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"Multiple Contracting in Insurance Markets: Cross-Subsidies and Quantity Discounts"

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Multiple Contracting in Insurance Markets: Cross-Subsidies and Quantity Discounts*

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Abstract

We study a nonexclusive insurance market with adverse selection in which insurers compete through simple contract offers. Multiple contracting endogenously emerges in equilibrium. Different layers of coverage are priced fairly according to the types of insurees who purchase them, giving rise to cross-subsidies between types. Riskier insurees demand greater total coverage at an increasing unit price, but the contracts offered by insurers feature quantity discounts in equilibrium. Our policy implications emphasize the need to regulate the supply side of nonexclusive insurance markets, leaving insurees free to choose their optimal level of coverage.

Keywords: Insurance Markets, Multiple Contracting, Adverse Selection.
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1 Introduction

Markets where risks are traded face the threat of adverse selection: insurees may be better informed about their risk characteristics than insurers are, and, because these characteristics directly matter to the profits of the latter, such informational asymmetries may impede trade, as emphasized by the well-known theoretical analyses of Akerlof (1970) and Rothschild and Stiglitz (1976). The possibility of this phenomenon raises the question of public intervention, and each reform of an insurance market is soon followed by discussions on how to best tackle the adverse selection problem. To take but one example, the recent reform wave of health insurance has led to a revival of important policy debates related to adverse selection: on whether basic coverage should be made compulsory; and on how competition between insurance providers should be organized, especially when it comes to policies that propose optional, additional coverage.

In this paper, we argue that answering such questions requires to take into account the possibility that a given insuree buys several insurance policies from several distinct insurers. Indeed, multiple contracting turns out to be a widespread phenomenon. In health-insurance markets, it is quite common for consumers in Europe and in the US to complement a basic coverage with an additional insurance policy. For example, 92% of the French population have both a mandatory coverage and a private coverage,[1] while in the US, 10 million out of the 42 million individuals covered by Medicare opt for buying an additional, private coverage (Medigap).[2] The US life-insurance market also typically allows for multiple contracting,[3] and so does the UK annuity market.[4]

In spite of the empirical relevance of multiple contracting, the theoretical literature has so far mostly relied on models à la Rothschild and Stiglitz (1976) and Wilson (1977), which postulate exclusive relationships between insurers and insurees, so that multiple contracting is a priori excluded. The literature on nonexclusive competition in insurance markets is not very conclusive either: as shown by Attar, Mariotti, and Salanié (2014a), existence of pure-strategy equilibria requires demanding assumptions, such equilibria when they exist do not feature the empirically prevalent distinction between basic and additional coverage, and, moreover, equilibrium allocations can often be sustained by exclusive relationships on the equilibrium path.

[1] See Thompson, Osborne, Squires, and Jun (2013).

[2] See http://www.protectmedigap.org.

[3] As stated in Cawley and Philipson (1999), "multiple contracting is highly prevalent" and represents roughly one quarter of agents with at least one insurance policy.

[4] See Finkelstein and Poterba (2004), in particular Footnote 2: "[...] we view the exclusivity condition as unlikely to be satisfied in annuity markets," and the end of Section IV.A.

This paper proposes a model of an insurance market in which multiple contracting is allowed for, and in which it indeed emerges as an equilibrium phenomenon. As in the Rothschild–Stiglitz–Wilson setting, first contracts are offered by insurers, and then each type of consumer chooses which contract(s) to accept—the key difference in our model being that the insuree may accept several contracts from several distinct insurers. As in the original Rothschild and Stiglitz (1976) model, each insurer is restricted to offer a single contract. We consider the two-type case, and we show that pure-strategy equilibria exists under reasonably general assumptions; moreover, the equilibrium aggregate allocation is unique. This model may thus be helpful for evaluating market reforms. It also yields several insights that we now detail.

In models that postulate exclusivity, competition bears on the pricing of the aggregate coverage bought by each insuree. Allowing for nonexclusivity changes the focus: from the insuree's viewpoint, each additional contract represents an additional layer of coverage, and the study naturally focuses on the pricing of such additional layers. The unique equilibrium aggregate outcome has a nice and natural structure: both low- and high-risk types buy the same basic coverage, and additionally the high-risk type buys a complementary coverage. Competition ensures that each contract makes zero expected profit: being sold to both low- and high-risk types, basic coverage is priced at the average cost, whereas complementary coverage is priced at marginal cost, reflecting that it is only sold to the high-risk type. In short, our equilibrium allocation offers a natural generalization of Akerlof (1970) pricing to divisible coverage, as each additional layer is priced at the expected cost associated to the set of insurees who buy this layer. A novel contribution of our analysis is to show how this allocation, originally studied by Jaynes (1978), Hellwig (1988), and Glosten (1994), naturally emerges in a game in which insurers compete in simple contract offers.[5]

It is worth noting that multiple contracting must take place in equilibrium for the high-risk type: no insurer would be ready to sell her aggregate coverage at the equilibrium price, as this would make losses. Thus the high-risk type type must buy basic coverage from one insurer, and complementary coverage from another insurer. As a result, the contracts bought by this type are qualitatively different: multiple contracting emerges because competition deals with layers of coverage that are sold independently.[6]

[5]Unlike Jaynes (1978, 2011) and Hellwig (1988), our construction does not rely on explicit communication between firms, in the spirit of decentralized markets. Unlike Glosten (1994), we show how this allocation can be sustained in an equilibrium of a competitive game with a finite number of strategic insurers. Properties of the Jaynes–Hellwig–Glosten allocation are also studied in Attar, Mariotti, and Salanié (2014a, 2014b).

[6]Notice also that in our setting multiple contracting is not related to the insuree splitting her aggregate demand between identical insurers. (See Biais, Martimort, and Rochet (2000, 2013) and Back and Baruch

In equilibrium, the low-risk type subsidizes the high-risk type: this is in contrast to the Rothschild and Stiglitz (1976) allocation, in which profits extracted from each type are exactly zero, but is reminiscent of the Wilson (1977), Miyazaki (1977), and Spence (1978) allocations. The fact that cross-subsidies survive cream-skimming deviations relies on two features. Recall first that the associated tariff for aggregate coverage has been proven by Glosten (1994) to be the only tariff that makes entry unprofitable. We reinforce this result by showing that no incumbent insurer can deviate by offering a single contract. Such deviations are made unprofitable by the existence of a latent contract (Arnott and Stiglitz (1993)), which is inactive on the equilibrium path but efficiently deters cream-skimming deviations. Second, each insurer is only allowed to offer a single contract. Otherwise, and as shown by Attar, Mariotti, and Salanié (2014a), an incumbent insurer could profitably deviate by offering two different contracts designed so as to dump high risks on other insurers.[7] We thus acknowledge that our equilibrium allocation may be vulnerable to strategic behavior on the insurers' part; as we argue in Section 5 below, this calls for some regulation of this side of the market. By contrast, there is no need to constraint insurees' choices, for instance by requiring basic coverage to be mandatory. This contrasts with standard policy recommendations from exclusive-competition models of insurance markets under adverse selection.

An important feature of our setting is that, compared to models that postulate exclusivity, it allows to draw a distinction between the set of contracts offered by the insurers and the set of equilibrium aggregate coverage levels chosen by an insuree by combining some of these contracts. Under a standard single-crossing condition, the latter has a familiar structure: a riskier type buys a higher aggregate coverage, at a unit price which is increasing with coverage. Yet, the former set has a very different structure; indeed, we show that, under the very same assumptions that ensure the existence of an equilibrium, the supply of coverage involves a unit price that is decreasing with coverage. That is, the contracts offered by insurers exhibit quantity discounts. The empirical predictions of our model are thus richer than in the exclusive case, and may be more in line with empirical findings; we comment on these points at more length in Section 6. In particular, both the predictions of a unit price that increases with coverage, and of a positive correlation between riskiness and coverage,[8] may be reversed when multiple contracting takes place, depending on whether data originate

(2013) for common-value environments in which multiple contracting arises for precisely this reason.)

[7]More precisely, the first contract would be profitable by attracting only the low-risk type on a basic coverage. The second contract would offer additional coverage at a small loss, so as to attract only the high-risk type, the key point being that this type would find it profitable to buy the basic coverage from other insurers.

[8]See Chiappori and Salanié (2000), and Chiappori, Jullien, Salanié, and Salanié (2006) for general results.

from the demand side or the supply side of the market.

The paper is organized as follows. Section 2 describes the model. Section 3 characterizes equilibrium aggregate trades. Section 4 shows how to construct equilibria sustaining such trades for a special class of preferences. Section 5 shows that the equilibria constructed in Section 4 are robust in the sense that similar equilibria exist for any preferences in a neighboring class. Section 6 draws the main theoretical, empirical, and normative lessons from our analysis.

2 The Model

We consider a buyer who can simultaneously trade a divisible good of uncertain quality with several identical sellers. A typical example is an insurance market in which a privately informed risk-averse agent can buy coverage from several insurance companies. The key difference with the nonexclusive-competition models studied in Biais, Martimort, and Rocher (2000, 2013) and Attar, Mariotti, and Salanié (2011, 2014a) is that sellers are restricted to make take-it-or-leave-it offers to the buyer. In the case of an insurance market, this means that each insurance company can issue only a single contract, that is, a single vector of state-contingent payments, as in Rothschild and Stiglitz's (1976) analysis.

2.1 The Buyer

The buyer is privately informed of her preferences. She may be of two types, $i = 1, 2$, with positive probabilities m_1 and m_2 such that $m_1 + m_2 = 1$. Type i has preferences over aggregate quantity-transfer bundles (Q, T) in some consumption set $X \subset \mathbb{R}_+ \times \mathbb{R}$, the precise specification of which depends on the interpretation of the model. We require that X contain the no-trade point $(0, 0)$, that it be convex with a nonempty interior, and that it be comprehensive in the sense that $(Q, T') \in X$ if $(Q, T) \in X$ and $T' < T$. Type i's preferences over X are taken to be representable by a function U_i defined over an open, convex, and comprehensive neighborhood V of X.[9] We assume that U_i is twice continuously differentiable, with $\partial U_i/\partial T < 0$, and that U_i is strictly quasiconcave.[10] Hence type i's marginal rate of substitution of the good for money

$$\tau_i \equiv -\frac{\partial U_i/\partial Q}{\partial U_i/\partial T} \tag{1}$$

[9]This last assumption is a standard technical trick that allows us to define marginal rates of substitution over the boundary of X (see, for instance, Mas-Collel (1985, Definition 2.3.17)).

[10]We do not assume that U_i is monotone in quantities, as, indeed, need not be the case for the quadratic specification of Section 2.4.2.

is well defined over V and strictly decreasing along her indifference curves. The following single-crossing property is key to our results.

Assumption SC *For each* $(Q,T) \in V$, $\tau_2(Q,T) > \tau_1(Q,T)$.

Geometrically, an indifference curve for type 2 crosses an indifference curve for type 1 only once, from below. Consequently, type 2 is more eager to increase her purchases than type 1 is.

2.2 The Sellers

There are n identical sellers, with n large but finite. If a seller provides type i with a quantity q and obtains a transfer t in return, he earns a profit $t - v_i q$, where v_i is the cost of serving type i. The following common-value assumption will be maintained throughout the analysis.

Assumption CV $v_2 > v_1$.

Combined with Assumption SC, Assumption CV implies that, whereas type 2 is more willing to trade at the margin than type 1 is, she faces sellers who are more reluctant to trade with her than with type 1. These assumptions are satisfied in the main examples presented below. We let $v \equiv m_1 v_1 + m_2 v_2$ be the average cost of serving the buyer, so that $v_2 > v > v_1$.

2.3 The Trading Game

As in Biais, Martimort, and Rochet (2000, 2013) and Attar, Mariotti, and Salanié (2011, 2014a), no seller can control, and, a fortiori, contract on the trades that the buyer makes with his competitors. The novel feature of our analysis is that sellers compete to serve the buyer by proposing bilateral contracts, that is, quantity-transfer bundles, rather than menus of such contracts. Therefore, the timing of our trading game is as follows:

1. Each seller k proposes a contract $(q^k, t^k) \in \mathbb{R}_+ \times \mathbb{R}$.[11]

2. After privately learning her type, the buyer selects which contracts to trade with the sellers, if any.

Given a vector of contract offers $((q^1, t^1), \ldots, (q^n, t^n))$, type i's problem is then

$$\max \left\{ U_i \left(\sum_{k \in K} q^k, \sum_{k \in K} t^k \right) : K \subset \{1, \ldots, n\} \text{ and } \left(\sum_{k \in K} q^k, \sum_{k \in K} t^k \right) \in X \right\}, \tag{2}$$

[11] The null contract is $(0,0)$. A contract (q^k, t^k) with $q^k > 0$ has unit price t^k / q^k.

with $\sum_{\emptyset} = 0$ by convention. Note that, because, by assumption, $(0, 0) \in X$, the feasible set in type i's problem is always nonempty. We use perfect Bayesian equilibrium as our equilibrium concept. Throughout the paper, we focus on pure-strategy equilibria.

2.4 The Main Examples

We now present a family of specifications of the above model that plays a key role in our analysis. The buyer's preferences in these specifications satisfy two further restrictions.

First, each type i has quasilinear preferences,

$$U_i(Q, T) = u_i(Q) - T, \tag{3}$$

where the utility function u_i is twice continuously differentiable, with $\partial^2 u_i < 0$. The marginal rate of substitution $\tau_i(Q, T) = \partial u_i(Q)$ of type i is then independent of T, so that all her indifference curves are vertical translates of each other. Assumption SC amounts to $\partial u_2(Q) > \partial u_1(Q)$ for all Q.

Second, and less standardly, there is a positive constant Q_0 such that

$$\partial u_2(Q) = \partial u_1(Q - Q_0) \tag{4}$$

for all $Q \geq Q_0$. Hence, in terms of the buyer's preferences, everything happens as if type 1 were identical to type 2, except that she had already traded a quantity Q_0. (Note, however, that, unlike in a private-value environment, this is not the only difference between types 1 and 2, because, by Assumption CV, the costs of serving them are not the same.)

Geometrically, properties (3)–(4) imply that any pair of indifference curves for types 1 and 2 are, over the relevant domain, oblique or horizontal translates of each other. The translating vector connects the points of these indifference curves where type 1 and type 2 have equal marginal rates of substitution, as illustrated in Figure 1. This vector defines a contract stipulating a positive quantity and a transfer.

We shall illustrate these properties by means of two examples.

2.4.1 The CARA Example

Our first example is an insurance model in line with Rothschild and Stiglitz (1976). A risk-averse agent can purchase coverage from several insurance companies. She faces a binomial risk on her wealth, which can take two values (W_B, W_G), with probabilities $(v_i, 1 - v_i)$ that define her type. Here $W_G - W_B$ is the positive monetary loss that the agent incurs in the bad state and v is the average probability of a loss. The agent's preferences have an expected-utility representation with constant absolute risk aversion α.[12] An insurance contract specifies a reimbursement q to be paid in the bad state, along with a premium t, implying an expected profit $t - v_i q$ for the insurance company that trades it with the agent. In the aggregate, the agent purchases at a price $T \equiv \sum_k t^k$ a reimbursement $Q \equiv \sum_k q^k$ in the bad state. Type i's preferences over aggregate quantity-transfer bundles $(Q, T) \in X \equiv \mathbb{R}_+ \times \mathbb{R}$ are then represented by (3), with

$$u_i(Q) \equiv -\frac{1}{\alpha} \ln(v_i \exp(-\alpha(W_B + Q)) + (1 - v_i) \exp(-\alpha W_G)). \tag{5}$$

For each type i, the marginal rate of substitution (1) of reimbursements for premia is

$$\partial u_i(Q) = \frac{1}{1 + [(1 - v_i)/v_i] \exp(-\alpha(W_G - W_B - Q))}. \tag{6}$$

By Assumption CV, type 2 has a higher probability of incurring a loss than type 1. This, in turn, implies that Assumption SC holds: type 2 is more eager to buy larger amounts of insurance than type 1. According to (6), condition (4) holds for

$$Q_0 \equiv \frac{1}{\alpha} \ln\left(\frac{(1 - v_1)/v_1}{(1 - v_2)/v_2}\right) > 0.$$

The first-best level of trade is the same for type 1 as for type 2 and entails full insurance, $Q_1^* = Q_2^* = W_G - W_B$.

2.4.2 The Quadratic Example

Our second example is a pure-trade model in line with Biais, Martimort, and Rochet (2000, 2013) and Back and Baruch (2013). In these market-microstructure models, a risk-averse insider with constant absolute risk aversion α trades shares of an asset with several market makers partly for informational and partly for hedging purposes, while facing Gaussian noise with variance σ^2. Here v_i is the expected value of the asset conditional on the insider's type i. Type i's preferences over aggregate quantity-transfer bundles $(Q, T) \in X \equiv \mathbb{R}_+ \times \mathbb{R}$ are then represented by (3), with

$$u_i(Q) \equiv \theta_i Q - \frac{\alpha\sigma^2}{2} Q^2. \tag{7}$$

For each type i, the marginal rate of substitution (1) of shares for money is

$$\partial u_i(Q) = \theta_i - \alpha\sigma^2 Q, \tag{8}$$

[12]That is, we abstract from income effects associated to price changes. A similar assumption underlies the estimation of the welfare cost of adverse selection proposed by Einav, Finkelstein, and Cullen (2010).

so that Assumption SC requires that $\theta_2 > \theta_1$. According to (8), condition (4) holds for

$$Q_0 \equiv \frac{\theta_2 - \theta_1}{\alpha\sigma^2} > 0.$$

When dealing with this example, we assume that $\theta_2 - v_2 > \theta_1 - v_1 > 0$. That is, there are always gains from trade between the insider and the market makers, and these gains are higher for type 2 than for type 1. As a result, the first-best level of trade is higher for type 2 than for type 1, $Q_2^* = (\theta_2 - v_2)/(\alpha\sigma^2) > (\theta_1 - v_1)/(\alpha\sigma^2) = Q_1^*$. Given Assumption SC, this is a standard responsiveness condition (Caillaud, Guesnerie, Rey, and Tirole (1988)) that ensures that first-best quantities are implementable.

3 Equilibrium Characterization

In this section, we show that any equilibrium aggregate outcome of our trading game is of the form predicted by Jaynes (1978), Hellwig (1988), and Glosten (1994). We give conditions under which equilibria feature multiple contracting, in that both types first trade the same basic amount, which type 2 complements by conducting additional trades. All contracts are priced fairly given the types who trade them. As a result, equilibria involve zero expected profit for the sellers and cross-subsidies between types. We also provide a necessary condition for the existence of an equilibrium.

3.1 Jaynes–Hellwig–Glosten Pricing

Let us fix an equilibrium, and let (Q_1, T_1) and (Q_2, T_2) be the equilibrium aggregate trades of types 1 and 2. According to Assumption SC together with the fact that marginal rates of substitutions are well defined for each type, we know that $Q_2 \geq Q_1$. Our first result describes the price structure of equilibrium.

Theorem 1 *Suppose that there are at least three sellers. Then, in any equilibrium,*

$$T_1 = vQ_1, \tag{9}$$

$$T_2 - T_1 = v_2(Q_2 - Q_1). \tag{10}$$

Moreover, any traded contract is issued at unit price v or v_2 and makes zero expected profit.

Hence, in the aggregate, type 2 first trades a quantity Q_1 at unit price v, just as type 1 does, and then, on top of this, a quantity $Q_2 - Q_1$ at unit price v_2. This implies that sellers earn zero expected profit. Theorem 1 also gives us information about the price of traded contracts. As in Jaynes (1988), Hellwig (1988), and Glosten (1994), any marginal quantity is bought at a unit price equal to the expected cost of providing it, given the types who trade it: basic contracts that are traded by both types have unit price v, and thus involve cross-subsidies between types, whereas complementary contracts that are only traded by type 2 have unit price v_2. In the case of insurance, this corresponds to a situation in which the agent can purchase basic coverage at a relatively low premium rate v, which she can complement by further coverage at a relatively high premium rate v_2. Premium rates reflect in a fair way the composition of the pool of types trading each policy.

The characterization of equilibrium aggregate trades in Theorem 1 differs from earlier results in the literature in several ways. Unlike in Jaynes (1978, 2011) and Hellwig (1988), it does not rely on the possibility of inter-seller communication or on a specific timing of the sellers' offers. Unlike in Glosten (1994), it does not result from the derivation of an entry-proof tariff, but rather from the analysis of the sellers' deviations. In that respect, a novel insight of Theorem 1 is that one only needs to consider single-contract deviations to obtain Jaynes–Hellwig–Glosten pricing in equilibrium.

3.2 Gains from Trade

Hereafter, and to focus on the most interesting case, we restrict parameter values to be such that type 1 would be ready to buy a positive quantity at unit price v.

Assumption 1-$\boldsymbol{v}$ *$\tau_1(0,0) > v$.*

That is, $W_G - W_B > \ln([(1-v_1)/v_1]/[(1-v)/v])/\alpha$ in the CARA example and $\theta_1 > v$ in the quadratic example. If Assumption 1-v did not hold, then, according to Theorem 1, we would have $Q_1 = 0$, so that type 1 would be excluded from trade in any equilibrium.[13] Under Assumption 1-v, it follows as a corollary to Theorem 1 that type 1 trades the optimal quantity at unit price v, which pins down the value of Q_1:

$$\tau_1(Q_1, vQ_1) = v. \tag{11}$$

That is, $Q_1 = W_G - W_B - \ln([(1-v_1)/v_1]/[(1-v)/v])/\alpha < W_G - W_B = Q_1^*$ in the CARA example and $Q_1 = (\theta_1 - v)/(\alpha\sigma^2) < (\theta_1 - v)(\alpha\sigma^2) = Q_1^*$ in the quadratic example: type 1 purchases a positive but suboptimal quantity at a unit price v strictly higher than the cost v_1 of serving her.

[13]It should be noted that this somewhat degenerate outcome is the only one consistent with equilibrium when sellers can compete through menus of contracts (Attar, Mariotti, and Salanié (2014a)).

We also restrict parameter values to be such that type 2, having already bought a quantity Q_1 at unit price v, would be ready to buy an additional positive quantity at unit price v_2.

Assumption 2-$\boldsymbol{v_2}$ $\tau_2(Q_1, vQ_1) > v_2$.

This assumption is always satisfied in the CARA example because type 2 wants to obtain full coverage at the fair premium rate v_2, whereas, as seen above, type 1 gets less than full coverage at the premium rate v. In the quadratic example, this assumption amounts to $\theta_2 - v_2 > \theta_1 - v$, which is automatically satisfied under the responsiveness condition $\theta_2 - v_2 > \theta_1 - v_1$. If Assumption 2-$v_2$ did not hold, then, according to Theorem 1, we would have $Q_1 = Q_2$, so that a pooling outcome would emerge. Under Assumption 2-v_2, it follows as a corollary to Theorem 1 that type 2 trades, on top of (Q_1, T_1), the optimal quantity complement at unit price v_2, which pins down the value of Q_2:

$$\tau_2(Q_2, vQ_1 + v_2(Q_2 - Q_1)) = v_2. \tag{12}$$

That is, $Q_2 = W_G - W_B = Q_2^*$ in the CARA example and $Q_2 = (\theta_2 - v_2)/(\alpha\sigma^2) = Q_2^*$ in the quadratic example: type 2 purchases her first-best quantity at a unit price $v_2 - (v_2 - v)Q_1/Q_2$ strictly lower than the cost of serving her. This property also holds in any model in which type 2 has quasilinear preferences or in any insurance model in which her preferences have an expected-utility representation, or more generally, exhibit second-order risk aversion (Segal and Spivak (1990)).

It should be noted that Assumptions 1-v and 2-v_2 were implicit in the insurance models of Jaynes (1978) and Hellwig (1988). The following corollary summarizes the aggregate features of candidate equilibria.

Corollary 1 *Under Assumptions 1-v and 2-v_2, any equilibrium satisfies (9)–(12).*

The corresponding Jaynes–Hellwig–Glosten outcome is illustrated in Figure 2.

3.3 Indispensability and Incentive Compatibility

A key feature of equilibrium is that, as in standard Bertrand competition, no seller can be indispensable in providing either type 1 or type 2 with their equilibrium trades; otherwise, he could earn a strictly positive expected profit by slightly increasing his price. This means, in particular, that the buyer has the opportunity to trade more than the quantity Q_1 at the relatively low price v. Whereas, according to (11), this opportunity is of no value for type 1, it could attract type 2, thereby destabilizing the equilibrium. Hence, a necessary condition for the existence of equilibrium is that these additional trades would be of such a magnitude that type 2 would not be willing to make them, given her convex preferences. An additional condition is that these trades cannot be profitably exploited by a deviating seller. These two conditions can be formulated as follows.

Corollary 2 *Under Assumptions 1-v and 2-v_2, any equilibrium satisfies*

$$U_2(Q_2, T_2) \geq U_2(2Q_1, 2T_1), \tag{13}$$

$$2Q_1 > Q_2. \tag{14}$$

Conditions (13)–(14) are most easily understood when only two sellers issue contracts at unit price v. Then, by the dispensability property, each of them must offer a contract equal to type 1's entire equilibrium aggregate trade (Q_1, T_1). Thus the incentive-compatibility condition (13) must hold, expressing that type 2 is not willing to trade (Q_1, T_1) twice on the equilibrium path. Now, if condition (14) were not satisfied, then some other seller could attract type 2 by proposing her to trade the quantity $Q_2 - 2Q_1$ at a unit price slightly above v_2. Indeed, combined with $(2Q_1, 2T_1)$, such a contract would allow type 2 to buy the same quantity Q_2 as in equilibrium, in exchange for a transfer decreased by almost $(v_2 - v)Q_1$. Such a deviation would clearly be profitable, thereby upsetting the equilibrium.[14] This logic easily extends when more than two sellers issue contracts at unit price v. The economic implications of condition (14) are discussed at greater length in Section 6.2.

Geometrically, conditions (13)–(14) state that the aggregate trade $(2Q_1, 2T_1)$ is located in the lower contour set of (Q_2, T_2) for type 2, to the right of (Q_2, T_2). As $T_1 = vQ_1$ according to Theorem 1, this implies that

$$v > \tau_2(2Q_1, 2T_1). \tag{15}$$

Conditions (13)–(14) are satisfied whenever the complement $Q_1 - Q_2$ that type 2 wants to trade at unit price v_2 is sufficiently small relative to the basic quantity Q_1 that both types want to trade at unit price v. In the case of insurance, this holds whenever type 1 wants to purchase some insurance at the premium rate v (Assumption 1-v) and type 1 and type 2's risk characteristics are not too far apart.

4 Equilibrium Existence: The Main Examples

In Section 3, we characterized the basic structure of aggregate and individual trades in

[14]This argument presumes that $2Q_1 \neq Q_2$, which is necessarily the case, for, otherwise, type 2 would trade $(2Q_1, 2T_1)$ instead of (Q_2, T_2) on the equilibrium path, because one would then have $2T_1 = 2vQ_1 < vQ_1 + v_2(Q_2 - Q_1) = T_2$.

any candidate equilibrium. Henceforth, we investigate whether and how these trades can effectively be sustained in equilibrium through appropriate contract offers. We start in this section with the class of examples presented in Section 2.4. Our construction relies on two kinds of contracts: first, contracts that can be traded in equilibrium, and, second, contracts that are not meant to be traded in equilibrium and the sole role of which is to deter deviations by the sellers. We describe these in turn.

4.1 Basic and Complementary Contracts

Let us first describe the contracts we use to reach the Jaynes–Hellwig–Glosten outcome that must prevail in equilibrium. Our construction involves two such contracts, a basic contract $c \equiv (Q_1, T_1)$, and a complementary contract $c' \equiv (Q_2 - Q_1, T_2 - T_1)$. According to Theorem 1, c has unit price v, whereas c' has higher unit price v_2. Type 1 reaches her equilibrium aggregate trade (Q_1, T_1) by trading a single contract c, while type 2 reaches her equilibrium aggregate trade (Q_2, T_2) by trading a contract c along with a contract c'. Thus types 1 and 2 do not trade the same contracts and type 2 trades two different contracts. As no seller can be indispensable in providing either of these contracts, we begin our construction of an equilibrium by imposing that two sellers offer the contract c and that two sellers offer the contract c'.

Lemma 1 *Suppose that Assumptions 1-v and 2-v_2 and conditions (13)–(14) are satisfied. Then, if two sellers offer the contract c and two sellers offer the contract c', it is a best response for both types to trade a contract c with the same seller, and for type 2 to additionally trade a contract c' with some other seller.*

Although the contracts c and c' lead to the desired aggregate trades, they are in general not sufficient to sustain an equilibrium. To clarify this point, consider the configuration illustrated in Figure 3. We have assumed that the only available contracts are c and c', with two sellers offering each of them, and that the trade $2c$ is strictly less preferred by type 2 than $c + c'$. Now, consider the contract $\tilde{c}$ close to c as shown. Contract $\tilde{c}$ allows the buyer to purchase a quantity less than Q_1 at a price lower than v. This contract certainly attracts type 1, and it yields a strictly positive profit to a deviating seller proposing it if it does not attract type 2 along the way. To see that this is indeed the case, observe that combining $\tilde{c}$ with c, c', or any combination of these contracts, leaves type 2 with a strictly lower utility than trading a contract c along with a contract c', which remains feasible following any seller's unilateral deviation. This is because $\tilde{c} + c$ is close to $2c$ and thus is strictly less preferred by type 2 than $c + c'$, just as $2c$, and because $\tilde{c} + c'$ is below the line of slope v_2 passing through c and thus is strictly less preferred by type 2 than $c + c'$. Thus, for any seller, offering contract $\tilde{c}$ is a successful cream-skimming deviation: it cannot be blocked by the contracts c and c' and it is profitable.

4.2 Deterring Cream-Skimming Deviations

To construct an equilibrium, we must supplement the contracts c and c' by further contracts preventing sellers from offering profitable deviations such as $\tilde{c}$. This role will be played by a single contract, denoted c'', defined as the contract which, combined with c, allows type 2 to reach a point on her equilibrium indifference curve $\mathcal{I}_2$ where her marginal rate of substitution is equal to v; that is, $U_2(c + c'') = U_2(c + c')$ and $\tau_2(c + c'') = v$. This contract exists and is unique if conditions (13)–(14) are satisfied. One can verify that c'' stipulates a quantity strictly larger than $Q_2 - Q_1$, at a unit price strictly between v and v_2.[15]

As c'' is not meant to be traded in equilibrium, we first check that its introduction does not cause the buyer to modify her trades on the equilibrium path.

Lemma 2 *Suppose that Assumptions 1-v and 2-v_2 and conditions (13)–(14) are satisfied. Then, if two sellers offer the contract c and two sellers offer the contract c', and if the other sellers offer either the contract c'' or the null contract, it remains a best response for both types to trade a contract c with the same seller, and for type 2 to additionally trade a contract c' with some other seller.*

The intuition for this result is simple. As for type 1, she cannot improve her utility by trading contracts other than c, as these contracts are issued at a price higher than v and she can buy her optimal quantity at unit price v by trading a contract c. Turning to type 2, observe first that she is indifferent between trading a contract c along with a contract c' and trading a contract c along with a contract c'' . Moreover, because her marginal rate of substitution at $c + c''$ is v, the optimal way for type 2 to trade c'' along with some of the offered contracts consists in combining it with a contract c. Thus type 2 is not made strictly better off when contract c'' is introduced on top of contracts c and c'.

We next investigate the sellers' deviations. We first show that no profitable deviation can attract type 2; that is, only cream-skimming deviations may create a problem for equilibrium

[15] The quantity stipulated by c'' must be larger than $Q_2 - Q_1$ because any point on $\mathcal{I}_2$ to the left of (Q_2, T_2) is such that the marginal rates of substitution for type 2 is higher than $\tau_2(Q_2, T_2) = v_2$ and thus, a fortiori, higher than v. For the same reason, the unit price of c'' must be strictly lower than v_2. Finally, the unit price of c'' must be strictly higher than v; otherwise, $\mathcal{I}_2$ would lie entirely above the line with slope v going through (Q_1, T_1) and thus could not go through (Q_2, T_2).

existence.

Lemma 3 *Suppose that Assumptions 1-v and 2-v_2 and conditions (13)–(14) are satisfied. Then, if two sellers offer the contract c and two sellers offer the contract c', and if the other sellers offer either the contract c'' or the null contract, there is no profitable deviation by a seller that at least attracts type 2.*

To understand this result, observe first that no profitable deviation can attract both types, as such a contract would need to have a unit price higher than v and type 1 can buy her optimal quantity at unit price v by trading a contract c. Furthermore, no profitable deviation can only attract type 2, as such a contract would need to have a unit price higher than v_2 and type 2 can trade, on top of c, the optimal complement c' at unit price v_2. Moreover, as noted above, type 2 would be strictly worse off combining such a contract with a contract c''.

Observe that Lemmas 1–3 hold independently of whether the buyer's preferences satisfy conditions (3)–(4). For instance, they would go through in a standard insurance setting à la Rothschild and Stiglitz (1976) with nonconstant absolute risk aversion.

It follows from Lemma 3 that the only remaining possibility for a profitable deviation is a cream-skimming deviation that only attracts type 1, as in the example illustrated in Figure 3. The following result shows that, when sufficiently many sellers offer the contract c'', such deviations are ruled out for preferences in the class discussed in Section 2.4.

Lemma 4 *Suppose that the buyer's preferences satisfy (3)–(4) and that Assumptions 1-v and 2-v_2 and conditions (13)–(14) are satisfied. Then, if two sellers offer the contract c and two sellers offer the contract c', and if sufficiently many sellers offer the contract c'', there is no profitable deviation by a seller that only attracts type 1.*

To understand how the contract c'' succeeds in deterring cream-skimming deviations, recall that, for preferences that satisfy (3)–(4), the buyer's indifference curves, whatever her type, are, over the relevant range, all translates of each other. This is in particular true of the equilibrium indifference curves $\mathcal{I}_1$ and $\mathcal{I}_2$ of types 1 and 2; moreover, as $\tau_2(c+c'') = v = \tau_1(c)$, the vector that translates $\mathcal{I}_1$ into $\mathcal{I}_2$ corresponds to the contract c''. Now, consider a potential cream-skimming deviation such as $\tilde{c}$ in Figure 4. This contracts certainly attracts type 1, like in Figure 3. However, because of the translation property, type 2 would also increase her utility by trading $\tilde{c}$ along with c''. Thus $\tilde{c}$ attracts both types. Because $\tilde{c}$ must have a unit price at most equal to v to attract type 1, this deviation cannot be profitable. More generally, any attempt by a seller at attracting and making profits with type 1 while leaving the other sellers to trade with type 2 is doomed to fail because, if type 1 can increase her utility by trading with the deviator, so can type 2 by mimicking type 1 and trading an additional contract c''. This is why Lemma 4 requires that there be enough sellers offering the contract c''; in any case, at least two of them. Indeed, c'' must remain available for type 2 to trade following a deviation, taking into account that some contracts c'' might be traded by type 1 in those circumstances.

The central result of this section is a direct implication of Lemmas 1–4.

Theorem 2 *Suppose that the buyer's preferences satisfy (3)–(4), that Assumptions 1-v and 2-v_2 and conditions (13)–(14) are satisfied, and that there are sufficiently many sellers. Then an equilibrium exists.*

We know from Theorem 1 and Corollary 1 that, under Assumptions 1-v and 2-v_2, the Jaynes–Hellwig–Glosten outcome is the only candidate equilibrium outcome. Moreover, according to Corollary 2, conditions (13)–(14) are necessary for an equilibrium to exist. What Theorem 2 shows is that these conditions are actually sufficient provided there are sufficiently many sellers and the buyer's preferences satisfy conditions (3)–(4).

5 Robustness

In this section, we investigate to which extent Theorem 2 can be extended to a more general class of preferences for the buyer than those satisfying conditions (3)–(4). We shall restrict attention to equilibria that only rely on the contracts c, c', and c'' introduced above. Because Lemmas 1–3 do not require conditions (3)–(4) to hold, the analysis can focus on Lemma 4, that is, on the possibility of deterring cream-skimming deviations through the single contract c''. It is easy to convince oneself that a sufficient condition for c'' to deter any profitable deviation that would attract type 1 is that the translate of the upper contour set of c for type 1 along the vector c'' lies in the upper contour set of $c + c'$ for type 2. Preference specifications that satisfy conditions (3)–(4) correspond to the knife-edge case where these two sets coincide. We first provide, in the quasilinear case, an economically meaningful condition on the demand function of the two types of buyer such that this property is upheld. In a second step, we relax the quasilinearity assumption and show that there is, in an appropriate sense, an open set of preferences for the buyer such that an equilibrium involving the same strategies as in our main examples exists.

5.1 Quasilinear Preferences

The desired translation property for upper contour sets is a special case of the property that, if type 1 is ready to trade a bundle (q, t), given that she can already buy any quantity at unit price p, then type 2 facing the same options would also choose to trade the bundle (q, t).[16] Theorem 3 below provides, in the quasilinear case, a condition on both types' demand functions that ensures that this is so, and thus that an equilibrium can be constructed along the lines of Theorem 2.

Formally, let type i's preferences over $X \equiv \mathbb{R}_+ \times \mathbb{R}$ be represented by (3). Then, for each p in the relevant range, the demand of type i at price p is given by

$$D_i(p) \equiv (\partial u_i)^{-1}(p).$$

Because $\partial^2 u_i < 0$, the demand functions D_i are strictly decreasing, that is, $\partial D_i < 0$ as long as $D_i > 0$. Moreover, because, by Assumption SC, $\partial u_2 > \partial u_1$, they are strictly ordered, that is, $D_2 > D_1$ as long as $D_2 > 0$. The following result shows that a strengthening of this property is sufficient to ensure the existence of an equilibrium.

Theorem 3 *Suppose that the buyer's preferences satisfy (3), that Assumptions 1-v and 2-v_2 and conditions (13)–(14) are satisfied, and that there are sufficiently many sellers. Then, if, for any price p in the relevant range,*

$$|\partial D_2(p)| > |\partial D_1(p)|, \tag{16}$$

an equilibrium can be constructed along the lines of Theorem 2.

Observe that (16) represents a strengthening of the single-crossing property: not only is type 2 always more willing to buy than type 1 at any price p, but the demand of type 2 is always more sensitive to price increases than that of type 1. The primary use of Theorem 3 is to show that perturbations of our leading examples admit similar equilibria. For instance, in the insurance example of Section 2.4.1, we maintain that the agent has constant absolute risk aversion and, thus, quasilinear preferences, but we perturb her preferences by making the low-risk agent more risk averse than the high-risk agent.

Example 1 *Type i has preferences represented by (3), with*

$$u_i(Q) = -\frac{1}{\alpha_i} \ln(v_i \exp(-\alpha_i(W_B + Q)) + (1 - v_i) \exp(-\alpha_i W_G)),$$

where $v_2 > v_1$, $\alpha_1 > \alpha_2$, and $[(1 - v_1)/v_1]/[(1 - v_2)/v_2] > \exp((\alpha_1 - \alpha_2)(W_G - W_B))$.

[16] Observe that the quantity q could be negative, so that the bundle (q, t) need not be a contract.

In the pure-trade example of Section 2.4.1, we perturb the seller's quadratic cost function.

Example 2 *Type i has preferences represented by (3), with*

$$u_i(Q) \equiv \theta_i Q - C(Q),$$

where $\theta_2 > \theta_1$, $\partial^2 C > 0$, $\partial^3 C < 0$, and $\lim_{Q \to \infty} \partial C(Q) = \infty$.

A noticeable feature of the functions (u_1, u_2) satisfying condition (16) is that they form a contractible subspace of the pairs of twice continuously differentiable and strictly concave utility functions over $\mathbb{R}_+$; that is, it can be continuously shrunk into a point. This property notably implies that this subspace is simply connected: it has no "holes." This means that, between two examples such as Examples 1–2, one can construct one, and essentially only one, path of similar examples connecting them.

5.2 General Preferences

Examples 1–2 show that the existence of an equilibrium of the kind constructed in Section 4 is not confined to preference specifications for the buyer that satisfy (3)–(4). We now establish that these examples are robust, in the sense that the desired translation property for upper contour sets is satisfied for any choice of preferences that are "close enough" to those in these examples. That is, we show that Theorem 2 holds for an open set of preferences, including preferences that are not quasilinear.

5.2.1 A Geometrical Condition

We first provide a geometrical interpretation of condition (16). Recall from Debreu (1972) or Mas-Colell (1985, Proposition 2.5.1) that the (Gaussian) curvature κ_i of type i's indifference curve at an arbitrary point of V is

$$\kappa_i \equiv \frac{1}{\|\partial U_i\|^3} \begin{vmatrix} -\partial^2 U_i & \partial U_i \\ -\partial U_i' & 0 \end{vmatrix}. \tag{17}$$

For quasilinear preferences represented by (3), simple algebra using $\tau_i(D_i(p), pD_i(p)) = p$ and $D_i(p) = (\partial u_i)^{-1}(p)$ yields that, at type i's optimal demand at price p,

$$\kappa_i(D_i(p), pD_i(p)) = -\frac{\partial^2 u_i(D_i(p))}{(1+p^2)^{3/2}} = \frac{1}{|\partial D_i(p)|(1+p^2)^{3/2}}. \tag{18}$$

Hence an alternative way of stating (16) is that

$$\tau_1(Q,T) = \tau_2(Q',T') \quad \text{implies} \quad \kappa_1(Q,T) > \kappa_2(Q',T'). \tag{19}$$

That is, the curvature of any indifference curve of type 1 is strictly higher than that of any indifference curve of type 2 at any pair of points where they have equal slopes. Note that (19), unlike (16), does not require preferences to be quasilinear. This property has a natural interpretation in terms of variations of marginal rates of substitution. Indeed, from (17),

$$\left.\frac{d\tau_i}{dQ}\right|_{U_i=\text{const}} = -(1+\tau_i^2)^{3/2}\kappa_i. \tag{20}$$

Together with (19), this identity implies that, at any pair of points (Q,T) and (Q',T') where type 1's and type 2's marginal rates of substitution coincide, type 1's marginal rate of substitution at (Q,T) decreases faster than type 2's marginal rate of substitution at (Q',T') along the corresponding indifference curves. As a result, the demand of type 1 reacts less to changes in prices than the demand of type 2, which in the quasilinear case is expressed by (16). It is clear from (20) that preferences which satisfy (19) also satisfy the desired property for deterring cream-skimming deviations: the translate of the upper contour set of c for type 1 along the vector c'' lies in the upper contour set of $c+c'$ for type 2.

5.2.2 The Openness Result

To establish our openness result, we first specify a set $\mathbf{P}_{sc}$ of strictly convex preferences for each type of the buyer, endowed with a suitable topology. This set, which we construct in Appendix B, can be identified to a subspace of $C^2(V)$, the space of real-valued C^2 functions over V endowed with the topology of uniform convergence over compact subsets of V of functions and of their derivatives up to the order 2. In line with the assumptions made in Section 2.1, any preference in $\mathbf{P}_{sc}$ can be represented by a strictly quasiconcave utility function $U \in C^2(V)$ such that $\partial U/\partial T < 0$.

Then, the following openness result holds.

Theorem 4 *For buyer's preferences in an open subset of* $\mathbf{P}_{sc} \times \mathbf{P}_{sc}$*, an equilibrium can be constructed along the lines of Theorem 2.*

The proof proceeds by showing that, around the preferences constructed in Examples 1–2, there is an open set of preferences in $\mathbf{P}_{sc} \times \mathbf{P}_{sc}$ such that condition (19) uniformly holds over a large enough compact subset of V. Key to this logic is that condition (19) involves a strict comparison of curvatures, and that marginal rates of substitution and curvatures vary continuously with preferences in the topology of $\mathbf{P}_{sc}$.

Theorem 4 in turn sheds light on the main examples of Section 2.4: as the indifference curves for type 1 and type 2 are then translates of each other, preferences in these examples do not satisfy condition (19), but instead a limit condition, namely

$$\tau_1(Q,T) = \tau_1(Q',T') \quad \text{implies} \quad \kappa_1(Q,T) = \kappa_2(Q',T').$$

This suggests that such preferences lie on the boundary of the set of preferences in $\mathbf{P}_{sc}$ such that the contract offers considered in Theorem 2 are consistent with equilibrium. Indeed, it can be shown that a necessary condition for preferences to belong to that set is

$$\kappa_1(c) \geq \kappa_2(c + c''). \tag{21}$$

Condition (21) provides a simple test to rule out certain equilibrium configurations.

Example 3 *Consider preferences as in Example 2, where $\theta_2 > \theta_1$, $\partial^2 C > 0$, and $\partial^3 C > 0$. Then an equilibrium cannot be constructed along the lines of Theorem 2.*

Thus Theorem 2 applies to $C(Q) = Q^\gamma/\gamma$ for $1 < \gamma < 2$, but not for $\gamma > 2$. This, of course, does not prejudge the possibility of alternative equilibrium constructions.

6 Discussion

6.1 Theoretical Insights

In our model, an equilibrium exists because sellers are only allowed to make take-it-or-leave-it offers to the buyer. This assumption is often made for simplicity, as in Rothschild and Stiglitz's (1976) canonical analysis of exclusive insurance markets. Indeed, under exclusivity, whether one allows sellers to compete through menus of contracts or through take-it-or-leave-it offers does not fundamentally alter the characterization of equilibrium: the same allocations emerge in both cases, although existence conditions become more stringent in the former case.[17] This insight does not carry over to nonexclusive competition. Attar, Mariotti, and Salanié (2014a) provide a full characterization of equilibrium allocations in a generalized version of Rothschild and Stiglitz's (1976) model in which sellers can offer arbitrary menus of nonexclusive contracts. They show that, in any pure-strategy equilibrium, a type of the buyer may trade only if the other type does not trade at all, a type of market breakdown reminiscent of that emphasized by Akerlof (1970).[18] By contrast, the present paper shows the existence of an equilibrium in which both types trade. In that respect, nonexclusive competition thus appears to be qualitatively different from exclusive competition.

[17] See, for instance, Hahn (1978), Fagart (1996), and Farinha Luz (2012).

[18] This result applies to the version of their model in which in which the buyer is restricted to trade nonnegative quantities, as in the present paper (see Attar, Mariotti, and Salanié (2014a, Section 5.1)).

That a restriction on the sellers' contractual opportunities is instrumental to sustain the Jaynes–Hellwig–Glosten outcome in equilibrium is also a novel contribution of our analysis. In contrast with Hellwig (1988) and Jaynes (2011), who rely on extensive forms with multiple rounds of communication between sellers to sustain this equilibrium outcome, we show that an equilibrium exists in a standard competitive game without communication. Explicit communication about a seller attempting to deal with type 1 only is replaced by the single latent contract c'', the presence of which makes it also profitable for type 2 to trade with this seller, thereby defeating any cream-skimming deviation.

From a game-theoretical viewpoint, our work is closely related to the common-agency literature, in which principals compete through mechanisms to attract a single agent. In that literature, a well-known result, sometimes acknowledged as a failure of the Revelation Principle, is that some outcomes are sustainable in equilibrium in the game relative to an arbitrary set of indirect mechanisms, but not in the corresponding direct-mechanism game.[19] In the present paper, sellers (principals) make take-it-or-leave-it offers to a single buyer (agent) who can be of two types. That is, the set of mechanisms available to each principal is even smaller than the set of direct mechanisms. Despite this restriction, existence of a pure-strategy equilibrium obtains. Moreover, expanding the sellers' strategy sets by allowing for arbitrary indirect mechanisms would lead, under Assumption 1-v, to nonexistence, as shown by Attar, Mariotti, and Salanié (2014a). The intuition for this counterintuitive result is that, when sellers are restricted to take-it-or-leave-it offers, none of them can exploit cross-subsidization between contracts at the deviation stage. As a consequence, a larger set of equilibrium allocations can be sustained.

Finally, the restriction to contracts with nonnegative quantities is crucial for our existence result. If negative quantities were allowed, a seller not trading with type 1 could use the fact that type 2 can buy the quantity Q_1 twice at a relatively low price v on the equilibrium path to offer re-buying a positive quantity from her at a price higher than v, but still lower than v_2. The corresponding quantity can moreover be chosen in such a way that it is attractive for type 2 but not for type 1 to seize this opportunity, giving rise to a profitable reverse cream-skimming deviation. Notice that this problem arises in our setting because the buyer can combine several contracts, in contrast with models of exclusive competition.

6.2 Testable Predictions

When applied to the insurance sector, the equilibrium allocation we have characterized is

[19] See, among others, Peck (1997), Peters (2001), and Martimort and Stole (2002).

natural and simple. Both types first buy the same basic coverage, priced at average cost v; the high-risk type 2 then buys an additional coverage at the fair price v_2, which is higher. In spite of its simplicity, this allocation has rich, and sometimes unanticipated, properties. First, one may notice from the zero-profit result that there are no cross-subsidies between contracts. Still, there are cross-subsidies between types: the low-risk type subsidizes the high-risk type in the pricing of basic coverage. In other words, there is pooling on one contract, in contrast to the classical, separating allocation that prevails under exclusivity. Second, there is multiple contracting, as the same consumer sometimes buys two different contracts from two different insurance companies. This is in line with many existing or proposed health insurance systems, as we noticed in the introduction.

In this section, we discuss more precisely the testable predictions of our analysis. Recall that, under exclusivity, the standard predictions of the Rothschild and Stiglitz (1976) model are that the unit price of coverage should increase with coverage, and that there should be a positive correlation between the aggregate coverage bought by an agent and this agent's risk. Empirically, these properties may be tested by surveying consumers to get data on their total coverage and total insurance premium. One may alternatively choose to gather information on the contracts offered by firms. Under exclusivity, these two approaches make no difference, as the aggregate demand of a consumer must be supplied by a single contract offered by a single firm. However, under nonexclusivity, the second approach yields strikingly different results, as we now argue.

Recall that three contracts are offered in equilibrium: a basic contract $c = (Q_1, T_1)$, a complementary contract $c' = (Q_2 - Q_1, T_2 - T_1)$, and a latent contract $c'' = (q'', t'')$. Under conditions (13)–(14), which are necessary for an equilibrium to exist, it is easily checked that one must have

$$Q_1 > q'' > Q_2 - Q_1.$$

Now, the unit price of coverage Q_1 is low, as it is bought by both types; the unit price of coverage $Q_2 - Q_1$ is high, as it is bought by the high-risk type only; and, as can be seen in Figure 4, the unit price of coverage q'' is intermediate. A testable prediction of our model is thus that the contracts offered by firms exhibit quantity discounts, whereas consumers end up paying quantity premia. This is a striking result that contrasts with a natural intuition, namely, that nonexclusivity should push consumers towards splitting their demands between firms (Chiappori (2000)). The reason why, in our competitive setting, firms end up proposing quantity discounts, is that basic coverage must be larger than complementary coverage to prevent high-risk consumers from purchasing several basic policies from different firms.[20] Each consumer then finds in her own interest to concentrate her trades on a minimum number of contracts. The key is that firms together only propose a few contracts, which the consumer may combine. The low-risk type then ends up trading a single contract, while the high-risk type ends up buying two different contracts.

Importantly, according to the above analysis, one should observe a positive correlation between risk and coverage when considering total coverage for each consumer: indeed, the single-crossing property is enough to ensure that riskier types buy more coverage. On the other hand, with data originating from a single firm, one should now observe a negative correlation between risk and coverage: the relatively small complementary coverage $Q_2 - Q_1$ is bought only by the riskiest type. Such remarks are useful when considering the empirical evidence, as exemplified by the work of Cawley and Philipson (1999) on life insurance or the work of Finkelstein and Poterba (2004) on annuities. Because the reference model in those papers is Rothschild and Stiglitz's (1976), the above distinction between demand- and supply-side approaches is overlooked. As a result, evidence of quantity discounts, or the absence of a positive correlation property, is interpreted as rejecting the presence of adverse selection on life-insurance or annuity markets. However, such markets being nonexclusive, one must be very careful when testing for the existence of quantity discounts, as one needs to observe the total insurance coverage and the total insurance premium paid by individuals. In particular, checking only the contracts offered by firms, or the contracts sold by a given firm, may be insufficient and even misleading. Such an empirical inquiry is beyond the scope of the present paper; but it would certainly be worth proceeding to this task, while taking all precautions to ensure that data are comprehensive.[21]

6.3 Normative Analysis: Constrained Efficiency

As noted in Section 3, a key implication of our analysis is that aggregate quantities and transfers in any equilibrium of our model are of the form predicted by Jaynes (1978), Hellwig (1988), and Glosten (1994). An important feature of the corresponding aggregate allocation is that it is constrained efficient, in a sense that we define now.

[20] This explanation for quantity discounts differs from that proposed by Chade and Schlee (2012), who consider a monopolistic insurance company as in Stiglitz (1977).

[21] At least one of the econometric treatments performed in Cawley and Philipson (1999) seems to escape this criticism, as it is based on a consumer survey (AHEAD) that includes information on aggregate demand. We acknowledge that the candidate explanation given above is not the end of the story: the quoted paper remains a sizable stone in our garden, and a precious contribution. For a recent and positive test for adverse selection using the same data, see He (2009).

Formally, an *aggregate allocation* specifies an aggregate trade $x_i \in X$ for each i. An aggregate allocation $(x_i)_{i=1,2}$ is *implemented* by a set of trades $\mathcal{M} \subset X$, $(0,0) \in \mathcal{M}$, if

$$x_i \in \arg\max \{U_i(x) : x \in \mathcal{M}\}.$$

for each i. The set $\mathcal{M}$ may correspond to a tariff, as in the standard taxation principle (Rochet (1985)), or, as in our model, may result from the combinations of contract offers made by competing sellers, as in (2). Then $\mathcal{M}$ typically contains more than two trades besides the no-trade point, unlike in standard definitions of incentive compatibility.

A set of trades $\mathcal{M} \subset X$, $(0,0) \in \mathcal{M}$, is *entry proof*, if there exists no set of trades $\mathcal{M}' \subset X$, $(0,0) \in \mathcal{M}'$, such that, for any optimal combination of trades in $\mathcal{M}$ and $\mathcal{M}'$ for each type i, the corresponding expected profit over trades in $\mathcal{M}'$ is positive; that is, for each i, there exist $x_i \in \mathcal{M}$ and $x'_i \in \mathcal{M}'$ such that

$$(x_i, x'_i) \in \arg\max \{U_i(x + x') : x \in \mathcal{M} \text{ and } x' \in \mathcal{M}'\}$$

and the expected profit for an entrant from trading x'_i with each type i is nonpositive; that is, letting $(Q'_i, T'_i) \equiv x'_i$,

$$m_1(T'_1 - v_1 Q'_1) + m_2(T'_2 - v_2 Q'_2) \leq 0.$$

Note that the criterion for a successful entry embedded in this definition is that it ensures a positive expected profit no matter the optimal choice of the buyer.

Our constrained-efficiency concept superimposes an entry-proofness constraint to the standard implementability constraint.

Definition 1 *An aggregate allocation $(x_i)_{i=1,2}$ is constrained efficient if it is implemented by an entry-proof set of trades, and any aggregate allocation $(x'_i)_{i=1,2}$ that strictly Pareto dominates $(x_i)_{i=1,2}$ from the two types of buyers' viewpoint either cannot be implemented by an entry-proof set of trades or generates negative expected profit for the sellers.*

Let us now see how these concepts apply to our context. Observe first that, according to Theorem 1 and Corollary 1, the equilibrium aggregate allocation $(x_i)_{i=1,2} \equiv (Q_i, T_i)_{i=1,2}$ is implemented by the set of trades $\mathcal{M}$ associated to the Glosten (1994) tariff, that is, the piecewise-linear convex tariff defined by

$$T(Q) \equiv 1_{\{Q \leq Q_1\}} vQ + 1_{\{Q > Q_1\}}[vQ_1 + v_2(Q - Q_1)]. \tag{22}$$

It follows from arguments paralleling Glosten (1994) that this tariff is entry proof. From this it is easily seen that $(Q_i, T_i)_{i=1,2}$ is constrained efficient. Suppose, for instance, that one wants to increase the utility of type 1. Because $T_1 = vQ_1$ and $\tau_1(Q_1, T_1) = v$, the new aggregate allocation $(x'_i)_{i=1,2} \equiv (Q'_i, T'_i)_{i=1,2}$ must satisfy $T'_1 < vQ'_1$. Moreover, the requirement that $(Q'_i, T'_i)_{i=1,2}$ be implemented by an entry-proof set of trades implies that the additional profit $T'_2 - T'_1 - v_2(Q'_2 - Q'_1)$ on the marginal trade $Q'_2 - Q'_1$ be nonpositive.[22] Otherwise, an entrant could make a profit by proposing a trade $(Q'_2 - Q'_1, T'_2 - T'_1 - \varepsilon)$ for some small positive number ε: this trade, in combination with (Q'_1, T'_1), profitably attracts type 2, and makes a further profit if it also attracts type 1. Overall, it follows that, if the aggregate allocation $(Q'_i, T'_i)_{i=1,2}$ strictly increases the utility of type 1 compared to $(Q_i, T_i)_{i=1,2}$ and is implemented by an entry-proof set of trades, the corresponding expected profit

$$m_1(T'_1 - v_1 Q'_1) + m_2(T'_2 - v_2 Q'_2) = T'_1 - vQ'_1 + m_2[T'_2 - T'_1 - v_2(Q'_2 - Q'_1)]$$

is negative. A similar argument can be used to show that one cannot increase the utility of type 2 and maintain that of type 1, while satisfying the entry-proofness and break-even constraints. Thus the equilibrium allocation $(Q_i, T_i)_{i=1,2}$ is constrained efficient in the sense of Definition 1.

It may be—rightly—objected to this line of reasoning that the tariff (22) cannot be generated by a finite number of contract offers, as must be the case in our model, owing to the assumption that each seller can only propose a single contract. One may then wonder why we did not instead consider the set of trades associated to the equilibrium constructed in Theorems 2–4. The answer is simple: this set is not entry proof in the above sense. The reason is that an entrant could propose two contracts: one designed to profitably attract type 1, and one designed to limit his losses with type 2.[23]

This objection seems to undermine our claim that the equilibrium aggregate allocation is constrained efficient, for it could be argued that such a claim should be interpreted relatively to a specific set of contractual instruments. The flip side of this observation, however, is that the concept of entry proofness should also be amended to reflect such restrictions. Indeed, if potential entrants are restricted to make take-it-or-leave-it offers, that is, if the set of trades $\mathcal{M}'$ in the above definitions must contain a single trade besides the no-trade point, then it can be shown along the lines of Lemmas 3–4 that the set of trades associated to the

[22]Observe that $Q'_2 \geq Q'_1$ by the single-crossing property together with the implementability constraint.

[23]The argument follows Attar, Mariotti, and Salanié (2014a, Proposition 3). In the case of quasilinear preferences, observe that, under the assumptions of Theorem 2, any contract $\tilde{c}_1$ that allows type 1 to increase her utility by a positive amount ε also allows type 2, through the use of the latent contract c'', to increase her utility by ε. However, the entrant may also propose a contract $\tilde{c}_2 = c' - (0, 2\varepsilon)$, which, coupled to a contract c offered by an incumbent, allows type 2 to increase her utility by 2ε. Choosing $\tilde{c}_1$ close enough to c and ε close enough to zero then enables the entrant to reap almost the entire aggregate profits $T_1 - v_1 Q_1$ associated with type 1, while making arbitrarily small losses when trading with type 2.

equilibrium constructed in Theorems 2–4 is robust to entry. These considerations call for a deeper discussion of the policy implications of restricting contractual offers, to which we now turn.

6.4 Policy Implications for Insurance Markets

In the context of insurance markets, especially health insurance, our analysis suggests that, when multiple contracting is allowed, coverage can be provided by the private sector in a constrained-efficient way (see Definition 1 above). Thus, strictly speaking, social security is not needed to provide such coverage. In a similar vein, notice that neither is there a need to make basic coverage mandatory: the state need not directly interfere with the choices of consumers, who can remain sovereign in their decisions to purchase insurance. This contrasts with policy recommendations from exclusive models of competitive insurance markets under adverse selection.[24] Similarly, there is no need for taxes or subsidies: competition is enough to select a unique equilibrium in which prices efficiently reflect costs—though this rule applies to successive layers of insurance, and not to the aggregate coverage bought by each type of insuree.[25]

However, whereas there is no need to tamper with demand in the framework we consider, an important insight of our analysis is that the supply side of the economy needs to be carefully regulated. Indeed, if firms were freely allowed to engage in strategic behavior, notably in cross-subsidizing between contracts, then the market would typically break down and equilibria with multiple contracting would fail to exist (Attar, Mariotti, and Salanié (2014a)). As a result, public intervention in nonexclusive insurance markets should aim at preventing cross-subsidies between contracts at the firm level so as to sustain cross-subsidies between types at the industry level.

In our model, the absence of cross-subsidies directly stems from the requirement that each insurer offers at most one contract; this extreme assumption may be relaxed as follows. Suppose that firms are allowed to offer several contracts, under the requirement that each contract be (weakly) profitable. Then it is easily seen that the situation described in Theorem 2 remains an equilibrium.[26] A public intervention that would prohibit firms from strategically

[24] Under exclusivity, mandatory insurance is evoked in Akerlof (1970), and has been the focus of much empirical work (Finkelstein (2004), Einav, Finkelstein, and Cullen (2010), Einav and Finkelstein (2011)). Wilson (1977), Dahlby (1981), and Crocker and Snow (1985a) show that making basic coverage mandatory and simultaneously allowing private insurers to compete on an extended coverage allows one to reach an informationally constrained efficient outcome. In the nonexclusive case, Villeneuve (2003) performs a similar analysis, however in a model that assumes linear pricing.

[25] See Crocker and Snow (1985b) for a study of taxes and subsidies under exclusivity.

[26] We sketch here a proof. A deviation with two contracts must attract the high-risk type on one contract

making losses on a contract in order to boost their profits by cream skimming on some other contract could take various forms. One could require that each insurance contract forms its own profit center in each insurance company, or rely on a monitoring of the level of risk borne by firms on each contract: for instance, one may prohibit firms from making large profits on basic coverage. An alternative approach that has been followed by several countries in the case of health insurance is to make the market for basic coverage entirely nonprofit;[27] but this imposes to define precisely what is basic coverage, something which is not required according to our analysis.

and the low-risk type on the other contract. Because the low-risk type still can buy his demand at price v after the deviation, this last contract must thus be sold at a unit price below v. Then, because the latent contract c'' is still available, it might also attract the low-risk type, a situation in which it would not be profitable. To avoid this, the former contract must allow the high-risk type to get strictly more than her equilibrium utility; but given the existing offer of contracts this means that it must be sold at a price below v_2, and thus would make losses.

[27]The case of Germany and Switzerland is discussed in Thomson, Osborne, Squires, and Jun (2013, pages 57 and 119).

Appendix A

Proof of Theorem 1. Subscripts i and j refer throughout to the buyer's possible types, with $i \neq j$ by convention. For all i and k, let (q_i^k, t_i^k) be the equilibrium trade of type i with seller k, with $(q_i^k, t_i^k) \equiv (0,0)$ by convention if type i does not trade with seller k. Denote by $b_i^k \equiv t_i^k - v_i q_i^k$ the corresponding profit for seller k; his expected profit then writes as $b^k \equiv m_1 b_1^k + m_2 b_2^k$. We let $B_i \equiv \sum_k b_i^k$ and $B \equiv \sum_k b^k$ be the corresponding aggregate type-by-type and expected profits. Define $s_i^k \equiv t_i^k - t_j^k - v_i(q_i^k - q_j^k)$ to be the profit from trading $(q_i^k - q_j^k, t_i^k - t_j^k)$ with type i, and similarly define $S_i \equiv \sum_k s_i^k = T_i - T_j - v_i(Q_i - Q_j)$ to be the profit from trading $(Q_i - Q_j, T_i - T_j)$ with type i. Because of the accounting identity $B = T_1 - vQ_1 + m_2 S_2$, establishing (9)–(10) amounts to proving that

$$B = S_2 = 0, \tag{23}$$

an implication of which, because sellers cannot make negative expected profits, is that they all earn zero expected profit in equilibrium, $b^k = 0$ for all k.

The proof of (23) goes through a series of steps, corresponding to various deviations for the sellers. We will thereby establish a number of intermediate results that are explicitly labelled as formulas (24)–(38) below.

As a preliminary remark, note that $S_i = B_i - B_j + (v_i - v_j)Q_j$, which, because $Q_2 \geq Q_1 \geq 0$ by Assumption SC and $c_2 > c_1$ by Assumption CV, implies that

$$S_2 \geq B_2 - B_1 \tag{24}$$

and

$$S_1 + S_2 \leq 0. \tag{25}$$

We can now proceed to the bulk of the argument.

Step 1 The first deviation we examine consists for any seller k in offering a contract $(Q_i, T_i - \varepsilon)$ for some positive number ε. This attracts type i for sure. If this only attracts type i, then one must have $b^k \geq m_i(B_i - \varepsilon)$ or, equivalently, $m_j B_j \geq B - b^k - m_i \varepsilon$. If this attracts both types, then one must have $b^k \geq T_i - vQ_i - \varepsilon$ or, equivalently, $m_j S_j \geq B - b^k - \varepsilon$. Letting ε go to zero, we get that, for each k,

$$m_1 \max\{B_1, S_1\} \geq B - b^k, \tag{26}$$

$$m_2 \max\{B_2, S_2\} \geq B - b^k. \tag{27}$$

Note that $B - b^k = \sum_{k' \neq k} b^{k'} \geq 0$ for all k. Several consequences follow.

First, the sellers' aggregate profits on type 1 are nonnegative,

$$B_1 \geq 0. \tag{28}$$

To see why, observe that, if $B_1 < 0$, then $S_1 \geq 0$ by (26), so that $S_2 \leq 0$ by (25), and hence $B_2 < 0$ by (24). But then the aggregate profits $B = m_1 B_1 + m_2 B_2$ are negative, a contradiction.

Second, $B_1 \geq S_1$, so that (26) reduces to

$$b^k \geq m_2 B_2 \tag{29}$$

for all k. To see why, observe that, if $B_1 < S_1$, then $S_1 > 0$ by (28), so that $S_2 < 0$ by (25), and hence $B_2 \geq 0$ by (27) and $S_2 + B_1 \geq 0$ by (24). But then, because, by assumption, $B_1 < S_1$, we get that $S_1 + S_2 > 0$, which contradicts (25).

Third, the profit from trading $(Q_2 - Q_1, T_2 - T_1)$ with type 2 is nonnegative,

$$S_2 \geq 0. \tag{30}$$

To see why, observe that, if $S_2 < 0$, then $B_2 \geq 0$ by (27), which reduces to $b^k \geq m_1 B_1$. Summing this inequality to (29), we get that $2b^k \geq B$. In turn, summing these inequalities over k, we get that $2B \geq nB$, which, as $n \geq 3$, implies that $B = 0$. But then, from (28) and $B_2 \geq 0$, we get that $B_1 = B_2 = B = 0$ and thus, by (24), that $S_2 \geq 0$, a contradiction.

Fourth, the profit from trading $(Q_1 - Q_2, T_1 - T_2)$ with type 1 is nonpositive,

$$S_1 \leq 0. \tag{31}$$

This follows at once from (25) and (30).

Fifth, $S_2 \geq B_2$, so that (27) reduces to

$$m_2 S_2 \geq B - b^k \tag{32}$$

for all k. To see why, observe that, if $S_2 < B_2$, summing the inequalities (27) over k yields $n m_2 B_2 \geq (n-1)B$, whereas summing the inequalities (29) over k yields $B \geq n m_2 B_2$. It follows that $n m_2 B_2 \geq n(n-1) m_2 B_2$ and thus, as $n \geq 3$, that $B_2 \leq 0$. But then, we get from (30) that $S_2 \geq B_2$, a contradiction.

Step 2 The second deviation we examine consists for any seller k in offering a contract $(q_i^k, t_i^k - \varepsilon)$ for some positive number ε. This attracts type i for sure, for instance along with the contracts $(q_i^{k'}, t_i^{k'})$, $k' \neq k$. If this only attracts type i, then one must have $b^k \geq m_i(b_i^k - \varepsilon)$ or, equivalently, $b_j^k \geq -(m_i/m_j)\varepsilon$. If this attracts both types, then one must have $b^k \geq t_i^k - vq_i^k - \varepsilon$ or, equivalently, $m_j s_j^k \geq -\varepsilon$. Letting ε go to zero, we get that, for each k,

$$\max\{b_1^k, s_1^k\} \geq 0, \tag{33}$$

$$\max\{b_2^k, s_2^k\} \geq 0. \tag{34}$$

A consequence of this is that sellers make nonnegative profits with type 1,

$$b_1^k \geq 0 \tag{35}$$

for all k. To see why, observe that, if $b_1^k < 0$ for some k, then $b_2^k > 0$ because seller k must earn nonnegative expected profit. But then we get that $s_1^k = b_1^k - b_2^k + (v_1 - v_2)q_2^k < 0$, which contradicts (33).

Step 3 The third deviation we examine consists for any seller k in offering a contract $(q_1^k + Q_2 - Q_1, t_1^k + T_2 - T_1 - \varepsilon)$ for some positive number ε. This attracts type 2 for sure, for instance along with the contracts $(q_1^{k'}, t_1^{k'})$, $k' \neq k$. If this only attracts type 2, then one must have $b^k \geq m_2[t_1^k + T_2 - T_1 - \varepsilon - v_2(q_1^k + Q_2 - Q_1)]$ or, equivalently, $m_1 b_1^k \geq m_2(S_2 - s_2^k - \varepsilon)$. If this attracts both types, then one must have $b^k \geq t_1^k + T_2 - T_1 - \varepsilon - v(q_1^k + Q_2 - Q_1)$ or, equivalently, $m_1 S_1 \geq m_2(S_2 - s_2^k) - \varepsilon$. Letting ε go to zero, we get that, for each k, $m_1 \max\{b_1^k, S_1\} \geq m_2(S_2 - s_2^k)$ and thus

$$m_1 b_1^k \geq m_2(S_2 - s_2^k) \tag{36}$$

by (31) and (35). Two consequences follow.

First, the sellers' aggregate profits on type 2 are nonpositive,

$$B_2 \leq 0. \tag{37}$$

To see why, observe that summing (36) over k yields $m_1 B_1 \geq (n-1)m_2 S_2$, whereas summing (32) over k yields $nm_2 S_2 \geq (n-1)B$ or, equivalently, $m_1 B_1 \leq -m_2 B_2 + [n/(n-1)]m_2 S_2$. Chaining these inequalities yields $m_2 B_2 \leq [n/(n-1) - n + 1]m_2 S_2$, from which (37) follows as $n \geq 3$ and $S_2 \geq 0$ by (30).

Second, the profit from trading $(q_2^k - q_1^k, t_2^k - t_1^k)$ with type 2 is nonnegative,

$$s_2^k \geq 0 \tag{38}$$

for all k. To see why, observe that, by (36), $m_2 b_2^k = b^k - m_1 b_1^k \leq b^k - m_2 S_2 + m_2 s_2^k$. Applying (32) to some $k' \neq k$ yields $m_2 S_2 \geq B - b^{k'} \geq b^k$ and hence $b^k - m_2 S_2 \leq 0$. Combining these inequalities yields $b_2^k \leq s_2^k$, from which (38) follows by (34).

Step 4 In this step, we investigate the structure of equilibrium in the hypothetical case where $S_2 > 0$. Then, $Q_2 > Q_1$ and, as each seller offers a single contract, some seller k must only trade with type 2. For any such k, $b_1^k = 0$ and $b_2^k = s_2^k$; hence, by (36), $b_2^k \geq S_2$.

Now, in this situation, it is not possible that all the sellers who trade with type 2, only trade with type 2. Indeed, in that case, we would have $B_2 \geq S_2$ according to the previous argument. But then, as $S_2 > 0$ by assumption, we would get that $B_2 > 0$, which contradicts (37). Thus, if $S_2 > 0$, at least one seller k must trade with both types, which implies that $Q_1 > 0$.

Let K_2 be the set of sellers who trade with type 2 but not with type 1, and let K_2^c be its complement. According to the above argument, if $S_2 > 0$, we know that $K_2 \neq \emptyset$, that $K_2^c \neq \emptyset$, and that $q_1^k = q_2^k > 0$ for some $k \in K_2^c$. We also have $s_2^k = b_2^k \geq S_2$ for all $k \in K_2$ and, therefore,

$$S_2 \geq S_2 - \sum_{k \in K_2^c} s_2^k \geq |K_2| S_2,$$

where the first inequality follows from (38). Hence, if $S_2 > 0$, there must be a single seller in K_2 who earns S_2 from trading with type 2. From the above inequality along with (38) and $|K_2| = 1$, we then get that $s_2^k = 0$ and thus $b^k = t_1^k - vq_1^k$ for all $k \in K_2^c$.

Let $k \in K_2^c$, so that either $q_1^k > 0$ or $q_1^k = q_2^k = 0$. The latter case, in which $b_1^k = s_2^k = 0$, is not possible, as we would have $m_2 S_2 \leq 0$ by (36), in contradiction with the assumption that $S_2 > 0$. Thus, if $S_2 > 0$, then $q_1^k > 0$ for all $k \in K_2^c$: all the sellers are thus active on the equilibrium path.

Write $K_2^c = K_1 \cup K_{12}$, where K_1 is the set of sellers who only trade with type 1, and K_{12} is the set of buyers who trade with both types. For each $k \in K_1$, we have $b^k = t_1^k - vq_1^k = m_1(t_1^k - v_1 q_1^k)$ and thus $t_1^k = v_2 q_1^k$; that is, whether type 2 is attracted by the contract (q_1^k, t_1^k) is irrelevant for any seller $k \in K_1$.

We already know that $K_{12} \neq \emptyset$. By (37), $B_2 \leq 0$. Hence, as the single seller in K_2 earns $S_2 > 0$ from trading with type 2, one must have $b_2^k < 0$ for some seller $k \in K_{12}$. Any seller $k \in K_{12}$ such that $b_2^k < 0$ can deviate by proposing $(q_1^k + \sum_{k' \in K_1} q_1^{k'}, t_1^k + \sum_{k' \in K_1} t_1^{k'} - \varepsilon)$ for some positive number ε. This attracts type 1 for sure, for instance along with the contracts $(q_1^{k'}, t_1^{k'})$, $k' \in K_{12}$, $k' \neq k$. Because $b_2^k = t_1^k - v_2 q_1^k < 0$ and, as observed in the previous paragraph, $t_1^{k'} = v_2 q_1^{k'}$ for all $k' \in K_1$, at worst this also attracts type 2. Letting ε go to zero, we get that

$$b^k \geq t_1^k - vq_1^k + \sum_{k' \in K_1} (t_1^{k'} - vq_1^{k'}).$$

As $b^k = t_1^k - vq_1^k$ and $t_1^{k'} = v_2 q_1^{k'}$ for all $k' \in K_1$, this implies that

$$(v_2 - v) \sum_{k' \in K_1} q_1^{k'} \leq 0$$

and thus $\sum_{k' \in K_1} q_1^{k'} = 0$. It follows that $K_1 = \emptyset$ and $K_{12} = K_2^c$.

For any seller $k \in K_{12}$, we have $b^k \geq B - m_2 S_2$ by (32). But $m_2 S_2$ is precisely the expected profit of the single seller in K_2. As $K_{12} = K_2^c$, we get that $b^k \geq \sum_{k' \in K_{12}} b^{k'}$ for all $k \in K_{12}$. Because $|K_{12}| = n - 1$ and $n \geq 3$, this implies that each seller in K_{12} earns zero expected profit and thus trades at unit price v.

Under the assumption that $S_2 > 0$, we now have a clear picture of the contracts offered and traded in equilibrium: $n - 1$ sellers offer contracts stipulating positive quantities at unit price v, which make up the aggregate trade (Q_1, T_1) of type 1 and are traded by both types, while the remaining seller offers a contract stipulating an additional positive quantity $Q_2 - Q_1$ at a unit price strictly higher than v_2, which is only traded by type 2 to overall reach her aggregate trade (Q_2, T_2).

Step 5 We now derive a contradiction from the assumption that $S_2 > 0$. We start with two standard observations.

As in Attar, Mariotti, and Salanié (2014a, Proof of Lemma 4), one can show that no seller in K_{12} can be indispensable in providing type 1 with her equilibrium utility. Otherwise, some seller $k \in K_{12}$ is indispensable and can therefore propose the quantity q_1^k at a unit price slightly above v. This attracts type 1 for sure, because trading only with the sellers other than k would yield her a lower utility. Moreover, this deviation yields seller k a positive profit even in the worst-case scenario where it attracts both types.

As in Attar, Mariotti, and Salanié (2014a, Proof of Lemma 3), one can also show that $\tau_1(Q_1, T_1) = v$. Otherwise, some seller $k \in K_{12}$ can deviate by proposing a quantity close to $Q_1 > 0$ at a unit price slightly above v. This attracts type 1 for sure. Moreover, this deviation yields seller k a positive profit even in the worst-case scenario where it attracts both types.

The desired contradiction follows from these two observations. Indeed, because, as shown in Step 4, all contracts are proposed at a unit price at least equal to v, the marginal condition $\tau_1(Q_1, T_1) = v$ implies that the equilibrium indifference curve of type 1 intersects his budget set $\{(\sum_{k \in K} q^k, \sum_{k \in K} t^k) : K \subset \{1, \ldots, n\}\} \cap X$ only at the point (Q_1, T_1). Because no seller in K_{12} is indispensable in providing type 1 with her equilibrium utility, this means that the aggregate trade (Q_1, T_1) remains available if any such seller k withdraws his contract; but this seller may then propose to trade the quantity $Q_2 - Q_1$ at a unit price between v_2 and

$(T_2 - T_1)/(Q_2 - Q_1)$, the latter number being strictly larger than v_2 if $S_2 > 0$. This attracts type 2 for sure, for instance along with the aggregate trade (Q_1, T_1). Moreover, this deviation yields seller k a positive profit even in the worst-case scenario where it only attracts type 2. This, however, is impossible as the sellers in K_{12} earn zero expected profit in equilibrium according to Step 4.

We are now ready to complete the proof of Theorem 1. That $S_2 = 0$ follows from (30) along with Step 5. Together with (32), this implies that $b^k = B = 0$ for all k and hence that (23) holds. That all traded contracts must be issued at unit price v or v_2 follows from considering the families of sellers K_1, K_2, and K_{12} introduced in Step 4. Because sellers earn zero expected profit, sellers in K_2 and K_{12} must respectively trade at unit prices v_2 and v. Now, if one had $K_1 \neq \emptyset$, then type 1 should trade the corresponding contract(s) at unit price $v_1 < v$. However, as $T_1 = vQ_1$, this implies that she should trade some contract(s) at a unit price strictly above v. But the only contracts satisfying this property are traded at price v_2 between type 2 and sellers in K_2. Therefore, $K_1 = \emptyset$. Hence the result. ■

Proof of Corollary 1. To show that the marginal condition (11) holds under Assumption 1-v, one can follow Attar, Mariotti, and Salanié (2014a, Proof of Lemma 3) as we did in Step 4 of the proof of Theorem 1. The only thing to be checked is that $Q_1 > 0$. To see why, observe that, otherwise, any seller can deviate and issue a contract stipulating a small quantity at a unit price between v and $\tau_1(0, 0)$. This attracts type 1 for sure. Moreover, this deviation yields a positive profit even in the worst-case scenario where it attracts both types.

The proof that (12) holds under Assumption 2-v_2 is similar. Arguing as in Step 4 of the proof of Theorem 1, we first get that the aggregate trade (Q_1, T_1) remains available if any seller withdraws his contract. Assumption 2-v_2 then implies that there are gains from trade between type 2 and any seller, who can propose additional trades to type 2 on top of the aggregate trade (Q_1, T_1) at a unit price between v_2 and $\tau_2(Q_1, vQ_1)$. As above, this implies that $Q_2 - Q_1 > 0$. The marginal condition (12) then follows from a standard argument along the lines of Attar, Mariotti, and Salanié (2014a, Proof of Lemma 6). Hence the result. ■

Proof of Corollary 2. We argued in the proof of Corollary 1 that the aggregate trade (Q_1, T_1) remains available if any seller withdraws his contract. One can more precisely show that the way in which (Q_1, T_1) thus remains available involves only contracts with unit price v. To see why, observe that, otherwise, some seller k offers a contract (q^k, t^k) stipulating a quantity $q^k < Q_1$ at unit price $t^k/q^k < v$. (This contract cannot be traded in equilibrium, according to Theorem 1.) But then any seller $k' \neq k$ can deviate and issue a contract stipulating a quantity $Q_1 - q^k$ at a unit price $v + \varepsilon$ for some positive number ε. Because $t^k/q^k < v$, we have

$$t^k + (v + \varepsilon)(Q_1 - q^k) < vQ_1 = T_1$$

when ε is small enough. Hence seller k''s offer attracts type 1 for sure, because combining it with the contract (q^k, t^k) allows her to purchase her equilibrium quantity Q_1 in exchange for a transfer lower than T_1. Moreover, this deviation yields a positive profit even in the worst-case scenario where it attracts both types.

Let K_v be the set of sellers issuing contracts (q^k, t^k) at unit price $t^k/q^k = v$ and, for each $k \in K_v$, let $\alpha^k \equiv q/Q_1$. Fix some $k \in K_v$ who is active in equilibrium, so that, in particular, $0 < \alpha^k \leq 1$. It follows from the previous reasoning that there exists $K_v^{-k} \subset K_v \setminus \{k\}$ such that $\sum_{k' \in K_v^{-k}} \alpha^{k'} = 1$ and, therefore,

$$1 < \sum_{k' \in K_v^{-k}} \alpha^{k'} + \alpha^k = 1 + \alpha^k \leq 2.$$

Note that the aggregate trade $(1 + \alpha^k)(Q_1, vQ_1)$ is available on the equilibrium path, and cannot be strictly more preferred by type 2 than $c + c'$. Moreover, $k'' \notin \{k\} \cup K_v^k$ for some seller k'' because some sellers must issue contracts at unit price v_2 according to Theorem 1 and Corollary 1.

Fix k as above. To conclude the proof, we only need to show that

$$(1 + \alpha^k)Q_1 > Q_2. \tag{39}$$

Indeed, along with the fact that the aggregate trade $(1 + \alpha^k)(Q_1, vQ_1)$ is not strictly more preferred by type 2 than $c + c'$ and that $1 + \alpha^k \leq 2$, (39) implies that the aggregate trade $(2Q_1, 2vQ_1) = (2Q_1, 2T_1)$ is also not strictly more preferred by type 2 than $c + c'$, which is (13), and that $2Q_1 > Q_2$, which is (14). To establish (39), let us first remark that we cannot have $(1 + \alpha^k)Q_1 = Q_2$, for, otherwise, type 2 could purchase the quantity Q_2 in exchange for a transfer vQ_2 strictly lower than her equilibrium aggregate transfer $T_2 = vQ_1 + v_2(Q_2 - Q_1)$, a contradiction. Let us then suppose that $(1 + \alpha^k)Q_1 < Q_2$. Then any seller $k'' \notin \{k\} \cup K_v^k$ can deviate by offering a contract stipulating a quantity $Q_2 - (1 + \alpha^k)Q_1$ at a unit price $v_2 + \varepsilon$ for some positive number ε. As $1 + \alpha^k > 1$, we have

$$v(1 + \alpha^k)Q_1 + (v_2 + \varepsilon)[Q_2 - (1 + \alpha^k)Q_1] < vQ_1 + v_2(Q_2 - Q_1) = T_2$$

Hence seller k'''s offer attracts type 2 for sure, because combining it with the contracts (q^k, t^k) and $(q^{k'}, t^{k'})$ for $k' \in K_v^{-k}$ allows her to purchase her equilibrium quantity Q_2 in exchange for a transfer strictly lower than T_2. Moreover, this deviation yields a positive profit even in the worst-case scenario where it only attracts type 2. Hence the result. ∎

Proof of Lemma 1. Consider first type 1. Because $\tau_1(c) = v$ by (11), c is her most preferred contract with unit price v. As all offered contracts have unit prices at least equal to v, trading a single contract c is therefore optimal for type 1. Consider next type 2. Because $\tau_2(c + c') = v_2$ by (12) and the unit price v_2 of c' is strictly higher than the unit price v of c, she is strictly worse off trading only contracts c' than trading a contract c along with a contract c'. Thus type 2 optimally trades at least one contract c. By (12) again, if she trades exactly one contract c, it is optimal for her to additionally trade exactly one contract c'. Hence we only need to prove that she cannot be strictly better off trading c twice. To see why, note that, according to (15), $\tau_2(2c)$ is lower than v and thus, a fortiori, lower than the unit price v_2 of c'. This, together with (13), implies that trading c twice, possibly along with one or two contracts c', cannot yield type 2 a higher utility than trading a contract c along with a contract c'. The result follows. ∎

Proof of Lemma 2. As for type 1, the proof follows along the lines of Lemma 1, observing that the unit price of c'' is strictly higher than the unit price v of c. Consider next type 2. As $U_2(c+c'') = U_2(c+c')$, it is sufficient to check that, if type 2 trades c'' once, the optimal thing for her to do is to combine this contract c'' with exactly one contract c. Indeed, because $\tau_2(c + c'') = v$, c is, among all contracts with unit price v, the best that type 2 can combine with c''. As all offered contracts have unit prices at least equal to v, trading a single contract c is, therefore, the unique optimal choice for type 2 once she has traded c''. The result follows. ∎

Proof of Lemma 3. First, we show that there is no profitable deviation for a seller that attracts both types. Indeed, to be profitable, the corresponding contract $\tilde{c}$ would need to have a unit price strictly higher than v. However, recall that $\tau_1(c) = v$ by (11) and that all offered contracts have unit prices at least equal to v. Trading $\tilde{c}$ would then yield type 1 a strictly lower utility than trading a single contract c, which remains feasible following any seller's unilateral deviation. Such a deviation is thus not possible.

Second, we show that there is no profitable deviation for a seller that only attracts type 2. Indeed, to be profitable, the corresponding contract $\tilde{c}$ would need to have a unit price strictly higher than v_2. However, recall that $\tau_2(c + c') = v_2$ by (12) and that, by (15), type 2 cannot gain from combining any contract with unit price strictly higher than v_2 with $2c$. Trading $\tilde{c}$, possibly along with some contracts c or c', would then yield type 2 a strictly lower utility than trading a contract c along with a contract c', which remains feasible following any seller's unilateral deviation. The possibility remains that type 2 combines c'' with $\tilde{c}$. However, as all offered contracts have unit prices at least equal to v, and strictly so for $\tilde{c}$, trading a single contract c is the unique optimal choice for type 2 once she has traded c'', see the proof of Lemma 2. Such a deviation is thus not possible. The result follows. ■

Proof of Lemma 4. The result requires that sufficiently many sellers offer the contract c''. A simple upper bound on the number of required sellers can be obtained as follows. Specifically, define

$$A_1 \equiv \{(q,t) \in \mathbb{R}_+ \times \mathbb{R} : q \leq Q_2 \text{ and } vq \geq t \geq v_1 q\},$$
$$A_2 \equiv \{Kc + K'c' : (K, K') \in \{0,1,2\} \times \{0,1,2\}\},$$
$$A_3 \equiv \{(Q,T) \in \mathbb{R}_+ \times \mathbb{R} : U_1(Q,T) \geq U_1(c)\}.$$

To interpret A_1, observe that if type 1 were attracted by a contract (q,t) with $q \geq Q_2$ issued by a deviating seller, then this would mean that by trading (q,t), possibly along with other available contracts, she could reach an aggregate trade (Q,T) with $Q \geq Q_2$, which she would weakly prefer to the aggregate trade $c + c'$ that remains available following any unilateral deviation. Because $c + c'$ is the equilibrium aggregate trade of type 2 and involves an aggregate quantity Q_2, it would follow from Assumption SC that type 2 would strictly prefer (Q,T) to her equilibrium aggregate trade $c+c'$, and thus would be strictly attracted by the contract (q,t). Therefore, we can safely restrict our quest for potential cream-skimming deviation to the set of contracts (q,t) such that $q \leq Q_2$. In addition, the contracts in A_1 imply no loss for the sellers when only traded by type 1 and have a unit price lower than v, so that they are potentially attractive for type 1, either per se or combined with other available contracts. Next, A_2 is the set of aggregate trades that can be made with four sellers, two of whom offer the contract c and two of whom offer the contract c'. Last, A_3 is the upper contour set of c for type 1. Then

$$K'' \equiv \max\{K \in \mathbb{N} : (A_1 + A_2 + Kc'') \cap A_3 \neq \emptyset\} \tag{40}$$

is the maximum number of contracts c'' type 1 may ever want to trade, if she were proposed a contract in A_1, which she could complement by aggregate trades in A_2 and as many contracts c'' as she wishes. Because A_1 is compact, $c \in A_1 \cap A_3$, $(0,0) \in A_2$, and $\tau_1(c) = v$ is strictly lower than v_2 and the unit price of c'', K'' is well defined and finite. Suppose now that two sellers offer the contract c, two sellers offer the contract c', and $\max\{K''+1, 2\}$ sellers offer the contract c''. Consider now a deviation that attracts type 1. Trading the corresponding contract $\tilde{c}$, possibly along with contracts c, c', and c'', must yield type 1 at least her equilibrium utility. However, because $\tau_2(c+c'') = \tau_1(c)$, under conditions (3)–(4) the equilibrium indifference curve $\mathcal{I}_2$ of type 2 is the translate of the equilibrium indifference curve $\mathcal{I}_1$ of type 1 along the vector c''. Thus type 2 could also weakly increase her utility by trading the same contracts as type 1, plus one additional contract c''. The definition (40) of K'' ensures that type 1 will never trade more than K'' contracts c'' following the deviation. If, as postulated, $\max\{K''+1, 2\}$ sellers offer the contract c'', a contract c'' remains available for type 2 to trade even after mimicking type 1. As a result, one can construct the buyer's best response in such a way that both types trade $\tilde{c}$ with the deviating seller, which, by Lemma 3, cannot be profitable for him. The result follows. ■

Proof of Theorem 3. According to our preliminary discussion, it is sufficient to show that, for all $p < \partial u_1(0)$ and $(q,t) \in (-D_1(p), \infty) \times \mathbb{R}$,

$$u_1(D_1(p)+q) - pD_1(p) - t \geq u_1(D_1(p)) - pD_1(p)$$

implies that

$$u_2(D_2(p)+q) - pD_2(p) - t \geq u_2(D_2(p)) - pD_2(p),$$

or, equivalently, that, for all $p < \partial u_1(0)$ and $q > -D_1(p)$,

$$\Delta(p,q) \equiv [u_1(D_1(p)) - u_1(D_1(p)+q)] - [u_2(D_2(p)) - u_2(D_2(p)+q)] \geq 0. \tag{41}$$

It follows from the definitions of the functions Δ, D_1, and D_2 that $\Delta(p,0) = 0$ and $(\partial\Delta/\partial q)(p,0) = 0$ for all $p < \partial u_1(0)$. A sufficient condition for $\Delta(p,\cdot)$ to reach a global minimum over $(-D_1(p),\infty)$ at $q = 0$, as required by (41), is thus that, for each $q > -D_1(p)$, $(\partial\Delta/\partial q)(p,q) = 0$ implies that $(\partial^2\Delta/\partial q^2)(p,q) > 0$. That is, for all $p < \partial u_1(0)$ and $q > -D_1(p)$,

$$\partial u_1(D_1(p)+q) = \partial u_2(D_2(p)+q)$$

implies that

$$\partial^2 u_2(D_2(p)+q) > \partial^2 u_1(D_1(p)+q).$$

But the first term of this implication simply says that there exists a price $p' < \partial u_1(0)$ such that $D_1(p)+q = D_1(p')$ and $D_2(p)+q = D_2(p')$. Hence a sufficient condition for (41) to hold is that, for each $p' < \partial u_1(0)$,

$$\partial^2 u_2(D_2(p')) > \partial^2 u_1(D_1(p')). \tag{42}$$

Differentiating the identity $\partial u_i(D_i(p')) = p'$ with respect to p' and using the fact that, for each i, $\partial^2 u_i < 0$ and $\partial D_i < 0$, shows that (16) and (42) are equivalent. Hence the result. ■

Proof of Example 1. The parameter restrictions imposed on v_1, v_2, α_1, and α_2 ensure that Assumption SC is satisfied. A direct computation yields

$$D_i(p) = \max\left\{W_G - W_B + \frac{1}{\alpha_i}\ \ln\left(\frac{(1-p)/p}{(1-v_i)/v_i}\right), 0\right\}.$$

Thus (16) holds over the relevant range if $\alpha_1 > \alpha_2$. ■

Proof of Example 2. A direct computation yields

$$D_i(p) = \max\left\{(\partial C)^{-1}(\theta_i - p), 0\right\}.$$

Thus (16) holds over the relevant range if $\partial^2 C \circ (\partial C)^{-1}$ is decreasing, that is, because $\partial^2 C > 0$, if $\partial^3 C < 0$. ■

Proof of the Contractibility of the Set of Preferences Satisfying (16). Observe first that the equation $D(p) = (\partial u)^{-1}(p)$ and its inverse $u(Q) = \int_0^Q D^{-1}(q)\,dq$ define, over the relevant range, a homeomorphism between the set of twice continuously differentiable and strictly concave utility functions u over $\mathbb{R}_+$ and the space of continuously differentiable and strictly decreasing demand functions D. The latter space is itself contractible, as is easily seen by taking convex combinations $\lambda D + (1-\lambda)\overline{D}$ of demand functions, for some fixed demand function $\overline{D}$. Because condition (16) is, over the relevant range, linear in the strictly negative functions ∂D_1 and ∂D_2, it follows that the set of pairs (D_1, D_2) satisfying (16) is contractible and, hence, by homeomorphism, that so is the corresponding space of utility functions (u_1, u_2). ■

Proof of (20). It follows from the implicit function theorem that the analytic expression for $d\tau_i/dQ|_{U_i=\text{const}}$ is

$$-\frac{[(\partial^2 U_i/\partial Q^2) + (\partial^2 U_i/\partial Q\partial T)\tau_i](\partial U_i/\partial T) - [(\partial^2 U_i/\partial Q\partial T) + (\partial^2 U_i/\partial T^2)\tau_i](\partial U_i/\partial Q)}{(\partial U_i/\partial T)^2}. \quad (43)$$

Moreover, using the fact that $\partial U_i/\partial T < 0$, it is straightforward to check from (17) that

$$\kappa_i = \frac{(\partial^2 U_i/\partial Q^2) + 2(\partial^2 U_i/\partial Q\partial T)\tau_i + (\partial^2 U_i/\partial T^2)\tau_i^2}{(\partial U_i/\partial T)(1+\tau_i^2)^{3/2}}. \quad (44)$$

Using (44) to simplify (43) yields (20). ■

Proof of Theorem 4. According to Lemmas 1–3, we only need to show that Assumptions 1-v and 2-v_2 and conditions (13)–(14) and (19) hold for preferences for types 1 and 2 in an open subset of $\mathbf{P}_{sc} \times \mathbf{P}_{sc}$. We focus on the last condition, the proof for the first set of assumptions and conditions following in a similar way. Because a cream-skimming deviation must not attract type 2, it is sufficient to prove that (19) uniformly holds over a large enough compact subset K of V, independent of the particular pair of preferences chosen in the required set. To do so, take any pair of preferences $(\succeq_1, \succeq_2) \in \mathbf{P}_{sc} \times \mathbf{P}_{sc}$ with representation (3) over $X \equiv \mathbb{R}_+ \times \mathbb{R}$ as in Examples 1–2. (For instance, one may take $C(Q) = Q^\gamma/\gamma$ for $1 < \gamma < 2$ in Example 2, assuming in addition that the inequality in (13) is strict.) Then condition (19) is satisfied for this pair of preferences. Suppose, by way of contradiction, that condition (19) is not satisfied over K for preferences in a neighborhood of $(\succeq_1, \succeq_2)$ in $\mathbf{P}_{sc} \times \mathbf{P}_{sc}$. Then there exists a sequence $\{(\succeq_1^n, \succeq_2^n)\}$ converging to $(\succeq_1, \succeq_2)$ in $\mathbf{P}_{sc} \times \mathbf{P}_{sc}$ and sequences $\{(Q_n, T_n)\}$ and $\{(Q'_n, T'_n)\}$ in K such that, for each n,

$$\tau_{1,n}(Q_n, T_n) = \tau_{2,n}(Q'_n, T'_n) \quad \text{and} \quad \kappa_{1,n}(Q_n, T_n) \leq \kappa_{2,n}(Q'_n, T'_n). \tag{45}$$

Because $\{(\succeq_1^n, \succeq_2^n)\}$ converges to $(\succeq_1, \succeq_2)$ in $\mathbf{P}_{sc} \times \mathbf{P}_{sc}$, it follows from the definition of the topology of $\mathbf{P}_{sc}$ that, for each i, the sequences $\{\tau_{i,n}\}$ and $\{\kappa_{i,n}\}$ converge uniformly to τ_i and κ_i over K. Because K is compact, we can assume without loss of generality that the sequences $\{(Q_n, T_n)\}$ and $\{(Q'_n, T'_n)\}$ converge in K to some (Q, T) and (Q', T'). Taking limits in (45) then yields that property (19) is not satisfied for $(\succeq_1, \succeq_2)$ at (Q, T) and (Q', T'), a contradiction. Hence the result. ■

Proof of the Necessity of (21). Suppose that the contracts offered are those of Theorem 2, and that $\kappa_2(c+c'') > \kappa_1(c)$. We show that a profitable cream-skimming deviation consists in offering a contract $\tilde{c}$ in the upper contour set of c for type 1, close enough to c, and above the line with slope v_2 that passes through c and $c + c'$. As each type's utility remains available if any seller deviates unilaterally, we only need to show that type 1 is strictly better off trading $\tilde{c}$ and that type 2 would be strictly worse off trading c. The first point is clear. To show the second point, observe first that, if $\tilde{c}$ is close enough to c and (13) is strict, type 2 would be strictly worse off trading $\tilde{c}$ along with one (or two) c contract(s) instead of trading c along with c'. Second, as $\tilde{c}$ is above the line with slope v_2 that passes through c and $c+c'$, and as $\tau_2(c+c') = v_2$, type 2 would be strictly worse off combining $\tilde{c}$ with c' contract(s) only. It thus only remains to show that type 2 would be strictly worse off trading $\tilde{c}$ along with c''; indeed, because $\tilde{c}$ is close to c and $c + c''$ gives type 2 her maximum utility if she only trades contracts other than $\tilde{c}$, and because $\tau_2(c + c'') = v$ and no contract other than $\tilde{c}$ is issued at a unit price lower than v, trading further contracts would clearly be suboptimal for type 2.

To show this, it is enough to show that, for any small enough ε,

$$\tau_1(Q_1+\varepsilon, \mathcal{T}_1(Q_1+\varepsilon)) \gtrless \tau_2(Q_1+q''+\varepsilon, \mathcal{T}_2(Q_1+q''+\varepsilon)) \quad \text{if} \quad \varepsilon \gtrless 0,$$

where the mappings $Q \mapsto \mathcal{T}_1(Q)$ and $Q \mapsto \mathcal{T}_2(Q)$ stand for the analytic expressions of the indifference curves $\mathcal{I}_1$ and $\mathcal{I}_2$, and q'' is the quantity specified by c''. A sufficient condition for this to hold is that

$$\left.\frac{d\tau_1}{dQ}\right|_{U_1=U_1(c)}(Q_1) > \left.\frac{d\tau_2}{dQ}\right|_{U_2=U_2(c+c'')}(Q_1+q''),$$

which, according to (20) and to the fact that $\tau_2(c+c'') = \tau_1(c)$, amounts to $\kappa_2(c+c'') > \kappa_1(c)$, as postulated. Thus (21) is a necessary condition to construct an equilibrium along the lines of Theorem 2. ■

Proof of Example 3. By construction, $\theta_1 - \partial C(Q_1) = \tau_1(c) = v = \tau_2(c+c'') = \theta_2 - \partial C(Q_1+q'')$, where q'' is the quantity specified by c''. By quasilinearity, $D_1(v) = Q_1$ and $D_2(v) = Q_1 + q''$. Hence, according to (18) and to the fact that, for each i, $-\partial^2 u_i = \partial^2 C$, comparing the curvature of $\mathcal{I}_1$ at c to that of $\mathcal{I}_2$ at $c+c''$ just amounts to compare $\partial^2 C(Q_1)$ to $\partial^2 C(Q_1+q'')$. Because $\partial^3 C > 0$ and $q'' > 0$, one has $\partial^2 C(Q_1+q'') > \partial^2 C(Q_1)$, so that $\kappa_2(c+c'') > \kappa_1(c)$ and (21) is violated. ■

Appendix B

In this appendix, we formally construct the preference space $\mathbf{P}_{sc}$. In line with Section 2.1, we consider regular preference relations $\succeq$ over an open, convex, and comprehensive set V that contains the no-trade point $(0, 0)$. We first impose the following restrictions on $\succeq$:

(i) $\succeq$ is closed relative to $V \times V$.

(ii) $\succeq$ is strictly monotone in transfers: if $(Q, T) \in V$ and $T' > T$, then $(Q, T) \succ (Q, T')$.

(iii) $\succeq$ is convex: if $(Q, T) \succeq (Q', T')$ and $\lambda \in [0, 1]$, then $\lambda(Q, T)+(1-\lambda)(Q', T') \succeq (Q', T')$.

(iv) $\succeq$ has closed upper contour sets relative to $\mathbb{R} \times \mathbb{R}$.

(v) $\succeq$ has a boundary in $V \times V$ that is a C^2 manifold.

Conditions (i) and (iii) are standard. Condition (ii) requires monotonicity of preferences in transfers, but not necessarily in quantities. Condition (iv) is a convenient boundary condition. Condition (v) is our basic regularity condition.

Our first task is to characterize the set $\mathbf{P}$ of preferences $\succeq$ that satisfy conditions (i)–(v). The following notation will be useful. Let $\mathcal{U}_{(Q,T)}$ and $\mathcal{L}_{(Q,T)}$ be the upper and lower contour sets of (Q, T) for $\succeq$, and let $\mathcal{I}_{(Q,T)} \equiv \mathcal{U}_{(Q,T)} \cap \mathcal{L}_{(Q,T)}$ be the indifference set of (Q, T) for $\succeq$. Also denote by cl and ∂ the closure and boundary operators relative to V or $V \times V$, depending on the context. We start with two technical lemmas.

Lemma 5 *If $\succeq$ satisfies (i)–(ii), then, for each $(Q, T) \in V$,*

- *$\mathcal{U}_{(Q,T)}$ has a nonempty interior relative to $\mathbb{R} \times \mathbb{R}$.*
- *$\mathcal{I}_{(Q,T)} = \partial\mathcal{U}_{(Q,T)}$.*

Proof. To prove the first claim, observe that $V \setminus \mathcal{L}_{(Q,T)}$ is open relative to V by (i), and thus relative to $\mathbb{R} \times \mathbb{R}$ as V is an open subset of $\mathbb{R} \times \mathbb{R}$. Hence, as $V \setminus \mathcal{L}_{(Q,T)}$ is nonempty by (ii), $\mathcal{U}_{(Q,T)} \supset V \setminus \mathcal{L}_{(Q,T)}$ has a nonempty interior relative to $\mathbb{R} \times \mathbb{R}$. To prove the second claim, observe that, as $\succeq$ is closed relative to $V \times V$ by (i), $\mathcal{U}_{(Q,T)}$ and $\mathcal{L}_{(Q,T)}$ are closed relative to V. Therefore, we have

$$\partial\mathcal{U}_{(Q,T)} \equiv \mathrm{cl}(\mathcal{U}_{(Q,T)}) \cap \mathrm{cl}(V \setminus \mathcal{U}_{(Q,T)}) = \mathcal{U}_{(Q,T)} \cap \mathrm{cl}(V \setminus \mathcal{U}_{(Q,T)}) \subset \mathcal{U}_{(Q,T)} \cap \mathcal{L}_{(Q,T)} = \mathcal{I}_{(Q,T)}.$$

The reverse inclusion holds if $\mathcal{I}_{(Q,T)} \subset \mathrm{cl}(V \setminus \mathcal{U}_{(Q,T)})$, which is obviously true because, for each $(Q', T') \in \mathcal{I}_{(Q,T)}$, $(Q', T' + \varepsilon) \in V$ for any small enough $\varepsilon > 0$ by openness of V, and $(Q', T') \succ (Q', T' + \varepsilon)$ for any such ε by (ii). The result follows. ■

Lemma 6 *If $\succeq$ satisfies (i)–(iv), then, for each $(Q,T) \in V$, $\partial \mathcal{U}_{(Q,T)}$ is connected.*

Proof. We first show that $\mathcal{U}_{(Q,T)}$ does not contain a vertical line $\{(Q',T') : T' \in \mathbb{R}\}$. Indeed, if this were the case then, by (iii), $\mathcal{U}_{(Q,T)}$ would contain all the points of the form $(1-1/T')(Q,T)+(1/T')(Q_1,T')$ for $T'>1$. Letting T' go to infinity, this would imply by (i) that $\mathcal{U}_{(Q,T)}$ contains the point $(Q,T+1)$, which is ruled out by (ii). Hence the claim. We next show that, if $\mathcal{U}_{(Q,T)}$ contains a nonvertical line $\{(Q',T') : T' = aQ'+b\}$, then $\mathcal{U}_{(Q,T)}$ is a union of closed half-spaces of the form

$$\bigcup_{(\tilde{Q},\tilde{T})\in\mathcal{U}_{(Q,T)}} \{(Q',T') \in \mathbb{R}\times\mathbb{R} : T' \leq \tilde{T} + a(Q'-\tilde{Q})\}. \tag{46}$$

Indeed, in that case, it follows from (iii) that, for each $(\tilde{Q},\tilde{T}) \in \mathcal{U}_{(Q,T)}$ and for each $(Q',Q'') \in \mathbb{R}\times\mathbb{R}$ such that $(Q'-\tilde{Q})(Q''-\tilde{Q})>0$ and $|Q'-\tilde{Q}|<|Q''-\tilde{Q}|$, $\mathcal{U}_{(Q,T)}$ contains the point $[(Q''-Q')/(Q''-\tilde{Q})](\tilde{Q},\tilde{T})+[(Q'-\tilde{Q})/(Q''-\tilde{Q})](Q'',aQ''+b)$. Letting $|Q''|$ go to infinity and taking advantage of (i), we obtain that $\mathcal{U}_{(Q,T)}$ contains the point $(Q',\tilde{T}+a(Q'-\tilde{Q}))$, and hence, by (ii), all the points (Q',T') with $T' \leq \tilde{T}+a(Q'-\tilde{Q})$. Hence the claim as $(\tilde{Q},\tilde{T})$ is arbitrary. It follows from (46) that $\mathcal{U}_{(Q,T)}$ is either the entire space $\mathbb{R}\times\mathbb{R}$, or a closed half-space in $\mathbb{R}\times\mathbb{R}$, and thus also relative to V. The former alternative is impossible as $\mathcal{U}_{(Q,T)}$ does not contain a vertical line. Thus the latter alternative holds, so that $\partial\mathcal{U}_{(Q,T)}$ is a line and, therefore, is connected. Finally, suppose that $\mathcal{U}_{(Q,T)}$ does not contain a line. By Lemma 5, $\mathcal{U}_{(Q,T)}$ has a nonempty interior in $\mathbb{R}\times\mathbb{R}$. Moreover, because V is comprehensive, $\mathcal{U}_{(Q,T)}$ is unbounded by (ii) and, according to (iv), $\mathcal{U}_{(Q,T)}$ is closed relative to $\mathbb{R}\times\mathbb{R}$. Thus, as $\mathcal{U}_{(Q,T)}$ does not contain a line, its boundary in $\mathbb{R}\times\mathbb{R}$ is homeomorphic to $\mathbb{R}$, and is therefore connected (Bourbaki (2003, Chapter II, §2, Exercise 19) a)). Because V is open in $\mathbb{R}\times\mathbb{R}$ and $\mathcal{U}_{(Q,T)}$ is closed in $\mathbb{R}\times\mathbb{R}$ by (iv), the boundary of $\mathcal{U}_{(Q,T)}$ in $\mathbb{R}\times\mathbb{R}$ is nothing but $\partial\mathcal{U}_{(Q,T)}$. The result follows. ■

Lemmas 5–6 imply the following representation result.

Lemma 7 *$\succeq$ satisfies (i)–(v) if and only if it admits a quasiconcave C^2 utility function U such that $\partial U/\partial T < 0$ and such that $U^{-1}((-\infty,v])$ is closed in $\mathbb{R}\times\mathbb{R}$ for all $v \in \mathbb{R}$.*

Proof. (Direct part.) Suppose that $\succeq$ is representable by U. Then $\succeq$ trivially satisfies (i)–(iii). Next, because $U^{-1}((-\infty,v])$ is closed in $\mathbb{R}\times\mathbb{R}$ for all $v \in \mathbb{R}$, $\succeq$ satisfies (iv). Finally, because U clearly has no critical point, it follows as in Mas-Colell (1985, Proposition 2.3.5) that $\succeq$ satisfies (v).

(Indirect part.) By (ii), $\succeq$ is locally nonsatiated, and by (v), $\partial \succeq$ is a C^2 manifold in $V \times V$. Hence $\succeq$ is of class C^2 (Mas-Colell (1985, Definition 2.3.4)). Moreover, by Lemmas 5–6, $\succeq$ has connected indifference sets $\mathcal{I}_{(Q,T)}$. Hence it admits a C^2 utility function U over V with no critical point, that is, $\partial U \neq 0$ over V (Mas-Colell (1985, Proposition 2.3.9)). That U is quasiconcave follows from (iii). To show that $\partial U/\partial T < 0$, observe first from (ii) that $\partial U/\partial T \leq 0$. Now, if $(\partial U/\partial T)(Q,T) = 0$, then $(\partial U/\partial Q)(Q,T) \neq 0$ as U has no critical point. Thus the line through (Q,T) orthogonal to $\partial U(Q,T)$ which supports the convex set $\mathcal{U}_{(Q,T)}$ is vertical. It follows then that the strict upper contour set of (Q,T) for $\succeq$, $\mathcal{U}_{(Q,T)} \setminus \mathcal{L}_{(Q,T)}$, lies either to the left or to the right of this line, which violates (ii). Finally, that $U^{-1}((-\infty, v])$ is closed in $\mathbb{R} \times \mathbb{R}$ for all $v \in \mathbb{R}$ follows from (iv). The result follows. ■

Let $\mathbf{U}$ be the set of quasiconcave C^2 functions $U : V \to \mathbb{R}$ such that $\partial U/\partial T < 0$ and such that $U^{-1}((-\infty, v])$ is closed in $\mathbb{R} \times \mathbb{R}$ for all $v \in \mathbb{R}$. We know from Lemma 7 that any preference relation in $\mathbf{P}$ can be represented by some function in $\mathbf{U}$ and, conversely, that any function $\mathbf{U}$ represents a preference relation in $\mathbf{P}$. For each $U \in \mathbf{U}$, let $P(U) \in \mathbf{U} \times \mathbf{U}$ be the preference relation represented by U. In line with Mas-Colell (1985, Chapter 2, Section 4), a topology on $\mathbf{P}$ can be constructed as follows. Note that $\mathbf{U}$ is a subspace of $C^2(V)$, the Polish space of real-valued C^2 functions over V endowed with the topology of uniform convergence over compact subsets of V of functions and of their derivatives up to the order 2 (Mas-Colell (1985, Chapter 1, K.1.2)). Then endow $\mathbf{P}$ with the identification topology from P, that is, let O be open in $\mathbf{P}$ if $P^{-1}(O)$ is open in $\mathbf{U}$. It will be convenient to work with a normalized space of utility functions, $\mathbf{U}_d \equiv \{U \in \mathbf{U} : u(0,T) = -T \text{ for all } T \text{ such that } (0,T) \in V\}$. We are now ready to complete the characterization of $\mathbf{P}$.

Lemma 8 $\mathbf{U}_d$ *and* $\mathbf{P}$ *are homeomorphic under the natural map* P.

Proof. We must prove that P restricted to $\mathbf{U}_d$ is one-to-one, onto, continuous, and open.

(One-to-one.) Let U and U' in $\mathbf{U}_d$ such that $P(U) = P(U')$. Then $U = \xi \circ U'$, where $\xi : U'(V) \to \mathbb{R}$ is C^2, increasing, and regular (Mas-Colell (1985, Proposition 2.3.11)). But for each $v \in U'(V)$, $\xi(v) = \xi(U'(0,-v)) = U(0,-v) = v$, so that $U = U'$.

(Onto.) Let $\succeq \in \mathbf{P}$, and let $U \in \mathbf{U}$ such that $\succeq = P(U)$ and $\text{range}(U(0,\cdot)) = \mathbb{R}$. Define $U' : V \to \mathbb{R}$ implicitly by $U(Q,T) = U(0, -U'(Q,T))$. Clearly $P(U') = \succeq$. We now check that $U' \in \mathbf{U}_d$. As $\partial U/\partial T < 0$, $U'(0,T) = -T$ for all T such that $(0,T) \in V$. That U' is quasiconcave follows from the observation that $\{(Q,T) \in V : U'(Q,T) \geq v\} = \{(Q,T) \in V : U(Q,T) \geq U(0,v)\}$ for all $v \in \mathbb{R}$; this also implies that $(U')^{-1}((-\infty, v])$ is closed in $\mathbb{R} \times \mathbb{R}$ for any such v. That U' is C^2 follows from the implicit function theorem along with the fact that $\partial U/\partial T \neq 0$. That $\partial U'/\partial T < 0$ follows from $(\partial U/\partial T)(Q,T) = -(\partial U/\partial T)(0, -U'(Q,T))(\partial U'/\partial T)(Q,T)$, using again the fact that $\partial U/\partial T < 0$. Hence $U' \in \mathbf{U}_d$, as claimed.

(Continuous.) This follows from the definition of the topology of $\mathbf{P}$.

(Open.) Mimic the proof of Mas-Colell (1985, Proposition 2.4.2)). The result follows. ■

Preferences in $\mathbf{P}$ are not necessarily strictly convex. Thus we must add this as a further restriction:

(vi) $\succeq$ is strictly convex: if $(Q,T) \succeq (Q',T')$, $(Q,T) \neq (Q',T')$, and $\lambda \in (0,1)$, then $\lambda(Q,T) + (1-\lambda)(Q',T') \succ (Q',T')$.

Finally, to obtain a topologically complete space of preferences, we require preferences to be nonlinear, even in a local sense. To do so, observe that because a utility function $U \in \mathbf{U}_d$ representing a preference $\succeq \in \mathbf{P}$ has no critical point, the curvature $\kappa(Q,T)$ of the indifference curve passing through any point (Q,T) of V is well defined and given by formula (17). The last restriction we impose on preferences is that this curvature nowhere vanishes.

(vii) Any point of V is regular for $\succeq$, that is, $\kappa \neq 0$ over V.

Preferences that satisfy conditions (vi)–(vii) are said to be differentiably strictly convex (Mas-Colell (1985, Definition 2.6.1)). We can now define our fundamental space of preferences as the space $\mathbf{P}_{sc}$ of preferences over V that satisfy conditions (i)–(vii). According to Lemma 8, $\mathbf{P}_{sc}$ can be seen as a subset of $\mathbf{U}_d$ and, hence, of $C^2(V)$. Our final result is that $\mathbf{P}_{sc}$ is topologically complete, as desired, and that it is contractible.

Lemma 9 $\mathbf{P}_{sc}$ *is a contractible Polish space.*

Proof. We first prove that $\mathbf{P}_{sc}$ is a Polish space. Let $\{T_n\}$ be a sequence in $\mathbb{R}$ increasing to $\sup\{T \in \mathbb{R} : (0,T) \in V\}$, and let $\{K_n\}$ be a countable collection of compact convex sets covering V. Then $\mathbf{P}_{sc}$ is the intersection of the following countable families of open sets:

$$\left\{U \in C^2(V) : \frac{\partial U}{\partial T}(Q,T) < 0 \text{ for all } (Q,T) \in K_n\right\},$$

$$\Big\{U \in C^2(V) : \text{ there exists } \varepsilon > 0 \text{ such that } U(Q,T) < U(0,T_n)$$

$$\text{if } (Q,T) \in K_n \text{ and } \inf\{\|(Q',T') - (Q,T)\| : (Q',T') \in \mathbb{R}\times\mathbb{R}\setminus V\} \leq \varepsilon\Big\},$$

$$\left\{U \in C^2(V) : \max\{|U(0,T) + T| : T \in [-n,n] \text{ and } (0,T) \in V\} < \frac{1}{n}\right\},$$

$$\Big\{U \in C^2(V): \text{ there exists } \xi : U(V) \to \mathbb{R} \text{ such that } \partial\xi > 0 \text{ over } U(V)$$
$$\text{and } \partial^2(\xi \circ U) \text{ is negative definite over } K_n\Big\}.$$

The first family deals with the monotonicity in transfers (condition (ii)), the second family with the boundary condition (condition (iv)), the third family with the normalization, and the fourth family with the differential strict convexity of preferences (conditions (vi)–(vii)), bearing in mind that differentiably strictly convex preferences that can be represented by a C^2 function with no critical point can be represented over any compact convex set K by a C^2 utility function U with no critical point such that $\partial^2 U$ is negative definite over K (Mas-Colell (1985, Proposition 2.6.4)). Hence $\mathbf{P}_{sc}$ is a G_δ in the Polish space $C^2(V)$ and thus, by Alexandrov's lemma (Mas-Colell (1985, Chapter 1, A.3.4)), a Polish space itself in the relative topology.

To prove that $\mathbf{P}_{sc}$ is contractible, we exhibit a contraction $h : \mathbf{P}_{sc} \times [0,1] \to \mathbf{P}_{sc}$, that is, we show that the identity function on $\mathbf{P}_{sc}$ is homotopic to a constant function. The proof follows Mas-Colell (1985, Proposition 2.6.7), with some adjustments. Pick an arbitrary $\overline{\succsim} \in \mathbf{P}_{sc}$ with corresponding utility function $\overline{U} \in \mathbf{U}_d$. To each $(U,\xi) \in \mathbf{U}_d \times [0,1]$ we associate a utility function $U_\xi \in \mathbf{U}_d$ as follows. We first let $U_0 \equiv U$ and $U_1 \equiv \overline{U}$. For all $\xi \in (0,1)$ and $(Q,T) \in V$, we then let $\mu_\xi(Q,T) \in (0, 1/\xi)$ be the unique solution to

$$\overline{U}(Q, \mu_\xi(Q,T)T) = U\left(Q, \left[\frac{1-\xi\mu_\xi(Q,T)}{1-\xi}\right]T\right)$$

and we let $U_\xi(Q,T) \equiv \overline{U}(Q, \mu_\xi(Q,T)T)$. What this transformation does is that, to each T' such that $(0,T') \in V$, it assigns an indifference curve $U_\xi^{-1}(-T')$, which is the vertical convex combination of $\overline{U}^{-1}(-T')$ and $U^{-1}(-T')$ with weights ξ and $1-\xi$. (Bear in mind that, by normalization, $\overline{U}(0,T') = U(0,T') = -T'$.) One can then verify that the mapping $h : (U,\xi) \mapsto U_\xi$ is the desired contraction. The result follows. ∎

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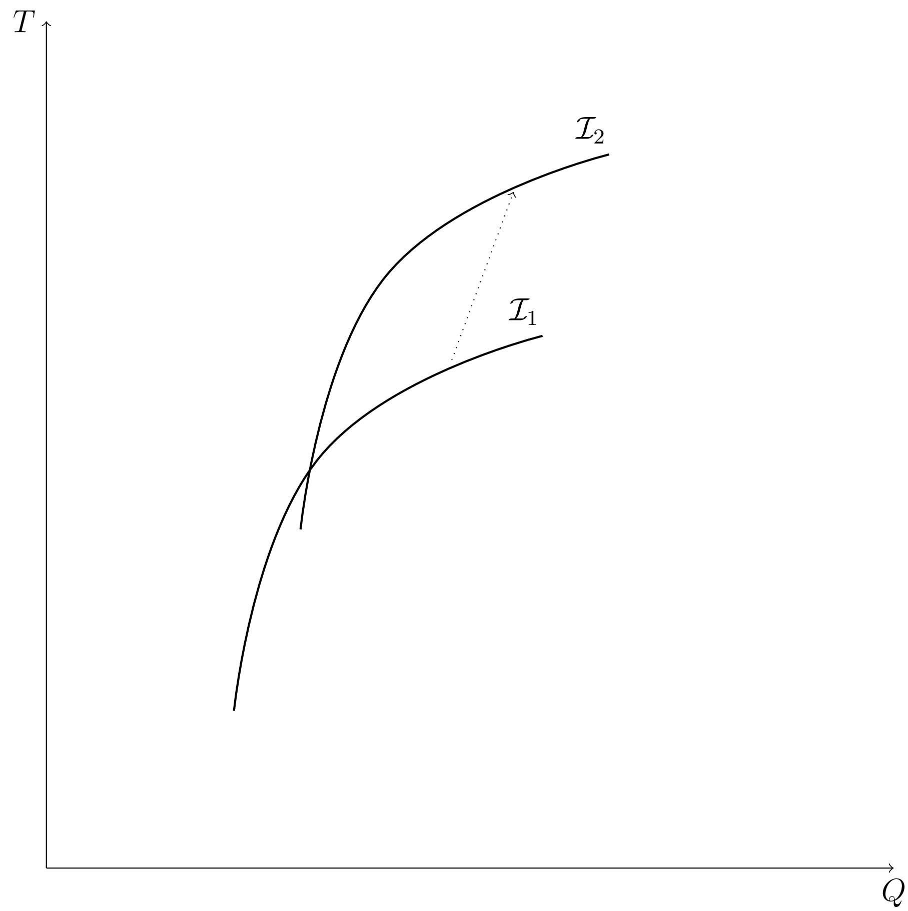

Figure 1 The translation property.

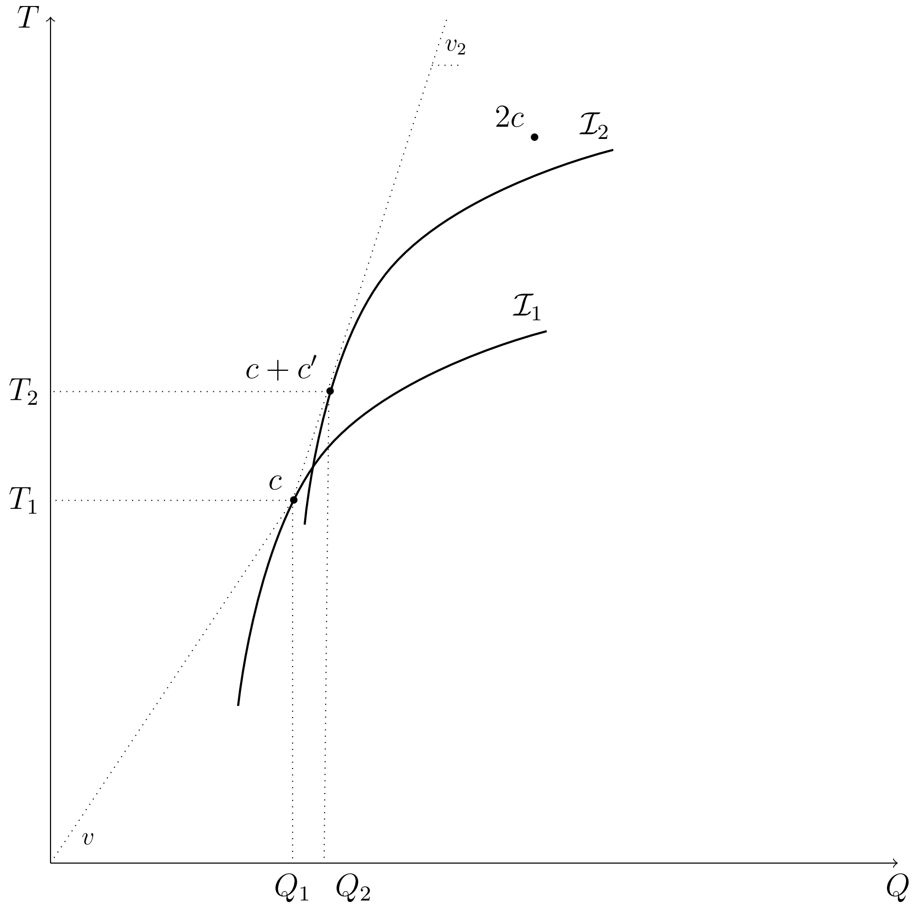

Figure 2 The Jaynes–Hellwig–Glosten outcome.

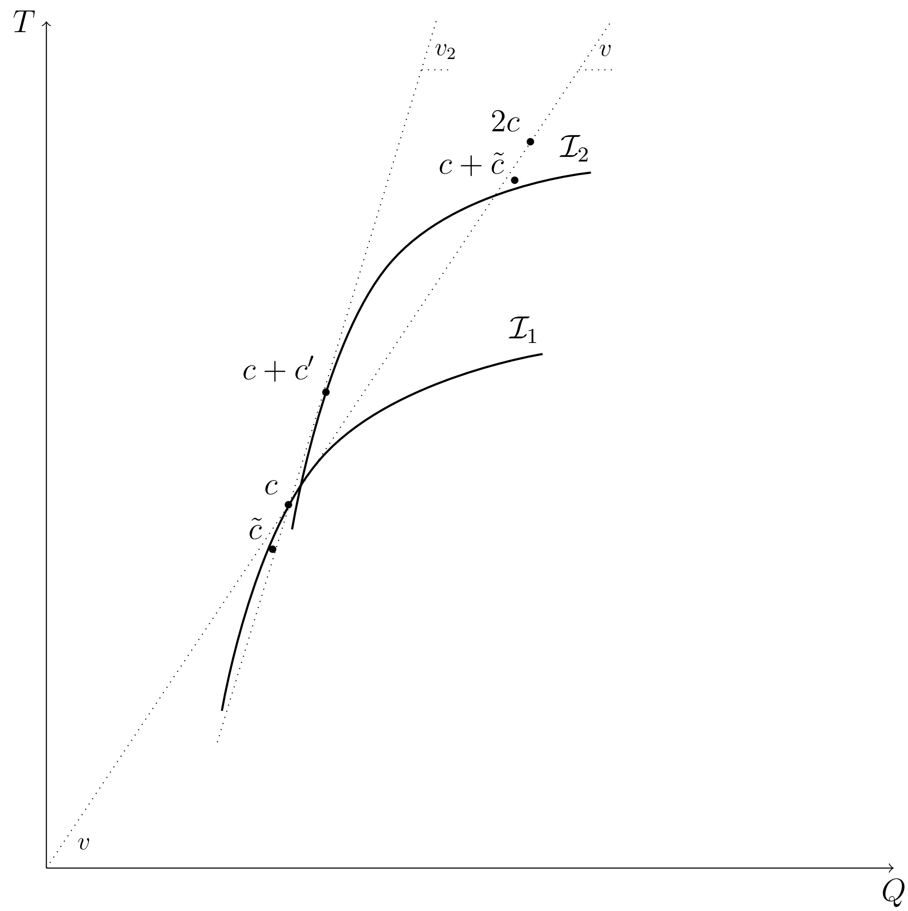

Figure 3 A cream-skimming deviation.

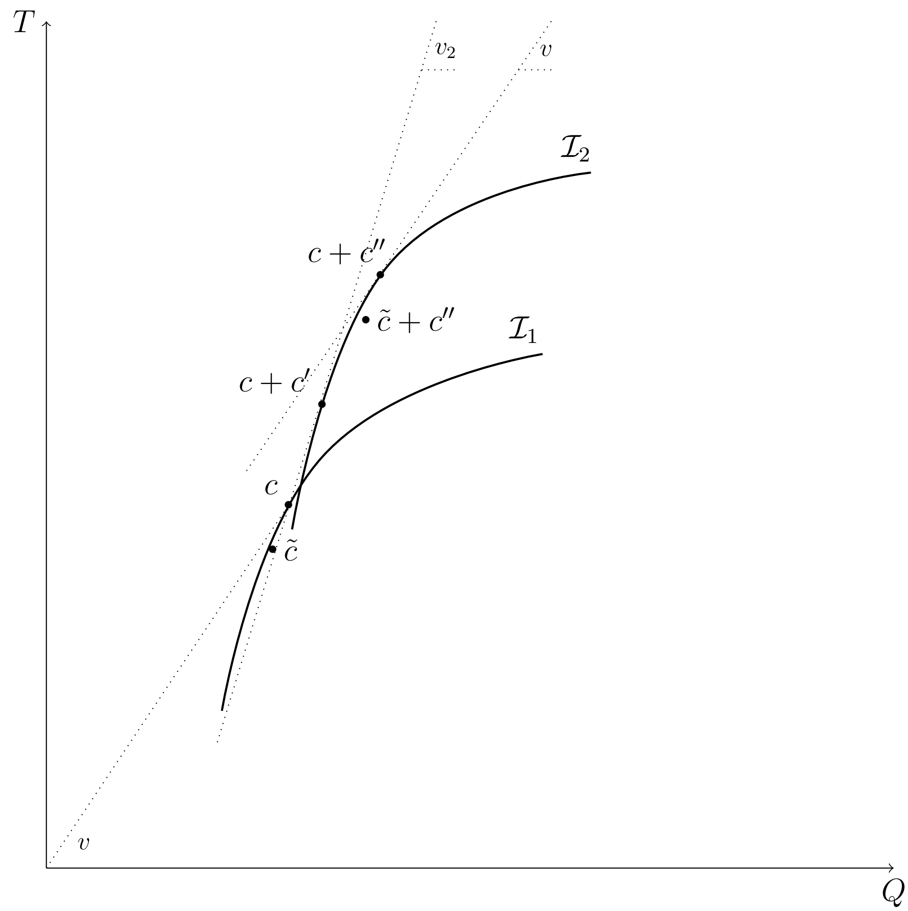

Figure 4 Deterring cream-skimming deviations with the contract c''.