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Abstract

This paper addresses the issue of agricultural production under both output level and output price risks, in a context of random climatic conditions affecting forage used in beef production. It contributes to the empirical literature by applying the framework proposed by Isik (2002) to derive estimating equations from a structural production model with two sources of risks. Flexible functional forms for risk preferences and production technology allow us to identify attitudes toward risk and compute marginal effects of inputs and climate on expected output and production risk. The model is applied on a panel of French cattle farms and estimation results suggest that cattle farmer exhibit strong risk aversion of the CRRA form, and that climate has a significant impact on the performance of animal feeding strategies.

Keywords

Just and Pope, production and price uncertainty, beef cattle, risk aversion, FIML

Introduction

Suckler cow systems[1] are an important feature of the French agriculture, representing more than one third of all European suckler cows and supplying around 60% of French beef production. These systems also participate to rural development as few economical alternatives to livestock farming exist in these areas and help maintaining large areas under grassland, hence favoring biodiversity and limiting pollution and soil erosion (Gibon 2005). However, they rely mainly on a relatively extensive management of forage, implying that beef production risk may be enhanced by the sensitivity of those crops to weather variability. According to Boyer (2008), the first recipients of the French fund for agricultural calamities are herbivorous farms, mainly because of drought on forage crops. In addition, these farmers have to face another source of risk associated to output market price. Because of the increasing liberalization of the world agricultural markets and of climate change, these risks are likely to have a more profound impact on farm income, while at the same time, the sustainability of the actual public farm payment schemes (calamity compensation, direct payments, etc.) remains uncertain.

In analyzing risk management in agriculture, issues related to production technology, market structure and public support policies are important to consider for several reasons. First, farmers develop individual risk-management strategies that can supplement or replace public compensation policies. Such strategies entail crop and product diversification, contracts and futures to hedge against price risk, and also the use of risk-reducing inputs. In the latter case, technology obviously matters in the definition of risk-increasing versus risk-decreasing inputs (pesticide, irrigation, etc.).

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[1] Suckler cow systems consist in raising calves with their mother's milk in order to produce meat.

Second, the level of risk on output price depends on market conditions and design, i.e., whether products are sold on a competitive market or are subsidized, whether contract-based supply chains are part of the market, etc. Third, from the farmer perspective, public compensation policies are accounted for when deciding on optimal production plans in the presence of risk. More precisely, the proportion of total income exposed to risk will determine the degree to which risk-hedging strategies will differ from those when no risk is present, together with farmer preferences toward risk (Hennessy 1998).

Beside the existence of public support payments, farmers rely on input management to alleviate the impact of random events on production and profit. The design of an optimal input mix is frequently seen as a way of implementing individual risk management strategies, in which the contribution of each input to expected output is weighed against its contribution to production risk (usually measured by the variance of output). In the vast literature devoted to risk management in agriculture, the approach based on conditional moment of production output has proved rather popular, mainly because the way inputs could modify expected output level or production risk is directly estimated from observations on inputs and output (Just and Pope 1978, 1979).

However, approaches along the lines of Just and Pope suffer from several drawbacks: the model is limited to the first two moments of the output distribution, input levels are likely to be endogenous in the production function, risk preferences are not identified and price risk is not considered. Antle (1983a) has suggested a way of dealing with some of the above criticisms to produce estimates of Arrow-Pratt and Downside Risk coefficients from higher moments of the profit distribution (see also Antle 1983b; Antle and Capalbo 2001). However, in Just and Pope as in Antle, no structural model exist that could explicitly represent endogeneity of inputs. Love and Buccola (1991), Saha, Shumway and Talpaz (1994), and Kumbhakar and Tveteras (2003) discuss the need for estimating jointly equations related to production function and to farmers' optimization program, in order to obtain a consistent and efficient model of production. Another limitation of Antle's approach is the fact that technological substitution patterns between inputs cannot be directly recovered, as the relationship between expected profit and production is not available.

In order to overcome the above-mentioned limitations and to extend the empirical analysis to the case of two sources of risks, we consider the production model of Isik (2002). Output price and output level may be correlated, farmers are risk-averse and are characterized by a Just-and-Pope production function, and the maximization of the expected utility of profit in the single-product case leads to a system of structural equations (first-order conditions) that represent explicitly the relationship between (endogenous) inputs and exogenous variables (prices, etc.). Risk preferences can in principle be estimated together with expected output and the variance of production, the limitation of the model being that higher-order moments of the distribution of output are not considered.

We conduct in this article an empirical application of the model proposed by Isik (2002), in the case of beef production (from suckler cows). We focus on input decisions when random climate may affect one of the two inputs, namely forage crop, which is complement or substitute to the other input, animal feed. Our econometric framework allows us to quantify the impact of weather variability on output level, production risk, input choice relationships, and to identify farmer preferences toward risk.

This article makes several contributions to the empirical literature on production under uncertainty and risk aversion. First, if some studies have estimated weather impacts on production distribution, they have dealt exclusively with crop production (Chen and Chang 2005; Isik and Devadoss 2006). In suckler cow production however, the weather-dependent variable –forage production and weather conditions may impact not only the distribution of production but also input/output relationships (through, as mentioned above, substitution effects between forage and animal feed), as farmers may adjust their decisions with partial observation of stochastic events.

Second, to the best of our knowledge, Just and Pope production function and risk preferences have not yet been estimated under two sources of risks. We provide, through the empirical implementation of Isik's framework, a structural model that can be consistently estimated to produce output and risk functions parameters, and risk preference coefficients, when both output price and level are random. To simplify the empirical analysis however, we do not consider correlated risks, such assumption being also justified by the price-taker behavior of cattle farmers, so that correlation between (aggregate) market price and output level is likely to be small.

We first present the production model with output and price risks, and discuss the specification of farmer preferences toward risk, and the farmer's optimization program. We then discuss econometric considerations regarding the structural simultaneous-equation model and the Full Information Maximum Likelihood estimation method are presented. The empirical application is based on a panel of 65 cattle farmers observed over the period 1987-2005 and specialized in Charolais suckler cow production in the centre of France. Finally, we jointly estimate the production model and risk-aversion parameters under two sources of risk and discuss estimation results by paying a particular attention to climatic variations and animal feed substitution patterns.

A production model with output and price risk

Although it has been criticized because it overlooks higher moments of output (e.g., skewness and kurtosis, see Antle 1983b; Groom et al. 2008), the mean-variance approach proposed by Just and Pope (1978, 1979) is still very popular in agricultural production analysis. This approach provides a simplified specification of production technology with output risk, where inputs affect not only expected output but also its variance (a measure of production risk), allowing marginal effects of inputs to be computed for both moments.

Production risk: a mean-variance approach

Denote y the level of output, defined as:

(1) $y = f(x,\beta) + h(x,\gamma)\varepsilon$,

where x is a K vector of inputs, β and γ are vectors of parameters, and ε is an error term with mean 0 and variance σ_ε^2. Function *f(.)* represents the expected output level, while $\sigma_\varepsilon^2 h^2(\bullet)$ is the variance of output (with the restriction $h(x;\beta) > 0,\ \forall x, \forall \beta$).

Estimating β and γ is typically performed by weighted least squares, using first-stage estimates of ε (from initial estimation of f*(.)*) to construct the function *h* (see Just and Pope 1978). The main problem regarding this estimation method is the fact that inputs x are likely to be endogenous in (1), i.e., correlated with the unpredicted output level, if explanatory variables are omitted for instance. Besides, farmer preferences giving rise to optimal decisions regarding input mix (and consequently, an optimal production plan for y) are not present in the model. It is well known however that risk aversion may modify the optimal input mix, or in other terms, that marginal effects of inputs depend on risk preferences. To see this, write the general program of a risk-averse farmer as:

(2)
$$\begin{aligned} &\max_x EU(\Pi) \equiv EU\left[Pf(x,\beta) + Ph(x,\gamma)\varepsilon - rx)\right] \\ &\Leftrightarrow \frac{r}{P} = \frac{\partial f(x)}{\partial x} + \frac{\partial h(x)}{\partial x} \times \frac{E\left(U'\varepsilon\right)}{E\left(U'\right)}, \end{aligned}$$

where *U(.)* is the von Neuman-Morgenstern utility function (increasing, concave in profit), *Π* is profit, *P* is the unit output price and *r* is the vector of unit input prices. In the case of a risk-neutral farmer, *U'* would be constant so that the last term in the equation above is equal to zero, and marginal productivity equals the price ratio *r/P*, which is the usual efficiency condition for production. If, on the other hand, the farmer is risk averse with $U'' \neq 0$, then the optimal input mix x

cannot be determined from the production function *f(.)* alone. Distortion due to risk aversion is reflected by the term $\frac{\partial h(x)}{\partial x} \times \frac{E(U'\varepsilon)}{E(U')}$; depending on the sign of $\frac{\partial h(x)}{\partial x}$, an input may be either risk-increasing or risk-decreasing. In the first case, the farmer would tend to decrease the use of such input, and in the second case the farmer would increase it (see Antle 1983b).

Little attention has been paid in the empirical literature on producer behavior in the presence of a dual source of risk, namely output jointly with production (or input) price. Denote *P* the random unit output price, with $P = \bar{P} + \theta,$ where $\bar{P}$ is the expected output price and θ is a random term with $E(\theta)=0$. The random profit under risk neutrality is then

(3) $$\Pi = (\bar{P} + \theta)\left[f(x;\beta) + h(x;\beta)\varepsilon\right] = E(\Pi) + \bar{P}h(x;\beta)\varepsilon + f(x;\beta)\theta,$$

where $E(\Pi) = \bar{P}f(x;\beta) + h(x;\beta)Cov(\theta,\varepsilon),$ since $E(\varepsilon) = E(\theta) = 0$.

As a consequence, the expected profit may be over- or under-estimated if the covariance (correlation) between both risks is not zero, even if the farmer is risk neutral.

The conclusion of this section is first, that a structural model is needed to represent in an adequate fashion risk preferences together with the presence of price and production risks. Second, the dual source of risk (output level and output price) has to be accounted for to produce consistent estimates of expected profit. The Just and Pope framework is considered only because it provides a simplified specification of the production model with output risk, and such simplification is required when dealing with two sources of risk.

Specifying farmer's risk preferences

A more risk-averse farmer will be willing to forgo a larger reduction in risk, this forgone part of expected profit being the marginal risk premium. Risk aversion is reflected by the curvature of the utility function, with the Arrow-Pratt absolute risk-aversion coefficient defined as follows:

$$(4) \qquad \phi_a(\bar{W}) = -\frac{U''(\bar{W})}{U'(\bar{W})},$$

where *U'* and *U''* denote respectively the first and second derivative of the utility function and $\bar{W}$ is the expected (post-risk) wealth.

The absolute risk aversion (ARA) may decrease (DARA), be constant (CARA) or increase (IARA) with wealth. The Constant, Increasing of Decreasing relative risk aversion (RRA) measures (CRRA, IRRA or DRRA) make explicit the way farmer decisions are affected if all payoffs are multiplied by a positive constant:

$$(5) \qquad \phi_r(\bar{W}) = \bar{W} * \phi_a(\bar{W}).$$

Note that CRRA functions are necessarily also DARA. The functional form chosen for the utility function has obviously strong implications for the structure of risk preferences (Hardaker et al, 2004). For instance, the mean-variance utility function or the negative exponential form used by Love and Buccola (1991), Shankar and Nelson (2003) or Antle (2003) imply a constant absolute risk aversion. In this paper we do not impose a priori restrictions on the structure of preferences (e.g., risk neutrality, CARA or CRRA/DARA). Rather, we specify a flexible form (in a way somewhat similar to Chavas and Holt, 1996) for the absolute risk aversion function as a function of post-risk wealth $\bar{W}$:

$$(6) \qquad \phi_a\left(\bar{W}\right) = \phi_0 + \phi_1 \bar{W} + \phi_2 \frac{\bar{W}^2}{2},$$

where ϕ_0, ϕ_1 and ϕ_2 are parameters to be estimated jointly with technology parameters. We therefore require that $\phi_1^2 - 2\phi_0\phi_2 > 0$ and $\left(\frac{\phi_1}{2\phi_2}\right)\left[1+\left(1-\frac{2\phi_0\phi_2}{\phi_1^2}\right)^{1/2}\right] < 0$ for parameters to be consistent with positive ex post wealth. Note that expected post-risk wealth is farmer-specific but may also be time-varying, as it implicitly depends on (non-random) input and output prices.

The farmer optimization program

Having specified the production function and the risk preferences, we now proceed to the derivation of the farmer's optimization program, following Isik (2002). Consider a risk-averse farmer facing both production uncertainty (typically related to weather uncertainty) and output price uncertainty. The objective of the farmer is to maximize the expected utility of wealth, where the expectation must be taken with respect to the distribution of all random variables. Wealth can be considered the sum of the initial non random wealth plus the random current period profit (Chavas and Pope 1985; Coyle 1999) or simply as the profit for the current period. However, there is no consensus on what an appropriate measure of initial wealth might be (Shankar and Nelson). The problem of the farmer is:

(7) $$\max_x EU\{W\} = EU\{W_0 + (\bar{P}+\theta)\cdot(f(x)+h(x)\varepsilon - rx\}.$$

Assuming that the second-order conditions are satisfied, the optimal level of x is given by the necessary first-order condition:

(8) $$\frac{\partial U}{\partial x} = EU\{U_W(W)[(\bar{P}+\theta)\cdot(f_x(x)+h_x(x)\varepsilon - r\} = 0,$$

where f_x and h_x are the derivatives of f and h with respect to x respectively. Next, we approximate U_W (partial derivative of U to W) around the expected post-risk wealth using a Taylor series expansion. We get:

(9) $U_W(W) = U_W(\bar{W}) + \left\{\bar{P}h(x)\varepsilon + \theta \cdot (f(x) + h(x)\varepsilon)\right\} U_{WW}(\bar{W})$.

Combining (8) and (9), we can rewrite the first-order condition as:

$$\frac{1}{U_W(\bar{W})}\frac{\partial U}{\partial x} = E\left\{\left[1 - \phi_a\left(\bar{P}h(x)\varepsilon + \theta \cdot (f(x) + h(x)\varepsilon)\right)\right] \cdot \left[(\bar{P} + \theta)\left(f_x(x) + h_x(x)\varepsilon\right) - r\right]\right\}, \tag{10}$$

which corresponds to equation (4) in Isik (2002). Rearranging this condition gives:

$$\begin{aligned} \frac{1}{U_W(\bar{W})}\frac{\partial U}{\partial x} = {} & \bar{P}f_x + h_x E(\theta\varepsilon) - r - \phi_a \left\{\bar{P}^2 h h_x \sigma_\varepsilon^2 + f f_x \sigma_\theta^2 + E(\theta\varepsilon)\left[2\bar{P}hf_x + \bar{P}fh_x - hr\right]\right. \\ & \left. + E(\varepsilon^2\theta)\left[2\bar{P}hh_x\right] + E(\varepsilon\theta^2)\left[fh_x + f_x h\right] + E(\varepsilon^2\theta^2)\left[hh_x\right]\right\} \end{aligned}. \tag{11}$$

Using the Bohrnstedt and Golberger's (1969) method for the covariance of products of random variables on the previous condition leads to:

$$\begin{aligned} \frac{1}{U_W(\bar{W})}\frac{\partial U}{\partial x} = {} & \bar{P}f_x - h_x C(\varepsilon,\theta) - r - \phi_a \left\{\bar{P}^2 h h_x \sigma_\varepsilon^2 + f f_x \sigma_\theta^2\right. \\ & \left. + C(\varepsilon,\theta)\left[-2Phf_x - Pfh_x + hr\right] + hh_x\left(\sigma_\varepsilon^2\sigma_\theta^2 - 2C(\varepsilon,\theta)^2\right)\right\} \end{aligned}. \tag{12}$$

Assuming statistical independence between ε and θ leads to a simplified necessary first-order condition:

$$\bar{P}\ f_x(x;\beta) = \frac{\phi_a h(x;\gamma) h_x(x;\gamma)\sigma_\varepsilon^2\left(\bar{P}^2 + \sigma_\theta^2\right) + r}{1 - \left[\phi_a \sigma_\theta^2 f(x;\beta)/\bar{P}\right]}, \tag{13}$$

where f_x and h_x respectively denote derivatives of *f* and *h* with respect to x, *r*, is input price and *P* is output price with *Var (P)*=σ^2_θ.

Econometric considerations

A well-known problem in primal production analysis, that must be addressed by the estimation method, is the likely endogeneity of inputs *x*. There are two main reasons as discussed in Shankar and Nelson (2003) for inputs to be correlated with production error terms. First, some inputs may be correlated with farmer's unobserved heterogeneity contained in the error term (reflecting, e.g., differences in land quality, farmer's management ability, or omitted variable inputs). The unobserved heterogeneity in technical efficiency of farmers may be somewhat reduced by incorporating ''environmental variables '' in the production function, but the homogeneity of parameters β (in the expected output function) and γ (in the variance of output) may then have to be relaxed. Second, in agriculture, some input-related decisions taken by farmers can be viewed as sequential within a season (see Antle, 1983a). In suckler cow systems for example, animal feed can compensate a shortage in on-farm forage crops due to adverse weather conditions. Consequently, omitting the sequentiality of decisions in the choice of inputs can be a source of endogeneity in this case.

The structural econometric model

Correction for endogeneity bias can be made by using instrumental variable methods (Wooldridge, 2002), by introducing panel error components to the stochastic term as in Griffith and Anderson (1982) or by adding exogenous variables (Kumbhakar and Tverteras 2003). In order to reduce the likely effect of unobserved heterogeneity on parameter consistency, we specify the production parameters as explicitly depending on exogenous, observed variables denoted z, which partly control for heterogeneous technologies and contemporaneous weather conditions. The model we consider has therefore output y and inputs x as endogenous variables, prices (P, r) and variables z as exogenous variables. Moreover, joint estimation of a structural model (instead of using Instrumental Variable techniques) can reduce endogeneity problems, as mentioned by Love and Buccola (1991). The structural model consists of the following system of equations, for farmer i at time period t:

$$(14a) \qquad E(y_{it}|x_{it}) = f\left[x_{it};\beta\left(z_{it}\right)\right],$$

$$(14b) \qquad E\left\{y_{it} - f\left[x_{it};\beta\left(z_{it}\right)\right]|x_{it}\right\}^2 = \sigma_\varepsilon^2 h\left[x_{it};\gamma\left(z_{it}\right)\right],$$

$$(14c) \qquad E(P_{it}) = \bar{P}_{it},$$

$$(14d) \qquad E\left(P_{it} - \bar{P}_{it}\right)^2 = \sigma_\theta^2,$$

$$(14e) \qquad E\left\{\left[r_{it} - \bar{P}_{it} f_x\left[x_{it};\beta\left(z_{it}\right)\right]\left\{1 - \left[\phi_{ai}\sigma_\theta^2 f\left[x_{it};\beta\left(z_{it}\right)\right]/\bar{P}_{it}\right] - \phi_{ai} h\left[x_{it};\gamma\left(z_{it}\right)\right] h_x\left[x_{it};\gamma\left(z_{it}\right)\right]\sigma_\varepsilon^2\left(\bar{P}_{it}^2 + \sigma_\theta^2\right)\right\}\right]\right\}$$

$$= E\left\{M_j\left[\left(r_{it}, \bar{P}_{it}, x_{it};\beta\left(z_{it}\right), \gamma, \phi, \bar{W}, \sigma_\theta^2, \sigma_\varepsilon^2\right)\right]\right\} = 0, \qquad j = 1,2,\ldots,K,$$

$$(14f) \qquad E\left(\Pi_{it}\right) = P_{it} f\left[x_{it};\beta\left(z_{it}\right)\right] - r_1 x_1 - r_2 x_2,$$

i=1, 2, …, N ; t=1, 2, …, T, where the absolute Arrow Pratt coefficient is

$\phi_{ai}\left(\bar{W}_i\right) = \phi_0 + \phi_1 \bar{W}_i + \phi_2 \dfrac{\bar{W}_i^2}{2}$ and j is the index of the variable input x_k.

We assume here that expected ex-post risk is a farmer-specific measure that does not vary through time. The set of equations (14e) can be combined for two different inputs to form a single first-order condition, for example:

$$(15) \qquad \begin{aligned} & r_{1it}\left[\bar{P}_{it} f_{x_{2,it}}\left(1 - \phi_{ai}\sigma_\theta^2 f_{it}/\bar{P}_{it}\right) - \left(\phi_{ai}\sigma_\varepsilon^2 h_{it} h_{x_{2,it}}\right)\left(\bar{P}_{it}^{\,2} + \sigma_\theta^2\right)\right] \\ & - r_{2it}\left[\bar{P}_{it} f_{x_1,it}\left(1 - \phi_{ai}\sigma_\theta^2 f_{it}/\bar{P}_{it}\right) - \left(\phi_{ai}\sigma_\varepsilon^2 h_{it} h_{x_1,it}\right)\left(\bar{P}_{it}^{\,2} + \sigma_\theta^2\right)\right] = 0 \end{aligned}.$$

Equations (14a)-(14b) and (14c)-(14d) refer to the first- and second-order moments of the distribution of output level and output price respectively. Equation (14e) is a structural equation that can be considered a homogeneous nonlinear restriction between endogenous and exogenous variables in the system. Equation (14f) corresponds to the first order moment of the profit distribution.

We specify a quadratic functional form (linear in the parameters) for *f(.)* in terms of two variable inputs denoted x_1 and x_2, and three variables capturing parameter heterogeneity as discussed above, z_1, z_2 and z_3.

$$(16) \qquad f\left[x_{it};\beta\left(z_{it}\right)\right]= \begin{array}{l}\beta_0+\beta_1x_1+\beta_2x_2+\frac{1}{2}\beta_{11}x_1^2+\frac{1}{2}\beta_{22}x_2^2+\beta_{12}x_1x_2+\beta_{z1}z_1+\beta_{z11}z_1x_1\\ +\beta_{z12}z_1x_2+\beta_{z2}z_2+\beta_{z21}z_2x_1+\beta_{z22}z_2x_2+\beta_{z3}z_3+\beta_{z31}z_3x_1+\beta_{z32}z_3x_2\end{array}$$

As for the standard deviation of output, we specify a Translog functional form to ensure positiveness of *h(.)*:..

$h\left[x_{it};\gamma(z_{it})\right]=$

$$(17) \qquad \exp\left(\begin{array}{l}\gamma_0+\gamma_1\log(x_1)+\gamma_2\log(x_2)+\frac{1}{2}\gamma_{11}\left[\log(x_1)\right]^2+\frac{1}{2}\gamma_{22}\left[\log(x_2)\right]^2+\gamma_{12}\log(x_1)\log(x_2)\\ +\gamma_{z1}\log(z_1)+\gamma_{z11}\log(z_1)\log(x_1)+\gamma_{z12}\log(z_1)\log(x_2)+\gamma_{z2}\log(z_2)+\gamma_{z21}\log(z_2)\log(x_1)\\ +\gamma_{z22}\log(z_2)\log(x_2)+\gamma_{z3}\log(z_3)+\gamma_{z31}\log(z_3)\log(x_1)+\gamma_{z32}\log(z_3)\log(x_2)\end{array}\right)$$

Note that variables z are introduced additively and interact with each x but not with the x^2's. This means that only first-order terms in expected output and the standard deviation of output are assumed to have heterogeneous parameters. It is clear from this quadratic specification that $Var(\varepsilon)=\sigma_\varepsilon^2$ is not identified and can be normalized to 1, so that $Var(y_{it})=h^2\left[x_{it};\gamma(z_{it})\right]$. As for the price function, we choose a simple linear, first-order autoregressive process:

$$(18) \qquad P_{it}=\overline{P}_{it}+\theta_{it}=\alpha_0+\alpha_1P_{t-1}+\theta_{it},$$

where $\alpha_0>0,\ \alpha_1\in[0,1)$ to ensure price stationarity.

Estimation method

In view of the system of equations (14), the GMM (Generalized Method of Moments) seems to be a particularly interesting estimation procedure to consider, because the GMM estimator is constructed from first -and second- order moment conditions easily obtained from the system's equations (Wooldridge 2002). In small samples however, results are generally sensitive to the choice of instruments. In practice, even with an appropriate set of instruments such that the model specification was not rejected by the Hansen over-identifying restriction test, parameter estimates for our model were neither robust nor very significant. A possible explanation is the fact that, when variable inputs x are treated as endogenous, the total number of endogenous variables is much more important than the number of structural equations in the system, because of the quadratic functional forms specified for *f(.)* and *h(.)* (cross-products of x's and between the x's and the z's). For this reason, the required number of instruments increases dramatically with the number of variable inputs (and the dimension of the vector z), and it may prove difficult to find enough admissible instruments to over-identify the model. Nonlinear maximum likelihood estimators such as FIML (Full Information Maximum Likelihood) can then be considered an alternative to GMM. Although these estimators are more efficient under the assumption of normality, they are generally not consistent when this assumption is not valid (although some special cases exist where such consistency holds, see Phillips 1982). There is no need to obtain a closed-form solution for the endogenous solutions in the system, as the Jacobian transformation computes directly the required transformation during likelihood maximization. This is particularly important for the first-order conditions (14) or (15) that are highly nonlinear in the endogenous variables x. For this reason, we turn to Nonlinear FIML, whose estimation principle is as follows (see Davidson and MacKinnon, 1993).

Consider the system of structural equations in general form:

(19) $$h_i(Y_i, X_i, \Delta) = U_i, \quad U_i \sim N(0, \Sigma),$$

where i indexes observations, h_i is a G vector of nonlinear functions (as many as there are structural equations in the system) and U_i is a G vector of normally-distributed error terms. Y_i and X_i respectively denote the vector of endogenous and exogenous variables, and Δ is a vector of structural parameters.

The density of Y_i can be written

$$(2\pi)^{-G/2}\left|\det J_i\right|\left|\Sigma\right|^{-1/2}\exp\left(-\frac{1}{2}h_i(Y_i,X_i,\Delta)\Sigma^{-1}h_i'(Y_i,X_i,\Delta)\right), \tag{20}$$

where $\left|\det J_i\right| = \left|\det\left(\frac{\partial h_i(Y_i,X_i,\Delta)}{\partial Y_i}\right)\right|$ is the Jacobian of the transformation from U_i to Y_i.

The log-likelihood of the sample is

$$\log L(\Delta,\Sigma) = -\frac{NG}{2}\log(2\pi) + \sum_{i=1}^{N}\log\left|\det J_i\right| - \frac{N}{2}\log\left|\Sigma\right| - \frac{1}{2}\sum_{i=1}^{N}h_i(Y_i,X_i,\Delta)\Sigma^{-1}h_i'(Y_i,X_i,\Delta) \tag{21}$$

and the concentrated log-likelihood, when *Σ* is replaced by its estimate obtained from maximizing $\log L(\Delta,\Sigma)$ with respect to *Σ*, is

$$\log L^C(\Delta,\Sigma) = -\frac{NG}{2}\left[\log(2\pi)+1\right] + \sum_{i=1}^{N}\log\left|\det J_i\right| - \frac{1}{2}\log\left|\frac{1}{N}\sum_{i=1}^{N}h_i'(Y_i,X_i,\Delta)h_i(Y_i,X_i,\Delta)\right|. \tag{22}$$

Because joint identification of all structural parameters proved difficult, we proceed in two steps. We first use Equations (14c) and (14d) (output price and price risk equations) to estimate parameters α_1 and σ_θ^2. We then set the variance of output price equal to its estimate in a second sep, where we estimate equations (14b), (14c), (14f), (15) (the combination of conditions (14e) over the two variable inputs x_1 and x_2), and the expression for the variance of sales, which is

$$Var(Py_{it}) = \pi_0 + \bar{P}^2 h_{it}^2\,\sigma_\varepsilon^2 + f_{it}^2\sigma_\theta^2 + h_{it}^2\sigma_\theta^2\sigma_\varepsilon^2\ , \tag{23}$$

because independence between ε and θ is assumed.

Since parameter σ_θ^2 in Equation (14d) is already estimated in a first stage, we perform the usual correction for the variance-covariance matrix of parameters in two-step estimation, as second-step estimating equations (α,β,γ,ϕ) depend on the first-step estimated parameter (σ_θ^2) (see Greene, 1997). Let $L_1(\sigma_\theta^2)$ and $L_2(\Delta\left|\sigma_\theta^2\right.)$ denote the likelihood functions corresponding to equations (14c)-

(14d) and the system (14b)-(14c)-(14f)-(15) respectively. The second-step maximum-likelihood estimator as the following asymptotic variance-covariance matrix:

$$Var\left(\hat{\Delta}\right)+Var\left(\hat{\Delta}\right)\left[C\,Var\left(\hat{\sigma}_{\theta}^{2}\right)C'-R\,Var\left(\hat{\sigma}_{\theta}^{2}\right)C'-C\,Var\left(\hat{\sigma}_{\theta}^{2}\right)R'\right]Var\left(\hat{\Delta}\right),$$

(24)

where $C=E\left[\left(\frac{\partial\log L_{2}\left(\Delta\middle|\sigma_{\theta}^{2}\right)}{\partial\Delta}\right)\left(\frac{\partial\log L_{2}\left(\Delta\middle|\sigma_{\theta}^{2}\right)}{\partial\sigma_{\theta}^{2\prime}}\right)\right]$, $R=E\left[\left(\frac{\partial\log L_{2}\left(\Delta\middle|\sigma_{\theta}^{2}\right)}{\partial\Delta}\right)\left(\frac{\partial\log L_{1}\left(\sigma_{\theta}^{2}\right)}{\partial\sigma_{\theta}^{2\prime}}\right)\right]$,

and $\left(\hat{\sigma}_{\theta}^{2},Var(\hat{\sigma}_{\theta}^{2})\right)$ and $\left(\hat{\Delta},Var\left(\hat{\Delta}\right)\right)$ are estimated from maximizing $L_{1}(\sigma_{\theta}^{2})$ and $L_{2}(\Delta|\sigma_{\theta}^{2})$ respectively.

Empirical application to beef cattle in France

Data

We use an original panel dataset from the INRA research department in livestock economics located in Clermont-Ferrand-Theix, France. This database contains yearly records on farm structure, income, production and costs, as well as on farm characteristics related to land and herd management. The sample we use consists of 65 individual farmers observed over the period 1987-2005 (19 years). Output y is the quantity of beef meat produced (in kg per hectare of fodder area), and thus takes into account both the stocking rate per hectare of fodder crop and the meat production rate per animal. This indicator actually assesses the gain in weight of the herd and includes animals that are not sold during the current year. A weighted average meat price is computed every year for each farm, over the various animal products. The quantity of meat produced is driven essentially by animal diets. Two kinds of feeds are considered in the present study: on-farm forage crops and other animal feed. The quantities of harvested and consumed forage crops are not recorded in the database. We use for input x_1 the quantity of fertilizer applied on forage crops. It is considered an indicator of farmers' intent to increase the yield of forage crops. Price r_1 of nitrogen unit per farm and per year comes from data base records. Endogenous input x_2 corresponds to off-farm forage crops and mainly comprises concentrate feeds such as barley and commercial feeds. Its unit price r_2 is computed as a weighted average of the different kinds of feed purchased by the farmer.

Input efficiency is supposed to be affected according to farm characteristics and weather conditions. Indeed, the actual forage production does not only result from forage crop inputs but also from exogenous factors such as climate conditions or agronomic quality of pasture. We assume that average cereal yield of each farm is related to soil quality and can be taken as a proxy (denoted z_1). In addition, farmers' technical ability can alter input-output relationships. Variable z_2 then corresponds to the average number of calves weaned per cow over the period since monitoring of reproduction process requires important technical skills. A proxy of climate conditions is introduced through variable z_3, computed from yearly regional forage production records.

Concerning the farmer-specific risk-aversion coefficient ϕ_{ai}, we hypothesize that farm-specific average profit (annual beef gross margin per worker unit) is a correct approximation of farm ex post wealth (as in particular, this variable includes public subsidies related to land). We therefore consider the individual mean (over time periods) of profit per farm as a proxy for ex post expected wealth, $\overline{W}_i$.

Descriptive statistics of the variables used in the econometric application are displayed in Table 1.

(Insert Table 1 here)

Estimation results

The FIML estimation results are presented in Table 2. Most parameter estimates are significant at the 1% level. Our estimation procedure fits the data quite well as revealed by the adjusted R^2 calculated for average production, price and profit equations (R^2 greater than 0.62). However, a rather low R^2 is found for the structural equation on input price r_1, while the one with first-order input price r2 is much higher (R^2=0.46). Price is found to be a stationary process, with a slope less than 1.

(Insert Table 2 here)

(Insert Table 3 here)

Parameter estimates of squared inputs (see Table 2) indicate that expected output is concave in both x_1 and x_2, in accordance with producer theory. Marginal effects of inputs on expected production and output risk are presented in Table 3, together with elasticities with respect to x and z. The elasticity of expected output with respect to feed purchased is slightly higher than the one with respect to nitrogen application, probably because concentrate feed is directly available to animals whereas nitrogen application on pasture crops is only an indirect input. Elasticities of output with respect to endogenous inputs (Table 3) are below 0.25, which indicates that the impact of inputs on expected output and on output variance is limited. It is indeed possible to raise animals even without applying mineral fertilizer on forage crops or using concentrate feed in the studied beef production systems. In addition, live weight variation per animal is bounded by biological constraints related to intake capacity, potential of growth and health. Efficiency of inputs is therefore limited. The analysis of elasticities of output risk with respect to inputs reveals that if feed purchased proves to have no significant impact on production risk, nitrogen application appears as a risk-reducing input. Fertilizer is generally found to increase crop production risk among the literature (Love and Buccola 1991; Just and Pope 1979; Shankar and Nelson 2003). However, applying more fertilizer on pasture favors early spring growth which is less sensitive to weather variation than summer and autumn growths. Hence, applying fertilizer on forage crops may reduce variations in the quality and quantity of pasture produced over the year,

Regarding elasticities of expected production with respect to exogenous variables, they appear positive whereas elasticities of output variance are negative. Favorable climatic conditions, farmer skills and good agronomic potential clearly favor enhanced output levels and warrant more regular outputs. A satisfactory level of know-how in cattle reproduction (z_2) has the highest impact on expected production and variance, because it is directly related to the quantity of animals that can be sold. The impact of climatic conditions on expected production and on production risk is lower. However, when inspecting estimates of cross products between the proxy for climatic condition and endogenous inputs, we find positive and highly significant interactions in the mean production function. As climatic conditions are adverse, input efficiency decreases, reflecting the trade-off between forage production and the expected effect of fertilizer. In addition, farmers need to use more feed to maintain their meat production at the required level. In this latter case, adjustment decisions of input levels subsequent to weather conditions observations are clearly identified. Significant and negative estimates of cross products between the weather indicator and endogenous inputs in the production variance function indicate that farmers who relied less on forage production to feed the herd are naturally found to have a less sensitive production risk to climatic conditions. Interaction estimates between agronomic potential, know-how in cattle production proxies and endogenous inputs are all significant. They are positive in the expected output function (except for ‘‘animal productivity times feed purchase’’ estimates, which is not significant), and negative in the output variance function. As a result, a fertile land and a skilled farmer enhance inputs efficiency and improve production control.

Finally, based on FIML parameter estimates, we compute the absolute and relative risk aversion parameters ϕ_a and ϕ_r. The absolute risk aversion coefficient is around 0.05, while the relative risk aversion coefficient is estimated at 2.38, indicating a strong risk aversion for farmers in our sample, although these estimated values are in the range of previous empirical studies, see Lins, Gabriel and Sonka (1981), Saha, Shumway and Talpaz (1994), Bar-Shira, Just and Zilberman (1997), Chavas and Holt (1990), Raskin and Cochran (1986). The derivatives of these risk measures with respect to ex post wealth (see Table 4) clearly indicate that farmers are characterized by CRRA and DARA preferences, which is also in line with several previous studies. The CARA assumption, as well as risk neutrality, is strongly rejected by our test statistics.

(Insert Table 4 here)

Conclusion

This article investigates the role of production and price risks on farmers' decisions regarding cattle feeding, when weather variability impacts the level of a particular input (forage crops). The application contributes to the empirical literature on production under risk in several ways. We estimate by nonlinear FIML a structural production model embedding a Just and Pope production function, to estimate technology parameters (expected output and production variance) and to identify farmers' risk preferences using flexible functional forms. To our knowledge, few studies have carried out such structural empirical estimation, and none has considered two sources of risks (production and output price risk). By following the framework proposed by Isik (2002), we consider a fully structural model that allows dealing with endogeneity issues related to variable inputs in both expected production and output variance functions. Technology parameters related to variable inputs have been specified as explicitly depending on exogenous variables which partly control for heterogeneous technologies and contemporaneous weather conditions. This method presents in addition the great advantage to enable us to measure the impact of climatic conditions, not only on the distribution of production but also on input-output relationships.

The main results of our empirical application are the following. Suckler cow farmers appear to be strongly risk adverse, preferences towards risk being of the CRRA form. Estimated interactions between exogenous structural farm characteristics and variable inputs indicate a significant degree of heterogeneity in technology efficiency across farms. Introducing a weather indicator in both expected output and production variance underlines the fact that the impact of weather on input level is sizable. Sequential decisions regarding non-forage feed and the joint effect of fertilizer and weather on forage production and on subsequent beef production are consequently brought into light. Favorable climatic conditions clearly improve average input efficiency and decrease the variability of this efficiency. These results highlight the fact that, in order to provide consistent prediction of the impact of weather or market change on producer decisions, farmers' risk preferences have to be taken into account through a structural production model.

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Table 1. Descriptive Statistics

Variable	Mean	Standard deviation
y (average meat output in kg per ha)	399.22	91.26
x_1 (forage input in nitrogen unit)	34.96	27.87
x_2 (animal feed input proxy in kg)	794.42	439.15
z_1 (average cereal yield in 100kg per ha)	50.89	9.69
z_2 (animal productivity proxy in calves weaned per cow)	87.74	4.15
z_3 (average climate proxy in 100 kg of dry matter per ha)	45.95	8.18
P (output price in €/kg)	1.89	0.38
r_1 (unit price of input x1 in €)	2.23	5.72
r_2 (unit price of input x2 in €)	0.20	0.07
W (average farm land in ha)	139.09	58.92
Gross margin in € per ha	557.78	175.37
$\bar{W}$ (farm individual average of beef gross margin per worker unit including subsidies, in € / 1000)	44.1217	2.4848

Notes: Gross margin is computed as $Py - r_1x_1 - r_2x_2$.

Table 2. FIML Estimation Results

Expected output and variance of production

	Parameter	Estimate	*t* statistic	Parameter	Estimate	*t* statistic
intercept	β_0	-16.5117	(-4.04)	γ_0	-14.4823	(-6.22)
x_1	β_1	-0.6981	(-7.60)	γ_1	2.1774	(30.66)
x_2	β_2	0.0734	(10.99)	γ_2	3.1119	(8.81)
$x_1\ x_2$	β_{12}	0.0006	(53.49)	γ_{12}	-0.1411	(-50.96)
x_1^2	β_{11}	-0.0199	(-107.82)	γ_{11}	-0.0521	(-32.05)
x_2^2	β_{22}	-0.0007	(-84.89)	γ_{22}	-0.0416	(-25.67)
$z_1\ x_1$	β_{z11}	0.0083	(13.44)	γ_{z11}	-0.0709	(-22.36)
$z_1\ x_2$	β_{z12}	0.0009	(33.70)	γ_{z12}	-0.0177	(-0.69)
$z_2\ x_1$	β_{z21}	0.0147	(13.83)	γ_{z21}	-0.1941	(-15.04)
z2 x2	β_{z22}	0.0002	(2.53)	γ_{z22}	-0.1988	(-3.10)
$z_3\ x_1$	β_{z31}	0.0097	(16.78)	γ_{z31}	-0.0357	(-11.26)
$z_3\ x_2$	β_{z32}	0.0003	(7.90)	γ_{z32}	-0.4252	(-24.62)
z_1	β_{z1}	-0.7179	(-19.26)	γ_{z1}	0.3446	(1.87)
z_2	β_{z2}	3.6001	(63.77)	γ_{z2}	0.5971	(1.44)
z_3	β_{z3}	-0.1811	(-8.20)	γ_{z3}	2.9723	(27.23)

Table 2. FIML Estimation Results (continued)

Output Price			
Intercept	α_0	0.3706	(13.29)
$\overline{P}$	α_1	0.7839	(60.11)
Variance of error	σ_θ^2	0.1388	(11.67)
Risk preferences			
Intercept	ϕ_0	-0.0709	(-4.49)
W	ϕ_1	0.0067	(9.42)
W^2	ϕ_2	-0.0002	(-10.80)

Notes. 1106 observations. σ_θ^2 is estimated from a first-step FIML.

R^2 on structural equations: 0.62 (expected output), 0.85 (output price), 0.63 (expected profit), 0.03 (first-order condition on x_1), 0.46 (first-order condition on x_2)

Table 3: Estimates of Marginal Effects and Elasticities

	Estimate	t-statistic
	Marginal effect on f (expected output)	
x_1	1.6384	103.54
x_2	0.0579	56.80
	Marginal effect on h (variance of output)	
x_1	-0.0416	-67.18
x_2	-0.00006	-1.06
	Elasticity of f.	
x_1	0.1729	104.30
x_2	0.2111	61.39
z_1	0.17	41.4
z_2	0.76	53.4
z_3	0.08	6.55
	Elasticity of h.	
x_1	-0.1884	-47.45
x_2	-0.0100	-1.06
z_1	-0.07	-2.70
z_2	-1.69	-16.5
z_3	-0.43	-12.5

Table 4. Estimated Absolute and Relative Risk Measures

	ϕ_a	ϕ_r	$\partial\phi_r / \partial\bar{W}$	$\partial\phi_a / \partial\bar{W}$	elasticity of ϕ_a with respect to $\bar{W}$
Estimate	0.0536	2.3790	0.0050	-0.0011	-0.9060
	(51.09)	(51.09)	(0.99)	(-9.68)	(-9.60)

Notes. *t* statistics are in parentheses.