TSE WP working paper series

09-063

Research Group: *Environmental Economics and Natural Resources*

July 16, 2009

Climate Change Mitigation Options and Directed Technical Change: A Decentralized Equilibrium Analysis

ANDRÉ GRIMAUD, GILLES LAFFORGUE AND BERTRAND MAGNÉ

Ecole d'économie de Toulouse (TSE) - Manufacture des Tabacs
Aile Jean-Jacques Laffont - 21, allée de Brienne - 31000 TOULOUSE
Tél : +33 (0) 5 61 12 85 89 - Fax : +33 (0) 5 61 12 86 37 - **www.TSE-fr.eu** - contact@TSE-fr.eu

Toulouse School of Economics

Climate change mitigation options and directed technical change: A decentralized equilibrium analysis

André Grimaud[a], Gilles Lafforgue[b*] and Bertrand Magné[c]

[a] *Toulouse School of Economics (IDEI and LERNA) and Toulouse Business School, France*
[b] *Toulouse School of Economics (INRA-LERNA), France*
[c] *International Energy Agency, Paris, France*

July 16, 2009

Abstract

The paper considers a growth model with climate change and three R&D sectors dedicated to energy, backstop and CCS (Carbon Capture and Storage) efficiency. First, we characterize the set of decentralized equilibria: to each vector of public tools, a carbon tax and a subsidy to each R&D sector, a particular equilibrium is associated. Second, we solve the first-best optimum problem and we implement it by computing the vector of optimal tools. In addition, we focus on the dymanic properties of the optimal carbon tax. Finally, we illustrate the theoretical model using some calibrated functional specifications. In particular, we investigate the effects of various combinations of public policies (including the optimal ones) by determining the deviation of each corresponding equilibrium from the "laisser-faire" benchmark.

JEL classification: H23, O32, Q43, Q54, Q55.

Keywords: Climate change, energy, CCS, directed technical change, carbon tax, R&D subsidies.

*Corresponding author. LERNA, 21 allée de Brienne, 31000 Toulouse – France. E-mail address: glafforg@toulouse.inra.fr. E-mail addresses of other authors: grimaud@cict.fr and Bertrand.MAGNE@iea.org

1 Introduction

As recommended by the IPCC, emerging green technologies reveal crucial for a cost-effective climate change mitigation policy. Such abatement technologies include for instance renewable energies, but also the possibilities to reduce the carbon emissions coefficient of fossil fuels. Among this second alternative, and according to the IPPC (2005), CCS (Carbon Capture and Storage) seems promising. As formulated by Hoffert et al. (2002), the decarbonisation, i.e. the reduction of the amount of carbon emitted per unit of primary energy, is intimately linked to sequestration. Carbon capture, sometimes referred to emissions control (see Kolstad and Toman, 2001), is the way of achieving this decarbonisation. This process consists in separating the carbon dioxide from other flux gases during the process of energy production. It is particularly adapted to large-scale centralized power stations but may also indirectly apply to non electric energy supply. Once captured, the gases are then being disposed into various reservoirs. The sequestration reservoirs include depleted oil and gas fields, depleted coal mines, deep saline aquifers, oceans, trees and soils. Those various deposits differ in their respective capacities, their costs of access or their effectiveness in storing the carbon permanently.

There exists a large strand of literature on economic growth, climate change and technological improvements (see for instance Bosetti et al., 2006; Bosetti et al., 2009, Edenhofer et al., 2005, 2006; Gerlagh 2006; Gerlagh and Van Der Zwaan 2006; Popp, 2004, 2006a, 2006b). In those models, the analysis usually focuses on the optimal trajectories and their comparison with the business-as-usual scenario. For many reasons that will be discussed below, it may be relevant to examine some intermediate cases between these two polar ones. Nevertheless, a decentralized economy framework is required to perform such an analysis. The objective of this paper is to complete the literature mentioned above by setting up a general equilibrium analysis that allows to compute any equilibrium in the decentralized economy.

In line with the "top-down" approach and based on the DICE and ENTICE-BR models (Nordhaus, 2008, and Popp, 2006a, respectively), we develop an endogenous growth model in which energy services can be produced from a polluting non-renewable resource as well as a clean backstop. Moreover, we assume that carbon emissions can be partially released by using a CCS (Carbon Capture and Storage) technology. We introduce three R&D sectors, the first one improving the efficiency of energy production, the second one, the efficiency of the backstop and the last one, the efficiency of the sequestration process. With this respect, we have to consider two types of market failures: the pollution associated with the carbon emissions that are not stockpiled and the research spillovers in each R&D sector. That is why, in the decentralized equilibrium, we introduce two kinds of economic policy instruments in accordance: an environmental tax on the carbon emissions and a research subsidy for the energy, backstop and CCS R&D sectors. There is an equilibrium associated to each vector of instruments. Clearly, when public instruments are optimally set, the equilibrium of the decentralized economy coincides with the first best optimum. In particular, we provide a full expression of the optimal carbon tax and we analyze its dynamic properties. We show that the tax can evolve non-monotonically over time and we characterize the driving forces that make it either growing or declining.

Next, we calibrate the model to fit the world 2005 data. As suggested by the theoretical findings, the optimal carbon tax reveals generally non-monotonic over time. We find that the implementation of this tax alone hardly provides any incentive to proceed with R&D activities. In order to provide enough R&D incentives, one needs first to correct for the externality by imposing a carbon tax and second by subsidizing the research sectors. Moreover, the introduction of some atmospheric pollution cap (or equivalently, an higher level of tax) reinforces: i) the recourse to CCS option in the middle run to prevent ceiling exceeding, and ii) the necessity to subsidy research to improve CCS efficiency.

A full description of the set of equilibria offers several advantages. Under a positive point of view, it allows to examine how the economy reacts to policy changes. We can thus look at the dissociated effect of a particular policy instrument as well as a particular subset of them, the other ones being unchanged. This will give some insights on the complementarity/substitutability of public tools. For instance, we show empirically that the simultaneous implementation of a carbon tax and appropriate R&D subsidies can strengthen the role of the backstop. Under a normative point of view, as usual, this approach allows for the computation of the economic instruments that restore the first-best optimum. However, because of budgetary, socioeconomic or political constraints, the enforcement of first-best optimum can be difficult to achieve for the policy-maker that would rather implement second-best solutions. Finally, another advantage is the possibility to compare the outcome of a cost-benefit analysis in a partial equilibrium approach with the one coming from a general equilibrium framework (Gerlagh et al., 2008).

A difficulty inherent to the characterization of the decentralized equilibrium in endogenous growth models lies in the way the research activity is modelled, in particular the type of innovation goods which are developed as well as their valuation. In the standard endogenous growth theory (Aghion and Howitt, 1998; Romer, 1990...), the production of an innovation is associated with a particular intermediate good. Research is funded by the monopoly profits of intermediate producers who benefit from an exclusive right, like a patent, for the production and the sale of these goods. However, embodying knowledge into intermediate goods usually becomes inextricable in more general computable endogenous growth models with pollution and/or natural resources such as the ones previously mentioned. In addition, those technical difficulties are emphasized when dealing with several research sectors, i.e. when there are several types of specific knowledge, each of them being dedicated to a particular input (resource, labor, capital, backstop...) as it is proposed in Acemoglu (2002).

To circumvent those obstacles, we assume the absence of tangible intermediate goods in research sectors, as it is done for instance by Gerlagh and Lise (2005), Edenhofer et al. (2006) and Popp (2004, 2006a). To provide innovations with an appropriate valuation, we adopt the shortcut proposed by Grimaud and Rougé (2008)[1] in the case of growth models with polluting resources and environmental concerns. This shortcut is based on the comparison between the socially optimal value of innovations, i.e. the income received by the innovator that would provides incentives to optimally produce innovations, with the value she effectively perceives at the decentralized equilibrium. Some empirical studies (Jones, 1995; Jones and Williams, 1998; Popp, 2004, 2006a) find that this last value is lower than the former one. This is justified in the standard literature by the presence of several failures that prevent the decentralized equilibrium to implement the first-best optimum[2]. In the present paper, we assume that the effective value of innovations is in fact equal to a given proportion (here, 30%) of the socially optimal one[3]. As already mentioned above, some R&D subsidies can be enforced in order to reduce the gap between these social and effective values[4].

[1] See also Grimaud and Tournemaine (2007).

[2] Jones and Williams (2000) exhibit four of them. i) *the duplication effect*: the R&D sector does not account for the redundancy of some research projects; ii) *the intertemporal spillover effect*: inventors do not account for that ideas they produce are used to produce new ideas; iii) *the appropriability effect*: inventors appropriate only a part of the social value they create; iv) *the creative-destruction effect*.

[3] Popp (2006a) takes 25%, and Jones and Williams (2000), 33%.

[4] According to the OECD Science, Technology and R&D Statistics, publicly-funded energy R&D in 2004 among OECD countries amounted to 9.72 billion US$, which represented 4% of overall public R&D budgets. In the United States, energy investments from the private sector have shrunk during the last

The article is organized as follows. Section 2 presents the decentralized economy and studies the behavior of agents in each sector. In section 3, we characterize both the decentralized equilibrium and the first-best optimum solutions by two sets of conditions. Next, by comparing these two sets of characterizing conditions, we show how the optimum can be implemented by appropriate public tools. In section 4, we derive a selection of numerical results and we conclude in section 5.

2 The decentralized economy

The model is mainly based on the DICE and ENTICE-BR models (Nordhaus, 2008 and Popp, 2006a, respectively). We consider a worldwide economy containing four production sectors: final output, energy services, fossil fuel and carbon-free backstop. The fossil fuel combustion process releases CO_2 flows into the atmosphere. Accumulation of those emissions acts to increase average temperatures, which implies feedbacks on the economic system that are captured by a damage function. This function measures the continuous and gradual losses in terms of final output (i.e. the direct losses in world product induced by global warming). Moreover, an atmospheric carbon concentration cap can be eventually introduced in order to take into account the high level of uncertainty and irreversibility that is generally avoided by the standard damage function. Industrial emissions can be partly sequestered and stored in carbon reservoirs owing to a CCS device. The production of final energy services, backstop and CCS require specific knowledge provided by three specific R&D sectors. We assume that all sectors, except R&D sectors, are perfectly competitive. Finally, in order to correct the two types of distortions involved by the model – pollution and research spillovers – we introduce two types of policy tools: an environmental tax on the fossil fuel use and a subsidy for each R&D sector. Note that, because of CCS, the tax applies on the sole part of the carbon emissions which are released into the atmosphere after sequestration. In that sense, carbon taxation is disconnected from the fossil resource use.

The model is sketched in Figure 1. Specific functional forms and calibration details are described in section 4. The following subsection derives the individual behaviors.

decade; governmental funding currently represents 76% of total US energy R&D expenditures (Nemet and Kammen, 2007).

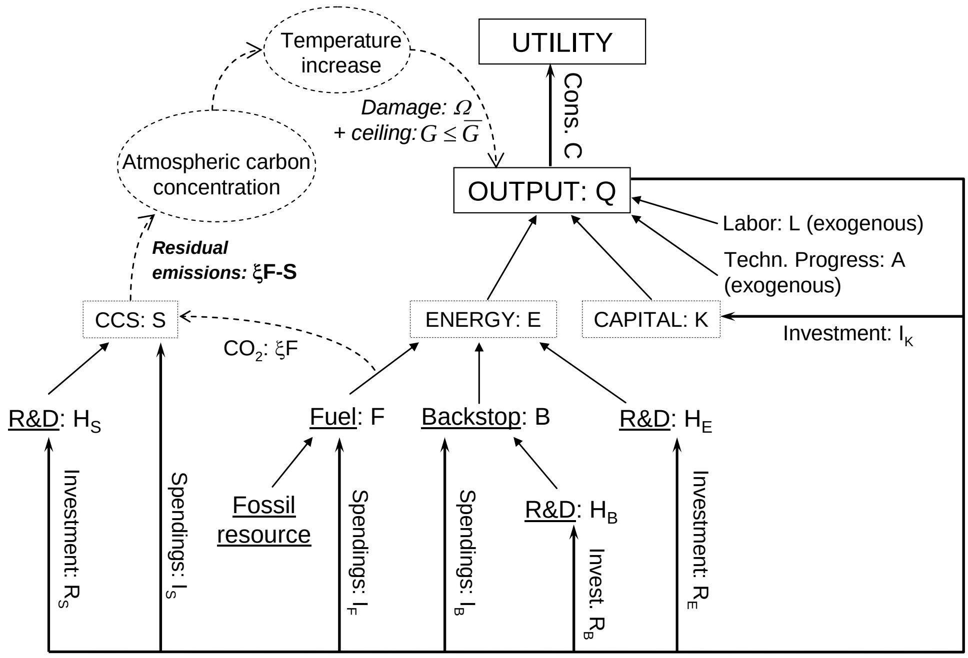

Figure 1: Description of the model

2.1 Behavior of agents

2.1.1 The final good sector

The production of a quantity Q_t of final good depends on three endogenous elements: capital K_t, energy services E_t, and a scaling factor Ω_t which accounts from climate-related damages, as discussed below. It also depends on exogenous inputs: the total factor productivity A_t and the population level L_t, growing at exogenous rates $g_{A,t}$ and $g_{L,t}$ respectively. We write $Q_t = Q(K_t, E_t, L_t, A_t, \Omega_t)$, where the production function $Q(.)$ is assumed to have the standard properties (increasing and concave in each argument).

We normalize to one the price of the final output and we denote by $p_{E,t}$, w_t, r_t and δ, the price of energy services, the real wage, the interest rate[5] and the depreciation rate of capital, respectively. Thus, the instantaneous profit of producers is $\Pi_t^Q = Q_t - p_{E,t}E_t - w_tL_t - (r_t+\delta)K_t$. Maximizing this profit function with respect to K_t, L_t and E_t, we obtain

[5] We assume here that the representative household holds the capital and rents it to firms at a rental price R_t. Standard arbitrage conditions imply $R_t = r_t + \delta$.

the following first-order conditions:

$$Q_K - (r_t + \delta) = 0 \quad (1)$$

$$Q_L - w_t = 0 \quad (2)$$

$$Q_E - p_{E,t} = 0 \quad (3)$$

where J_X stands for the partial derivative of function $J(.)$ with respect to X.

2.1.2 The energy-CCS sector

At each time t, the amount E_t of energy services is produced from two imperfect substitute primary energies: a fossil fuel, F_t, and a backstop energy source, B_t. Energy efficiency can be improved by a stock $H_{E,t}$ of specific knowledge (see Popp, 2006a). The energy technology writes $E_t = E(F_t, B_t, H_{E,t})$, where $E(.)$ is increasing and concave in each argument.

The economic and climatic systems are linked in the model by anthropogenic CO_2 emissions, generated by fossil fuel burning. Let ξ be the unitary carbon content of fossil fuel such that, without CCS, the carbon flow released into the atmosphere would be equal to ξF_t. We postulate that, at each date t, the CCS device allows a reduction of those emissions by an amount S_t, $0 \leq S_t \leq \xi F_t$ and, for the sake of simplicity, that CCS activities are part of the energy sector. To change emissions into stored carbon, the sequestration device needs specific investment spendings, $I_{S,t}$, and knowledge, $H_{S,t}$. The CCS technology writes $S_t = S(F_t, I_{S,t}, H_{S,t})$, where function $S(.)$ is assumed to be increasing and concave in each argument[6]. Note that in our model, we consider neither limited capacity of carbon sinks nor leakage problems. Those questions are addressed, for instance, by Lafforgue et al. (2008) and Keller et al. (2007) respectively.

Denoting by $p_{F,t}$ and $p_{B,t}$ the prices of fossil fuel and backstop, and by τ_t the unit carbon tax on the flow of carbon emissions $(\xi F_t - S_t)$, the energy producer chooses F_t, B_t and $I_{S,t}$ that maximizes $\Pi_t^E = p_{E,t}E_t - p_{F,t}F_t - p_{B,t}B_t - I_{S,t} - \tau_t(\xi F_t - S_t)$, where Π_t^E is the instantaneous profit before payments of innovations (we will come back on this point

[6] In a model "à la Romer" with tangible intermediate goods, the energy and CCS production functions would write $E_t = E\left[F_t, B_t, \int_0^{H_{E,t}} f^E(x_{j,t}^E)dj\right]$ and $S_t = S\left[F_t, I_{S,t}, \int_0^{H_{S,t}} f^S(x_{j,t}^S)dj\right]$ respectively, where $x_{j,t}^n$ is the j^{th} intermediate good and $f^n(.)$ is an increasing and strictly concave function, for $n = \{E, S\}$.

in section 2.1.4 below). The first order conditions write:

$$
\begin{aligned}
p_{E,t}E_F - p_{F,t} - \tau_t(\xi - S_F) &= 0 && (4)\\
p_{E,t}E_B - p_{B,t} &= 0 && (5)\\
-1 + \tau_t S_{I_S} &= 0 && (6)
\end{aligned}
$$

Condition (6) equalizes the private cost of one unit of stockpiled carbon, $1/S_{I_S}$, with the carbon tax. Moreover, from the expression of the profit function given above, the extended unit cost of fossil fuel use, denoted by $c_{F,t}$, includes the fuel price, the environmental penalty and the sequestration cost:

$$
c_{F,t} = p_{F,t} + \frac{\tau_t(\xi F_t - S_t)}{F_t} + \frac{I_{S,t}}{F_t} \tag{7}
$$

2.1.3 The primary energy sectors

At each time t, the extraction flow F_t of fossil resource depends on specific productive investments $I_{F,t}$ and on the cumulated past extraction. As in Popp (2004) or in Gerlagh and Lise (2005), we do not explicitly model an initial fossil resource stock that is exhausted, but we focus on the increase in the extraction cost as the resource is depleted. We denote by Z_t the amount of resource extracted from the initial date up to t:

$$
Z_t = \int_0^t F_s ds \Leftrightarrow \dot{Z}_t = F_t \tag{8}
$$

The fossil fuel extraction function writes $F_t = F(I_{F,t}, Z_t)$, where $F(.)$ is increasing and concave in I_F, decreasing and convex in Z. The instantaneous profit of the fuel producer is then $\Pi_t^F = p_{F,t}F_t - I_{F,t}$ and its program consists in choosing $\{I_{F,t}\}_{t=0}^{\infty}$ that maximizes $\int_0^\infty \Pi_t^F e^{-\int_0^t r_s ds} dt$, subject to (8). Denoting by η_t the multiplier associated with (8), the static and dynamic first-order conditions are:

$$
\begin{aligned}
(p_{F,t}F_{I_F} - 1)e^{-\int_0^t r_s ds} + \eta_t F_{I_F} &= 0 && (9)\\
p_{F,t}F_Z e^{-\int_0^t r_s ds} + \eta_t F_Z &= -\dot{\eta}_t && (10)
\end{aligned}
$$

Combining these two equations, and using the transversality condition $\lim_{t\to\infty} \eta_t Z_t = 0$, we get the following fossil fuel price expression:

$$
p_{F,t} = \frac{1}{F_{I_F}} - \int_t^\infty \frac{F_Z}{F_{I_F}} e^{-\int_t^s r_x dx} ds \tag{11}
$$

Differentiating (11) with respect to time, it comes:

$$
\dot{p}_{F,t} = r_t \left(p_{F,t} - \frac{1}{F_{I_F}} \right) + \frac{1}{F_{I_F}} \left(F_Z - \frac{\dot{F_{I_F}}}{F_{I_F}} \right) \tag{12}
$$

which reads as a generalized version of the Hotelling rule in the case of an extraction technology given by function $F(.)$. In particular, if the marginal productivity of investment spendings coincides with the average productivity, i.e. if $F_{I_F} = F(.)/I_F$, then it is easy to see that (12) reduces to $\dot{p}_{F,t} = r_t(p_{F,t} - 1/F_{I_F})$. This last equation corresponds to the standard Hotelling rule which is obtained if the marginal extraction cost is equal to the average cost of extraction, i.e. to I_F/F in our model.

The backstop resource production requires specific investment spendings, $I_{B,t}$, and knowledge, $H_{B,t}$. The backstop technology writes $B_t = B(I_{B,t}, H_{B,t})$, where $B(.)$ is increasing and concave in I_B and H_B.[7] Maximization of the profit $\Pi_t^B = p_{B,t}B(I_{B,t}, H_{B,t}) - I_{B,t}$ (here also, Π_t^B denotes the profit before innovation expenditures) yields the following first-order condition:

$$p_{B,t}B_{I_B} - 1 = 0 \tag{13}$$

2.1.4 The R&D sectors

There are three stocks of knowledge, each associated with a specific R&D sector (i.e. the energy, the backstop and the CCS ones). We consider that each innovation is a non-rival, indivisible and infinitely durable piece of knowledge (for instance, a scientific report, a data base, a software algorithm...) which is simultaneously used by the sector which produces the good i and by the R&D sector i, $i = \{B, E, S\}$.

Here, an innovation is not directly embodied into tangible intermediate goods and thus, it cannot be financed by the sale of these goods. However, in order to fully describe the equilibrium, we need to find a way to assess the price received by the inventor for each piece of knowledge. We proceed as follows: i) In each research sector, we determine the *social* value of an innovation. Since an innovation is a non-rival good, this social value is the sum of marginal profitabilities of this innovation in each sector using it. If the inventor was able to extract the willingness to pay of each user, he would receive this social value and the first-best optimum would be implemented. ii) In reality, there are some failures that constrain the inventor to extract only a part of this social value[8]. This implies that the *effective* value which is received by innovators in the absence of research subsidy is

[7] Again, in a model with tangible intermediate goods, the backstop technology would write $B_t = B\left[I_{B,t}, \int_0^{H_{B,t}} g(x_{i,t}^B)di\right]$.

[8] For instance, Jones and Williams, 1998, estimate that actual investment in research are at least four times below what would be socially optimal; on this point, see also Popp, 2006a.

lower than the social one. iii) The research sectors are eventually subsidized in order to reduce the gap between these two values.

Let us apply this three-steps procedure to the backstop R&D sector for instance. Each innovation produced by this sector is used by this R&D sector itself as well as by the backstop production sector. Thus, at each date t, the instantaneous social value of this innovation is $\bar{v}_{B,t} = \bar{v}_{B,t}^{B} + \bar{v}_{B,t}^{H_B}$, where $\bar{v}_{B,t}^{B}$ and $\bar{v}_{B,t}^{H_B}$ are the marginal profitabilities of this innovation in the backstop production sector and in the backstop R&D sector, respectively. The social value of this innovation at t is $\bar{V}_{B,t} = \int_t^{\infty} \bar{v}_{B,s} e^{-\int_t^s r_x dx} ds$. Remark that $\bar{V}_{B,t}$ reads as the optimal value of an infinitely lived patent. The same procedure applies for any R&D sector i, $i = \{B, E, S\}$. We denote by γ_i, $0 < \gamma_i < 1$, the rate of appropriability of the innovation value by the market, i.e. the share of the social value which is effectively paid to the innovator, and by $\sigma_{i,t}$ the subsidy rate that government can eventually apply. Note that if $\sigma_{i,t} = 1 - \gamma_i$, the effective value matches the social one. The instantaneous effective value (including subsidy) is:

$$v_{i,t} = (\gamma_i + \sigma_{i,t})\bar{v}_{i,t} \tag{14}$$

and the intertemporal effective value at date t is:

$$V_{i,t} = \int_t^{\infty} v_{i,s} e^{-\int_t^s r_x dx} ds \tag{15}$$

Differentiating (15) with respect to time leads to the usual arbitrage relation:

$$r_t = \frac{\dot{V}_{i,t}}{V_{i,t}} + \frac{v_{i,t}}{V_{i,t}}, \quad \forall i = \{B, E, S\} \tag{16}$$

which equates the rate of return on the financial market to the rate of return on the R&D sector i.

We can now analyze the R&D sector behavior. We assume that the dynamics of the stock of knowledge in sector i is governed by the following innovation function $H^i(.)$:

$$\dot{H}_{i,t} = H^i(R_{i,t}, H_{i,t}) \tag{17}$$

where $R_{i,t}$ is the R&D investment into sector i. Function $H^i(.)$ is assumed to be increasing and concave in each argument[9]. At each time t, sector i supplies the flow of innovations $\dot{H}_{i,t}$ at price $V_{i,t}$ and demands some specific investments $R_{i,t}$, so that the profit function –

[9] As previously, in a model with tangible intermediate goods, (17) would be replaced by $\dot{H}_{i,t} = H^i\left[R_{i,t}, \int_0^{H_{i,t}} h(x_{i,t}^H) di\right]$.

before payments of innovations – to be maximized is $\Pi_t^{H_i} = V_{i,t} H^i(R_{i,t}, H_{i,t}) - R_{i,t}$. The first-order condition implies:

$$V_{i,t} = \frac{1}{H^i_{R_i}} \tag{18}$$

The marginal profitability of innovations in the R&D sector i is:

$$\bar{v}^{H_i}_{i,t} = \frac{\partial \Pi_t^{H_i}}{\partial H_{i,t}} = V_{i,t} H^i_{H_i} = \frac{H^i_{H_i}}{H^i_{R_i}} \tag{19}$$

Finally, in order to determine the social and the effective values of an innovation for each R&D sector, we need to know the marginal profitability of innovations in each production sector using them. From the expressions of Π_t^B and Π_t^E, those values are given respectively by $\bar{v}^B_{B,t} = \partial \Pi_t^B / \partial H_{B,t} = B_{H_B}/B_{I_B}$, $\bar{v}^E_{E,t} = \partial \Pi_t^E / \partial H_{E,t} = E_{H_E}/E_B B_{I_B}$ and $\bar{v}^E_{S,t} = \partial \Pi_t^E / \partial H_{S,t} = \tau_t S_{H_S}$. Therefore, the instantaneous effective values (including subsidies) of innovations are:

$$\begin{aligned} v_{B,t} &= (\gamma_B + \sigma_{B,t}) \left(\frac{B_{H_B}}{B_{I_B}} + \frac{H^B_{H_B}}{H^B_{R_B}} \right) && (20) \\ v_{E,t} &= (\gamma_E + \sigma_{E,t}) \left(\frac{E_{H_E}}{E_B B_{I_B}} + \frac{H^E_{H_E}}{H^E_{R_E}} \right) && (21) \\ v_{S,t} &= (\gamma_S + \sigma_{S,t}) \left(\tau_t S_{H_S} + \frac{H^S_{H_S}}{H^S_{R_S}} \right) && (22) \end{aligned}$$

2.1.5 The household and the government

Denoting by C_t the consumption at time t, by $U(.)$ the instantaneous utility function (assumed to have the standard properties) and by $\rho > 0$ the pure rate of time preferences, households maximize the welfare function $W = \int_0^\infty U(C_t) e^{-\rho} dt$ subject to the following dynamic budget constraint:

$$\dot{K}_t = r_t K_t + w_t L_t + \Pi_t - C_t - T_t^a \tag{23}$$

where Π_t is the total profits gained in the economy and T_t^a is a lump-sum tax (subsidy-free) that allows to balance the budget constraint of the government. This maximization leads to the following condition:

$$\rho - \frac{\dot{U}'(C_t)}{U'(C_t)} = r_t \Rightarrow U'(C_t) = U'(C_0) e^{\rho t - \int_0^t r_s ds} \tag{24}$$

which is no other than the standard Keynes-Ramsey rule, i.e. $\rho + \epsilon_t g_{C,t} = r_t$, where ϵ_t denotes the inverse of the elasticity of intertemporal substitution of consumption, and $g_{C,t}$ is the instantaneous growth rate of consumption.

Assuming that the government's budget constraint is balanced at each time t (i.e. the sum of the various taxes equals R&D subsidies), then we have:

$$T_t^a + \tau_t(\xi F_t - S_t) = \sum_i Sub_{i,t}, \quad i = \{B, E, S\} \tag{25}$$

where $Sub_{i,t}$ denotes the amount of subsidy distributed to R&D sector i:

$$Sub_{i,t} = \left[\int_t^{\infty} \left(\frac{\sigma_{i,s}}{\gamma_i + \sigma_{i,s}} \right) v_{i,s} e^{-\int_t^s r_x dx} ds \right] H^i(R_{i,t}, H_{i,t}) \tag{26}$$

Finally, the balance equation of the final output writes:

$$Q_t = C_t + I_{F,t} + I_{B,t} + I_{S,t} + I_{K,t} + R_{E,t} + R_{B,t} + R_{S,t} \tag{27}$$

where $I_{K,t}$ is the instantaneous investment in capital, given by:

$$I_{K,t} = \dot{K}_t + \delta K_t \tag{28}$$

Hence, in our worldwide economy, the final output is devoted to aggregated consumption, fossil fuel production, backstop production, CCS, capital accumulation and R&D.

2.2 The environment and damages

Let G_0 be the stock of carbon in the atmosphere at the beginning of the planning period, G_t the stock at time t and ζ, $\zeta > 0$, the natural rate of decay. The increase in atmospheric carbon concentration drives the global mean temperature away from a given state, here the 1900 level. The difference between this state and the present global mean temperature, denoted by T_t, is taken here as the index of anthropogenic climate change. The climatic dynamic system under reduced form can be captured by the following two state equations:

$$\dot{G}_t = \xi F_t - S_t - \zeta G_t \tag{29}$$

$$\dot{T}_t = \Phi(G_t) - m T_t, \quad m > 0 \tag{30}$$

Function $\Phi(.)$ links the atmospheric carbon concentration to the dynamics of temperature and is assumed to be increasing and concave in G. It is in fact the reduced form of a more complex radiative forcing function that takes into account the inertia of the climate dynamics[10].

[10] In the analytical treatment of the model, we assume, for the sake of clarity, that the carbon cycle through atmosphere and oceans as well as the dynamic interactions between atmospheric and oceanic temperatures, are captured by the reduced form (29) and (30). Goulder and Mathai (2000), or Kriegler and Bruckner (2004), have recourse to such simplified dynamics. From DICE-99, the formers estimate parameters ξ and ζ that take into account the inertia of the climatic system. They state that only 64% of current emissions actually contribute to the augmentation of atmospheric CO_2 and that the portion of current CO_2 concentration in excess is removed naturally at a rate of 0.8% per year. However, in the numerical simulations, we adopt the full characterization of the climate module from the last version of DICE (Nordhaus, 2008).

Global warming generates economic damages. By convention, those damages are measured in terms of final output losses through the scaling factor $\Omega(T_t)$, with $\Omega'(.) < 0$. In addition to the damage reflected by Ω_t, we will possibly be induced to impose a stabilization cap on the carbon pollution stock that society can not overshoot (see for instance Chakravorty et al., 2006):

$$G_t \leq \bar{G}, \quad \forall t \geq 0 \tag{31}$$

This additional constraint can be justified by assuming that the social damage function is not able to reflect the entire environmental damages, but only part of it. In reality, uncertainty in the climatic consequences of global warming can imply some discontinuities in the damage, such as natural disasters or other strong irreversibilities, that are not taken into account by the standard functional representation of the damage.

3 Decentralized equilibrium and welfare analysis

3.1 Characterization of the decentralized equilibrium

From the previous analysis of individual behaviors, we can now study the set of equilibria in the decentralized economy. A particular equilibrium is associated with each quadruplet of policies $\{\sigma_{B,t}, \sigma_{E,t}, \sigma_{S,t}, \tau_t\}_{t=0}^{\infty}$. It is defined as a vector of quantity trajectories $\{Q_t, K_t, E_t, ...\}_{t=0}^{\infty}$ and a vector of price profiles $\{r_t, p_{E,t}, ...\}_{t=0}^{\infty}$ such that: i) firms maximize profits, ii) the representative household maximizes utility, iii) markets of private (i.e. rival) goods are perfectly competitive and cleared, iv) in each R&D sectors i, innovators receive a share $(\gamma_i + \sigma_{i,t})$ of the social value of innovations. Such an equilibrium is characterized by the set of equations given by Proposition 1 below. Clearly, as analyzed in the following subsection, if the policy tools are set to their optimal levels, those equations also characterize the first-best optimum together with the system of prices that implements it.

Proposition 1 *At each time t, for a given quadruplet of policies $\{\sigma_{B,t}, \sigma_{E,t}, \sigma_{S,t}, \tau_t\}_{t=0}^{\infty}$, the equilibrium in the decentralized economy is characterized by the following seven-equations* *system:*

$$
\begin{aligned}
Q_E E_F - \tau_t(\xi - S_F) - \frac{1}{F_{I_F}} &= \frac{-1}{U'(C_t)} \int_t^\infty \frac{F_Z}{F_{I_F}} U'(C_s) e^{-\rho(s-t)} ds && (32)\\
Q_E E_B &= \frac{1}{B_{I_B}} && (33)\\
\frac{1}{S_{I_S}} &= \tau_t && (34)\\
Q_K - \delta &= \rho + \epsilon_t g_{C,t} && (35)\\
(\gamma_B + \sigma_{B,t}) \left(\frac{B_{H_B} H_{R_B}^B}{B_{I_B}} + H_{H_B}^B \right) - \frac{\dot{H}_{R_B}^B}{H_{R_B}^B} &= \rho + \epsilon_t g_{C,t} && (36)\\
(\gamma_E + \sigma_{E,t}) \left(\frac{E_{H_E} H_{R_E}^E}{E_B B_{I_B}} + H_{H_E}^E \right) - \frac{\dot{H}_{R_E}^E}{H_{R_E}^E} &= \rho + \epsilon_t g_{C,t} && (37)\\
(\gamma_S + \sigma_{S,t}) \left(\tau_t S_{H_S} H_{R_S}^S + H_{H_S}^S \right) - \frac{\dot{H}_{R_S}^S}{H_{R_S}^S} &= \rho + \epsilon_t g_{C,t} && (38)
\end{aligned}
$$

The associated system of prices $\left\{ r_t^*, w_t^*, p_{E,t}^*, p_{F,t}^*, p_{B,t}^*, V_{i,t}^* \right\}_{t=0}^\infty$ *is obtained from the equations (1), (2), (3), (11), (13) and (18), respectively.*

Proof. See Appendix A1.

Equation (32) is an arbitrage condition that equalizes the marginal net profit in terms of output due to the increase of the fossil fuel extraction by one unit (LHS) to the total marginal gain if there is no additional extraction (RHS)[11]. Equation (33) tells that the marginal productivity of the backstop (LHS) equals its marginal cost (RHS). As already mentioned, equation (34) formalizes the incentive effect of the carbon tax on the decision to invest in CCS. Equation (35) characterizes the standard trade-off between capital K_t and consumption C_t. Equation (36) (resp. (37) and (38)) characterizes the same kind of trade-off between specific investment into backstop R&D sector, $R_{B,t}$ (resp. energy R&D sector, $R_{E,t}$, and CCS R&D sector, $R_{S,t}$) and consumption. Obviously, the marginal return of each specific stock of knowledge H_i depends on the associated rate of subsidy σ_i.

3.2 First-best optimum and implementation

The social planner problem consists in choosing $\{C_t, I_{B,t}, I_{F,t}, I_{S,t}, R_{B,t}, R_{E,t}, R_{S,t}\}_{t=0}^\infty$ that maximizes the social welfare W, subject to the various technological constraints, the output

[11] If extraction increases by one unit, the associated revenue is $Q_E E_F$ and firms face two kinds of costs: the extraction cost, $1/F_{I_F}$, and the pollution cost, $\tau(\xi - S_F)$. Conversely, if no more fossil resource is extracted during the time interval dt, this generates an instantaneous gain in terms of output due to the diminution in specific investment spending I_F corresponding to $(dI_F/dt)/F|_{dF=0} = -F_Z/F_{I_F}$. Multiplying this term by the marginal utility and integrating from t to ∞ with the discount rate ρ gives the total gain in terms of utility. Finally, dividing by $U'(C)$, this expression gives the gain in terms of output as specified in the RHS of (32).

allocation constraint (27), the state equations (8), (17), (28), (29) and (30), and finally, the environmental constraint (31). After eliminating the co-state variables, the first-order conditions leads to Proposition 2 below.

Proposition 2 *At each time t, an optimal solution is characterized by the following seven-equations system:*

$$
\begin{aligned}
Q_E E_F - \frac{(\xi - S_F)}{S_{I_S}} - \frac{1}{F_{I_F}} &= \frac{-1}{U'(C_t)} \int_t^{\infty} \frac{F_Z}{F_{I_F}} U'(C_s) e^{-\rho(s-t)} ds && (39) \\
Q_E E_B &= \frac{1}{B_{I_B}} && (40) \\
\frac{1}{S_{I_S}} &= \frac{-1}{U'(C_t)} \int_t^{\infty} \left[\Phi'(G_s) J_s - \varphi_{G,s} e^{\rho s}\right] e^{-(\zeta+\rho)(s-t)} ds && (41) \\
Q_K - \delta &= \rho + \epsilon_t g_{C,t} && (42) \\
\frac{B_{H_B} H_{R_B}^B}{B_{I_B}} + H_{H_B}^B - \frac{\dot{H}_{R_B}^B}{H_{R_B}^B} &= \rho + \epsilon_t g_{C,t} && (43) \\
\frac{E_{H_E} H_{R_E}^E}{E_B B_{I_B}} + H_{H_E}^E - \frac{\dot{H}_{R_E}^E}{H_{R_E}^E} &= \rho + \epsilon_t g_{C,t} && (44) \\
\frac{S_{H_S} H_{R_S}^S}{S_{I_S}} + H_{H_S}^S - \frac{\dot{H}_{R_S}^S}{H_{R_S}^S} &= \rho + \epsilon_t g_{C,t} && (45)
\end{aligned}
$$

where $J_s = \int_s^{\infty} Q_\Omega \Omega'(T_x) U'(C_x) e^{-(m+\rho)(x-s)} dx \leq 0$ and $\varphi_{G,s} \geq 0$, with $\varphi_{G,s} = 0$ for any s such that $G_s < \bar{G}$.

Proof. See Appendix A2.

The interpretation of those conditions are almost the same than the ones formulated for Proposition 1, excepted that, now, all the trade-offs are socially optimally solved. Note that, in equation (41), $\varphi_{G,t}$ denotes the Lagrange multiplier associated with the ceiling constraint and then we have $\varphi_{G,t} \geq 0$, with $\varphi_{G,t} = 0$ for any t such that $G_t < \bar{G}$.

Recall that, for a given set of public policies, a particular equilibrium is characterized by conditions (32)-(38) of Proposition 1. This equilibrium will be said to be optimal if it satisfies the optimum characterizing conditions (39)-(45) of Proposition 2. By analogy between these two sets of conditions, we can show that there exists a single quadruplet $\{\sigma_{B,t}, \sigma_{E,t}, \sigma_{S,t}, \tau_t\}_{t=0}^{\infty}$ that implements the optimum. These findings are summarized in Proposition 3 below.

Proposition 3 *The equilibrium characterized in Proposition 1 is optimal if and only if $\left\{\sigma_{B,t}, \sigma_{E,t}, \sigma_{S,t}, \tau_t\right\}_{t=0}^{\infty} = \left\{\sigma_{B,t}^o, \sigma_{E,t}^o, \sigma_{S,t}^o, \tau_t^o\right\}_{t=0}^{\infty}$, where $\sigma_{i,t}^o = 1 - \gamma_i$ $\forall t \geq 0$, for $i =$*

$\{B, E, S\}$, *and where* τ_t^o *is given by:*

$$\tau_t^o = \frac{-1}{U'(C_t)} \int_t^\infty \left[\Phi'(G_s) \int_s^\infty Q_\Omega \Omega'(T_x) U'(C_x) e^{-(m+\rho)(x-s)} dx - \varphi_{G,s} e^{\rho s} \right] e^{-(\zeta+\rho)(s-t)} ds \tag{46}$$

Proof. First, if $\tau_t = \tau_t^o$, then conditions (39) and (41) are satisfied by using (32) and (34). Second, (40) and (42) are identical to (33) and (35), respectively. Third, if $\sigma_{i,t} = 1 - \gamma_i$, for $i = \{B, E, S\}$, then (43), (44) and (45) are identical to (36), (37) and (38), respectively.

First, Proposition 3 states that, in any R&D sector, the optimal subsidy rate must be equal to the share of the social value of innovations which is not captured by the market, in order to entirely fill the gap between the value received by the innovator and this social value. Since the γ_i's are assumed to be constant over time, then the σ_i^o's are also constant. In the empirical part and according to Jones (1995), we will postulate that $\gamma_i = 0.3$, thus implying $\sigma_i^o = 0.7$ for $i = \{B, E, S\}$.

Second, it provides the carbon tax optimal trajectory, as characterized by (46). Since $\Omega'(T_t) < 0$, we have $\tau_t^o \geq 0$ for any $t \geq 0$. This expression can read as the ratio between the marginal social cost of climate change – the marginal damage in terms of utility coming from the emission of an additional unit of carbon – and the marginal utility of consumption. In other words, it is the environmental cost of one unit of carbon in terms of final good. This carbon tax can be expressed as the sum of two components. The first one depends on the damage function and on the dynamics of the atmospheric carbon stock and temperatures. It gives the discounted sum of marginal damages from t to ∞ coming from the emission of an additional unit of carbon at date t. The second one is only related to the ceiling constraint and depends on φ_G. It gives the social cost at t of one unit of carbon in the atmosphere due to a tightening in the ceiling constraint. Then, the sum of these two components is the instantaneous total social cost of one unit of carbon.

Log-differentiating (46) gives us the optimal growth rate of the tax:

$$\frac{\dot{\tau}_t}{\tau_t} = \zeta + \rho + \epsilon_t g_{C,t} - \frac{\left[\Phi'(G_t) J_t - \varphi_{G,t} e^{\rho t}\right]}{\int_t^\infty \left[\Phi'(G_s) J_s - \varphi_{G,s} e^{\rho s}\right] e^{-(\zeta+\rho)(s-t)} ds} \tag{47}$$

As we can see in (47), the dynamics of the optimal carbon tax results in the combination of four components. In order to analyze each of them, let us assume that one unit of carbon is emitted at date t and let us consider the impact of this emission on a consumer living at any date s, $s > t$. First, along the time interval $\Delta = s - t$, this unit of carbon gradually depreciates at rate ζ per unit of time. As t increases, the length of Δ diminishes, thus rising the impact of the unit of carbon on the utility of household at s and contributing to an increase in the tax. This impact, captured by ζ, can be designated as the *decay effect*. Second, we use the rate of time preferences ρ to get at date t the impact on utility generated at date s and mentioned above. As t increases, this discounted value increases, thus involving an increase in the carbon tax. This is the *discount effect*. Third, in equation (46), the optimal tax is expressed in terms of final goods since $U'(C)$ appears at the denominator of this expression. As t increases, consumption increases at rate g_C due to economic growth, $U'(C)$ decreases because of concavity of the utility function (whose curvature is captured by ϵ_t here) and then $1/U'(C)$ increases, thus also accounting for a rise in the tax. This effect is referred in the literature as the *wealth effect*, which reflects the fact that, since future generations are expected to be richer than the present ones, it will become more and more expensive to compensate them for the emission of one unit of carbon today. Finally, as t increases, the integration interval in (46) is reduced, meaning that the number of people which will be harmed by the carbon emission decreases. Then, this effect, which we call the *harmed generations effect*, involves a decrease in the optimal carbon tax.

To sum up, we have four effects; the three first ones act to increase the carbon tax over time, whereas the last one leads to a decrease of this tax. As a result, the shape of the tax over time is ambiguous. In the following section, we illustrate this point by depicting some monotonous or non-monotonous trajectories depending on the relative weights of those effects.

4 Empirical results

4.1 Analytical specifications and calibration

Functional forms are mainly provided by the last version of the DICE model (Nordhaus, 2008) for the climate module, the final output, the social preferences, the feedbacks on economic productivity from climate change, the total factor productivity and demographic dynamics. The energy production and R&D systems come from the ENTICE-BR model (Popp, 2006a). For the incorporation of the CCS technology in the model, we use a specification derived from the sequestration cost function used in the DEMETER model

(Gerlagh and van der Zwaan, 2006)[12]. All those analytical specifications are listed below:

$$
\begin{aligned}
Q(K_t, E_t, L_t, A_t, \Omega_t) &= \Omega_t A_t K_t^{\gamma} E_t^{\beta} L_t^{1-\gamma-\beta}, \quad \gamma, \beta \in (0,1) \\
L_t &= L_0 e^{\int_0^t g_s^L ds}, \quad A_t = A_0 e^{\int_0^t g_s^A ds} \\
g_{jt} &= g_{j0} e^{-d_j t}, \quad d_j > 0, \forall j = \{A, L\} \\
E(F_t, B_t, H_{E,t}) &= \left[(F_t^{\rho_B} + B_t^{\rho_B})^{\frac{\rho_H}{\rho_B}} + \alpha_H H_{E,t}^{\rho_H} \right]^{\frac{1}{\rho_H}}, \quad \alpha_H, \rho_H, \rho_B \in (0,1) \\
F(I_{F,t}, Z_t) &= \frac{I_{F,t}}{c_F + \alpha_F (Z_t/\bar{Z})^{\eta_F}}, \quad c_F, \alpha_F, \eta_F > 0 \\
B(I_{B,t}, H_{B,t}) &= \alpha_B I_{B,t} H_{B,t}^{\eta_B}, \quad \alpha_B, \eta_B > 0 \\
S(F_t, I_{S,t}, H_{S,t}) &= \kappa(\xi F_t) \left[\left(1 + \frac{2 I_{S,t} H_{S,t}}{\kappa(\xi F_t)} \right)^{1/2} - 1 \right], \quad \kappa > 0 \\
H^i(R_{i,t}, H_{i,t}) &= a_i R_{i,t}^{b_i} H_{i,t}^{\phi_i}, \quad a_i > 0, \, b_i, \phi_i \in [0,1], \forall i = \{B, E, S\} \\
W &= v_1 \int_0^{\infty} L_t \frac{(C_t/L_t)^{1-\epsilon}}{(1-\epsilon)} e^{-\rho} dt + v_2, \quad v_1, v_2 > 0 \\
\Omega(T_t) &= \left[1 + \alpha_T T_t^2 \right]^{-1}, \quad \alpha_T > 0
\end{aligned}
$$

Next, let us provide some calibration details here. The starting year is the year 2005. According to IEA (2007), world carbon emissions in 2005 amounted to 17.136 MtCO2. We retain 7.401 GtCeq as the initial fossil fuel consumption, given in gigatons of carbon equivalent. In addition, carbon-free energy produced out of renewable energy, excluding biomass and nuclear, represented 6% of total primary energy supply. We thus retain another 0.45 GtCeq as the initial amount of backstop energy use. We retain the Gerlagh's assumption for the cost of CCS that is worth 150US$/tC. According to IEA (2006), the cumulative CO_2 storage capacity is in the order of 184 million tons per year. This value serves as a seed value for sequestration level, S_0, in the initial year, which is then fixed at 0.05 GtC. The cost of CCS sequestration and the initial storage level allow for the calibration of the initial sequestration effort using the following relation: $I_{S,0}/S_0$ =CCS cost, which implies $I_{S,0}$ = 0.05GtC×150$/tC=7.5G$. The total factor productivity has been adjusted so as to produce a similar pattern of GWP development until 2100 to the one

[12] In our model, we replace the cost function of fossil fuel and backstop from Popp (2006a) and the cost function of sequestration from Gerlagh (2006) by their corresponding production functions in order to derive an utility/technology canonical model. With our notations, these unit cost functions are:

$$
\begin{aligned}
\frac{I_{F,t}}{F_t} &= c_F + \alpha_F \left(\frac{Z_t}{\bar{Z}} \right)^{\eta_F} \\
\frac{I_{B,t}}{B_t} &= \frac{1}{\alpha_B H_B^{\eta_B}} \\
\frac{I_{S,t}}{S_t} &= \frac{1}{H_{S,t}} \left(1 + \frac{S_t}{2\kappa(\xi F_t)} \right)
\end{aligned}
$$

from DICE-08. The rates of return on both R&D spending and knowledge accumulation have been set to 0.3 and 0.2 respectively so as provide long term sequestration in line with IPCC (2007) projections. Without loss of generality, the initial stock of knowledge dedicated to CCS is set equal to 1. Calibration of the other parameters come from DICE or ENTICE-BR and we defer their assignment to Appendix A3, Table 3.

4.2 Scenarii

To study numerically the effects of policy instruments on the decentralized equilibrium, we first run the benchmark case in which neither environmental tax nor R&D subsidies are implemented. Next, we solve the equilibrium for various values of τ_t and σ_i, $i = \{B, E, S\}$. The selected cases are listed in Table 1.

Case	τ_t	$\sigma_{i,t}$	Comment
A	0	0	Laisser-faire
B	τ_t^o	0	Optimal tax, no R&D subsidy
C	0	σ_i^o	Optimal subsidy rates, no tax
D	τ_t^o	σ_i^o	Optimum without ceiling
E	τ_t^{550}	σ_i^o	Optimum with a 550ppm ceiling
F	τ_t^{450}	σ_i^o	Optimum with a 450ppm ceiling

Table 1: Summary of the various scenarii

Case A refers to the laisser-faire equilibrium. In case B, we study the effect on the equilibrium of an environmental tax, for instance by setting it equal to its first-best optimal level τ^o. Similarly, we analyze the impact of optimal R&D subsidy rates in case C, by assuming $\gamma_i = 0.3$ and thus $\sigma_i^o = 0.7$ for $i = \{B, E, S\}$ and for any scenario[13]. Case D refers to the first-best optimum without ceiling on carbon concentration. Finally, two stabilization caps of 450 and 550ppm, which are enforced owing to the specific tax trajectories τ_t^{550} and τ_t^{450} respectively, are also studied (cases E and F).

[13]Although the optimal subsidy rates are the same in all scenarii, the amount of subsidies that are distributed among R&D sectors may differ. Formally, in the case of constant subsidy rates, expression (26) becomes: $Sub_{i,t} = \sigma_i V_{i,t} H^i(R_{i,t}, H_{i,t})$.

4.3 Numerical results

4.3.1 Summary of results

We adopt the following notations that will help us pointing at various facts when describing the effects of the various policy combinations. $\Delta X|_{A\to D}$ stands for the change in variable X due to a simultaneous increase of τ from 0 to τ^o and of σ_i from 0 to σ_i^o, for $i = \{B, E, S\}$. Those changes are illustrated in the following figures by a shift from the "LF" trajectories to the "Optimum" trajectories. $\Delta X|_{A\to B}$ is the change of X due to an increase in τ from 0 to τ^o, given $\sigma_i = 0$ (i.e. shifts from "LF" to "Opti tax" on the figures). Symmetrically, given $\tau = 0$, $\Delta X|_{A\to C}$ denotes the change in variable X due to a simultaneous increase of the σ_i from 0 to σ_i^o (i.e. shifts from "LF" to "Opti subs."). Finally, $\Delta X|_{D\to E/F}$ measures the change in X due to an increase in the tax level, given the optimal enforcement of the R&D policies (i.e. shifts from "Optimum" to "Optimum 550" or "Optimum 450"). Table 2 summarizes the findings from our sensitivity analysis conducted consequently, i.e. provides the signs of the Δ for the main variables of interest (where insignificant changes are depicted by $\sim$).[14]

4.3.2 Optimal carbon tax and energy prices

The optimal tax levels required for the restoration of the first-best optimum and the stabilization of carbon atmospheric carbon are depicted in Figure 2. The first-best tax level starts from 49\$/tC and follows a quite linear increase to reach 256\$/tC by 2100. The stabilization to 550 and 450 requires much higher tax levels. Starting from respectively 73 and 172\$/tC, they increase sharply, reach some high 550\$/tC and 735\$/tC in 2075 and 2055, before declining once the concentration ceiling has been reached. Naturally, the rate of increase of the carbon prices for the 450ppm target is more rapid than that of the 550ppm case. Those carbon prices prove slightly higher than Nordhaus (2008) estimates for similar climate strategies.

In any case, the tax pace evolves non-monotonically over time. This means that, from a particular future date, the last component in equation (47), that reflects the harmed generation effect, overcomes the sum of the three first ones. Moreover, in the case where a carbon target is introduced, this component is strengthened by the Lagrange multiplier φ_G associated to the ceiling constraint. As long as the ceiling is not reached, this multiplier

[14] We do not discuss here about the dissociated effects of the R&D subsidies. It can be showed that cross effects are very weak, i.e. an R&D policy in a particular sector has no crowding out impact on the others sectors.

X	$\Delta X\|_{A\to D}$	$\Delta X\|_{A\to B}$	$\Delta X\|_{A\to C}$	$\Delta X\|_{D\to E}$
p_F	$-$	$-$	$\sim$	$-$
c_F	$+$	$+$	$\sim$	$+$
p_B	$-$	$\sim$	$-$	$-$
p_E	$+$	$+$	$-$	$+$
V_{H_B}	$+$	$\sim$	$+$	$+$
V_{H_E}	$-$	$\sim$	$-$	$\sim$
V_{H_S}	$+$	$+$	$\sim$	$+$
F	$-$	$-$	$-$(weak)	$-$
B	$+$	$+$(weak)	$+$	$+$
E	$-$	$-$	$+$	$-$
S	$+$	$+$(weak)	$\sim$	$+$
H_B	$+$	$\sim$	$+$	$+$
H_E	$+$	$\sim$	$+$	$\sim$
H_S	$+$	$+$(weak)	$\sim$	$+$
R_B	$+$	$\sim$	$+$	$+$
R_E	$+$	$\sim$	$+$	$\sim$
R_S	$+$	$+$(weak)	$\sim$	$+$
Sub_B	$+$		$+$	$+$
Sub_E	$+$		$+$	$\sim$
Sub_S	$+$		$+$	$+$
Q_B	$+$	$+$	$+$	$+$
Q_F	$-$	$-$	$-$(weak)	$-$
Q_S	$+$	$+$(weak)	$\sim$	$+$
G, T	$-$	$-$	$-$(weak)	$-$
Q	$-$ then $+$	$-$ then $+$	$+$	$-$ then $+$
C	$+$	$-$ then $+$	$+$	$-$ then $+$

Table 2: Summary of economic policy effects

is nil and it becomes positive at the moment the constraint is binding. That is why the date at which the tax starts to decline and the date at which the carbon stabilization cap is reached are closed.

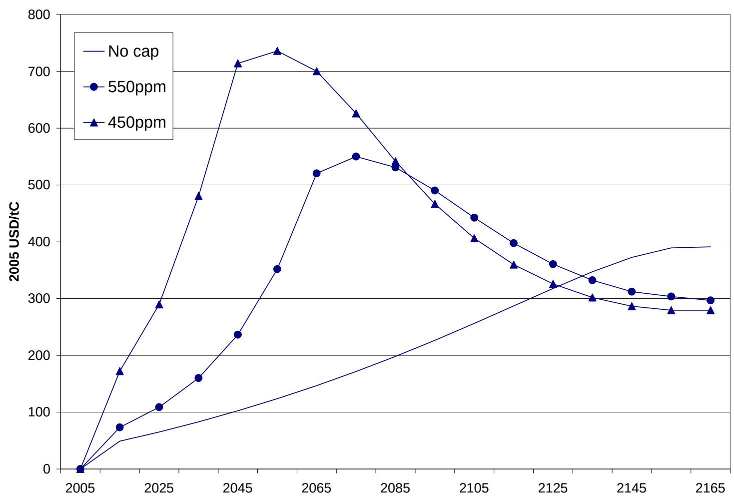

Figure 2: Optimal environmental taxes

Let us now analyze the effect of those tax trajectories on the prices of primary energies. First, the fossil fuel market price increases only slowly due to the relative flatness of our fossil fuel supply curve (see Figure 3-a). The implementation of a carbon tax reduces the producer price which induces substantial rent transfers from extractive industries to governments. In 2105, the revenues losses for the fossil energy producer amount to 55% and 52% when carbon caps are set at 550 and 450ppm, respectively. The concerns of oil-rich countries towards stringent climate mitigation commitments has already been commented and assessed in the literature (see for example Bergstrom, 1982, or Sinn, 2008). Moreover, an increment in the R&D subsidy rates has no effect on the fossil fuel price, thus illustrating the absence of crossed effects in this case.

Simultaneously, introducing a carbon tax implies obviously a rise in the unit user cost of the fossil fuel (cf. $c_{F,t}$ as defined by (7)), as observed by comparing the upper trajectories of cases a to d in Figure 4. When carbon emissions are penalized, this creates an incentive for energy firms to store a part of those emissions so that their cost of using fossil fuel is obtained by adding two components to the fossil fuel market price: i) the tax on the emissions released in the atmosphere and ii) the unit cost of CCS. Such a decomposition

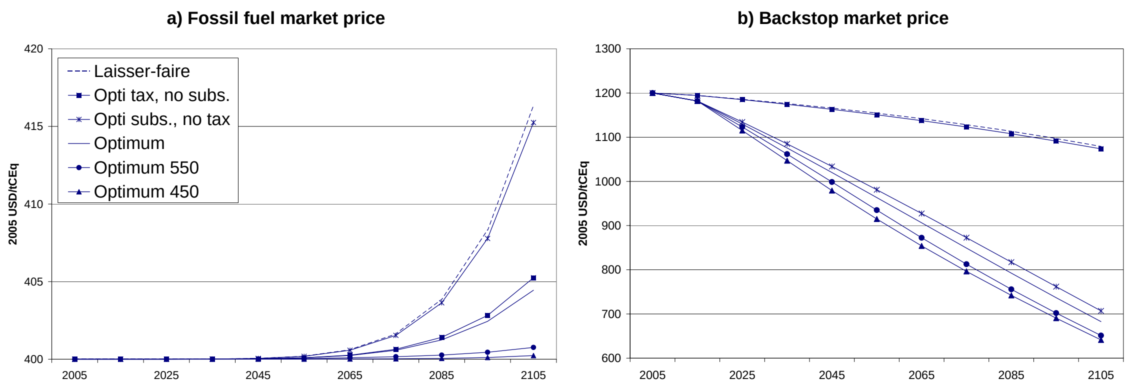

Figure 3: Fossil fuel and backstop prices

is depicted in Figure 4. The incentives to use CCS devices, and thus the CCS unit cost, are contingent to an high level of tax, or equivalently to a constraining carbon target.

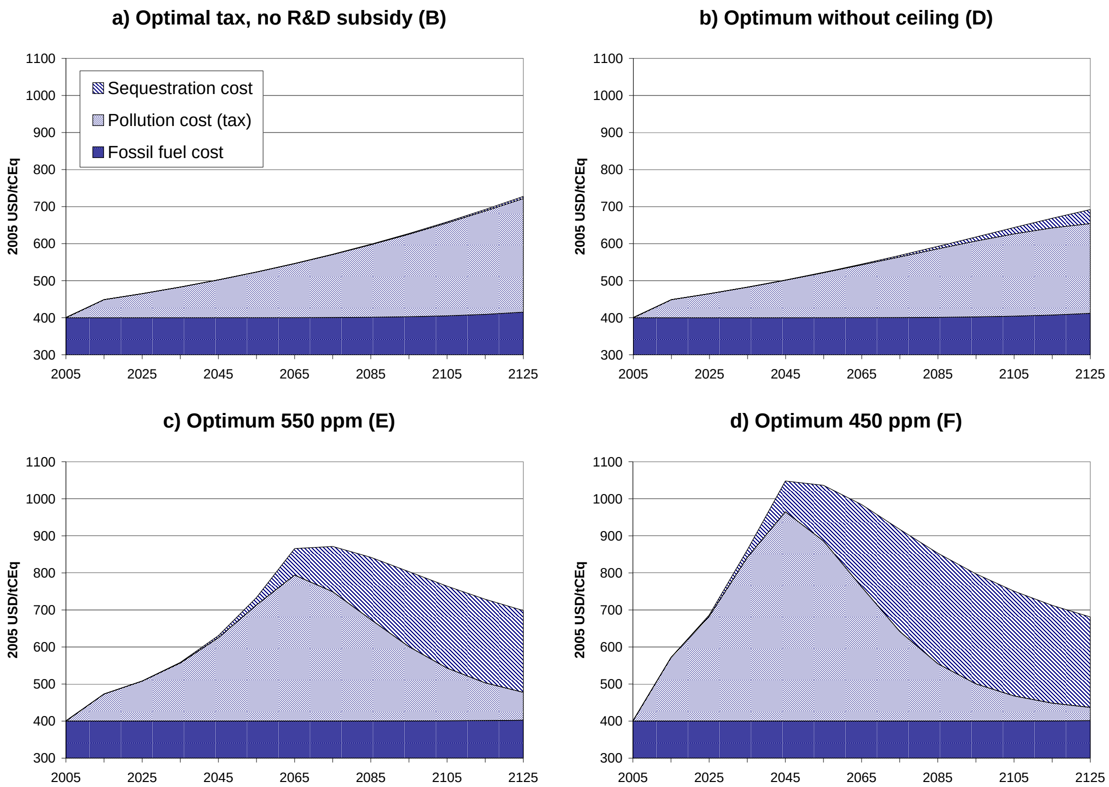

Figure 4: Decomposition of the unit cost of fossil fuel use

Second, the decreasing market price of the backstop energy reveals largely affected by the introduction of research subsidies, as can be seen from Figure 3-b. Such subsidies stimulate backstop research, thereby increasing its productivity and then, reducing production cost. They allow the backstop price to be cut by half by 2105. Moreover, two different streams of trajectories can be identified. The higher ones are drawn for cases A and B, i.e. when backstop R&D is not granted at all whereas the lower ones imply some positive σ_B. Then, R&D subsidies mainly matter to explain a decrease in the backstop price whereas the level of tax has only a weak depressive effect. Again, there is no crossed effect.

4.3.3 R&D

The effects of directed technical change can be portrayed by examining the effective value of an innovation in both CCS and backstop R&D, V_B and V_S, as depicted in Figure 5.[15]

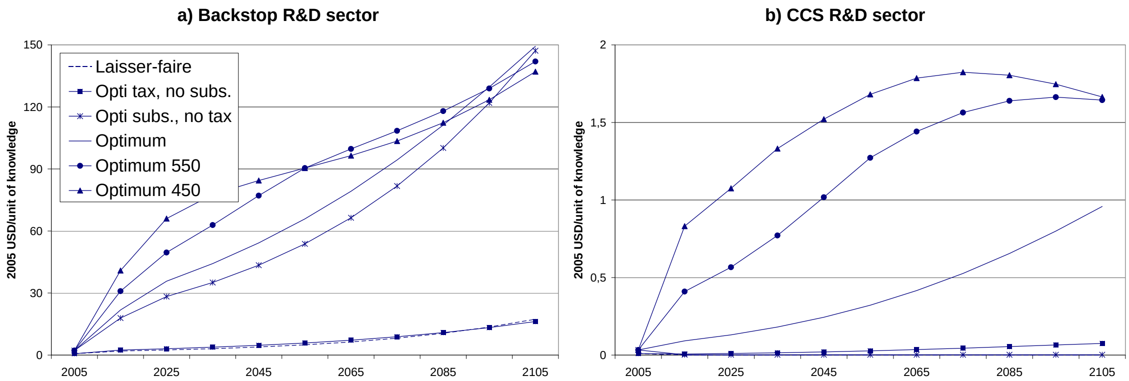

Figure 5: Effective innovation values in backstop and CCS R&D

The behavior of those innovation values provide insights on the allocation and the direction of R&D funding over time. First, the rising values demonstrate that the innovation activity grows strongly during the century, with the exception of the laisser-faire case which does not provide incentive for investing in CCS. Second, the increase in innovation values is strongly governed by the stringency of climate policy. Clearly, the introduction of a carbon ceiling induces the fastest increase in the effective value of innovations. Third, the role of each mitigation option can be inferred from the time-path of both CCS and backstop innovation values: CCS innovation value grows fast from the earliest periods, reaches a peak by around 2075 and starts declining thereafter. On the contrary, the backstop innovation value keeps on rising over time, though at a slow pace initially. A simple supply-demand argument is necessary to understand those behaviors. As the innovation activity is growing fast, due to the urgent need of developing carbon-free energy supply, and as the expected returns on CCS R&D are the highest initially because of relatively low

[15]Results on energy R&D are less of interest and are not discussed here.

cost of technology improvement relative to the backstop, a "technology push" in favor of CCS cause its innovation value to rise fast. In the longer run, backstop energy offers larger deployment potential and thus takes over CCS investments. Its value then develops at a faster pace while the CCS innovation is becoming less valued as its development shrinks.

Those innovation values drive the R&D expenses flowing to each research sector. Figure 6 depicts such R&D investments for our major cases. In the polar laisser-faire case, hardly any R&D budget is dedicated to research and CCS R&D is not financed at all. A similar outcome occurs when an optimal tax is set while research subsidies are nil. When all research subsidies are optimally set without carbon tax, R&D allowances do not profit the CCS sector but mainly the backstop research sector that receives similar amounts to the first-best optimal case. The first-best optimum restoration calls for a continuous increase in R&D budgets that will mainly benefit the development of the backstop technology. By the end of the century, overall R&D budgets will then have been multiplied by a factor of roughly 10, amounting to slightly less than 1 billion USD. The energy efficiency sector and the CCS sector receive respectively 13 and 17% of total R&D budgets in 2100. Looking at the two stabilization cases, one notices drastic changes in R&D budgets allocation and volumes. By the end of century, the overall R&D budgets exceed the ones obtained when restoring the first-best solution. The necessity of curbing quickly the net polluting emissions flow leads to substantial investments in CCS R&D that constitutes the cheapest mid-term mitigation option. The more stringent the carbon target, the higher is the share of CCS R&D spending.

Two conclusions can be drawn so far. The implementation a carbon tax alone hardly provides any incentive to proceed with R&D activities. In order to provide enough R&D incentives, one needs first to correct for the externality by imposing a carbon tax and second by subsidizing the research sectors. Moreover, short term investment in carbon-free R&D, namely in CCS activities, can become relevant when imposing a stringent cap on carbon accumulation, or equivalently, an higher level of tax.

Additionally, results depicted in Figure 7 clearly demonstrate how important the subsidies become to both CCS and backstop research sectors when a cap on carbon accumulation is set. We have seen that subsidies flow massively to each sector by the middle of the century when the climate change adverse effects need to be urgently mitigated.

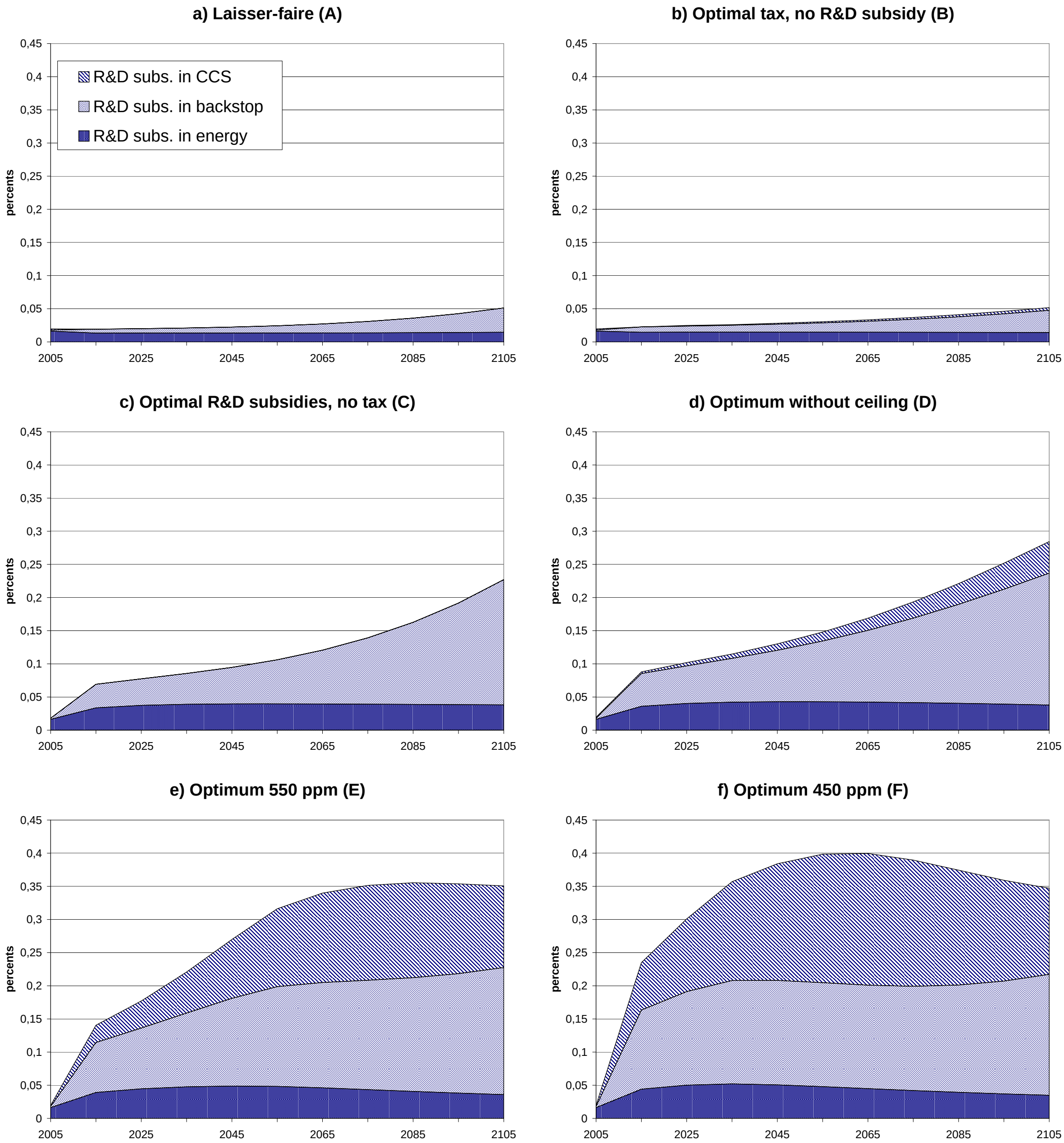

Figure 6: Intensity of dedicated R&D investments (i.e. R_i/Q)

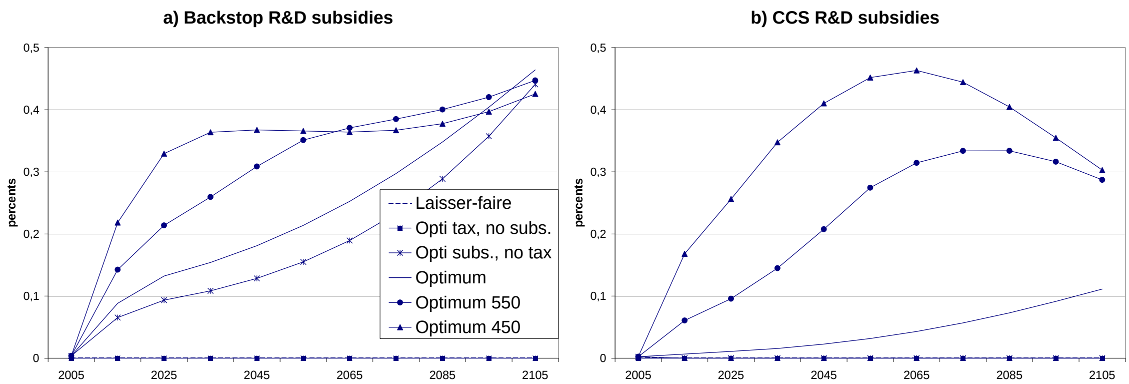

Figure 7: Backstop and CCS R&D subsidies (in percents of final output)

4.3.4 Impacts on the energy mix

Let us now turn to the development of primary energy use throughout the century. As seen from Figure 8, the laisser-faire case induces a five-fold increase in energy use over the century, driven by strong economic growth and the absence of policy restrictions. Because of the lack of incentive (no carbon tax), the CCS technology is not utilized at all. In addition, despite the fossil fuel price growth over time, the backstop technology remains marginal because it is not competitive enough.

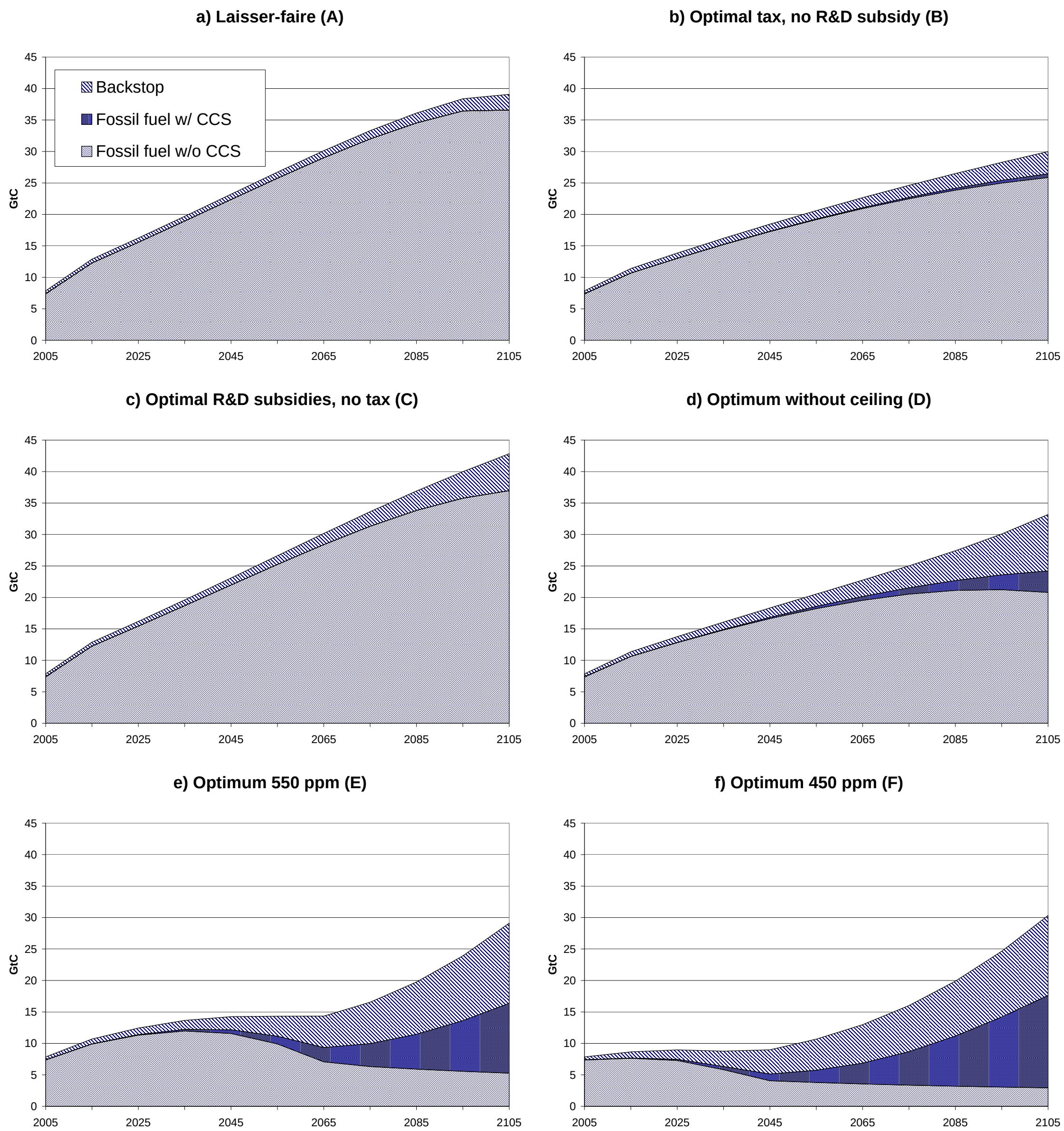

Figure 8: Primary energy use

When moving from case A to case B, the implementation of the optimal carbon tax alone does not result in substantial carbon sequestration, and/or backstop penetration. However, the fossil fuel share, and then the total primary energy use, are strongly reduced. Symmetrically, the implementation of research policies alone (i.e. moving from case A to case C) does not affect the fossil fuel use, but it slightly stimulates the backstop.

The simultaneous implementation of all optimal instruments (i.e. from case A to case D) reveals a *complementarity effect between research grants and carbon taxation*. Indeed, this scenario reinforces the effect of the tax on the fossil fuel use as observed in case B, and it increases the fraction of carbon emissions that are effectively sequestered (up to 4% of total carbon emissions in 2100). In addition, such a policy mix strengthens the role of backstop.

Finally, the two stabilization cases induce radical changes in world energy supply because of the sharp increase of carbon prices. This results in strong reductions of fossil fuel use, and thus of energy use, especially in the short-term where substitution possibilities with carbon-free energy are not yet available. By 2050, energy demand will have been reduced by 47% in the 550 ppm case, and by 60% in the 450 ppm case, as compared with the unconstrained optimum. In addition, the large amounts of R&D budgets allocated to CCS and backstop research sectors produce the expected benefits and allow for a deep mitigation of climate change owing to the decarbonisation of the economy both via the massive introduction of sequestration and via the backstop. When those carbon-free alternatives become economical, energy use rises again to reach similar levels to the laisser-faire ones in 2100. By that time, the backstop energy supplies 46% and 42% of total energy consumption. In the 550 and 450ppm cases, the CCS-based fossil fuel use accounts for 40% and 49% of total energy use in the 550 and 450ppm cases respectively. Therefore the lower the carbon target, the higher is the share of emission-free fossil fuel use.

4.3.5 Climate feedbacks on output

The environmental consequences of alternative scenarii are represented in Figure 9-a. The decentralized market outcome without any policy intervention involves a more intensive energy use without CO_2 removal and thus a faster carbon accumulation above to some dangerous 1000ppm level (IPCC, 2007). The implementation of optimal instruments limits the increase of atmospheric carbon accumulation to 800ppm by 2100. The implementation of the sole optimal tax without further R&D subsidies leads a slightly higher level of 850ppm. Notice that the sole optimal subsidies without CO_2 pricing just prove as inefficient from the environmental point of view.

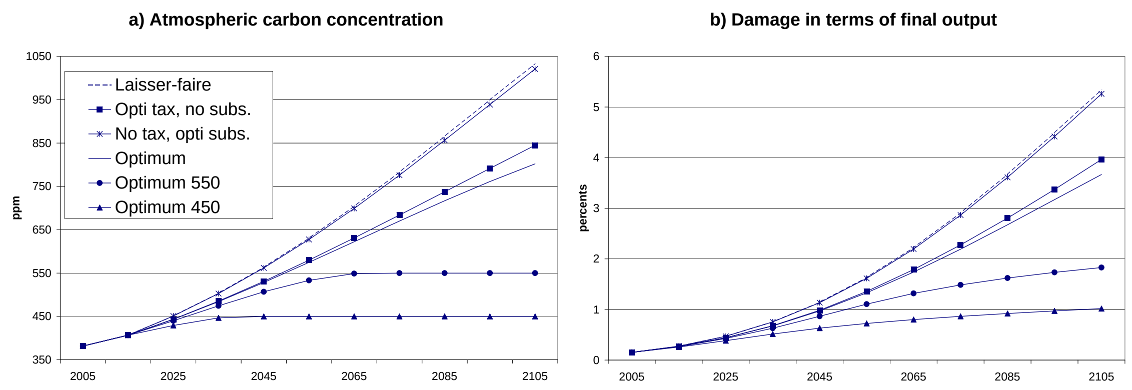

Figure 9: Atmospheric carbon concentration and damages

Figure 9-b shows the feedbacks of those atmospheric carbon concentrations on the economic damages, as measured in terms of final output. Policy inaction would lead to 5% of gross world product (GWP) losses per year by 2100, which is slightly lower then the forecasts established by Stern (2006). At the opposite, the implementation of the more stringent carbon cap, i.e. 450 ppm, limits those damages to 1% of GWP by 2100. Between these two extreme cases, the ranking of the trajectories among the various scenarii is the same than the one depicted in Figure 9-a.

Figure 10-a gives the GWP time-development as a percentage of the one from the laisser-faire case. The sole implementation of optimal subsidies improves the GWP at any date. The implementation of the optimal tax alone reveal costly until the end of the century. More importantly, setting economic instruments to their optimal values leads to further GWP losses in the short and mid term compared to the market outcome without intervention. In the longer run though, GWP increases significantly again and catches up the laisser-faire trajectory by 2095, to reach even higher gains eventually, up to 8% in 2145. To sum up: i) The presence of a carbon tax implies some GWP losses for the earlier generations, and some gains for the future ones. In other words, The long run economic growth is always enhanced when climate change issue is addressed with a carbon tax. ii) The larger the tax is, i.e. the lower the carbon ceiling is, the stronger the initial losses but also the higher the long run gains.

Figure 10-b depicts the same kind of variations, but now applied to consumption, and thus to welfare. Except for the optimal case D, this figure drives to the same conclusions

than the previous one. However, we observe now that *the simultaneous implementation of the optimal public instruments allows to avoid the losses in consumption for the first generations.*

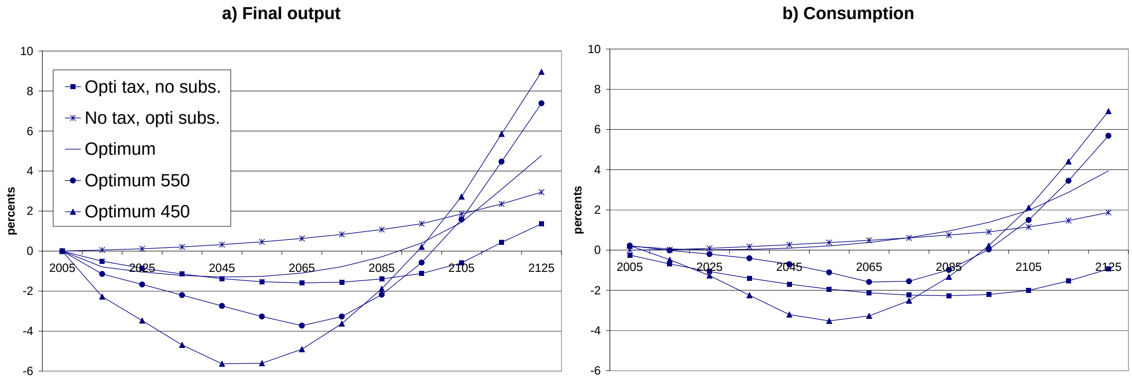

Figure 10: Final output and consumption variations as compared with the laisser-faire

5 Conclusion

Our analysis consisted first in decentralizing the "top-down" ENTICE-BR model (Popp, 2006a) in order to characterize the set of all equilibria. In addition to the backstop, we have also considered a second abatement possibility by adding to the original model a CCS sector, together with an associated dedicated R&D activity. Simultaneously, in order to account for further climate change damages that are not integrated in the damage function, we imposed a cap on the atmospheric carbon accumulation. Since the economy faces two types of market failures, global warming and R&D spillovers, the regulator uses two types of public tools to correct them, a carbon tax and a subsidy for each R&D sector. Obviously, a particular equilibrium is associated with each vector of instruments and there exists a unique vector that implements the first-best optimum. We have analytically computed the optimal tax and subsidies and we have investigated their dynamic properties. In particular, we have shown that the optimal time profile of the tax can be non-monotonic over time and we identified the four effects that drive this dynamics. In brief, three effects leads to a positive growth rate (the decay effect, the discount effect and the wealth effect), when the fourth one implies a negative growth rate (the harmed generation effect) so that the full effect is, a priori, undetermined.

In a second step, we have used a calibrated version of the theoretical model based on

DICE 2007 (Nordhaus, 2008), ENTICE-BR (Popp, 2006a) and DEMETER (Gerlagh et al., 2006), to assess the environmental and economic impacts of various climate change policies. In addition to the standard comparison of the first-best outcome with the laisser-faire, we also provide some intermediate scenarii in which we analyze the dissociated impacts of the policy tools. Our main findings are the following.

i) The optimal carbon tax is generally non-monotonous over time. In particular, under a carbon stabilization constraint, the harmed generation effect overrides the other ones and the tax declines when the ceiling is reached.

ii) Our results do not exhibit relevant crossed effects in the sense that the implementation of a carbon tax alone hardly provides any incentive to proceed with R&D activities and backstop production, when R&D policies used alone have only weak effects on the fossil fuel and CCS sectors.

iii) The implementation of the first-best optimum reveals a complementarity effect between research grants and carbon taxation (the simultaneous use of the two types of tools reinforces the dissociated effects of each one used alone).

iv) The first-best case (without ceiling) does not result in substantial carbon sequestration.

v) A carbon cap reinforces the role of CCS as a mid-term option for mitigating the climate change. In the longer term, if the policy-maker aims at stabilizing the climate, the massive introduction of backstop is necessary.

References

[1] Acemoglu, D., 2002. Directed technical change. Review of Economic Studies 69(4), 781-809.

[2] Aghion, P., Howitt, P., 1998. Endogenous growth theory. Cambridge, MA: MIT Press.

[3] Bergstrom, T.C., 1982. On capturing oil rents with a national excise tax. American Economic Review 72, 194-201.

[4] Bosetti, V., Carraro, C., Graleotti, M., 2006. The dynamics of carbon and energy intensity in a model of endogenous technical change. The Energy Journal, Special issue, 191-206.

[5] Bosetti, V., Carraro, C., Massetti, E., Sgobbi, A., Tavoni, M., 2009. Optimal energy investment and R&D strategies to stabilize atmospheric greenhouse gas concentrations. Resource and Energy Economics, forthcoming.

[6] Chakravorty, U., Magné, B., Moreaux, M., 2006. A Hotelling model with a ceiling on the stock of pollution, Journal of Economic Dynamics and Control 30, 2875-2904.

[7] Edenhofer, O., Bauer, N., Kriegler, E., 2005. The impact of technological change on climate protection and welfare: Insights from the model MIND. Ecological Economics 54, 277-292.

[8] Edenhofer, O., Lessman, K., Bauer, N., 2006. Mitigation strategies ans costs of climate protection: The effects of ETC in the hybrid model MIND. The Energy Journal, Special issue, 207-222.

[9] Farzin, Y.H., Tahvonen, O., 1996. Global carbon cycle and the optimal time path of carbon tax. Oxford Economic Papers 48, 515-536.

[10] Fischer, C., Parry, I., Pizer, W.A., 2003. Instrument choice for environmental protection when technological innovation is endogenous. Journal of Environmental Economics and Management 45, 523-545.

[11] Gerlagh, R., Lise, W., 2005. Carbon taxes: A drop in the ocean, or a drop that erodes the stone? The effect of carbon taxes on technological change. Ecological Economics 54, 241-260.

[12] Gerlagh, R., 2006. ITC in a global growth-climate model with CCS: The value of induced technical change for climate stabilization. The Energy Journal, Special issue, 223-240.

[13] Gerlagh, R., van der Zwaan, B.C.C., 2006. Options and instruments for a deep cut in CO_2 emissions: Carbon capture or renewables, taxes or subsidies? The Energy Journal 27, 25-48.

[14] Gerlagh, R., Kverndokk, S., Rosendahl, K.E., 2008. Linking Environmental and Innovation Policy. FEEM working paper 53.2008.

[15] Goulder, L.H., Mathai, K. 2000. Optimal CO_2 abatement in the presence of induced technological change. Journal of Environmental Economics and Management 39, 1-38.

[16] Grimaud, A., Tournemaine, F., 2007. Why can an environmental policy tax promote growth through the channel of education? Ecological Economics 62(1), 27-36.

[17] Grimaud, A., Rougé, L., 2008. Environment, directed technical change and economic policy. Environmental and Resource Economics 41(4), 439-463.

[18] Hart, R., 2007. The timing of taxes on CO_2 emissions when technological change is endogenous. Journal of Environmental Economics and Management, forthcoming.

[19] Hoffert, M., et al. 2002. Advanced technology paths to global climate stability: Energy for a greenhouse planet. Science 298, 981-987.

[20] International Energy Agency (IEA), 2006. Energy Technology Perspectives, IEA Publications, Paris.

[21] Intergovernmental Panel on Climate Change (IPCC), 2005. Special Report on Carbon Dioxide Capture and Storage, Working Group III.

[22] Jones, C.I., 1995. R&D-based models of economic growth. Journal of Political Economy 105, 759-784.

[23] Jones, C.I., Williams, J.C., 1998. Measuring the social return to R&D. The Quarterly Journal of Economics 113(4), 119-1135.

[24] Jones, C.I, Williams, J.C., 2000. Too much of a good thing? The economics of investment in R&D. Journal of Economic Growth 5(1), 65-85.

[25] Keller, K., McInerney, D., Bradford, D.F., 2008. Carbon dioxide sequestration: how much and when? Climatic Change 88, 267-291.

[26] Kolstad, C.D., Toman, M. 2001. The economics of climate policy. Resources For the Future, Discussion Paper n#00.40REV.

[27] Kriegler, E., Bruckner, T., 2004. Sensitivity analysis of emissions corridors for the 21^{st} century. Climatic change 66, 345-387.

[28] Lafforgue, G., Magné, B., Moreaux, M., 2008. Energy substitutions, climate change and carbon sinks. Ecological Economics 67(4), 589-597.

[29] Nemet, G.F., Kammen, D.M., 2007. U.S. energy research and development: Declining investment, increasing need, and the feasibility of expansion. Energy Policy 35(1), 746-55.

[30] Nordhaus, W.D., 2006. After Kyoto: Alternative mechanisms to control global warming. American Economic Review 96(2), 31-4.

[31] Nordhaus, W.D., 2007. A Review of the Stern Review on the Economics of Global Warming. Journal of Economic Literature 45(3), 686-702.

[32] Nordhaus, W.D., 2008. A question of balance: Weighing the options on global warming policies. Yale University Press.

[33] Popp, D., 2004. ENTICE: Endogenous technological change in the DICE model of global warming. Journal of Environmental Economics and Management 48, 742-768.

[34] Popp, D., 2006a. ENTICE-BR: The effects of backstop technology R&D on climate policy models. Energy Economics 28, 188-222.

[35] Popp, D., 2006b. Comparison of climate policies in the ENTICE-BR model. The Energy Journal, Special issue, 163-174.

[36] Quah, D.T., 2001. The weightless economy in economic development. In M. Pohjola (Ed.), *Information technology, productivity and economic growth: International evidence*. Oxford: Oxford University Press.

[37] Romer, P.M., 1990. Endogenous technical change. Journal of Political Economy 98(5), 71-102.

[38] Scotchmer, S., 2005. Innovations and incentives. Cambridge, MA: MIT Press.

[39] Sinn, H.-W., 2008. Public policies against global warming: A supply side approach. International Tax and Public Finance 15, 360-394.

Appendix

A1. Proof of Proposition 1

Integrating (10) and using (9) and the transversality condition on Z_t, we find:

$$\eta_t = \int_t^\infty \frac{F_Z}{F_{I_F}} e^{-\int_0^s r_x dx} ds.$$

The first characterizing condition (32) is obtained by replacing η into (9) by the expression above, and by noting that $p_F = Q_E E_F - (\xi - S_F)/S_{I_S}$ from (3), (4) and (6), and that $\exp(-\int_0^t r ds) = U'(C)\exp(-\rho t)/U'(C_0)$ from (24). Combining (3), (5) and (13) leads to condition (33). Condition (34) directly comes from (6). Next, using (1) and (24), we directly get condition (35). Finally, the differentiation of (18) with respect to time leads to:

$$\frac{\dot{V}_i}{V_i} = -\frac{\dot{H}^i_{R_i}}{H^i_{R_i}}, \quad i = \{B, E, S\}.$$

Substituting this expression into (16) and using (18) again, it comes:

$$r = -\frac{\dot{H}^i_{R_i}}{H^i_{R_i}} + v_i H^i_{R_i}, \quad \forall i = \{B, E, S\}.$$

We thus obtain the three last characterizing conditions (36), (37) and (38) by replacing into this last equation v_B, v_E and v_S by their expressions (20), (21) and (22), respectively.

A2. Proof of Proposition 2

Let H be the discounted value of the Hamiltonian of the optimal program (we drop time subscripts for notational convenience):

$$\begin{aligned} H &= U(C)e^{-\rho t} + \lambda \left\{ Q\left[K, E(B,F,H_E), \Omega(T)\right] - C - I_F - I_B - I_S - \delta K - \sum_i R_i \right\} \\ &\quad + \sum_i \nu_i H^i(R_i, H_i) + \eta F(I_F, Z) + \mu_G \left\{\xi F(I_F, Z) - S[F(I_F,Z), I_S, H_S] - \zeta G\right\} \\ &\quad + \mu_T[\Phi(G) - mT] + \varphi_G(\bar{G} - G). \end{aligned}$$

The associated first order conditions are:

$$\begin{aligned}
\frac{\partial H}{\partial C} &= U'(C)e^{-\rho t} - \lambda = 0 &(48)\\
\frac{\partial H}{\partial I_F} &= \lambda(Q_E E_F F_{I_F} - 1) + \eta F_{I_F} + \mu_G F_{I_F}(\xi - S_F) = 0 &(49)\\
\frac{\partial H}{\partial I_B} &= \lambda(Q_E E_B B_{I_B} - 1) = 0 &(50)\\
\frac{\partial H}{\partial I_S} &= -\lambda - \mu_G S_{I_S} = 0 &(51)\\
\frac{\partial H}{\partial R_i} &= -\lambda + \nu_i H^i_{R_i} = 0, \quad i = \{B, E, S\} &(52)\\
\frac{\partial H}{\partial K} &= \lambda(Q_K - \delta) = -\dot{\lambda} &(53)\\
\frac{\partial H}{\partial H_i} &= \lambda Q_E E_{H_i} + \nu_i H^i_{H_i} = -\dot{\nu}_i, \quad i = \{B, E\} &(54)\\
\frac{\partial H}{\partial H_S} &= \nu_S H^S_{H_S} - \mu_G S_{H_S} = -\dot{\nu}_S &(55)\\
\frac{\partial H}{\partial Z} &= \lambda Q_E E_F F_Z + \eta F_Z + \mu_G F_Z(\xi - S_F) = -\dot{\eta} &(56)\\
\frac{\partial H}{\partial G} &= -\zeta\mu_G + \mu_T\Phi'(G) - \varphi_G = -\dot{\mu}_G &(57)\\
\frac{\partial H}{\partial T} &= \lambda Q_\Omega \Omega'(T) - m\mu_T = -\dot{\mu}_T &(58)
\end{aligned}$$

The complementary slackness condition is:

$$\varphi_G(\bar{G} - G) = 0, \quad \text{with } \varphi_G \geq 0, \forall t \geq 0 \tag{59}$$

and the transversality conditions are:

$$\begin{aligned}
\lim_{t\to\infty} \lambda K &= 0 &(60)\\
\lim_{t\to\infty} \nu_i H_i &= 0, \quad i = \{B, E, S\} &(61)\\
\lim_{t\to\infty} \eta Z &= 0 &(62)\\
\lim_{t\to\infty} \mu_G G &= 0 &(63)\\
\lim_{t\to\infty} \mu_T T &= 0 &(64)
\end{aligned}$$

From (49), we find that $\eta = -\mu_G(\xi - S_F) - \lambda(Q_E E_F - 1/F_{I_F})$. Replacing this expression into (56) and using (48) leads to the following differential equation: $\dot{\eta} = -(F_Z/F_{I_F})U'(C)\exp(-\rho t)$. Integrating this expression and using the transversality condition (62), we obtain:

$$\eta = \int_t^\infty \frac{F_Z}{F_{I_F}} U'(C)e^{-\rho s} ds. \tag{65}$$

Replacing into (49) λ, μ_G and η by their expressions coming from (48), (51) and (65), respectively, gives us the equation (39) of Proposition 2. Equation (40) directly comes from condition (50). From (48) and (58), we have: $\dot{\mu}_T = m\mu_T - Q_\Omega \Omega'(T)U'(C)\exp(-\rho t)$. Using the transversality condition (64), the solution of such a differential equation is given by:

$$\mu_T = \int_t^\infty Q_\Omega \Omega'(T)U'(C)e^{-[m(s-t)+\rho s]}ds. \tag{66}$$

Next, using the transversality condition (63), we determine the solution of the differential equation (57) as:

$$\mu_G = \int_t^\infty \left[\mu_T \Phi'(G) - \varphi_G\right] e^{-\zeta(s-t)}ds \tag{67}$$

where μ_T is defined by (66) and φ_G must be determined by looking at the behavior of the economy once the ceiling have been reached. Condition (41) is then obtained by replacing into (51) λ and μ_G by their expressions coming from (48) and (67), respectively.

Log-differentiating (48) with respect to time implies:

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{U}'(C)}{U'(C)} - \rho = \epsilon g_C - \rho \tag{68}$$

Condition (42) is a direct implication of equations (53) and (68). Finally, the log-differentiation of (52) with respect to time yields:

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\nu}_i}{\nu_i} + \frac{\dot{H}^i_{R_i}}{H^i_{R_i}}. \tag{69}$$

Conditions (43) and (44) come from (52), (54), (68), (69) and from (50) by using $Q_E E_B = 1/B_{I_B}$. Similarly, condition (45) is obtained from (51), (52), (55), (68) and (69).

A3. Calibration of the model

Param.	Value	Description	Source
γ	0.3	Capital elasticity in output prod.	Nordhaus (2008)
β	0.07029	Energy elasticity in output prod.	Nordhaus (2008)
α_T	0.0028388	Scaling param. on damage	Nordhaus (2008)
ρ_B		Elasticity of subs. for backstop	Calibrated
ρ_E	0.38	Elasticity of subs. for energy	Popp (2006a)
α_H	0.336	Scaling param. of H_E on energy	Popp (2006a)
F_0	7.401	2005 fossil fuel use in GtC	IEA (2007)
c_F	400	2005 fossil fuel price in USD	Computed from IEA (2007)
α_F	700	Scaling param. on fossil fuel cost	Popp (2006a)
η_F	4	Exponent in fossil fuel prod.	Popp (2006a)
B_0	0.45	2005 backstop use in GtC	IEA (2007)
α_B	1200	2005 backstop price in USD	Nordhaus (2008)
η_B		Exponent in backstop prod.	Calibrated
a_B	0.0122	Scaling param. in backstop innovation	Popp (2006a)
a_E	0.0264	Scaling param. in energy innovation	Popp (2006a)
b_B	0.3	Rate of return of backstop R&D	Popp (2006a)
b_E	0.2	Rate of return of energy R&D	Popp (2006a)
c_S	150	Sequestration cost in 2005 USD/tC	Gerlagh (2006)
S_0	0.05	2005 sequestration in GtC	IEA (2006)
$Q_{S,0}$	7.5	2005 sequestration effort in bill. USD	Calibrated
$H_{S,0}$	1	2005 level of knowledge in CCS	
$R_{S,0}$	0.5	2005 R&D investment in CCS in bill. USD	
a_S	0.5	Scaling param. in CCS innovation	
b_S	0.3	Rate of return of CCS R&D	
ϕ_S	0.2	Elasticity of knowledge in CCS innovation	
Φ_i	0.54	Elasticity of knowledge in innovation	Popp (2006a)
ϵ	2	Elasticity of intertemporal subst.	Nordhaus (2008)
A_t		Total factor productivity trend	Nordhaus (2008)
L_t		World population trend	Nordhaus (2008)
ρ_t		Time preference rate	Nordhaus (2008)
...		Other param. and initial values	Calibrated

Table 3: Calibration of parameters