuitive since from (20), the extra competition in terms of fixed fee, generated by our access pricing rule, becomes more intensive as κ increases.

The equilibrium number of subscribers per firm is thus found by solving the system of equations (21) and (10).

Proposition 6 *Consider the retail benchmarking rules $\lambda(a, \kappa)$ defined by (17). Then, for $|a - c_0|$ small and any $\kappa \leq 1$,*

1. Each firm chooses on-net price equal to on-net marginal cost (c) and off-net price equal to off-net marginal cost ($c + a - c_0$).

2. The symmetric equilibrium is characterized by (21) and (10). In the equilibrium, the fixed fee strictly decreases with κ.

Corollary 1 *From a social welfare point of view, the retail benchmarking approach dominates the approach using a fixed reciprocal access charge for two reasons.*

1. For a given fixed access charge, it is possible to increase the number of subscribers by increasing κ from zero.

2. While in the case of fixed access charge, a distortion in off-net price is necessary to increase the number of subscribers, in the case of retail benchmarking, it is possible to increase the number of subscribers (by increasing κ from zero) while maintaining both on-net and off-net call prices equal to the marginal cost c.

Basically, our retail benchmarking rule creates an extra policy instrument that is absent in the fixed access charge approach. Namely, the regulator can increase the intensity of retail competition by increasing κ the intensity of the feedback from retail prices to access payment.

When $a = c_0$, (21) becomes

$$F = f - \frac{\wp(1 - \kappa\wp)}{\frac{\partial \wp_i}{\partial F_i}}.$$

At symmetric equilibrium, we have

$$\frac{\partial \wp_i}{\partial F_i} = -\frac{\wp}{\mu} \frac{(1 - \wp)\mu - \wp(1 - 2\wp)v}{\mu - 2\wp(1 - 2\wp)v}.$$

Therefore, we have

$$F = f + \frac{\mu(1 - \kappa\wp)(\mu - 2\wp(1 - 2\wp)v)}{(1 - \wp)\mu - \wp(1 - 2\wp)v}. \tag{22}$$

Hence, $F > f$ for $\kappa \leq 1$ as long as $F > f$ for $\kappa = 0$, which suggests that each firm realizes a positive profit for $\kappa \leq 1$. On the other hand, (10) is given by

$$F = 2\wp v - V_0 - \mu \ln \left[\frac{\wp}{1 - 2\wp} \right]. \tag{23}$$

Note that (23) does not depend on κ. The equilibrium $(F, \wp)$ is determined by the two equations (22) and (23). Clearly, as κ increases, the equilibrium $(F, \wp)$ moves down following the curve of (23) and therefore, the fixed fee decreases while the number of subscribers increases.

We also note that even though the regulator does not know whether $\mu < (1-2\wp^*)v$ holds or not, she can choose some $k \in (0, 1]$ to expand the number of subscribers. By contrast, in the case of the standard approach based on a fixed access charge, the regulator must know whether $\mu < (1 - 2\wp^*)v$ holds or not in order to choose between $a > c_0$ and $a < c_0$.

Furthermore, when there is a net network externality effect, it is also in the interest of firms to have the retail benchmarking approach with $a = c_0$ and with $\kappa > 0$. More precisely, when $a = c_0$, as the equilibrium fixed fee decreases with κ, there is $\kappa^m > 0$ such that the equilibrium fixed fee for $\kappa = \kappa^m$ is exactly equal to the fixed fee that would be chosen by a monopolist owning both networks. On the other hand, if there is a net business stealing effect, firms would like to have an access charge $\tilde{a} < c_0$ in order to soften competition. However, there exists a retail benchmarking rule that allows the firms to make even higher profits but leave consumer surplus unaffected. Namely, there exists a retail benchmarking rule $\lambda(a', \kappa)$ with $\tilde{a} < a' < c_0$ and $\kappa < 0$ such that the rule leads to exactly the same market penetration and consumer surplus as the fixed access charge $\tilde{a}$. Since the retail benchmarking rule leads to less distorted call volumes than the fixed access charge $\tilde{a}$, it is more efficient, leads to higher social welfare and thus also to higher profits. Therefore,

Corollary 2 *(i) When $\mu < (1-2\wp^*)v$, firms prefer the retail benchmarking approach (17) with $a = c_0$ and $\kappa = \min\{\kappa^m, 1\}$ to any fixed access charge.*

(ii) When $\mu > (1 - 2\wp^*)v$, firms prefer some fixed access charge $\tilde{a} < c_0$ to $a = c_0$ in order to soften competition. However, there exists a retail benchmarking rule $\lambda(a', \kappa)$ with $\tilde{a} < a' < c_0$ and $\kappa < 0$ such that it leads to exactly the same market penetration and consumer surplus as the fixed access charge $\tilde{a}$. The retail benchmarking rule leads to less distorted call volumes and thus to higher profits (and higher social welfare).

4.3 Interpretation of the retail benchmarking rule

We now provide an economic interpretation of our access pricing rule. For this purpose, we consider $a = c_0$. Then, (17) is equivalent to

$$a_i - c_0 = \kappa(\wp_i + \wp_j)\left[s_i \frac{q_i^{on}}{q_i^{off}}(AP_i^{on} - c) + s_j(AP_i^{off} - c)\right] + \kappa(1 - \wp_i - \wp_j)\frac{F_i}{q_i^{off}} \tag{24}$$

where

$$s_i = \frac{\wp_i}{\wp_i + \wp_j}, AP_i^{on} = \frac{p_i q_i^{on} + F_i}{q_i^{on}}, AP_i^{off} = \frac{\widehat{p}_i q_i^{off} + F_i}{q_i^{off}}.$$

In other words, s_i is firm i's market share and AP_i^{on} (respectively, AP_i^{off}) is firm i's average on net price (off-net price).

To explain the rule (24), we consider some specific cases. First, without termination-based price discrimination and with full subscription of all consumers (i.e. $\wp_i + \wp_j = 1$), we are back to the rule (16) that we considered in Jeon and Hurkens (2008). Therefore, (24) generalizes (16) in two directions: termination-based price discrimination and partial participation.

Second, under full subscription but with termination-based price discrimination, (24) becomes

$$a_i - c_0 = \kappa\left[s_i \frac{q_i^{on}}{q_i^{off}}(AP_i^{on} - c) + s_j(AP_i^{off} - c)\right]$$

In other words, our rule linearly links the access charge mark up to a weighted average retail price mark up in which the average price is a weighted sum of on-net average price and off-net average price and the weights depend on market shares (and are equal to market shares if $q_i^{on} = q_i^{off}$).

Third, under partial participation but without termination-based price discrimination, (24) becomes

$$a_i - c_0 = \kappa\left[(\wp_i + \wp_j)\frac{p_i q_i + F_i - c q_i}{q_i} + (1 - \wp_i - \wp_j)\frac{F_i}{q_i}\right]$$

Still our rule linearly links the access charge mark up to a weighted average retail price mark up in which the weights used are the fraction of subscribers and the fraction of non-subscribers. For the subscribers, the average retail price markup is computed as usual $(p_i q_i + F_i - c q_i)/q_i$; for the non-subscribers, the average retail price markup is given by putting the volume equal to zero in the numerator of the previous formula.

5 Retail Benchmarking Approach and the Ramsey Outcome

In this section, we assume that both the regulator and the firms have the same information (i.e. all of them know demand and cost structures) and show that there is a simple modification of our access pricing rule (17) that achieves the Ramsey outcome as a Nash equilibrium. Our aim is not so much to promote this modified access pricing rule but to illustrate the power of the retail benchmarking approach with respect to the approach based on a fixed access charge. Note that the Ramsey outcome is achieved when the firms charge the prices equal to the costs (i.e. $p_i = \widehat{p}_i = c$, $F_i = f$ for $i = 1, 2$) and this outcome can never be achieved under the approach based on a fixed access charge.

Let $\wp^R$ be each network's number of subscribers in the Ramsey outcome. In a Logit model with duopoly, we have

$$0 < \wp^R < 1/2.$$

Since the regulator knows demand and cost structure, the regulator knows $\wp^R$. Define κ^* by $1 - \kappa^* \wp^R = 0$; hence $\kappa^* > 2$. Let π_i denote network i's retail profit per customer gross of the fixed cost when $a = c_0$;

$$\pi_i \equiv \wp_i(p_i - c)q(p_i) + \wp_j(\widehat{p}_i - c)q(\widehat{p}_i) + F_i.$$

Then, for $a = c_0$, the access pricing rule (17) is given by

$$(a_i - c_0)\, q(\widehat{p}_i) = \kappa \pi_i \tag{25}$$

We modify it as follows:

$$(a_i - c_0)\, q(\widehat{p}_i) = \kappa^* \max\{\pi_i, f\} \tag{26}$$

In (26), we choose $\kappa = \kappa^*$ and add the max operator such that firm i cannot realize any further reduction of its access payment by pricing below costs. If i's access payment does not depend on its retail prices, firm i has no incentive to choose retail prices that give him a retail profit per customer below the fixed cost per customer. However, under our retail benchmarking approach, firm i may have an incentive to choose very low retail prices only to reduce its access payment such that its net access revenue more than covers its net retail loss. By adding the max operator, (26) makes such a deviation not profitable.

We now introduce one additional assumption:

Assumption 2 *An increase in F_i increases the number of subscribers to firm j.*

Assumption 2 is satisfied if μ is large enough. For instance, in a symmetric equilibrium with $p_i = \widehat{p}_i = c$, it holds if $\mu > (1 - 2\wp)v$ where $\wp$ is the number of subscribers to a firm in the symmetric equilibrium. In other words, the assumption holds if there is a net business-stealing effect.

Then, we have:

Proposition 7 *Suppose that the regulator proposes the access pricing rule (26). Then, under assumption 2, the Ramsey outcome can be implemented as a Nash Equilibrium: in the equilibrium, firm i chooses $p_i = \widehat{p}_i = c$, $F_i = f$ for $i = 1, 2$.*

Proof. See Appendix for the proof.

6 Conclusion

We studied how access pricing affects network competition when consumers' subscription demand is elastic and firms compete with non-linear tariffs and can apply termination-based price discrimination. We first considered the standard approach based on a fixed and reciprocal (per-minute) termination charge and found that both the firms and the regulator want to depart from cost-based termination charge (and hence want to distort call volumes) in order to affect market penetration. In particular, two opposing effects (softening competition and internalizing network externalities) are associated with a reduction in termination charge. The former decreases market penetration while the latter increases it. We find that firms always prefer having termination charge below cost for either motif while the regulator prefers termination charge below cost only if this boosts penetration.

After studying the standard approach, we investigated the retail benchmarking approach. We find that for a given fixed reciprocal termination charge, we can find a family of access pricing rules parameterized such that all the access pricing rules in the family induce firms to charge on-net price equal to on-net cost and off-net price equal to off-net cost but the equilibrium fixed fee decreases with the strength of the feedback from retail prices to access payments. The result implies that the regulator can boost market penetration without distorting call volumes. Our access pricing rules intensify retail competition since a firm can reduce its access payment to rival firm(s) by reducing its retail prices. In addition, we show that for any given fixed reciprocal termination charge, we can find a retail benchmarking rule that gives both higher social welfare and higher profits.

Appendix

Proof of Lemma 1

We prove that the fixed point of the mapping $\wp : \Delta^2 \to \Delta^2$ where $\wp(\beta_1, \beta_2) = (\wp_1, \wp_2)$ is unique for μ large enough. This can be done by looking at the index of zeros of the mapping $g(\beta) = \wp(\beta) - \beta$. The Jacobian of g is

$$D_\beta g = \begin{pmatrix} \frac{1}{\mu}[\beta_1(1-\beta_1)v(p_1) - \beta_1\beta_2 v(\hat{p}_2)] - 1 & \frac{1}{\mu}[\beta_1(1-\beta_1)v(\hat{p}_1) - \beta_1\beta_2 v(p_2)] \\ \frac{1}{\mu}[\beta_2(1-\beta_2)v(\hat{p}_2) - \beta_1\beta_2 v(p_1)] & \frac{1}{\mu}[\beta_2(1-\beta_2)v(p_2) - \beta_1\beta_2 v(\hat{p}_1)] - 1 \end{pmatrix}.$$

Let $d = \det D_\beta g$. In the case at hand, the index of a fixed point β is equal to $+1$ if $d > 0$ and equal to -1 if $d < 0$. The Poincaré-Hopf Theorem implies that the sum of indexes of all fixed points equals $+1$ (the Euler index of the simplex). It is clear that for large enough μ, $d > 0$ so that then every fixed point has index $+1$. This then implies that there is a unique fixed point. Thus for large enough μ rational expectations are uniquely defined for all tariff schedules. ■

Proof of Lemma 2

The proof uses elements from the proof of the previous Lemma in the special case of firms using perceived marginal cost pricing. Suppose first that $a = c_0$ and that firms set both on-net and off-net price equal to cost, but possibly $F_1 \neq F_2$ (and thus $\wp_1 \neq \wp_2$). Let d denote the determinant of the Jacobian used in the previous Lemma. In this case

$$d = \frac{\mu - (1 - \wp_1 - \wp_2)(\wp_1 + \wp_2)v(c)}{\mu} = \frac{\mu - \wp_0(1-\wp_0)v(c)}{\mu} > 0$$

where the inequality follows from Assumption 1. By continuity the determinant will be strictly positive also when a is close to c_0 and firms price at perceived marginal cost. This means that expectations are uniquely defined. ■

Proof of lemma 3

We continue to use the notation of the proof of Lemma 1. In particular, $g(\beta) = \wp(\beta) - \beta$.

Note that

$$D_{F_1}g = \begin{pmatrix} -\beta_1(1-\beta_1)/\mu \\ \beta_1\beta_2/\mu \end{pmatrix}.$$

This implies that an increase in the fixed fee of network 1, *everything else equal*, will decrease the number of subscribers to network 1 and will increase the subscribers to network 2. However, a change in F_1 also affects rational expectations and the total effect on the number of subscribers by a change in fixed fee F_1 is given by the implicit function theorem as

$$D_{F_1}\beta = -[D_\beta g]^{-1} D_{F_1} g.$$

The results follows immediately. ■

Proof of Lemma 4

Rational expectations are zeros of the mapping $\tilde{g}(\beta) = (\tilde{\wp}_1 - \beta_1, \tilde{\wp}_2 - \beta_2)$. The Jacobian of $\tilde{g}$ is

$$D_\beta \tilde{g} = \begin{pmatrix} \frac{1}{\mu}[\beta_1(1-2\beta_1)v(p_1)] - 1 & \frac{1}{\mu}[\beta_1(1-2\beta_1)v(\hat{p}_1)] \\ \frac{1}{\mu}[\beta_2(1-2\beta_2)v(\hat{p}_2)] & \frac{1}{\mu}[\beta_2(1-2\beta_2)v(p_2)] - 1 \end{pmatrix}.$$

Let $\tilde{d}$ denote the determinant of this Jacobian. For large enough μ, we have $\tilde{d} > 0$ and therefore rational self-fulfilling expectations are then unique. Relying again on continuous rational expectations one obtains again that firms will always set variable price equal to perceived marginal cost: $p_i = c$ and $\hat{p}_i = \hat{c}$. Note that

$$D_{F_1}\tilde{g} = \begin{pmatrix} -\beta_1(1-2\beta_1)/\mu \\ 0 \end{pmatrix}.$$

This implies that an increase in the fixed fee of network 1, *everything else equal*, will decrease the number of subscribers to network 1 and will keep the number of subscribers to network 2 constant. The latter illustrates the fact that there is no business stealing effect in this model. However, a change in F_1 does affect expectations and the total effect on the number of subscribers by a change in fixed fee F_1 is given by the implicit function theorem as

$$D_{F_1}\beta(F_1) = -[D_\beta \tilde{g}]^{-1} D_{F_1}\tilde{g}.$$

One thus verifies that

$$\frac{\partial \beta_1}{\partial F_1} = \frac{-1}{\tilde{d}\mu^2}\beta_1((1-2\beta_1)(\mu - \beta_2(1-2\beta_2)v)$$

and

$$\frac{\partial\beta_2}{\partial F_1} = \frac{-1}{\tilde{d}\mu^2}\beta_1\beta_2(1-2\beta_1)(1-2\beta_2)\hat{v}.$$

Thus, at a symmetric solution with rational expectations $\tilde{\wp}_1 = \tilde{\wp}_2 = \tilde{\wp}$

$$\frac{\partial\tilde{\wp}_1}{\partial F_1} = \frac{-1}{\tilde{d}\mu^2}\tilde{\wp}((1-2\tilde{\wp})(\mu-\tilde{\wp}(1-2\tilde{\wp})v) < 0 \tag{27}$$

and

$$\frac{\partial\tilde{\wp}_2}{\partial F_1} = \frac{-1}{\tilde{d}\mu^2}\tilde{\wp}^2(1-2\tilde{\wp})^2\hat{v} < 0. \tag{28}$$

■

Proof of Proposition 3

At the equilibrium (at $a = c_0$) per consumer profit equals

$$F - f = \frac{\mu(\mu - 2\wp^*(1-2\wp^*)v}{(1-2\wp^*)(\mu-\wp^*(1-2\wp^*)v)}.$$

The effect on total profit $H(c_0, \wp) = \wp\,(F-f)$ with respect to a change in $\wp$ is thus

$$\begin{aligned}\frac{\partial H}{\partial\wp}(c_0,\wp^*) &= H(c_0,\wp^*)/\wp^* + \wp^*\left[2v(c) - \frac{\mu}{\wp^*(1-2\wp^*)}\right] \\ &= \frac{\wp^* v(\mu - 2\wp^*(1-2\wp^*)v)}{\mu - \wp^*(1-2\wp^*)v} > 0\end{aligned}$$

Hence, profits increase along the rational expectations curve, in a neighborhood around $\wp^*$.

Next, we have to account for the fact that when a is varied, the rational expectations curve, and thus the equilibrium, will change. The partial effect on profits (keeping $\wp^*$ fixed) equals

$$\frac{\partial H}{\partial a} = (\wp^*)^2(a-c_0)q'(\hat{c}),$$

so that at $a = c_0$ a marginal change in a does not affect profits directly. The extra profit made through access revenues is just offset by the decrease in the fixed fee. However, profits are affected indirectly by a change in market penetration.

Therefore, we have:

$$\frac{dH}{da} = H_a + H_\wp \times \wp'(a) < 0;$$

profits are decreasing in a in a neighborhood around c_0. Firms thus indeed prefer an access fee below cost. This leads to higher market penetration, which means it is also good for consumer and social welfare. ■

Proof of Proposition 7

Suppose that firm j uses $F_j = f, p_j = \widehat{p}_j = c$. Then, we distinguish two cases depending on whether $\wp_i > \wp^R$ or $\wp_i < \wp^R$.

Case 1: when $\wp_i > \wp^R$. $\wp_i > \wp^R$ implies that $\pi_i < f$. Then, firm i's profit is

$$\begin{aligned}\Pi_i &= \wp_i [\pi_i - f] - \wp_i \wp_j \kappa^* [f - f] \\ &= \wp_i [\pi_i - f] < 0.\end{aligned}$$

Case 2: when $\wp_i < \wp^R$. $\wp_i < \wp^R$ implies, from assumption 2, $1 < \wp_j \kappa^*$. Consider first $\pi_i \geq f$. Then, firm i's profit becomes

$$\Pi_i = \wp_i (1 - \wp_j \kappa^*) [\pi_i - f] \leq 0.$$

Consider now $\pi_i < f$. Then, firm i's profit is

$$\begin{aligned}\Pi_i &= \wp_i [\pi_i - f] - \wp_i \wp_j \kappa^* [f - f] \\ &= \wp_i [\pi_i - f] \leq 0.\end{aligned}$$

■

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