Exclusive Dealing and Vertical Integration in Interlocking Relationships*

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Abstract

We develop a model of interlocking bilateral relationships between upstream manufacturers that produce differentiated goods and downstream retailers that compete imperfectly for consumers. Contract offers and acceptance decisions are private information to the contracting parties. We show that both exclusive dealing and vertical integration between a manufacturer and a retailer lead to vertical foreclosure, at the detriment of consumers and society. Finally, we show that firms have indeed an incentive to sign such contracts or to integrate vertically.

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1 Introduction

In this paper, we provide a tractable framework for the analysis of interlocking relationships in vertically related oligopolies, where upstream firms (manufacturers) produce differentiated goods, whereas downstream firms (retailers) decide which of these goods (if any) to offer to consumers. We allow for general contracts between upstream and downstream firms, the terms of which are private information to the contracting parties. We use this framework to analyze the (anti-)competitive effects of exclusive dealing and vertical integration. We show that firms have an incentive to sign exclusive dealing provisions or, alternatively, to integrate vertically and that this is at the expense of consumers and society.

Vertical relationships are said to be *interlocking* when each downstream firm deals with multiple upstream firms, and each upstream firm deals multiple downstream firms. Such interlocking relationships are endemic in both consumer goods and intermediate goods industries. For example, most supermarkets carry both Coca Cola and Pepsi Cola, and competing aircraft manufacturers procure components (e.g., avionics, wheels and brakes) from the same competing suppliers (such as Honeywell and Thales).

Despite the prevalence of interlocking relationships, so far much of the IO literature on vertically related markets has focused on upstream (or downstream) monopoly, or on "competing vertical structures," where each upstream firm deals with a distinct set of downstream firms (e.g., franchise networks).[1] The few papers that do allow for such interlocking vertical relationships have two types of limitations: they often restrict attention to particular types of (public) contracts such as linear tariffs (e.g., Dobson and Waterson, 2007) or two-part tariffs (e.g., Rey and Vergé, 2010), or they assume that the upstream firms produce a homogeneous good (e.g., de Fontenay and Gans, 2005; Nocke and White, 2007 and 2010). Moreover, most papers assume that contracts are publicly observable, giving rise to strategic commitment effects which may not seem very plausible.[2]

Section 2 describes our framework, which involves a successive duopoly. For the sake of exposition, we will refer to the upstream firms as manufacturers and to the downstream firms as retailers; it should however be clear that the analysis can be transposed to other types of vertically related industries. Manufacturers produce differentiated goods, and retailers can

[1] Papers featuring competing vertical structures include Bonanno and Vickers (1988), Rey and Stiglitz (1988, 1995), Gal-Or (1991), Jullien and Rey (2007), and Piccolo and Miklos-Thal (2012).

[2] Models of interlocking relationships with observable contracts may also run into equilibrium existence problems; see Schutz (2013).

choose which of the goods (if any) to stock and sell on to consumers. In contrast to much of the literature, we do not impose *ad hoc* restrictions on tariffs. Building on Hart and Tirole (1990) and McAfee and Schwartz (1994), we assume that manufacturers' contract offers as well as retailers' acceptance decisions are private information to the contracting parties, thereby discarding strategic effects.[3]

Section 3 provides a complete characterization of equilibria in the absence of any exclusive dealing or vertical integration. There always exists a range of equilibria, which all yield the same retail prices and quantities, but differ in the type of contracts being signed and on how manufacturers and retailers share profits. Manufacturers prefer the equilibrium outcome induced by two-part tariffs, which is also the unique equilibrium outcome when tariffs cannot involve any below-cost pricing on incremental units.

Section 4 studies exclusive dealing provisions. We characterize the unique equilibrium outcome (in terms of retail prices and quantities) under both single and pairwise exclusive dealing. We show that firms have an incentive to sign exclusive distribution contracts, and that this reduces both consumer surplus and social welfare.

Section 5 turns to vertical integration. Under single vertical integration, there exists an equilibrium that leads to the complete foreclosure of the independent downstream rival, and thus replicates the outcome (in terms of retail prices and quantities) of (single) exclusive dealing. Under pairwise vertical integration, the unique equilibrium outcome is identical to that under pairwise exclusive dealing, as each vertically integrated firm chooses to foreclose its rival. Importantly, firms have an incentive to integrate vertically, to the detriment of consumers and society.

Section 6 concludes.

2 The Framework

We consider a vertically related industry with two symmetrically differentiated manufacturers, M_A and M_B. Manufacturer M_i, $i \in \{A, B\}$, produces good i at constant unit cost $c > 0$. The manufacturers distribute their goods through two perfectly substitutable retailers, R_1 and R_2, each of whom faces the same constant unit cost γ. For notational simplicity, we henceforth set

[3] The seminal contribution of Hart and Tirole (1990) shows that a Coasian commitment problem arises when contract offers are private information to the contracting parties; see also O'Brien and Shaffer (1992), McAfee and Schwartz (1994). However, these papers have analyzed only upstream monopoly or quasi-monopoly settings, which has severely limited their impact on actual policy decisions.

$\gamma \equiv 0$.

For expositional simplicity consumer demand is assumed to be symmetric: The inverse demand for good $i = A, B$ is given by $P(Q_i, Q_j)$, $j \neq i \in \{A, B\}$, where $Q_i \equiv q_{i1} + q_{i2}$ denotes total consumption of good i, and $q_{ih} \geq 0$ the quantity of good i purchased from retailer R_h, $h \in \{1, 2\}$.[4] Throughout the paper, we further impose the following conditions:

(A.1) $P(0,0) > c$ and, for Q sufficiently large, $P(Q,0) < c$ and $P(0,Q) < c$.

(A.2) For any $(Q_i, Q_j) \geq 0$,[5]

$$\partial_1 P(Q_i, Q_j) \leq \partial_2 P(Q_i, Q_j) \leq 0,$$

with strict inequalities when $P(Q_i, Q_j) > 0$.

Condition (A.1) is essentially a viability assumption, whereas condition (A.2) simply asserts that goods A and B are (imperfect) substitutes.[6]

We confine attention to *vertical* contracts; that is, we do not allow for any kind of "horizontal" agreements such as, e.g., market-share contracts, and consider instead contracts purely based on the quantity traded: Formally, a contract between M_i and R_h is a tariff $\tau_{ih} : \Re_+ \to \Re$, where $\tau_{ih}(q)$ is the payment from R_h to M_i in return for a quantity q of good i.[7] We do not impose any further restriction, however, and thus allow for any nonlinear tariff; special cases of interest are:

- *Two-part tariff:* $\tau_{ih}(q) = F + wq$, where F is the fixed (or "franchise") fee, and $w \geq 0$ the marginal wholesale price; we will denote such a two-part tariff by (w, F).
- *Forcing contract:*

$$\tau_{ih}(q) = \begin{cases} \hat{T} & \text{if } q = \hat{q}, \\ \infty & \text{otherwise,} \end{cases}$$

where $\hat{q}$ is the "forced" quantity; we will denote such a forcing contract by $(\hat{q}, \hat{T})$.

[4] Demand symmetry implies $\partial_2 P(Q, Q') = \partial_2 P(Q', Q)$ $(= \partial_{12} U(Q, Q')$, where $U(\cdot, \cdot)$ denotes consumers' gross surplus), and thus $\partial_{12}^2 P(Q, Q') = \partial_{22}^2 P(Q', Q)$, for all Q and Q'.

[5] Throughout the paper, $\partial_n f$ denotes the partial derivative of the function f with respect to its n^{th} argument; likewise, $\partial_{nm}^2 f$ will denote the second-order partial derivative with respect to the n^{th} and m^{th} arguments.

[6] The condition on $P(Q, 0)$ follows from the others but is mentioned explicitly for the sake of exposition.

[7] For the sake of exposition, we will assume that parties contract on the quantity q *sold* to consumers, rather than the quantity *bought* from the manfacturer. The distinction becomes moot when the production cost is large enough, as then a retailer will not want to buy more than it needs in any relevant scenario.

The contracting terms between M_i and R_h are private information to the two parties. The timing is as follows:

Stage 1 Manufacturers simultaneously offer (secret) contracts to retailers.

Stage 2 Retailers simultaneously (and secretly): (i) accept or reject the offers; and (ii) for each accepted contract, choose how much to put on the final market.[8] The resulting prices are such that markets clear.

We will also consider variants involving exclusive dealing and vertical integration. An exclusive dealing provision restricts the set of partners to whom offers can be made (in case of exclusive distribution), or from whom they can be accepted (in case of single branding). When instead M_i and R_h are vertically integrated, they are assumed to maximize their joint profits, regardless of internal transfer prices. M_i and R_h moreover "share information" in the sense that, when making its acceptance and output decisions, R_h is informed about the offer that its upstream affiliate M_i has previously made to the rival retailer R_k. By contrast, as acceptance and output decisions are made simultaneously, when making its output decisions R_h cannot be informed of whether the rival R_k accepted M_i's offer.[9]

We will study Perfect Bayesian Equilibria with *passive* beliefs, in which retailers do not revise their beliefs about the offer made to the other retailer when receiving an out-of-equilibrium offer. As retailers compete downstream in quantities, these passive beliefs coincide with the "wary beliefs" introduced by McAfee and Schwartz (1994), as the contract signed with a retailer has no impact on a manufacturer's gains from trade with the other retailer.

3 Baseline Model

In this section, we characterize the equilibria of our baseline model. We first define the notion of a "cost-based" contract, in which the marginal input price coincides with the marginal cost of production, and show that unintegrated manufacturers sign cost-based contracts with every available retailer. Drawing on this insight, we then characterize the equilibria in the absence of exclusive dealing and vertical integration.

[8] As acceptance and output decisions are simultaneous, there is no role here for menus of contracts: Offering a menu of tariffs is *de facto* equivalent to offering the lower envelope of these tariffs.

[9] This avoids having to take a stand on how the integrated retailer would interpret an unexpected acceptance or rejection of the upstream affiliate's offer.

3.1 Independent Manufacturers

Throughout the analysis, we will use indices $i \neq j$ when referring to M_A and M_B, and $h \neq k$ when referring to R_1 and R_2. Let

$$\chi\left(q_{ik}, q_{jh}, q_{jh}\right) \equiv \arg\max_{q_{ih}} \left[P\left(q_{i1}+q_{i2}, q_{j1}+q_{j2}\right)-c\right] q_{ih}+P\left(q_{j1}+q_{j2}, q_{i1}+q_{i2}\right) q_{jh}$$

denote the set of bilaterally efficient values for the output q_{ih}, from the standpoint of the pair $M_i - R_h$, holding fixed all other outputs.[10] We will say that the equilibrium contract signed by M_i and R_h is "cost-based" if it induces a bilaterally efficient output, given the other equilibrium outputs:

Definition 1 *The equilibrium contract $\tau_{ih}(\cdot)$ between M_i and R_h is said to be* cost-based *if, when accepted, and given the outputs of the other channels $\left(q_{ik}, q_{jh}, q_{jh}\right)$, $\tau_{ih}(\cdot)$ induces a quantity $q_{ih} \in \chi\left(q_{ik}, q_{jh}, q_{jh}\right)$.*

The following lemma characterizes the equilibrium contracts signed by unintegrated manufacturers:

Lemma 1 *Suppose M_i is not vertically integrated (whereas M_j may or may not be vertically integrated). Then, in any equilibrium M_i signs a cost-based contract with every retailer R_h that is available, given the exclusive dealing provisions (with the convention that they sign a "null" contract if it is bilaterally efficient not to trade).*

Proof. See Appendix A. ■

The intuition is simple: Under passive beliefs, each R_h expects its rival R_k to stick to the equilibrium quantities even when receiving a deviant offer from an independent M_i. Moreover, such a deviant offer does not affect the profit that M_i makes on its contract with R_k. In equilibrium, the contract between M_i and R_h must therefore maximize the joint bilateral profit of the contracting parties, assuming that R_k sticks to its equilibrium quantities, which is achieved by signing a cost-based contract.

3.2 Non-Exclusive Relationships and Vertical Separation

We now characterize the set of equilibria in the absence of exclusive dealing and vertical integration (which we will index by the superscript "$^\circ$"). From Lemma 1, we know that *each* M_i must sign a cost-based contract with *each* R_h, implying the following result:

[10] Note that this set does not depend on the tariffs τ_{ik} andτ_{jh}.

Proposition 1 *In the absence of exclusive dealing and vertical integration, the set of equilibrium quantities* $(q^\circ_{A1}, q^\circ_{A2}, q^\circ_{B1}, q^\circ_{B2})$ *coincides with that a Cournot multiproduct duopoly in which the two firms* (1 *and* 2) *can produce the same two goods* (*A and B*) *at marginal cost c.*

Proof. This follows directly from Lemma 1. ■

For the sake of exposition, it is useful to pin down the equilibrium outcome; the following mild regularity conditions ensure that the equilibrium retail prices and quantities are uniquely defined and symmetric:

(A.3) For any $(Q_i, Q_j) \geq 0$ such that $P(Q_i, Q_j) > 0$, we have

$$2\partial_1 P(Q_i, Q_j) + \partial^2_{11} P(Q_i, Q_j) Q_i < \partial_2 P(Q_i, Q_j) + \partial^2_{12} P(Q_i, Q_j) Q_i < 0.$$

(A.4) For any $(Q_i, Q_j) \geq 0$ such that $P(Q_i, Q_j) > 0$, and for any $q_i \in [0, Q_i]$ and any $q_j \in [0, Q_j]$, we have

$$\begin{aligned} & 2\partial_1 P(Q_i, Q_j) + \partial^2_{11} P(Q_i, Q_j) q_i + \partial^2_{22} P(Q_j, Q_i) q_j \\ < \ & \partial_2 P(Q_i, Q_j) + \partial_2 P(Q_j, Q_i) + \partial^2_{12} P(Q_i, Q_j) q_i + \partial^2_{12} P(Q_j, Q_i) q_j \\ < \ & 0. \end{aligned}$$

Condition (A.3) ensures that a Cournot duopoly game, in which one firm sells good A and the other good B, would have a unique, stable equilibrium. Condition (A.4) further ensures that profits remain concave when both firms can sell goods A and B. In the case of linear demand, (A.3) and (A.4) boil down to $\partial_1 P < \partial_2 P < 0$, and are thus implied by (A.2).

We have:

Proposition 2 *In the absence of exclusive dealing and vertical integration, under Assumptions (A.3)-(A.4) the equilibrium quantities must satisfy* $q^\circ_{ih} = q^\circ$, *for* $i \in \{A, B\}$ *and* $h \in \{1, 2\}$, *where* q° *is the unique solution to:*

$$P(2q^\circ, 2q^\circ) - c + [\partial_1 P(2q^\circ, 2q^\circ) + \partial_2 P(2q^\circ, 2q^\circ)] q^\circ = 0.$$

Proof. See Appendix B. ■

Hence, any equilibrium must be bilaterally efficient, which under the regularity assumptions (A3)-(A4), implies that each channel must sell the same quantity q°. It follows that all equilibria generate the same industry-wide aggregate profit,

$$\Pi^\circ \equiv [P(2q^\circ, 2q^\circ) - c] \, 4q^\circ.$$

M_i's profit is then of the form $\pi_i^\circ = \pi_{i,1}^\circ + \pi_{i,2}^\circ$, where $\pi_{i,h}^\circ = \tau_{ih}^\circ(q^\circ) - cq^\circ$ is M_i's profit on its contract with R_h, whereas R_h's profit is $\pi_h^\circ = 2P\left(2q^\circ, 2q^\circ\right)q^\circ - \tau_{Ah}^\circ(q^\circ) - \tau_{Bh}^\circ(q^\circ)$.

To support such an equilibrium, contracts must be somewhat flexible: As shown in Appendix D, there is no equilibrium in which a manufacturer offers a single forcing contract. The intuition is as follows. In equilibrium, each retailer must be indifferent between accepting both manufacturers' offers, and only one (either one): If a retailer strictly preferred dealing with both manufacturers than with only one of them, then the rival manufacturer could profitably deviate by asking for a larger share of the profits. But if, say, M_i offers R_h a single forcing contract (q°, T_{ih}°), then M_i is also indifferent between whether or not R_h also accepts upstream rival M_j's offer; hence, the joint profit of M_i and R_h must be the same as what they would obtain if R_h were to deal exclusively with M_i. But in equilibrium, the sum of R_h's profit and of M_i's profit from its contract with R_h is given by

$$\begin{aligned}
\pi_{i,h}^\circ + \pi_h^\circ &= \left[\tau_{ih}^\circ\left(q^\circ\right) - cq^\circ\right] + \left[2P\left(2q^\circ, 2q^\circ\right) - \tau_{ih}^\circ\left(q^\circ\right) - \tau_{jh}^\circ\left(q^\circ\right)\right] \\
&= \left[\tau_{ih}^\circ\left(q^\circ\right) - cq^\circ\right] + \left[P\left(2q^\circ, q^\circ\right) - \tau_{ih}^\circ\left(q^\circ\right)\right] \\
&= \left[P\left(2q^\circ, q^\circ\right) - c\right] q^\circ.
\end{aligned}$$

Hence, under exclusivity M_i and R_h could generate more profit by replacing q° with

$$\hat{q} \equiv \arg\max_q \left\{\left[P\left(q^\circ + q, q^\circ\right) - c\right] q\right\},$$

and share the profit increase through an appropriate transfer $\hat{T}$; it follows that M_i could profitably deviate by offering the forcing contract $\left(\hat{q}, \hat{T}\right)$, thereby inducing R_h to "bump" the rival manufacturer (note that this deviation does not affect the profit that M_i obtains from dealing with the other retailer, R_k).

We now show that simple two-part tariffs suffice to support an equilibrium:

Proposition 3 *In the absence of exclusive dealing and vertical integration, under Assumptions (A.3)-(A.4) there exists an equilibrium in which each manufacturer signs with each retailer the cost-based two-part tariff* $(w^\circ, F^\circ) = (c, \Delta^\circ)$*, where*

$$\Delta^\circ \equiv \left[P\left(2q^\circ, 2q^\circ\right) - c\right] 2q^\circ - \max_q \left\{\left[P\left(q^\circ + q, q^\circ\right) - c\right] q\right\}$$

denotes each manufacturer's contribution to the profit generated by a retailer.

Proof. See Appendix C. ■

The intuition is as follows. First, cost-based two-part tariffs allow retailers to buy any marginal quantity at cost. It follows that: (i) as a manufacturer does not care about the level of trade chosen by retailers, it is indifferent as to whether retailers will deal with the rival manufacturer or not; and (ii) in both instances, the tariff is bilaterally efficient, whether or not the retailer deals with the other manufacturer. Second, the equilibrium fixed fees are such that each retailer is indifferent between dealing with both manufacturers, or with either one on an exclusivity basis. It follows that these tariffs eliminate the above "bumping" problem, as a manufacturer would have to give away some of its profit in order to convince a retailer to opt for exclusivity.

There exist many other equilibria, however; although all equilibria must rely on cost-based contracts, and thus generate the same industry profit, Π°, they can differ in the way firms share this profit:

Proposition 4 *In the absence of exclusive dealing and vertical integration, under Assumptions (A.3)-(A.4):*

- *Manufacturers' equilibrium profits are of the form* $\left(\pi_A^\circ = \pi_{A,1}^\circ + \pi_{A,2}^\circ, \pi_B^\circ = \pi_{B,1}^\circ + \pi_{B,2}^\circ\right)$, *where* $\pi_{i,h}^\circ \in [0, \Delta^\circ]$.
- *Any profile of profits in this range can be supported by an equilibrium in which each* M_i *offers each* R_h *a pair of forcing contracts; however, outcomes giving less than its contribution to a manufacturer (i.e., such that* $\pi_i^\circ < 2\Delta^\circ$ *for some* M_i*) can only be supported by tariffs that price some incremental quantity below cost.*

Proof. See Appendix D. ■

Proposition 4 shows that, without loss of generality, we can restrict attention to equilibria in which each M_i offers each R_h a menu of two forcing contracts: A cost-based contract (q°, T_{ih}°) "designed" for common agency, which R_h accepts along the equilibrium path, and a contract $\left(\tilde{q}_{ih}^\circ, \tilde{T}_{ih}^\circ\right)$ "designed" for exclusivity, where $\tilde{q}_{ih}^\circ > q^\circ$ and $c\left(\tilde{q}_{ih}^\circ - q^\circ\right) \geq \tilde{T}_{ih}^\circ - T_{ih}^\circ > 0$. In equilibrium, each retailer is indifferent between picking both (q°, T_{Ah}°) and (q°, T_{Bh}°), or picking only $\left(\tilde{q}_{ih}^\circ, \tilde{T}_{ih}^\circ\right)$, from either manufacturer $i = A, B$.

Finally, the division of profit varies substantially across equilibria. The intuition is as follows. Even though they are not accepted in equilibrium, the "exclusive deal" offers determine retailers' outside options, and thus how much profit is left for the manufacturers. In equilibrium, the two manufacturers' exclusive deal offers must be equally "generous." Moreover, each manufacturer must (weakly) prefer that the retailer does not choose the exclusive deal offer but rather the one designed for common agency: If a manufacturer were to prefer the retailer to accept the exclusive deal option over the common agency option, the manufacturer could profitably deviate by making the exclusive deal option slightly more attractive, thereby inducing the retailer to accept that option. This generates a multiplicity of equilibria in terms of how the profits are shared between the upstream and downstream levels: The more generous is one manufacturer, the more generous must be the other one. Only in the equilibrium with the least generous offers is each manufacturer indifferent between the retailer choosing the common agency contract (which is accepted in equilibrium) and the exclusive deal offered by the manufacturer (which is not accepted).

The analysis also identifies bounds on how profits can be shared. While Proposition 3 shows that cost-based two-part tariffs (where *any* incremental quantity is sold at cost) enables manufacturers to obtain exactly their contribution to industry profits, Δ°, Proposition 4 establishes that manufacturers cannot obtain more, and can obtain much less; in particular, there are equilibria in which retailers appropriate all profits – as well as equilibria in which one manufacturer obtains its contribution Δ° when the other one gets nothing. Note however that all these other equilibria rely on tariffs that price the incremental quantity for exclusivity below cost: At least one tariff τ_{ih}° must be such that $\tau_{ih}^\circ(\tilde{q}_{ih}^\circ) - \tau_{ih}^\circ(q^\circ) < c(\tilde{q}_{ih}^\circ - q^\circ)$, where $\tilde{q}_{ih}^\circ$ denotes the quantity that R_h would pick if it were to deviate and deal exclusively with M_i; M_i would thus obtain less profit if R_h were to move to exclusivity – by contrast, the equilibria giving manufacturers their contribution Δ° are such that each M_i is indifferent as well between R_h buying q° from both manufacturers, or $\tilde{q}_{ih}^\circ$ exclusively from M_i.

4 Exclusive Dealing

In this section, we analyze the effects of exclusive dealing provisions on the equilibrium outcome and on welfare. These provisions include *exclusive distribution* contracts, which preclude the manufacturer from selling to the rival retailer, and *single branding* contracts, which preclude the retailer from buying from the other manufacturer. We first consider the case of a single exclusive dealing provision that precludes trade between M_A and R_2, and then that of two exclusive dealing provisions that preclude trade between M_A and R_2 and between M_B and R_1. Next, we make an excursion by analyzing an associated duopoly game without any vertical aspects. We then use the insights from this game to study the incentives for firms to engage in exclusive dealing. Finally, we provide a welfare analysis of the effects of exclusive dealing.

4.1 Equilibrium Outcomes

We begin by analyzing the equilibrium effects of a (pre-existing) exclusion dealing provision that precludes trade between manufacturer M_A and retailer R_2. Such a provision may be either an exclusive distribution contract between M_A and R_1, or a single branding contract between M_B and R_2.

The following proposition provides a characterization of the equilibrium quantities (which we will index by the superscript "*"):

Proposition 5 *Suppose that a single exclusive dealing provision precludes trade between M_A and R_2 (i.e., $q^*_{A2} = 0$). Then, under Assumptions (A.3) and (A.4):*

- *There exists an equilibrium supported by cost-based two-part tariffs.*
- *In all equilibria, M_A signs a cost-based contract with R_1, and M_B signs cost-based contracts with both R_1 and R_2; the equilibrium quantities, $(q^*_{A1}, q^*_{B1}, q^*_{B2})$, are moreover positive and uniquely defined by:*

$$\begin{aligned} q^*_{A1} &= \arg\max_{q_{A1}} \left[P\left(q_{A1}, q^*_{B1} + q^*_{B2}\right) - c\right] q_{A1} + P\left(q^*_{B1} + q^*_{B2}, q_{A1}\right) q^*_{B1}, \\ q^*_{B1} &= \arg\max_{q_{B1}} \left[P\left(q_{B1} + q^*_{B2}, q^*_{A1}\right) - c\right] q_{B1} + P\left(q^*_{A1}, q_{B1} + q^*_{B2}\right) q^*_{A1}, \end{aligned}$$

and

$$q^*_{B2} = \arg\max_{q_{B2}} \left[P\left(q^*_{B1} + q_{B2}, q^*_{A1}\right) - c\right] q_{B2}.$$

Proof. See Appendix E. ■

The market outcome is thus that of an asymmetric duopoly, in which one firm offers both goods A and B, whereas the other offers only one of these goods. Following the same steps as for Proposition 4, it can be checked that there exist multiple equilibria, which differ on how R_1 shares its profit with the manufacturers. More precisely, let

$$\Pi^*_1 = \left[P\left(q^*_{A1}, q^*_{B1} + q^*_{B2}\right) - c\right] q^*_{A1} + \left[P\left(q^*_{B1} + q^*_{B2}, q^*_{A1}\right) - c\right] q^*_{B1}$$

denote the profit generated by R_1 in equilibrium (some of which may be captured by the two manufacturers), and

$$\begin{aligned} \Delta^*_{A,1} &= \Pi^*_1 - \max_{q_{B1}} \left\{\left[P\left(q_{B1} + q^*_{B2}, 0\right) - c\right] q_{B1}\right\}, \\ \Delta^*_{B,1} &= \Pi^*_1 - \max_{q_{A1}} \left\{\left[P\left(q_{A1}, q^*_{B2}\right) - c\right] q_{A1}\right\} \end{aligned}$$

M_A's and M_B's contributions to this profit, respectively. We then have $\Delta^*_{A,1} > \Delta^*_{B,1} > 0$[11] and:

- In equilibrium, M_B now always appropriates the profit generated by R_2.
- As regards sharing the profit generated by R_1, any tuple $\left(\pi^*_{B,1}, \pi^*_{A,1}\right)$ with $\pi^*_{B,1} \in \left[0, \Delta^*_{B,1}\right]$ for M_B and $\pi^*_{A,1} \in \left[\Delta^*_{A,1} - \Delta^*_{B,1}, \Delta^*_{A,1}\right]$ for M_A can be sustained as an equilibrium outcome.

Suppose now that (pre-existing) pairwise exclusive dealing provisions preclude trade between M_A and R_2 as well as between M_B and R_1. For example, M_A and R_1 as well as M_B and R_2 may have signed exclusive distribution contracts with each other, or M_A and R_2 as well as M_B and R_1 may have signed single branding contracts.

The following proposition characterizes the equilibrium (which we will index by the superscript "**").

Proposition 6 *Suppose that pairwise exclusive dealing provisions preclude trade between M_A and R_2 as well as between M_B and R_1 (i.e., $q^{**}_{A2} = q^{**}_{B1} = 0$). Then under Assumption (A.3):*

- *There exists an equilibrium supported by cost-based two-part tariffs.*
- *In all equilibria:*
  - *M_A and R_1 as well as M_B and R_2 sign cost-based contracts, and each manufacturer fully appropriates the profit generated by its good.*
  - *$q^{**}_{A1} = q^{**}_{B2} = Q^{**} > 0$, where Q^{**} is the unique solution to*

$$P(Q^{**}, Q^{**}) - c + \partial_1 P(Q^{**}, Q^{**})Q^{**} = 0.$$

Proof. See Appendix F. ■

The market outcome is thus that of a standard duopoly, with one firm offering good A and the other offering good B. This equilibrium outcome can for instance be supported by cost-based

[11] That M_B's contribution is positive stems from $q^*_{B1} > 0$, which implies

$$\begin{aligned} \Pi^*_1 &= \max_{q_{A1}, q_{B1}} \left\{ \left[P\left(q_{A1}, q_{B1} + q^*_{B2}\right) - c \right] q_{A1} + \left[P\left(q_{B1} + q^*_{B2}, q_{A1}\right) - c \right] q_{B1} \right\} \\ &> \max_{q_{A1}} \left\{ \left[P\left(q_{A1}, q^*_{B2}\right) - c \right] q_{A1} \right\}. \end{aligned}$$

That M_A's contribution is larger than M_B's follows from the fact that, from (A.2), $P\left(q, q^*_{B2}\right) > P\left(q + q^*_{B2}, 0\right)$ for any q such that $P\left(q, q^*_{B2}\right) > 0$. To see this, note that $\phi(x) = P\left(\frac{q+q^*_{B2}}{2} + x, \frac{q+q^*_{B2}}{2} - x\right)$ is decreasing in x from (A.2), and $P\left(q, q^*_{B2}\right) = \phi\left(\frac{q-q^*_{B2}}{2}\right)$ whereas $P\left(q + q^*_{B2}, 0\right) = \phi\left(\frac{q+q^*_{B2}}{2}\right)$.

two-part tariffs. Finally, in contrast to the previous cases, equilibrium profits are here unique as well, as manufacturers appropriate all profits.

4.2 Excursion: Associated Duopoly Game

Consider the following hypothetical duopoly game, denoted Γ_2. There are two players, firms 1 and 2, and two goods, A and B. Firm 1's strategy consists in choosing the quantity $q_{A1} \in [0, \infty)$ of good A to sell at the same time as firm 2 chooses the quantity $q_{B2} \in [0, \infty)$ of good B. In addition, firm 1 also sells an exogenous quantity $\hat{q}_{B1}$ of good B and firm 2 an exogenous quantity $\hat{q}_{A2}$ of good A, so that the profit functions of firms 1 and 2 are given by

$$\hat{\Pi}_1 \left(q_{A1}, q_{B2}; \hat{q}_{B1}, \hat{q}_{A2}\right) \equiv \left[P\left(q_{A1} + \hat{q}_{A2}, \hat{q}_{B1} + q_{B2}\right) - c\right] q_{A1} + \left[P\left(\hat{q}_{B1} + q_{B2}, q_{A1} + \hat{q}_{A2}\right) - c\right] \hat{q}_{B1}$$

and

$$\hat{\Pi}_2 \left(q_{A1}, q_{B2}; \hat{q}_{B1}, \hat{q}_{A2}\right) \equiv \left[P\left(\hat{q}_{B1} + q_{B2}, q_{A1} + \hat{q}_{A2}\right) - c\right] q_{B2} + \left[P\left(q_{A1} + \hat{q}_{A2}, \hat{q}_{B1} + q_{B2}\right) - c\right] \hat{q}_{A2},$$

respectively. In the special case where $\hat{q}_{A2} = \hat{q}_{B1} = 0$, this game simplifies to a standard differentiated goods Cournot duopoly, where each of the two goods is sold by only one firm.

For our main results, we will assume that the equilibrium of game Γ_2 has the following properties:

(P.1) Game Γ_2 has a unique Nash equilibrium $\left(\tilde{q}_{A1}\left(\hat{q}_{B1}, \hat{q}_{A2}\right), \tilde{q}_{B2}\left(\hat{q}_{B1}, \hat{q}_{A2}\right)\right)$.

(P.2) In equilibrium, the aggregate profit

$$\begin{aligned} & \Pi\left(\tilde{q}_{A1}\left(\hat{q}_{B1}, \hat{q}_{A2}\right) + \hat{q}_{A2}, \hat{q}_{B1} + \tilde{q}_{B2}\left(\hat{q}_{B1}, \hat{q}_{A2}\right)\right) \\ \equiv\ & \left[P\left(\tilde{q}_{A1}\left(\hat{q}_{B1}, \hat{q}_{A2}\right) + \hat{q}_{A2}, \hat{q}_{B1} + \tilde{q}_{B2}\left(\hat{q}_{B1}, \hat{q}_{A2}\right)\right) - c\right]\left(\tilde{q}_{A1}\left(\hat{q}_{B1}, \hat{q}_{A2}\right) + \hat{q}_{A2}\right) \\ & + \left[P\left(\hat{q}_{B1} + \tilde{q}_{B2}\left(\hat{q}_{B1}, \hat{q}_{A2}\right), \tilde{q}_{A1}\left(\hat{q}_{B1}, \hat{q}_{A2}\right) + \hat{q}_{A2}\right) - c\right]\left(\hat{q}_{B1} + \tilde{q}_{B2}\left(\hat{q}_{B1}, \hat{q}_{A2}\right)\right) \end{aligned}$$

is uniquely maximized for $\hat{q}_{B1} = \hat{q}_{A2} = 0$; that is,

$$\Pi\left(\tilde{q}_{A1}(0,0), \tilde{q}_{B2}(0,0)\right) > \Pi\left(\tilde{q}_{A1}\left(\hat{q}_{B1}, \hat{q}_{A2}\right) + \hat{q}_{A2}, \hat{q}_{B1} + \tilde{q}_{B2}\left(\hat{q}_{B1}, \hat{q}_{A2}\right)\right)$$

whenever $\hat{q}_{B1} + \hat{q}_{A2} > 0$.

(P.3) The equilibrium quantity $\tilde{q}_{A1}\left(\hat{q}_{B1}, \hat{q}_{A2}\right)$ (resp., $\tilde{q}_{B2}\left(\hat{q}_{B1}, \hat{q}_{A2}\right)$) is (weakly) decreasing in $\hat{q}_{A2}$ (resp., $\hat{q}_{B1}$) .

These properties are satisfied in the case of linear demand. In Appendix G, we provide more general sufficient conditions on demand that ensure that (P.1)-(P.3) do indeed hold.

4.3 Incentives to Engage in Exclusive Dealing

We now study firms' incentive to engage in exclusive dealing. We first show that at least one manufacturer-retailer pair benefits from signing an exclusive distribution contract (whether or not this induces the other pair to do the same), and the other pair then benefits from doing the same. Hence, absent any rule against exclusive dealing provisions, we would expect the emergence of pairwise exclusivity.

We then consider the case of single-branding contracts. The incentives for a manufacturer-retailer pair to sign such a contract are less clear, as doing so benefits the other pair but may reduce the joint profit of the signing pair. In addition, once such a contract is in place, the other manufacturer-retailer pair does not have an incentive to adopt any exclusivity provision; the first pair, however, has an incentive to move to pairwise exclusivity, by complementing its single-branding contract with an exclusive distribution provision.

4.3.1 Exclusive Distribution

To analyze firms' incentives to sign exclusive distribution contracts, we proceed as follows. First, starting from an environment without exclusive dealing, we ask whether there exists a manufacturer-retailer pair $M_i - R_h$ that could raise its joint profit by signing an exclusive distribution contract that prevents M_i from dealing with the rival retailer R_k. Second, starting from an environment in which the pair $M_i - R_h$ has already signed such an exclusive distribution contract, we ask whether the other pair, $M_j - R_k$, can increase its joint profit by signing an exclusive distribution contract that prevents M_j from dealing with R_h. Despite the multiplicity of equilibria (under either no or single exclusive dealing) in terms of rent shifting between manufacturers and retailers, we obtain a strong result: Firms do have an incentive to engage in exclusive dealing, no matter how profits are shared.

Proposition 7 *Assume (A.3)-(A.4) and (P.1)-(P.3). Then:*

- *In any equilibrium that arises in the absence of exclusivity, there exists a manufacturer-retailer pair $M_i - R_h$ that can strictly increase its joint profit by signing an exclusive distribution contract, regardless of:*
  - *which equilibrium is selected under single exclusivity; and*
  - *whether this induces the other pair to engage in exclusive dealing as well.*

- *In any equilibrium under single exclusive distribution between $M_i - R_h$, the other pair $M_j - R_k$ can strictly increase its joint profit by signing an exclusive distribution contract.*

Proof. See Appendix H. ■

The intuition builds upon two observations: (i) Industry profits are larger under pairwise exclusive dealing than under any other configuration; and (ii) a manufacturer-retailer pair, say $M_A - R_1$, obtains a larger joint profit when it is the only pair that engages in exclusive distribution, than under pairwise exclusive dealing. The first observation is intuitive and follows from (P.2).[12] The second observation follows from the fact, in a single exclusive dealing equilibrium, M_A and R_1 must at least obtain what they could achieve by deviating to pairwise exclusivity, and moreover face a less aggressive rival than in the pairwise exclusive dealing equilibrium: $q_{B2}^{*} < q_{B2}^{**}$, from (P.3).

By construction, in the absence of exclusivity at least one manufacturer-retailer pair obtains less than half of the equilibrium industry profit Π°, which itself is (strictly) smaller than the equilibrium industry profit Π^{**} achieved under pairwise exclusive dealing. It follows from the above observations that this pair would benefit from signing an exclusive distribution contract – whether or not this induces the other pair to engage in exclusivity as well.

It also follows from the above observations that under single exclusive dealing, the pair that is under exclusive distribution obtains a larger joint profit than the other pair. But then, this other pair has an incentive to engage in exclusivity as well, so as to earn a bigger share (namely, one-half) of a bigger pie (as the industry profit is maximal under pairwise exclusive dealing).

4.3.2 Single Branding

The above analysis suggests that the incentives to adopt single branding provisions are less clear. To see this, consider an environment without exclusivity, and suppose that M_j and R_k sign a single branding contract that prevents R_k from dealing with the rival manufacturer M_i. Intuitively, this eliminates intrabrand competition for good A, and may thus increase in this way total industry profit. However, the above analysis points out that the other manufacturer-retailer pair, say $M_i - R_h$, gets the bigger share of that profit; hence, even if total industry profit is increased, M_j and R_h may obtain too small a share of that bigger pie, making single branding unprofitable. Indeed, in the case of the linear demand considered at the end of Section 4.4, starting from a situation without exclusivity where the manufacturer-retailer pairs $M_i - R_h$

[12] Recall that (P.2) follows in turn from the regularity assumptions provided in Appendix G.

and $M_j - R_k$ share the industry profit equally, then none of them can increase its joint profit by opting for single branding; in the same vein, M_j and R_k cannot benefit from signing a single branding contract if this does not allow M_j to extract more profit from the other retailer, R_h (i.e., if $\pi^*_{j,h} \leq \pi^\circ_{j,h}$).

It also follows from the above analysis that, if one manufacturer-retailer pair, say $M_j - R_k$, opts for single branding, then the other pair, $M_i - R_h$, will not follow suit. Indeed, we have seen that M_i and R_h's joint profit is larger in the single exclusive dealing situation where M_i does not deal R_k, than in case of pairwise exclusivity. However, M_j and R_k would have an incentive to complement their single branding contract with an exclusive distribution provision, in order to move towards pairwise exclusivity, so as obtain again a bigger share of a bigger pie. By the same token, starting from an environment without exclusivity, at least one pair (any pair that does not get more than half of the industry profit) would have an incentive to engage in mutual exclusivity, that is, to sign a contract involving both exclusive distribution and single branding, in order to move towards pairwise exclusivity.

4.4 Welfare Effects of Exclusive Dealing

We now turn to the welfare effects of exclusive dealing. For given quantities Q_A and Q_B, consumer surplus can be expressed as

$$S(Q_A, Q_B) \equiv \int_0^{Q_A} P(q_A, Q_B) dq_A + \int_0^{Q_B} P(q_B, 0) dq_B - P(Q_A, Q_B) Q_A - P(Q_B, Q_A) Q_B,$$

aggregate profit as

$$\Pi(Q_A, Q_B) \equiv [P(Q_A, Q_B) - c]\, Q_A + [P(Q_B, Q_A) - c]\, Q_B,$$

and social welfare as

$$W(Q_A, Q_B) \equiv \int_0^{Q_A} [P(q_A, Q_B) - c]\, dq_A + \int_0^{Q_B} [P(q_B, 0) - c]\, dq_B.$$

Let $Q^\circ = 2q^\circ$ denote the aggregate output per good in the absence of exclusive dealing, $Q^{**} = q^{**}$ that under pairwise exclusive dealing, and $(Q^*_A = q^*_{A1}, Q^*_B = q^*_{B1} + q^*_{B2})$ the aggregate outputs when a single exclusive dealing provision precludes trade between M_A and R_2 .

For the second part of our first welfare result, we require the following technical assumption on demand (which holds with equality if demand is linear):

(A.5) For any $(Q_i, Q_j) \geq 0$ such that $P(Q_i, Q_j) > 0$, we have

$$\begin{aligned} & \partial_2 P(Q_i, Q_j) \left[Q_i \partial^2_{11} P(Q_i, Q_j) + Q_j \partial^2_{22} P(Q_j, Q_i) \right] \\ \geq\ & \partial_1 P(Q_i, Q_j) \left[Q_i \partial^2_{12} P(Q_i, Q_j) + Q_j \partial^2_{12} P(Q_j, Q_i) \right]. \end{aligned}$$

The first welfare result says that introducing a single exclusive dealing agreement raises aggregate industry profit at the expense of consumer surplus and social welfare:

Proposition 8 *Compared to the baseline case with no exclusive dealing, an exclusive dealing provision that precludes trade between M_A and R_2: i) under (A.4), reduces social welfare, $W(Q^*, Q^*) < W(Q^\circ, Q^\circ)$; and ii) under (A.5), reduces consumer surplus, $S(Q^*, Q^*) < S(Q^\circ, Q^\circ)$, and increases aggregate profit, $\Pi(Q^*, Q^*) \equiv \Pi^* > \Pi^\circ \equiv \Pi(Q^\circ, Q^\circ)$.*

Proof. See Appendix I. ■

The second welfare result says that pairwise exclusive dealing increases profits at the expense of consumer surplus and social welfare:

Proposition 9 *Compared to the baseline case with no exclusive dealing, exclusive dealing provisions that preclude trade between M_A and R_2 and between M_B and R_1: i) reduce both consumer surplus and social welfare: $S(Q^{**}, Q^{**}) < S(Q^\circ, Q^\circ)$ and $W(Q^{**}, Q^{**}) < W(Q^\circ, Q^\circ)$; and ii) under (A.4), increase aggregate profit: $\Pi(Q^{**}, Q^{**}) \equiv \Pi^{**} > \Pi^\circ \equiv \Pi(Q^\circ, Q^\circ)$.*

Proof. See Appendix J. ■

The intuition is straightforward. Exclusive dealing restricts the number of retailers selling any given good, leading to less intense competition, lower outputs and higher prices. This increases firms' profits (as output levels remain above monopoly levels), but obviously harms consumers and reduces total welfare (as prices remain above marginal cost).

Intuitively, we would expect the impact of exclusivity to be more important when goods A and B are more differentiated. Indeed, exclusive dealing has no effect in the limit case of perfect substitutes (as, in that case, the retailers do not care about whether they sell one good or both of them), and enables instead firms to achieve the industry-wide monopoly outcome when goods A and B face independent demands. To illustrate this, consider the case of linear demand:[13]

$$P\left(Q_A, Q_B\right) = 1 - \frac{Q_A + sQ_B}{1+s},$$

where s reflects the degree of substitution between A and B, and ranges from $s = 0$ (independent demands) to $s = 1$ (perfect substitutes). Normalizing the production cost to $c \equiv 0$, we have:

$$\begin{aligned} Q^{**} &= q^{**} = \frac{1}{2+s} < Q^\circ = 2q^\circ = \frac{2}{3}, \\ p^{**} &= P\left(Q^{**}, Q^{**}\right) = \frac{1}{2+s} > p^\circ = P\left(Q^\circ, Q^\circ\right) = \frac{1}{3}. \end{aligned}$$

[13] The demand is here normalized so as to ensure that its size remains constant (for symmetric configurations) when the degree of product differentiation varies. In particular, the benchmark monopoly quantity (summing over both goods), $Q^M = 1/2$, is independent of s.

Hence, pairwise exclusivity reduces output and raises prices, and all the more so as goods A and B become more differentiated: Q^{**} decreases, and p^{**} increases, as s decreases.

Intuitively, we would also expect each exclusive dealing provision to contribute to increasing profit, at the expense of consumers and allocative efficiency. This is indeed the case in the above linear model. When a single exclusive dealing provision precludes trade between M_A and R_2, the equilibrium prices and quantities are uniquely defined and given by:

$$\begin{aligned} q_{A1}^* &= \frac{1}{2} < Q^\circ = \frac{2}{3}, q_{B1}^* = \frac{2-s}{6} < q^\circ = \frac{1}{3} < q_{B2}^* = \frac{1+s}{3}, \\ p^{**} &= \frac{1}{2+s} > p_A^* = \frac{1}{2} - \frac{s}{6} > p_B^* = p^\circ = \frac{1}{3}. \end{aligned}$$

That is, starting from the baseline scenario with no exclusivity, shutting down the channel $M_A - R_2$ induces R_1 to sell more of good A, but not so much as to compensate for R_2's lost sales of good A; this also induces R_2 to sell more of good B, a move partially offset by R_1 reducing its own sales of that good (both because it faces a more aggressive rival R_2 for good B, and because R_1 itself sells more of the substitute good A). As one price increases, the other one remaining constant, consumer surplus and social welfare decrease, whereas industry profit increases.

It can be checked that each exclusivity provision increases industry profit, and reduces both consumer surplus and social welfare:

$$\begin{aligned} \Pi^{**} &= 2p^{**}Q^{**} = 2\frac{1+s}{(2+s)^2} > \Pi^* = p_A^* Q_A^* + p_B^* Q_B^* = \frac{17-s}{36} > \Pi^\circ = 2p^\circ Q^\circ = \frac{4}{9}, \\ S^{**} &= S(Q^{**}, Q^{**}) = \frac{(1+s)^2}{(2+s)^2} < S^* = S(Q_B^*, Q_A^*) = \frac{25+7s}{72} < S^\circ = S(Q^\circ, Q^\circ) = \frac{4}{9}, \\ W^{**} &= S^{**} + \Pi^{**} = \frac{3+4s+s^2}{(s+2)^2} < W^* = \Pi^* + S^* = \frac{59+5s}{72} < W^\circ = S^\circ + \Pi^\circ = \frac{8}{9}. \end{aligned}$$

Finally, the following figure shows that the impact of *each* exclusivity provision is also larger when the goods are more differentiated:

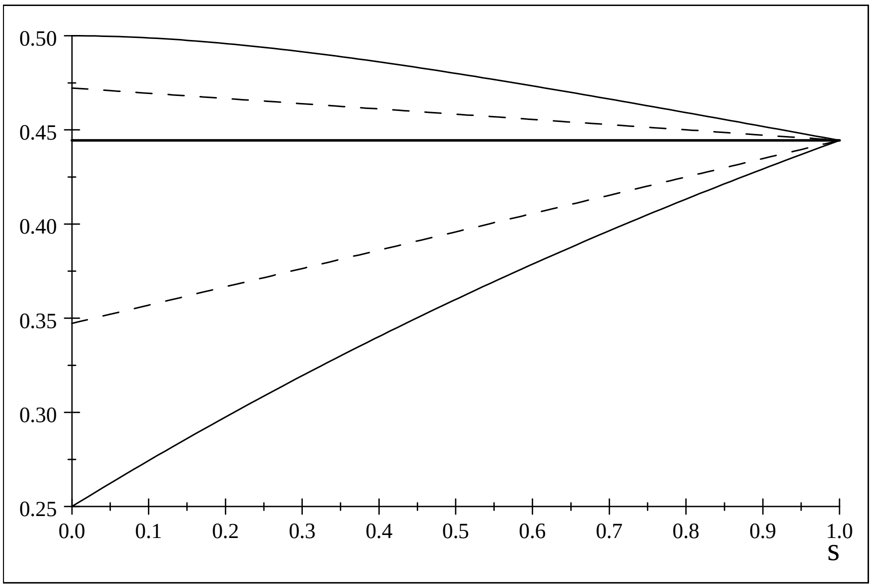

Figure 1

The figure shows the impact of exclusivity on industry profit (top curves) and consumer surplus (bottom curves). The solid curves represent the case of pairwise exclusivity; the dashed curves the case of single exclusivity, and the bold line the non-exclusivity benchmark.

5 Vertical Integration

In this section, we analyze the positive and normative effects of vertical integration. We begin by considering the case of a single vertically integrated upstream-downstream pair, and then turn to the case of pairwise vertical integration. We show that, under single vertical integration, there exists an equilibrium in which the integrated firm forecloses its downstream rival; this equilibrium thus replicates the outcome (in terms of retail prices and quantities) under single exclusive dealing. That is, a vertical merger leads to the foreclosure of the rival retailer. We also show that pairwise vertical integration yields a unique equilibrium outcome, in which each vertically integrated firm forecloses its rival. The welfare analysis of vertical integration therefore mirrors that of exclusive dealing: vertical integration reduces both consumer surplus and social welfare.

We begin by considering the case where a single upstream-downstream pair, $M_A - R_1$ say, is vertically integrated. Our previous analysis allows us to provide a very partial characterization of equilibrium:

Lemma 2 *Suppose that $M_A - R_1$ are vertically integrated whereas M_B and R_2 are vertically separated. Then, in equilibrium, the unintegrated manufacturer M_B signs a cost-based contract with each retailer. The vector of equilibrium quantities, $(q^*_{A1}, q^*_{A2}, q^*_{B1}, q^*_{B2})$, is thus such that*

$$q^*_{Bh} = \arg\max_{q_{Bh}} \left[P\left(q_{Bh} + q^*_{Bk}, q^*_{Ah} + q^*_{Ak}\right) - c\right] q_{Bh} + P\left(q^*_{Ah} + q^*_{Ak}, q_{Bh} + q^*_{Bk}\right) q^*_{Ah}$$

for all $h \neq k \in \{1, 2\}$.

Proof. This is an immediate implication of Lemma 1. ■

The following proposition shows that there exists an equilibrium in which the integrated firm will not supply its downstream rival.

Proposition 10 *Assume (P.1)-(P.3). Then, there exists an equilibrium in which:*

- *M_A offers R_2 a quantity-forcing contract, $\left(\hat{q}, \hat{T}\right)$, whereas M_B offers R_1 a (cost-based) quantity-forcing contract, (q^*_{B1}, T^*_{B1}), and offers R_2 to supply any quantity at cost (i.e., $\tau_{B2}(q) = cq$, for any $q \geq 0$).*
- *R_2 is indifferent between accepting both M_A's and M_B's contracts, or either one of them, and rejects M_A's offer;*
- *R_1 is indifferent between accepting and rejecting M_B's contract, and accepts it.*

Proof. See Appendix K. ■

The equilibrium under single vertical integration of $M_A - R_1$, characterized by the proposition, replicates the outcome in terms of retail prices and quantities that would obtain under an exclusive distribution contract between M_A and R_1 (or under a single branding contract between M_B and R_2). However, unlike in the case of exclusive dealing, the independent retailer R_2 here extracts some rents thanks to the competition between the two manufacturers for its business. While the integrated manufacturer M_A does not require access to R_2 to sell its good (any unit that M_A sells through R_2, it could instead sell through its own downstream affiliate R_1), it has an interest to have R_2 reject M_B's offer; in equilibrium, M_A does not achieve this goal, but for equilibrium to exist, M_A has to make an attractive offer to R_2 that gets rejected along the equilibrium path.

Proposition 10 extends the analysis of Hart and Tirole (1990) to the case of oligopolistic upstream competition. As in the case of a pure upstream bottleneck, the integrated manufacturer M_A completely forecloses the downstream rival R_2, and monopolizes the distribution of its product. By contrast with the case of a pure upstream bottleneck, the competition for R_2's business leads M_A to offer a contract guaranteeing some rents to R_2, which R_2 however chooses to reject. Interestingly, this can be contrasted with another situation considered by Hart and Tirole, in which M_A would face a less efficient competitive fringe of suppliers, offering the same input but facing a higher cost than M_A. In that situation, M_A would end up supplying R_2 (although on terms based on the higher cost of the competitive fringe). By contrast, here the other supplier offers a differentiated good, and each manufacturer is "more efficient" than its rival on the provision of its own input; as a result, M_A ends up *not* supplying R_2. Finally, the proposition also shows that, while the integrated manufacturer forecloses the downstream rival, the integrated retailer keeps dealing with the independent manufacturer.

We now turn to the case where there are two vertically integrated firms, $M_A - R_1$ and $M_B - R_2$. The following proposition shows that pairwise vertical integration leads to complete foreclosure of rivals, mirroring the outcome under pairwise exclusive dealing:

Proposition 11 *Suppose $M_A - R_1$ and $M_B - R_2$ are vertically integrated. If Properties (P.1)-(P.3) hold, then there exists a unique equilibrium, $(q_{A1}^{**}, q_{A2}^{**}, q_{B1}^{**}, q_{B2}^{**})$, in which moreover there is no cross-selling: $(q_{A1}^{**}, q_{A2}^{**}, q_{B1}^{**}, q_{B2}^{**}) = (Q^{**}, 0, 0, Q^{**})$, where $Q^{**} \equiv \arg\max_Q \left[P(Q, Q^{**}) - c\right] Q$.*

Proof. See Appendix L. ■

The proposition shows that pairwise vertical integration leads to a strong form of foreclosure, as each integrated firm refuses to deal with the other integrated firm. In particular, combined with Lemma 2, it shows that pairwise vertical integration is "less competitive" than single vertical integration. It follows from our previous welfare analysis that vertical integration harms consumers and society. In particular, under pairwise vertical integration both prices are higher (and consumer surplus as well as social welfare are thus lower) than under vertical separation.

From our analysis of exclusive dealing, it also follows that firms have an incentive to integrate vertically: If no firm is vertically integrated, there exists a manufacturer-retailer pair, say $M_A - R_1$, that can increase their joint profit by merging. Moreover, if $M_A - R_1$ are vertically integrated, the remaining manufacturer-retailer pair $M_B - R_2$ can also increase their joint profit by merging.

We conclude this section by noting that "complete foreclosure" arises here from the fact that a single retailer suffices to serve the entire market. If it were not the case, e.g., due to downstream capacity constraints or to differentiation among the retailers, then integrated manufacturers would still wish to deal with downstream rivals in order to expand market coverage or serve customer niches; in such situations, we would thus expect vertical integration to result into partial rather then complete foreclosure. By the same token, in such situations vertical integration (and partial foreclosure) is likely to be more profitable than exclusive dealing (and thus complete foreclosure).[14]

6 Conclusion

In this paper, we have developed a framework for the analysis of interlocking bilateral relationships in vertically related markets. Key features of the framework are that upstream firms are horizontally differentiated, contract offers and acceptance decisions are private information to the contracting parties, and there are no *ad hoc* restrictions on tariffs. In the absence of exclusive dealing provisions and vertical integration, all channels are active and involve cost-based (nonlinear) tariffs. Under mild regularity conditions, there exists a unique equilibrium outcome in terms of retail prices and quantities; we also provide a complete characterization of the equilibrium outcomes in terms of profit sharing between manufacturers and retailers. We use this framework to analyze the positive and normative effects of exclusive dealing and vertical integration, and show that vertical integration leads to foreclosure of rival retailers: Firms have an incentive to sign exclusive dealing provisions or to integrate vertically, at the expense of consumers and society.

There are exciting avenues for future research. First, it would be natural to extend the model to an arbitrary number of manufacturers and retailers, and to introduce upstream and/or downstream firm heterogeneity. Second, it would be interesting to allow retailers to be horizontally differentiated. We expect that, in this case, vertical integration no longer leads to complete foreclosure of rival retailers, unlike exclusive dealing provisions. Third, it seems important to extend the analysis to downstream price competition, which is however known to raise additional issues for the treatment of out-of-equilibrium beliefs.[15] Finally, and perhaps most importantly, the framework developed in this paper can be used to study the positive and normative effects of other contractual arrangements, such as “fidelity rebates” based on market shares, MFN clauses, or agency contracts.

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[14] See Rey and Tirole (2007) for an analysis of the impact of downstream differentiation on the extent of foreclosure in the case of an upstream monopoly.

[15] Passive beliefs appear to remain the most tractable approach in that case, but they become less compelling – in particular, they no longer coincide with wary beliefs – and equilibrium existence is no longer guaranteed; see Rey and Vergé (2004).

A Proof of Lemma 1

Fix a candidate equilibrium, with associated equilibrium quantities $(q^e_{ih})_{i=A,B,h=1,2}$ and acceptance decisions $(\delta^e_{ih})_{i=A,B,h=1,2}$, with the convention that $\delta^e_{ih} = 1$ if M_i and R_h are vertically integrated and, when they are independent, $\delta^e_{ih} = 1$ if the offer is accepted and $\delta^e_{ih} = 0$ if it is not (in which case $q^e_{ih} = 0$). Suppose that an unintegrated M_i deviates and offers R_h a cost-based two-part tariff $(c, \tilde{F}_{ih})$, where the fixed fee $\tilde{F}_{ih}$ is as follows:

- if R_h is vertically integrated with M_j, then:

$$\begin{aligned}\tilde{F}_{ih} &= \max_{q_{ih}} \left\{ \left[P\left(q_{ih} + q^e_{ik}, q^e_{jh} + q^e_{jk}\right) - c\right] q_{ih} \right.\\ &\quad \left. + \left[P\left(q^e_{jh} + q^e_{jk}, \tilde{q}_{ih} + q^e_{ik}\right) - c\right] q^e_{jh} + \delta^e_{jk}\left[\tau^e_{jk}\left(q^e_{jk}\right) - c q^e_{jk}\right]\right\} - \pi^e_{j-h}, \qquad (1)\end{aligned}$$

  where π^e_{j-h} denotes the profit of the integrated firm $M_j - R_h$ in the candidate equilibrium. The terms in curly brackets represent the profit that the vertically integrated firm $M_j - R_h$ would make if R_h accepted M_i's deviant offer and maintained the equilibrium quantity q^e_{jh}, and R_k maintained the equilibrium quantities q^e_{ik} and q^e_{jk}:

  - the first two terms are the profits generated by, respectively, the channels $M_i - R_h$ and $M_j - R_h$,
  - whereas the third term is the profit that M_j generates in equilibrium through the sales to the other, unintegrated retailer R_k.

- if instead R_h is not vertically integrated, then:

$$\begin{aligned}\tilde{F}_{ih} &= \max_{q_{ih}} \left\{ \left[P\left(q_{ih} + q^e_{ik}, \delta^e_{jh} q^e_{jh} + q^e_{jk}\right) - c\right] q_{ih} \right.\\ &\quad \left. + \delta^e_{jh}\left[P\left(q^e_{jh} + q^e_{jk}, q_{ih} + q^e_{ik}\right) q^e_{jh} - \tau^e_{jh}\left(q^e_{jh}\right)\right]\right\} - \pi^e_h, \qquad (2)\end{aligned}$$

  where π^e_h denotes the profit that the unintegrated R_h makes in equilibrium. The terms in curly brackets represent the profit that the unintegrated R_h would make if it accepted M_i's deviant offer and maintained its acceptance decision δ^e_{jh} vis-à-vis M_j's contract offer as well as the equilibrium quantity q^e_{jh}, and R_k maintained the equilibrium quantities q^e_{ik} and q^e_{jk}:

  - the first term is the profit generated by the channel $M_i - R_h$,

– whereas the second term is the profit that R_h makes on its contract with M_j.

We first claim that R_h is willing to accept the deviant offer $(c, \tilde{F}_{ih})$:

1. Having passive beliefs, at the acceptance stage R_h continues to believe that its downstream rival R_k has been offered the equilibrium contracts and will sell the equilibrium quantities q_{ik}^e and q_{jk}^e in the continuation game.

2. By accepting M_i's deviant offer, R_h can make the same profit as in the candidate equilibrium by sticking to its acceptance decision vis-à-vis M_j's nondeviant offer and maintaining the quantity q_{jh} at its equilibrium level q_{jh}^e, and can do only better by optimizing over these decisions.

3. If instead R_h rejects M_i's deviant offer, it obtains the same profit as in the continuation game following the rejection of M_i's equilibrium offer. By construction, this cannot exceed M_i's equilibrium profit: it constitutes the equilibrium profit if in equilibrium R_h rejects τ_{ih}^e, and must be (weakly) lower otherwise.

As R_h is willing to accept this deviant offer (and can be induced to do so, if needed, by slightly reducing the fixed fee $\tilde{F}_{ih}$), which gives M_i a profit equal to $\tilde{F}_{ih}$, this deviation is unprofitable only if $\tilde{F}_{ih} \leq \pi_{i,h}^e$, where

$$\pi_{i,h}^e = \delta_{ih}^e \left[\tau_{ih}^e \left(q_{ih}^e\right) - c q_{ih}^e\right]$$

denotes the equilibrium profit that M_i makes from selling through retailer R_h. But then:

- If R_h is vertically integrated with M_j (implying $\delta_{ih}^e = 1$), we can rewrite $\pi_{i,h}^e$ as follows:

$$\begin{aligned}\pi_{i,h}^e &= \left\{\left[P\left(q_{ih}^e + q_{ik}^e, q_{jh}^e + q_{jk}^e\right) - c\right] q_{ih}^e\right.\\ &\quad \left.+\left[P\left(q_{jh}^e + q_{jk}^e, q_{ih}^e + q_{ik}^e\right) - c\right] q_{jh}^e + \delta_{jk}^e\left[\tau_{jk}^e\left(q_{jk}^e\right) - c q_{jk}^e\right]\right\} - \pi_{j-h}^e.\end{aligned}$$

  Using (1), $\pi_{i,h}^e \geq \tilde{F}_{ih}$ then implies $q_{ih}^e \in \chi\left(q_{ik}^e, q_{jh}^e, q_{jh}^e\right)$.

- If instead R_h is unintegrated, we can rewrite $\pi_{i,h}^e$ as follows:

$$\begin{aligned}\pi_{i,h} &= \left\{\left[P\left(q_{ih}^e + q_{ik}^e, \delta_{jh}^e q_{jh}^e + q_{jk}^e\right) - c\right] q_{ih}^e\right.\\ &\quad \left.+\delta_{jh}^e\left[P\left(q_{jh}^e + q_{jk}^e, q_{ih}^e + q_{ik}^e\right) q_{jh}^e - \tau_{jh}^e\left(q_{jh}^e\right)\right]\right\} - \pi_h^e.\end{aligned}$$

  Using (2), $\pi_{i,h}^e \geq \tilde{F}_{ih}$ then implies $q_{ih}^e \in \chi\left(q_{ik}^e, q_{jh}^e, q_{jh}^e\right)$.

B Proof of Proposition 2

From Proposition 1, we know that the equilibrium quantities satisfy $q_{ih}^{\circ} \in \chi\left(q_{ik}^{\circ}, q_{jh}^{\circ}, q_{jk}^{\circ}\right)$. We first show that Assumptions (A.3)-(A.4) ensure that all quantities are positive; we then use first-order conditions to characterize the unique, symmetric equilibrium outcome.

B.1 Interior solution

To see that all quantities are positive, suppose that q_{B2}°, say, is zero.

Step 1: $q_{B1}^{\circ} > 0$. Suppose otherwise that $q_{B1}^{\circ} = 0$. By construction, we then have:

$$q_{Ah}^{\circ} = \arg\max_{q_{Ah}} \left[P\left(q_{Ah} + q_{Ak}^{\circ}, 0\right) - c\right] q_{Ah}.$$

Note that $q_{A1}^{\circ} = 0$ would imply $p_A^{\circ} = P\left(q_{A2}^{\circ}, 0\right) \leq c$, and thus $q_{A2}^{\circ} = 0$ as well;[16] but this would therefore require $P(0,0) \leq c$, contradicting the viability condition (A.1). Thus, we can assume that q_{A1}° is positive, and thus satisfies R_1's first-order condition which, using

$$\pi_1 = \left[P\left(q_{A1} + q_{A2}^{\circ}, q_{B1}\right) - c\right] q_{A1} + \left[P\left(q_{B1}, q_{A1} + q_{A2}^{\circ}\right) - c\right] q_{B1},$$

and $q_{B1}^{\circ} = 0$, is given by:

$$\left.\frac{\partial \pi_1}{\partial q_{A1}}\right|_{(q_{ih})=\left(q_{ih}^{\circ}\right)} = P\left(Q_A^{\circ}, 0\right) - c + \partial_1 P\left(Q_A^{\circ}, 0\right) q_{A1}^{\circ} = 0.$$

But then, a small increase in q_{B1} would increase R_1's profit:

$$\begin{aligned}
\left.\frac{\partial \pi_1}{\partial_{q_{B1}}}\right|_{(q_{ih})=\left(q_{ih}^{\circ}\right)} &= P\left(0, Q_A^{\circ}\right) - c + \partial_2 P\left(Q_A^{\circ}, 0\right) q_{A1}^{\circ} \\
&> P\left(Q_A^{\circ}, 0\right) - c + \partial_1 P\left(Q_A^{\circ}, 0\right) q_{A1}^{\circ} = 0,
\end{aligned}$$

where the inequality stems from (A.2) ($\partial_2 P > \partial_1 P$, which also implies $P(0, Q) > P(Q, 0)$ for any $Q > 0$).

Step 2: $q_{A2}^{\circ} > q_{A1}^{\circ}$. From Step 1, q_{B1}° is positive and therefore satisfies the first-order condition

$$\left.\frac{\partial \pi_1}{\partial_{q_{B1}}}\right|_{(q_{ih})=\left(q_{ih}^{\circ}\right)} = P\left(Q_B^{\circ}, Q_A^{\circ}\right) - c + \partial_1 P\left(Q_B^{\circ}, Q_A^{\circ}\right) q_{B1}^{\circ} + \partial_2 P\left(Q_A^{\circ}, Q_B^{\circ}\right) q_{A1}^{\circ} = 0. \tag{3}$$

[16] Otherwise, a slight reduction in q_{A2} would increase R_2's profit $\pi_2^{\circ} = \left(p_A^{\circ} - c\right) q_{A2}^{\circ}$, since then

$$\left.\frac{\partial \pi_2}{\partial q_{A2}}\right|_{(q_{ih})=\left(q_{ih}^{\circ}\right)} = p_A^{\circ} - c + q_{A2}^{\circ} \partial_1 P\left(q_{A2}^{\circ}, 0\right) < p_A^{\circ} - c \leq 0.$$

From (A.2), $\partial_1 P \leq 0$ and $\partial_2 P \leq 0$ and thus $P(Q_B^\circ, Q_A^\circ) \geq c > 0$; but (A.2) then implies $\partial_1 P(Q_B^\circ, Q_A^\circ) < 0$, which in turn yields $P(Q_B^\circ, Q_A^\circ) > c$.

Using

$$\pi_2 = \left[P\left(q_{A1}^\circ + q_{A2}, q_{B1}^\circ + q_{B2}\right) - c\right] q_{A2} + \left[P\left(q_{B1}^\circ + q_{B2}, q_{A1}^\circ + q_{A2}\right) - c\right] q_{B2},$$

the first-order condition for $q_{B2}^\circ = 0$ yields:

$$\left.\frac{\partial \pi_2}{\partial_{q_{B2}}}\right|_{(q_{ih})=\left(q_{ih}^\circ\right)} = P(Q_B^\circ, Q_A^\circ) - c + \partial_2 P(Q_A^\circ, Q_B^\circ)\, q_{A2}^\circ \leq 0. \tag{4}$$

Using (A.2) and $P(Q_B^\circ, Q_A^\circ) > c$, it follows that q_{A2}° is positive and thus satisfies the first-order condition

$$\left.\frac{\partial \pi_2}{\partial_{q_{A2}}}\right|_{(q_{ih})=\left(q_{ih}^\circ\right)} = P(Q_A^\circ, Q_B^\circ) - c + \partial_1 P(Q_A^\circ, Q_B^\circ)\, q_{A2}^\circ = 0,$$

implying $P(Q_A^\circ, Q_B^\circ) > c$.

Subtracting (3) from (4) yields:

$$\partial_2 P(Q_A^\circ, Q_B^\circ)\,(q_{A2}^\circ - q_{A1}^\circ) \leq \partial_1 P(Q_B^\circ, Q_A^\circ)\, q_{B1}^\circ,$$

where $\partial_1 P_A < 0$ and $\partial_2 P_B < 0$ (from (A.2), as both prices are positive), and $q_{B1}^\circ > 0$ (from Step 1); therefore, $q_{A2}^\circ > q_{A1}^\circ$.

Step 3: $q_{A1}^\circ > 0$. Suppose otherwise that $q_{A1}^\circ = 0$. In that case, $Q_A^\circ = q_{A2}^\circ$ and $Q_B^\circ = q_{B1}^\circ$ satisfy $Q_B^\circ = \hat{\chi}(Q_A^\circ)$ and $Q_A^\circ = \hat{\chi}(Q_B^\circ)$, where the best response function

$$\hat{\chi}(Q) \equiv \arg\max_{\hat{Q}} \left\{ \left[P\left(\hat{Q}, Q\right) - c\right] \hat{Q} \right\}$$

is characterized by the first-order condition:

$$P(\hat{\chi}(Q), Q) - c + \partial_1 P(\hat{\chi}(Q), Q)\, \hat{\chi}(Q) = 0.$$

Assumption (A.3) ensures that this response function satisfies

$$-1 < \hat{\chi}'(Q) < 0.$$

Therefore, we must have $Q_A^\circ = Q_B^\circ = \hat{Q}^\circ$, where $\hat{Q}^\circ$ is such that $\hat{Q}^\circ = \hat{\chi}\left(\hat{Q}^\circ\right)$, and thus satisfies:

$$P\left(\hat{Q}^\circ, \hat{Q}^\circ\right) - c + \partial_1 P\left(\hat{Q}^\circ, \hat{Q}^\circ\right) \hat{Q}^\circ = 0.$$

But then, each retailer would want to sell the other brand as well:

$$\left.\frac{\partial \pi_1}{\partial_{q_{A1}}}\right|_{(q_{ih})=\left(q_{ih}^\circ\right)} = \left.\frac{\partial \pi_2}{\partial_{q_{B2}}}\right|_{(q_{ih})=\left(q_{ih}^\circ\right)} = P\left(\hat{Q}^\circ, \hat{Q}^\circ\right) - c + \partial_2 P\left(\hat{Q}^\circ, \hat{Q}^\circ\right) \hat{Q}^\circ > 0$$

as $P\left(\hat{Q}^\circ, \hat{Q}^\circ\right) > c$ from above, and thus $\partial_1 P\left(\hat{Q}^\circ, \hat{Q}^\circ\right) < \partial_2 P\left(\hat{Q}^\circ, \hat{Q}^\circ\right) < 0$ from (A.2). Hence, $q^\circ_{A1} > 0$.

Step 4. It follows from the previous steps that q°_{A2}, q°_{A1} and q°_{B1} must all be positive, and thus satisfy the first-order conditions:

$$\left.\frac{\partial \pi_1}{\partial_{q_{A1}}}\right|_{(q_{ih})=\left(q^\circ_{ih}\right)} = P\left(Q^\circ_A, Q^\circ_B\right) - c + \partial_1 P\left(Q^\circ_A, Q^\circ_B\right) q^\circ_{A1} + \partial_2 P\left(Q^\circ_B, Q^\circ_A\right) q^\circ_{B1} = 0, \tag{5}$$

$$\left.\frac{\partial \pi_1}{\partial_{q_{B1}}}\right|_{(q_{ih})=\left(q^\circ_{ih}\right)} = P\left(Q^\circ_B, Q^\circ_A\right) - c + \partial_1 P\left(Q^\circ_B, Q^\circ_A\right) q^\circ_{B1} + \partial_2 P\left(Q^\circ_A, Q^\circ_B\right) q^\circ_{A1} = 0, \tag{6}$$

$$\left.\frac{\partial \pi_2}{\partial_{q_{A2}}}\right|_{(q_{ih})=\left(q^\circ_{ih}\right)} = P\left(Q^\circ_A, Q^\circ_B\right) - c + \partial_1 P\left(Q^\circ_A, Q^\circ_B\right) q^\circ_{A2} = 0, \tag{7}$$

whereas the first-order condition for $q^\circ_{B2} = 0$ yields:

$$\left.\frac{\partial \pi_2}{\partial_{q_{B2}}}\right|_{(q_{ih})=\left(q^\circ_{ih}\right)} = P\left(Q^\circ_B, Q^\circ_A\right) - c + \partial_2 P\left(Q^\circ_A, Q^\circ_B\right) q^\circ_{A2} \leq 0. \tag{8}$$

Subtracting (7) from (5) and (6) from (8) yields:

$$\begin{aligned} -\partial_1 P\left(Q^\circ_A, Q^\circ_B\right)\left(q^\circ_{A2} - q^\circ_{A1}\right) &= -\partial_2 P\left(Q^\circ_B, Q^\circ_A\right) q^\circ_{B1}, \\ -\partial_2 P\left(Q^\circ_A, Q^\circ_B\right)\left(q^\circ_{A2} - q^\circ_{A1}\right) &\geq -\partial_1 P\left(Q^\circ_B, Q^\circ_A\right) q^\circ_{B1}. \end{aligned}$$

The first condition yields $q^\circ_{A2} > q^\circ_{A1}$, and thus the two conditions can be rewritten as:

$$\frac{-\partial_1 P\left(Q^\circ_A, Q^\circ_B\right)}{-\partial_2 P\left(Q^\circ_B, Q^\circ_A\right)} = \frac{q^\circ_{B1}}{q^\circ_{A2} - q^\circ_{A1}} \leq \frac{-\partial_2 P\left(Q^\circ_A, Q^\circ_B\right)}{-\partial_1 P\left(Q^\circ_B, Q^\circ_A\right)}.$$

This, in turn, implies

$$\partial_1 P\left(Q^\circ_A, Q^\circ_B\right) \partial_1 P\left(Q^\circ_B, Q^\circ_A\right) \leq \partial_2 P\left(Q^\circ_B, Q^\circ_A\right) \partial_2 P\left(Q^\circ_A, Q^\circ_B\right),$$

a contradiction as $\partial_1 P < \partial_2 P < 0$ from (A.2). Hence, there is no equilibrium in which $q^\circ_{B2} = 0$.

B.2 The equilibrium outcome is unique and symmetric

It follows from the above analysis that all equilibrium quantities are positive and thus satisfy the first-order conditions. Adding the conditions for good A, namely:

$$\left.\frac{\partial \pi_1}{\partial_{q_{A1}}}\right|_{(q_{ih})=\left(q^\circ_{ih}\right)} = P\left(Q^\circ_A, Q^\circ_B\right) - c + \partial_1 P\left(Q^\circ_A, Q^\circ_B\right) q^\circ_{A1} + \partial_2 P\left(Q^\circ_B, Q^\circ_A\right) q^\circ_{B1} = 0,$$

$$\left.\frac{\partial \pi_2}{\partial_{q_{A2}}}\right|_{(q_{ih})=\left(q^\circ_{ih}\right)} = P\left(Q^\circ_A, Q^\circ_B\right) - c + \partial_1 P\left(Q^\circ_A, Q^\circ_B\right) q^\circ_{A2} + \partial_2 P\left(Q^\circ_B, Q^\circ_A\right) q^\circ_{B2} = 0,$$

implies that

$$P\left(Q_A^{\circ}, Q_B^{\circ}\right) \geq c > 0$$

and

$$Q_A^{\circ} = \tilde{\chi}\left(Q_B^{\circ}\right),$$

where $Q_A = \tilde{\chi}(Q_B)$ denotes the "best-response" function defined by

$$\phi\left(Q_A, Q_B\right) \equiv 2\left[P\left(Q_A, Q_B\right) - c\right] + \partial_1 P\left(Q_A, Q_B\right) Q_A + \partial_2 P\left(Q_B, Q_A\right) Q_B = 0.$$

Likewise, adding the first-order conditions for good B yields $P\left(Q_B^{\circ}, Q_A^{\circ}\right) \geq c > 0$ and $Q_B^{\circ} = \tilde{\chi}\left(Q_A^{\circ}\right)$. The derivatives of ϕ are given by:

$$\begin{aligned}
\partial_1 \phi\left(Q_A, Q_B\right) &= 3\partial_1 P\left(\tilde{\chi}(Q), Q\right) + \partial_{11} P\left(\tilde{\chi}(Q), Q\right) \tilde{\chi}(Q) + \partial_{22} P\left(Q, \tilde{\chi}(Q)\right) Q, \\
\partial_2 \phi\left(Q_A, Q_B\right) &= 2\partial_2 P\left(\tilde{\chi}(Q), Q\right) + \partial_2 P\left(Q, \tilde{\chi}(Q)\right) + \partial_{12}^2 P\left(\tilde{\chi}(Q), Q\right) \tilde{\chi}(Q) + \partial_{21} P\left(Q, \tilde{\chi}(Q)\right) Q.
\end{aligned}$$

Assumptions (A.2)[17] and (A.4) ensure that $\partial_1 \phi\left(Q_A, Q_B\right) < \partial_2 \phi\left(Q_A, Q_B\right) < 0$. Hence the reaction function $\tilde{\chi}(.)$ is uniquely defined and such that

$$\tilde{\chi}'(Q) = -\frac{\partial_2 \phi\left(Q_A, Q_B\right)}{\partial_1 \phi\left(Q_A, Q_B\right)} \in (-1, 0).$$

It follows that the equilibrium is symmetric: $Q_A^{\circ} = Q_B^{\circ} = Q^{\circ}$. The first-order conditions for R_h's quantity choices, for $h \in \{1, 2\}$, then yield:

$$\begin{aligned}
-\partial_1 P\left(Q^{\circ}, Q^{\circ}\right) q_{Ah}^{\circ} - \partial_2 P\left(Q^{\circ}, Q^{\circ}\right) q_{Bh}^{\circ} &= P\left(Q^{\circ}, Q^{\circ}\right) - c, \\
-\partial_1 P\left(Q^{\circ}, Q^{\circ}\right) q_{Bh}^{\circ} - \partial_2 P\left(Q^{\circ}, Q^{\circ}\right) q_{Ah}^{\circ} &= P\left(Q^{\circ}, Q^{\circ}\right) - c,
\end{aligned}$$

and thus

$$q_{Ah}^{\circ} = q_{Bh}^{\circ} = q^{\circ} \equiv -\frac{P\left(Q^{\circ}, Q^{\circ}\right) - c}{\partial_1 P\left(Q^{\circ}, Q^{\circ}\right) + \partial_2 P\left(Q^{\circ}, Q^{\circ}\right)}.$$

C Proof of Proposition 3

Consider a candidate equilibrium in which each manufacturer offers the cost-based two-part tariffs $(w, F) = (c, \Delta^{\circ})$, which accept it.

We first note that the continuation equilibrium is then such that both retailers sell $(q_{Ah}, q_{Bh}) = (q^{\circ}, q^{\circ})$, for $h = 1, 2$. From the proof of Proposition 2, this constitutes the unique candidate equilibrium, and it satisfies all first-order conditions. It thus suffices to check that retailers' profit

[17] Using demand symmetry (see footnote 2), (A.2) implies $\partial_1 P\left(Q_A, Q_B\right) < \partial_2 P\left(Q_A, Q_B\right) = \partial_2 P\left(Q_B, Q_A\right)$.

functions are concave. Anticipating that its rival will sell (q°, q°), by selling (q_A, q_B) a retailer obtains a profit (gross of the fixed fees) equal to

$$\Pi_R^\circ (q_A, q_B) \equiv [P(q_A + q^\circ, q_B + q^\circ) - c]\, q_A + [P(q_B + q^\circ, q_A + q^\circ) - c]\, q_B.$$

The second-order derivatives of $\Pi_R^\circ (\cdot, \cdot)$ are given by:

$$\begin{aligned}
\partial_{11}^2 \Pi_R^\circ (q_A, q_B) &= 2\partial_1 P(q_A + q^\circ, q_B + q^\circ) + \partial_{11}^2 P(q_A + q^\circ, q_B + q^\circ)\, q_A + \partial_{22}^2 P(q_B + q^\circ, q_A + q^\circ)\, q_B, \\
\partial_{22}^2 \Pi_R^\circ (q_A, q_B) &= 2\partial_1 P(q_B + q^\circ, q_A + q^\circ) + \partial_{11}^2 P(q_B + q^\circ, q_A + q^\circ)\, q_B + \partial_{22}^2 P(q_A + q^\circ, q_B + q^\circ)\, q_A, \\
\partial_{12}^2 \Pi_R^\circ (q_A, q_B) &= \partial_2 P(q_A + q^\circ, q_B + q^\circ) + \partial_2 P(q_B + q^\circ, q_A + q^\circ) \\
&\quad + \partial_{12}^2 P(q_A + q^\circ, q_B + q^\circ)\, q_A + \partial_{12}^2 P(q_B + q^\circ, q_A + q^\circ)\, q_B.
\end{aligned}$$

Assumption (A.4) then ensures that the profit function Π_R° is strictly concave, as $\partial_{11}^2 \Pi_R^\circ$ and $\partial_{22}^2 \Pi_R^\circ$ are both negative, and the determinant of the Hessian is positive: $\partial_{11}^2 \Pi_R^\circ \partial_{22}^2 \Pi_R^\circ > \left(\partial_{12}^2 \Pi_R^\circ\right)^2$. The assumption moreover yields $\partial_{12}^2 \Pi_R^\circ < 0$, which will be used in the proof of 4.

Next, we note that each retailer is willing to carry both goods. Indeed, the fee F° is such that, if its rival were to accept both offers and sell (q°, q°), then a retailer:

- Obtains the same profit, π_R°, by accepting both manufacturers' offers or only one of them:

$$\pi_R^\circ = 2\left\{[P(2q^\circ, 2q^\circ) - c]\, q^\circ - F^\circ\right\} = \max_q [P(q + q^\circ, q^\circ) - c]\, q^\circ - F^\circ;$$

- Strictly prefers securing this profit to rejecting both offers:

$$\begin{aligned}
\frac{\pi_R^\circ}{2} &= [P(2q^\circ, 2q^\circ) - c]\, q^\circ - F^\circ \\
&= \max_q [P(q^\circ + q, q^\circ) - c]\, q - [P(2q^\circ, 2q^\circ) - c]\, q^\circ \\
&> \max_q [P(q^\circ + q, q^\circ) - c]\, q - [P(2q^\circ, q^\circ) - c]\, q^\circ \\
&\geq 0.
\end{aligned}$$

Thus, if these contracts are offered, it is a continuation equilibrium for both retailers to accept both manufacturers' offers, and then to sell (q°, q°). We now show that manufacturers cannot profitably deviate from this candidate equilibrium. As the profit that a manufacturer achieves with a retailer is not affected by its relation with the other retailer, without loss of generality we can restrict attention to "one-sided" deviations, in which a manufacturer offers a deviating contract to one of the retailers. Furthermore, the above tariffs are profitable for the manufacturers:

$$\begin{aligned}
F^{\circ} &= \left[P\left(2q^{\circ},2q^{\circ}\right)-c\right]2q^{\circ}-\max_{q}\left[P\left(q+q^{\circ},q^{\circ}\right)-c\right]q \\
&= \max_{q_A,q_B}\left\{\left[P\left(q_A+q^{\circ},q_B+q^{\circ}\right)-c\right]q_A+\left[P\left(q_B+q^{\circ},q_A+q^{\circ}\right)-c\right]q_B\right\}-\max_{q}\left[P\left(q+q^{\circ},q^{\circ}\right)-c\right]q \\
&> 0,
\end{aligned}$$

where the second equality comes from the definition of q° and the inequality comes from the fact that the second optimization problem is more constrained than the first one (and the optimal q_A and q_B are indeed both positive, as they are equal to q°). It follows that a deviation cannot be profitable if it is not accepted by the retailer; and since the retailer can secure its equilibrium profit π_R° by accepting only the rival's offer, it must be the case that the deviation increases the joint profit of the manufacturer and of the retailer.

If the deviation induces the retailer to keep dealing with the other manufacturer, then the joint profit of the manufacturer and of the retailer (gross of the manufacturer's cost of supplying q° to the rival retailer, which is not affected by the deviation) cannot exceed

$$\max_{q_A,q_B}\left\{\left[P\left(q_A+q^{\circ},q_B+q^{\circ}\right)-c\right]q_A+\left[P\left(q_B+q^{\circ},q_A+q^{\circ}\right)-c\right]q_B\right\}=\left[P\left(2q^{\circ},2q^{\circ}\right)-c\right]2q^{\circ},$$

which the two parties already obtain in the candidate equilibrium. Therefore, such a deviation cannot be profitable.

If instead the deviation induces the retailer to reject the other manufacturer's offer, then the joint profit of the manufacturer and of the retailer (again gross of the manufacturer's cost of supplying q° to the rival retailer) cannot exceed

$$\begin{aligned}
\max_{q}\left\{\left[P\left(q+q^{\circ},q^{\circ}\right)-c\right]q\right\}+F^{\circ} &= \max_{q}\left\{\left[P\left(q+q^{\circ},q^{\circ}\right)-c\right]q\right\}+\left[P\left(2q^{\circ},2q^{\circ}\right)-c\right]2q^{\circ} \\
&\quad -\max_{q}\left[P\left(q+q^{\circ},q^{\circ}\right)-c\right]q \\
&= \left[P\left(2q^{\circ},2q^{\circ}\right)-c\right]2q^{\circ},
\end{aligned}$$

which is again what they obtain in the candidate equilibrium. Therefore, such a deviation cannot be profitable either, which concludes the argument.

D Proof of Proposition 4

Given the assumption on passive beliefs, we can focus on one particular retailer R, taking as given that the other retailer will sell (q°,q°). Let

$$\rho_R^{\circ}\left(q_A,q_B\right)\equiv P\left(q_A+q^{\circ},q_B+q^{\circ}\right)q_A+P\left(q_B+q^{\circ},q_A+q^{\circ}\right)q_B$$

denote the total revenue generated by R selling (q_A, q_B). As shown in the proof of Proposition 3, the associated profit

$$\Pi_R^\circ(q_A, q_B) = \rho_R^\circ(q_A, q_B) - c(q_A + q_B)$$

is strictly concave in (q_A, q_B) in the range where it is positive; this profit moreover reaches its maximum at $(q_A, q_B) = (q^\circ, q^\circ)$, where it is equal to half the aggregate industry profit, $\Pi_R^\circ(q^\circ, q^\circ) = \Pi^\circ/2$.

Let $\tau_{iR}^\circ(q_i)$ denote the equilibrium tariff that M_i offers R, $\pi_{i,R}^\circ \equiv \tau_{iR}^\circ(q^\circ) - cq^\circ$ denote M_i's equilibrium profit from supplying R, and $\pi_R^\circ \equiv \Pi^\circ/2 - \pi_{A,R}^\circ - \pi_{B,R}^\circ$ denote R's equilibrium profit. Also, let

$$\tilde{q}_{iR}^\circ \in \arg\max_{q_i} \rho_R^\circ(q_i, 0) - \tau_{iR}^\circ(q_i)$$

denote the output level that R would choose under exclusivity with M_i, and $\tilde{\pi}_{i,R}^\circ \equiv \tau_{iR}^\circ(\tilde{q}_{iR}^\circ) - c\tilde{q}_{iR}^\circ$ denote M_i's associated profit (the corresponding profit for R is thus $\Pi(\tilde{q}_{iR}^\circ, 0) - \tilde{\pi}_i^\circ$). We have:

Lemma 3 *Under Assumptions (A.2)-(A.4), the output and profit levels satisfy,* $\tilde{q}_{iR}^\circ > q^\circ$ *and*

$$0 \le \pi_{i,R}^\circ \le \Delta^\circ \equiv \frac{\Pi^\circ}{2} - \hat{\Pi}_R^\circ \tag{9}$$

for $i = A, B$, *where*

$$\hat{\Pi}_R^\circ \equiv \max_q \Pi_R^\circ(q, 0),$$

and:

$$\pi_R^\circ = \frac{\Pi^\circ}{2} - \pi_{A,R}^\circ - \pi_{B,R}^\circ = \Pi_R^\circ(\tilde{q}_{AR}^\circ, 0) - \tilde{\pi}_{A,R}^\circ = \Pi_R^\circ(\tilde{q}_{BR}^\circ, 0) - \tilde{\pi}_{B,R}^\circ > 0. \tag{10}$$

Proof. We first provide bounds on equilibrium payoffs, before turning to the comparison between $\tilde{q}_{iR}^\circ$ and q°.

By construction, we have $\pi_{i,R}^\circ \ge 0$ for $i = A, B$. Furthermore, if $\pi_{i,R}^\circ > \Delta^\circ$ for some $i \in \{A, B\}$, then the aggregate profit of R and the other supplier M_j (gross of the profit that M_j makes with the other retailer), is such that :

$$\pi_R^\circ + \pi_{j,R}^\circ = \frac{\Pi^\circ}{2} - \pi_{i,R}^\circ < \hat{\Pi}_R^\circ.$$

But then, M_j could profitably deviate to exclusivity by offering a forcing contract of the form $\left(\hat{q}, \hat{T}\right)$, where $\hat{q} \equiv \arg\max_q \Pi_R^\circ(q, 0)$ denote the bilaterally efficient output under exclusivity: by accepting this offer (and only that one), R would generate a bilateral profit of $\hat{\Pi}_R^\circ$, which can then be shared by an appropriate $\hat{T}$ so as to ensure that both M_j and R benefit from the deviation. It follows that $\pi^\circ_{A,R}, \pi^\circ_{B,R} \leq \Delta^\circ$, which in turn implies that the retailer obtains a positive profit:

$$\pi^\circ_R = \frac{\Pi^\circ}{2} - \pi^\circ_{A,R} - \pi^\circ_{B,R} \geq \frac{\Pi^\circ}{2} - 2\left(\frac{\Pi^\circ}{2} - \hat{\Pi}^\circ_R\right) = 2\hat{\Pi}^\circ_R - \frac{\Pi^\circ}{2} > 0,$$

where the inequality stems from the fact that, from Assumption (A.2), goods A and B are (imperfect) substitutes. Finally, (10) follows from the fact, already noted in the main text, that in equilibrium R must be indifferent between accepting both manufacturers' offers, or only one (either one).

We now establish $\tilde{q}^\circ_{iR} > q^\circ$. By a revealed preference argument, we have:

$$\begin{aligned} \rho^\circ_R(\tilde{q}^\circ_{iR}, 0) - \tau^\circ_{iR}(\tilde{q}^\circ_{iR}) &\geq \rho^\circ_R(q^\circ, 0) - \tau^\circ_{iR}(q^\circ), \\ \rho^\circ_R(q^\circ, q^\circ) - \tau^\circ_{iR}(q^\circ) &\geq \rho^\circ_R(\tilde{q}^\circ_{iR}, q^\circ) - \tau^\circ_{iR}(\tilde{q}^\circ_{iR}). \end{aligned}$$

Therefore:

$$\begin{aligned} & \rho^\circ_R(\tilde{q}^\circ_{iR}, 0) - \rho^\circ_R(q^\circ, 0) \geq \rho^\circ_R(\tilde{q}^\circ_{iR}, q^\circ) - \rho^\circ_R(q^\circ, q^\circ) \\ \Longleftrightarrow \quad & \int_{q^\circ}^{\tilde{q}^\circ_{iR}} \partial_1 \rho^\circ_R(x, 0)\, dx \geq \int_{q^\circ}^{\tilde{q}^\circ_{iR}} \partial_1 \rho^\circ_R(x, q^\circ)\, dx \\ \Longleftrightarrow \quad & \int_{q^\circ}^{\tilde{q}^\circ_{iR}} \int_0^{q^\circ} \partial^2_{12} \rho^\circ_R(x, y)\, dx dy \leq 0. \end{aligned}$$

As $q^\circ > 0$ and $\partial^2_{12}\rho^\circ_R = \partial^2_{12}\Pi^\circ_R < 0$ from the proof of Proposition 3, it follows that $\tilde{q}^\circ_{iR} \geq q^\circ$.

Assume now that $\tilde{q}^\circ_{iR} = q^\circ$, which implies $\tau^\circ_{iR}(q^\circ) = \tau^\circ_{iR}(\tilde{q}^\circ_{iR})$ and thus $\pi^\circ_{i,R} = \tilde{\pi}^\circ_{i,R}$; hence, from condition (10), both M_i and R are indifferent between R accepting both suppliers' equilibrium offers, or only M_i's offer. But then, M_i could profitably deviate to exclusivity by offering a forcing contract of the form $\left(\hat{q}, \hat{T}\right)$: by accepting this offer (and only that one), R would increase their bilateral profit from $\Pi^\circ_R(q^\circ, 0)$ to $\hat{\Pi}^\circ_R = \max_q \Pi^\circ_R(q, 0)$, which can then be shared by an appropriate $\hat{T}$ so as to ensure that both M_i and R benefit from the deviation. Therefore, $\tilde{q}^\circ_{iR} > q^\circ$. ■

Corollary 1 *There is no equilibrium in which a supplier offers a single forcing contract.*

Proof. This follows directly from Lemma 3, which implies that equilibrium contracts must offer at least two relevant options, q° and $\tilde{q}^\circ_{iR} \neq q^\circ$. ■

We next show that, for any equilibrium based on tariffs $\{\tau^\circ_{AR}(.), \tau^\circ_{BR}(.)\}$, there exists an equilibrium, yielding the same profits, in which each M_i offers a pair of forcing contracts:

Lemma 4 *Let* $\{\tau^\circ_{AR}(.), \tau^\circ_{BR}(.)\}$ *denote the tariffs signed by retailer* R *in a given equilibrium, with associated equilibrium profits* $\pi^\circ_{A,R}$*,* $\pi^\circ_{B,R}$ *and* $\pi^\circ_R = \Pi^\circ/2 - \pi^\circ_{A,R} - \pi^\circ_{B,R}$*, and let* $\tilde{\tau}^\circ_{iR} \equiv \{(q^\circ, \tau^\circ_{iR}(q^\circ)), (\tilde{q}^\circ_{iR}, \tau^\circ_{iR}(\tilde{q}^\circ_{iR}))\}$ *denote the corresponding pair of forcing contracts, respectively based on the equilibrium output level* q° *and on the output level* $\tilde{q}^\circ_{iR}$ *that* R *would choose under exclusivity with* M_i*. Then there exists an equilibrium in which each* M_i *offers the tariff* $\tilde{\tau}^\circ_{iR}$*, leading* R *to pick the forcing contract* $(q^\circ, \tau^\circ_{iR}(q^\circ))$*; this alternative equilibrium moreover yields the same profits* $\pi^\circ_{A,R}$*,* $\pi^\circ_{B,R}$ *and* π°_R*.*

Proof. Consider an equilibrium based on tariffs $\{\tau^\circ_{AR}(.), \tau^\circ_{BR}(.)\}$, and suppose that each supplier offers instead $\tilde{\tau}^\circ_{iR} = \{(q^\circ, \tau^\circ_{iR}(q^\circ)), (\tilde{q}^\circ_{iR}, \tau^\circ_{iR}(\tilde{q}^\circ_{iR}))\}$. By construction, R is willing to accept both offers, in which case it is willing to choose the "option" $(q^\circ, \tau^\circ_{iR}(q^\circ))$ from each $\tilde{\tau}^\circ_{iR}$; furthermore, from Lemma 3 R is indifferent between doing so and accepting only either M_i's offer, in which case it would choose the option $(\tilde{q}^\circ_{iR}, \tau^\circ_{iR}(\tilde{q}^\circ_{iR}))$. We now show that manufacturers have no incentive to deviate.

Without loss of generality, we can restrict attention to deviations in which the deviating manufacturer offers a single forcing contract. As $\pi^\circ_{i,R} \geq 0$ from Lemma 3, to be profitable the deviant offer must be accepted, either alone or in combination with one of the two options offered by M_j; as M_j's equilibrium contract offers contain, among other options, the options $\left(q^\circ, \tau^\circ_{jR}(q^\circ)\right)$ and $\left(\tilde{q}^\circ_j, \tau^\circ_{jR}\left(\tilde{q}^\circ_j\right)\right)$ this implies that the deviant offer would also be accepted in the original equilibrium, as R could then combine it with even more options. But then, as by construction there is no profitable deviation in the original equilibrium $\{\tau^\circ_{AR}(.), \tau^\circ_{BR}(.)\}$, there is no profitable deviation from $\{\tilde{\tau}^\circ_{AR}, \tilde{\tau}^\circ_{BR}\}$ either. ■

From now on, without loss of generality we will consider equilibria in which each M_i offers two options: that is, $\tau^\circ_{iR} = \left\{(q^\circ, T^\circ_{iR}), \left(\tilde{q}^\circ_{iR}, \tilde{T}^\circ_{iR}\right)\right\}$.

Lemma 5 *The contracts* $\left(\tau^\circ_i = \left\{(q^\circ, T^\circ_{iR}), \left(\tilde{q}^\circ_{iR}, \tilde{T}^\circ_{iR}\right)\right\}\right)_{i=A,B}$ *support an equilibrium if and only if the associated profits* $\left(\pi^\circ_{i,R} = T^\circ_{iR} - cq^\circ, \tilde{\pi}^\circ_{i,R} = \tilde{T}^\circ_{iR} - c\tilde{q}^\circ_{iR}\right)_{i=A,B}$ *and* $\pi^\circ_R = \Pi^\circ/2 - \pi^\circ_{A,R} - \pi^\circ_{B,R}$ *satisfy* (9), (10) *and*

$$\pi^\circ_{i,R} - \tilde{\pi}^\circ_{i,R} \leq \frac{\Pi^\circ}{2} - \Pi^r_R(\tilde{q}^\circ_{iR}), \tag{11}$$

where

$$\Pi^r_R(q_i) \equiv \max_q \Pi^\circ_R(q, q_i)$$

denotes the maximal aggregate profit that R *can generate, conditional on selling* q_i *units of good* i*.*

Proof. We first check that R is indeed willing to accept both contracts, and to pick the options $\{(q^\circ, T^\circ_{AR}), (q^\circ, T^\circ_{BR})\}$:

• From (10), R is willing to accept the offers, and is indifferent between accepting $\{(q^\circ, T^\circ_{AR}), (q^\circ, T^\circ_{BR})\}$, $\left\{\left(\tilde{q}^\circ_{AR}, \tilde{T}^\circ_{AR}\right)\right\}$, or $\left\{\left(\tilde{q}^\circ_{BR}, \tilde{T}^\circ_{BR}\right)\right\}$;

• In addition, R prefers accepting $\{(q^\circ, T^\circ_{AR}), (q^\circ, T^\circ_{BR})\}$, which yield π°_R, to accepting only (q°, T°_{iR}): This amounts to

$$\frac{\Pi^\circ}{2} - \pi^\circ_{A,R} - \pi^\circ_{B,R} > \Pi^\circ_R(q^\circ, 0) - \pi^\circ_{i,R},$$

or:

$$\pi^\circ_{j,R} < \frac{\Pi^\circ}{2} - \Pi^\circ_R(q^\circ, 0), \tag{12}$$

which follows from (9), as the RHS of (12) is strictly larger than Δ°.

• Using (10), the previous observation also implies that, if R were to accept M_i's contract only, then it would pick the option $\left(\tilde{q}^\circ_{iR}, \tilde{T}^\circ_{iR}\right)$ rather than (q°, T°_{iR}).

• Finally, R indeed prefers accepting $\{(q^\circ, T^\circ_{AR}), (q^\circ, T^\circ_{BR})\}$ to accepting $\left\{(q^\circ, T^\circ_{iR}), \left(\tilde{q}^\circ_j, \tilde{T}^\circ_{jR}\right)\right\}$: (11) implies that the joint profit of M_i and R is larger in the equilibrium configuration; as M_i is indifferent between the two scenarios (either way, it gets $\pi^\circ_{i,R} = T^\circ_{iR} - cq^\circ$), R must prefer sticking to $\{(q^\circ, T^\circ_{AR}), (q^\circ, T^\circ_{BR})\}$.

We now turn to deviations by the manufacturers:

• *M_i has no incentive to deviate by making an unacceptable offer (or no offer)*, as $\pi^\circ_{i,R} \geq 0$.

• *M_i has no incentive to deviate to exclusivity.* To see that, it suffices to note that, as R can secure its equilibrium profit by accepting M_j's offer only, to be profitable a deviation must increase the joint profit of M_i and R; but along the equilibrium path, this joint profit (gross of the profit that M_i makes with the other retailer) satisfies:

$$\pi^\circ_R + \pi^\circ_{i,R} = \frac{\Pi^\circ}{2} - \pi^\circ_{j,R} \geq \frac{\Pi^\circ}{2} - \Delta^\circ = \hat{\Pi}^\circ_R,$$

where $\hat{\Pi}^\circ_R$ is the maximal profit that can be achieved under exclusivity. Note that, as M_i could induce R to switch to exclusivity by slightly reducing $\tilde{T}^\circ_{iR}$, we must therefore have $\pi^\circ_{i,R} \geq \tilde{\pi}^\circ_{i,R}$.

• *M_i cannot profitably deviate by inducing R to combine the deviant offer with M_j's equilibrium option $\left(q^\circ, T^\circ_{jR}\right)$.* As the profit generated by R is maximal along the equilibrium path (that is, $\Pi^\circ/2 = \max_{q_A, q_B} \Pi^\circ_R(q_A, q_B)$), a deviation by M_i that induces R to combine the deviant offer with M_j's equilibrium option $\left(q^\circ, T^\circ_{jR}\right)$ cannot be profitable, as this would maintain M_j's profit at the equilibrium level (that is, M_j would obtain a $T^\circ_{jR} - cq^\circ = \pi^\circ_{j,R}$), and a deviation cannot lower R's profit either.

• *Finally, M_i cannot profitably deviate by inducing R to combine the deviant offer with M_j's alternative option* $\left(\tilde{q}_j^\circ, \tilde{T}_{jR}^\circ\right)$. As R can secure its equilibrium profit by accepting M_j's offer only, such a deviation could only be profitable if it increased the joint profit of M_i and R (gross of the profit that M_i makes with the other retailer), that is, only if :

$$\Pi_R^\circ\left(q_i, \tilde{q}_j^\circ\right) - \tilde{\pi}_{j,R}^\circ > \frac{\Pi^\circ}{2} - \pi_{j,R}^\circ,$$

which is ruled out by (11) (written for M_j). ■

We now turn to existence. We first note that relying on the bilateral efficient quantity $\tilde{q}_{iR}^\circ = \hat{q} = \arg\max_q \{\Pi_R^\circ(q, 0)\}$ for the "exclusive deal" option restricts somewhat the range of admissible profits, as (11) is then more demanding than (9): To see this, note that (10) yields

$$\tilde{\pi}_{i,R}^\circ = \Pi_R^\circ(\hat{q}, 0) + \pi_{A,R}^\circ + \pi_{B,R}^\circ - \frac{\Pi^\circ}{2},$$

so that (11) amounts to:

$$\begin{aligned} \pi_{i,R}^\circ &\leq \pi_{A,R}^\circ + \pi_{B,R}^\circ + \Pi_R^\circ(\hat{q}, 0) - \Pi_R^r(\hat{q}) \\ \Leftrightarrow \pi_{j,R}^\circ &\geq \Pi_R^r(\hat{q}) - \Pi_R^\circ(\hat{q}, 0) = \max_q \Pi_R^\circ(\hat{q}, q) - \Pi_R^\circ(\hat{q}, 0) > 0. \end{aligned}$$

We now show that M_j's equilibrium profit can however cover the full range $[0, \Delta^\circ]$ by relying on a "large enough" quantity $\tilde{q}_{iR}^\circ$ for M_i's exclusive deal option.

Lemma 6 *For any $\pi_{A,R}^\circ, \pi_{B,R}^\circ \in [0, \Delta^\circ]$, there exists an equilibrium yielding profits $\pi_{A,R}^\circ$, $\pi_{B,R}^\circ$ and $\pi_R^\circ = \Pi^\circ/2 - \pi_{A,R}^\circ - \pi_{B,R}^\circ$.*

Proof. We first note that the expression $\Pi_R^r(q) - \Pi_R^\circ(q, 0) = \max_{\tilde{q}} \Pi_R^\circ(q, \tilde{q}) - \Pi_R^\circ(q, 0)$ decreases as q increases. Using the envelope theorem, and letting $q^r(q) = \arg\max_{\tilde{q}} \Pi_R^\circ(q, \tilde{q})$ denotes R's "best response" to selling a quantity q of the other brand, we have:

$$\begin{aligned} \frac{d}{dq}\left[\Pi_R^r(q) - \Pi_R^\circ(q, 0)\right] &= \partial_1 \Pi_R^\circ(q, q^r(q)) - \partial_1 \Pi_R^\circ(q, 0) \\ &= \int_0^{q^r(q)} \partial_{12} \Pi_R^\circ(q, \tilde{q})\, d\tilde{q}, \end{aligned}$$

which is negative as long as $q^r(q) > 0$, as $\partial_{12}\Pi_R^\circ < 0$ from the proof of Proposition 3. From (A.1), $q^r(q) = 0$ for q large enough; let $\bar{q}$ denote the smallest such quantity,[18] which by construction is

[18] It is characterized by

$$P(q^\circ, q^\circ + \bar{q}) - c + \partial_2 P(q^\circ, q^\circ + \bar{q})\, \bar{q} = 0.$$

also the smallest quantity satisfying

$$\Pi_R^r(\bar{q}) = \Pi_R^\circ(\bar{q}, 0).$$

For $(\tilde{q}_{AR}^\circ, \tilde{q}_{BR}^\circ) \geq (\bar{q}, \bar{q})$, (10) yields

$$\tilde{\pi}_{i,R}^\circ = \Pi_R^\circ(\tilde{q}_{iR}^\circ, 0) + \pi_{A,R}^\circ + \pi_{B,R}^\circ - \frac{\Pi^\circ}{2}.$$

Hence (11) becomes:

$$\begin{aligned}
\pi_{i,R}^\circ - \tilde{\pi}_{i,R}^\circ &= \pi_{i,R}^\circ + \frac{\Pi^\circ}{2} - \left(\Pi_R^\circ(\tilde{q}_{iR}^\circ, 0) + \pi_{A,R}^\circ + \pi_{B,R}^\circ\right) = \frac{\Pi^\circ}{2} - \Pi_R^\circ(\tilde{q}_{iR}^\circ, 0) - \pi_{j,R}^\circ \leq \frac{\Pi^\circ}{2} - \Pi_R^r(\tilde{q}_{iR}^\circ) \\
\iff & \pi_{j,R}^\circ \geq \Pi_R^r(\tilde{q}_{iR}^\circ) - \Pi_R(\tilde{q}_{iR}^\circ, 0) = 0,
\end{aligned}$$

and thus follows from (9). Therefore, from Lemma 5, contracts of the form $\left(\tau_i^\circ = \left\{(q^\circ, T_{iR}^\circ), \left(\tilde{q}_{iR}^\circ, \tilde{T}_{iR}^\circ\right)\right\}\right)_{i=A,B}$, where $\tilde{q}_{iR}^\circ \geq \bar{q}$, support an equilibrium if and only if the associated profits satisfy conditions (9) and (10).

Conversely, for any $\pi_{A,R}^\circ, \pi_{B,R}^\circ \in [0, \Delta^\circ]$, the contracts $\left(\tau_i^\circ = \left\{(q^\circ, T_{iR}^\circ), \left(\tilde{q}_{iR}^\circ, \tilde{T}_{iR}^\circ\right)\right\}\right)_{i=A,B}$, where

$$\begin{aligned}
T_{iR}^\circ &= cq^\circ + \pi_{i,R}^\circ, \\
\tilde{q}_{iR}^\circ &\geq \bar{q}, \\
\tilde{T}_{iR}^\circ &= cq^\circ + \tilde{\pi}_{i,R}^\circ, \text{ where } \tilde{\pi}_{i,R}^\circ = \Pi_R^\circ(\tilde{q}_{iR}^\circ, 0) + \pi_{A,R}^\circ + \pi_{B,R}^\circ - \frac{\Pi^\circ}{2},
\end{aligned}$$

support an equilibrium with profits $\left(\pi_{A,R}^\circ, \pi_{B,R}^\circ\right)$ for the manufacturers and $\pi_R^\circ = \Pi^\circ/2 - \pi_{A,R}^\circ - \pi_{B,R}^\circ$ for R. ■

Finally, we have:

Lemma 7 *There exist equilibria in which each M_i, too, is indifferent between R accepting $\{(q^\circ, T_{AR}^\circ), (q^\circ, T_{BR}^\circ)\}$ or $\left\{\left(\tilde{q}_j^\circ, \tilde{T}_{jR}^\circ\right)\right\}$, and these equilibria yield the same equilibrium profits as the equilibrium in two-part tariffs, namely, M_A and M_B both obtain their full contribution to industry profits: $\pi_{A,R}^\circ = \pi_{B,R}^\circ = \Delta^\circ$.*

Proof. Suppose $\pi_{i,R}^\circ = \tilde{\pi}_{i,R}^\circ$. Together with conditions (10), this yields $\pi_R^\circ + \pi_{i,R}^\circ = \Pi_R^\circ(\tilde{q}_{iR}^\circ, 0)$ and $\pi_{j,R}^\circ = \Pi^\circ/2 - \Pi_R^\circ(\tilde{q}_{iR}^\circ, 0)$. The first equality implies $\Pi_R^\circ(\tilde{q}_{iR}^\circ, 0) = \hat{\Pi}_R$ (and thus $\tilde{q}_{iR}^\circ = \hat{q}$), as otherwise M_i would have a profitable deviation to exclusivity. The second equality then implies $\pi_{j,R}^\circ = \Pi^\circ/2 - \hat{\Pi}_R = \Delta^\circ$.

Conversely, the contracts $(\tau_i^\circ = \{(q^\circ, cq^\circ + \Delta^\circ), (\hat{q}, c\hat{q} + \Delta^\circ)\})_{i=A,B}$ support an equilibrium in which each M_i obtains $\pi_{i,R}^\circ = \tilde{\pi}_{i,R}^\circ = \Delta^\circ$. ■

E Proof of Proposition 5

We start with the characterization of the equilibrium contracts and quantities, before establishing the existence of an equilibrium in two-part tariffs.

That contracts are cost-based follows from Lemma 1. We now show that, under Assumptions (A.1)-(A.4), the equilibrium quantities $(q^*_{A1}, q^*_{B1}, q^*_{B2})$ are uniquely defined and all positive. The quantities are those that would be induced in a duopoly game where firm 1's problem is given by

$$\max_{q_{A1}, q_{B1}} \left[P(q_{A1}, q_{B1} + q_{B2}) - c\right] q_{A1} + \left[P(q_{B1} + q_{B2}, q_{A1}) - c\right] q_{B1},$$

and firm 2's problem by

$$\max_{q_{B2}} \left[P(q_{B1} + q_{B2}, q_{A1}) - c\right] q_{B2}.$$

The first order conditions are given by:

$$\begin{aligned} P(Q^*_A, Q^*_B) - c + q^*_{A1}\partial_1 P(Q^*_A, Q^*_B) + q^*_{B1}\partial_2 P(Q^*_A, Q^*_B) &\leq 0, \\ P(Q^*_B, Q^*_A) - c + q^*_{B1}\partial_1 P(Q^*_B, Q^*_A) + q^*_{A1}\partial_2 P(Q^*_B, Q^*_A) &\leq 0, \\ P(Q^*_B, Q^*_A) - c + q^*_{B2}\partial_1 P(Q^*_B, Q^*_A) &\leq 0, \end{aligned}$$

where the first-order condition of quantity q^*_{ik} holds with equality if $q^*_{ik} > 0$, and where $Q^*_A \equiv q^*_{A1}$ and $Q^*_B \equiv q^*_{B1} + q^*_{B2}$.

We first establish that all quantities are indeed positive, implying that the first-order conditions hold with equality.

(i) $P(Q^*_B, Q^*_A) > c$. To see this, suppose otherwise that $P(Q^*_B, Q^*_A) \leq c$. Then, the first-order condition of q^*_{B2} implies that $q^*_{B2} = 0$. Moreover, we must also have $q^*_{B1} = 0$; if not, firm 1 could profitably deviate by reducing q_{B1}. Hence, $Q^*_B = 0 \leq Q^*_A$. By (A.2), we thus have $P(Q^*_A, Q^*_B) \leq c$, implying that $q^*_{A1} = 0$ (otherwise, firm 1 could profitably deviate by reducing q_{A1}). Hence, $P(0, 0) \leq c$, contradicting (A.1).

(ii) $q^*_{B2} > 0$. As $P(Q^*_B, Q^*_A) > c$, firm 2 could otherwise profitably deviate by choosing $q_{B2} > 0$ small enough such that $P(Q^*_B + q_{B2}, Q^*_A) > c$.

(iii) $P(Q^*_A, Q^*_B) > c$. To see this, suppose otherwise that $P(Q^*_A, Q^*_B) \leq c$. It follows that $q^*_{A1} = 0$. (If not, firm 1 could profitably deviate by setting $q_{A1} = 0$; if $q^*_{B1} = 0$, firm could combine this deviation by choosing $q_{B1} > 0$ sufficiently small such that $P(Q^*_B + q_{B1}, Q^*_A - q^*_{A1}) > c$.) Hence, $Q^*_A = 0 \leq Q^*_B$. By (A.2), we thus have $P(Q^*_A, Q^*_B) \geq P(Q^*_B, Q^*_A) > c$, a contradiction.

(iv) $q_{A1}^* > 0$. Suppose otherwise that $q_{A1}^* = 0$. We have:

$$\begin{aligned} 0 &\geq P(Q_A^*, Q_B^*) - c + q_{B1}^* \partial_2 P(Q_A^*, Q_B^*) \\ &> P(Q_B^*, Q_A^*) - c + q_{B1}^* \partial_1 P(Q_B^*, Q_A^*), \end{aligned}$$

where the first inequality follows from $q_{A1}^* = 0$, and the second by (A.2) and $Q_B^* > Q_A^*$. It follows that $q_{B1}^* = 0$. But as $P(Q_B^*, Q_A^*) > c$, firm 1 could then profitably deviate by setting $q_{B1} > 0$ sufficiently small such that $P(Q_B^* + q_{B1}, Q_A^*) > c$.

(v) $q_{B1}^* > 0$. Suppose otherwise that $q_{B1}^* = 0$. The induced outcome thus coincides with the equilibrium outcome in a duopoly in which firm 1 sells only good A and firm 2 sells only good B. Under Assumption (A.3), this implies that $Q_A^* = Q_B^*$, as shown in Proposition 6. We thus have:

$$\begin{aligned} P(Q_B^*, Q_A^*) - c + Q_A^* \partial_2 P(Q_B^*, Q_A^*) &> P(Q_A^*, Q_B^*) - c + Q_A^* \partial_1 P(Q_A^*, Q_B^*) \\ &= 0, \end{aligned}$$

where the inequality follows from (A.2) and the equality from the first-order condition of $q_{A1}^* = Q_A^*$. But then firm 1 could profitably deviate by slightly raising q_{B1}.

Having established that all quantities $(q_{A1}^*, q_{B1}^*, q_{B2}^*)$ are strictly positive, implying that each of the three first-order conditions holds with equality, we now show that the quantities are unique.

Let

$$\hat{c} \equiv P(Q_A^*, Q_B^*) + q_{B2}^* \partial_1 P(Q_A^*, Q_B^*)$$

denote the marginal cost level that would induce firm 2 to produce just zero units of good A if it could produce that good at marginal cost $\widehat{c}$, given that aggregate outputs are (Q_A^*, Q_B^*). Equilibrium quantities are thus characterized by the following system of equations:

$$P(Q_A^*, Q_B^*) - c + q_{A1}^* \partial_1 P(Q_A^*, Q_B^*) + q_{B1}^* \partial_2 P(Q_B^*, Q_A^*) = 0, \tag{13}$$

$$P(Q_B^*, Q_A^*) - c + q_{B1}^* \partial_1 P(Q_B^*, Q_A^*) + q_{A1}^* \partial_2 P(Q_A^*, Q_B^*) = 0, \tag{14}$$

$$P(Q_B^*, Q_A^*) - c + q_{B2}^* \partial_1 P(Q_B^*, Q_A^*) = 0, \tag{15}$$

$$P(Q_A^*, Q_B^*) - \hat{c} + q_{B2}^* \partial_2 P(Q_B^*, Q_A^*) = 0. \tag{16}$$

Adding equations (14) and (15), we obtain:

$$\phi(Q_B^*, Q_A^*; c) \equiv 2\left[P(Q_B^*, Q_A^*) - c\right] + Q_B^* \partial_1 P(Q_B^*, Q_A^*) + Q_A^* \partial_2 P(Q_A^*, Q_B^*) = 0. \tag{17}$$

Similarly, adding equations (13) and (16) yields:

$$\phi(Q_A^*, Q_B^*; \widehat{c}) \equiv 2P(Q_A^*, Q_B^*) - c - \hat{c} + Q_A^* \partial_1 P(Q_A^*, Q_B^*) + Q_B^* \partial_2 P(Q_B^*, Q_A^*) = 0. \quad (18)$$

Let $R(Q_A)$ be such that $\phi(R(Q_A), Q_A; c) = 0$, and $\hat{R}(Q_B, \hat{c})$ be such that $\phi(\hat{R}(Q_B, \hat{c}), Q_B; \hat{c}) = 0$. Hence, aggregate equilibrium outputs (Q_A^*, Q_B^*) satisfy $Q_B^* = \hat{R}(Q_A^*, \hat{c})$ and $Q_A^* = R(Q_B^*)$. Using the implicit function theorem, we have for $i \neq j \in \{A, B\}$,

$$R'(Q_i) = \partial_1 \hat{R}(Q_i, \hat{c}) = -\frac{2\partial_2 P(Q_j, Q_i) + Q_j \partial_{12}^2 P(Q_j, Q_i) + \partial_2 P(Q_i, Q_j) + Q_i \partial_{12}^2 (Q_i, Q_j)}{3\partial_1 P(Q_j, Q_i) + Q_j \partial_{11}^2 P(Q_j, Q_i) + Q_i \partial_{22}^2 P(Q_i, Q_j)}.$$

(A.2) and (A.4) imply that $R'(Q_i) = \partial_1 \hat{R}(Q_i, \hat{c}) \in (-1, 0)$. It follows that the aggregate equilibrium outputs (Q_A^*, Q_B^*) are unique. From the first-order conditions the individual outputs are unique as well.

We now show that there exists an equilibrium in which all equilibrium contracts are cost-based two-part tariffs of the form $\left(w_{i,h}^*, F_{i,h}^*\right) = \left(c, \Delta_{i,h}^*\right)$, where $\Delta_{i,h}^*$ denotes M_i's contribution to the profit generated by R_h, namely:

$$\begin{aligned}
\Delta_{A,1}^* &= (p_A^* - c)\, q_{A1}^* + (p_B^* - c)\, q_{B1}^* - \max_{q_{B1}} \left\{ \left[P\left(q_{B1} + q_{B2}^*, 0\right) - c \right] q_{B1} \right\}, \\
\Delta_{B,1}^* &= (p_A^* - c)\, q_{A1}^* + (p_B^* - c)\, q_{B1}^* - \max_{q_{A1}} \left\{ \left[P\left(q_{A1}, q_{B2}^*\right) - c \right] q_{A1} \right\}, \\
\Delta_{B,2}^* &= (p_B^* - c)\, q_{B2}^*.
\end{aligned}$$

We first note that the continuation equilibrium is then such that R_1 sells $(q_{A1}, q_{B1}) = (q_{A1}^*, q_{B1}^*)$ and R_2 sells $q_{B2} = q_{B2}^*$. From the first part of this proof, this constitutes the unique candidate equilibrium, and it satisfies all first-order conditions. And going through the same steps as in the proof of Proposition 3, it is easy to check that retailers' profit functions are again concave.[19]

Next, we note that each retailer is willing to accept all offers made. Indeed, the fees are such that each, anticipating this behavior for its rival:

- R_1 is indifferent between accepting both manufacturers' offers or either one (either one);
- R_2 is indifferent between accepting M_B's offer or not.

[19] It suffices to note that retailers' profit functions are similar to the previous function $\Pi_R(q_A, q_B)$, replacing the rival's equilibrium quantities (q°, q°) with the new equilibrium quantities (q_{A1}^*, q_{B1}^*) or q_{B2}^*.

It thus suffices to check that R_1 is strictly better-off accepting the manufacturers' offers rather than rejecting both of them; indeed, we have:

$$\begin{aligned}
\pi_1^* &= (p_A^* - c)\, q_{A1}^* + (p_B^* - c)\, q_{B1}^* - \Delta_{A,1}^* - \Delta_{B,1}^* \\
&= \max_{q_{A1}} \{[P(q_{A1}, q_{B2}^*) - c]\, q_{A1}\} + \max_{q_{B1}} \{[P(q_{B1} + q_{B2}^*, 0) - c]\, q_{B1}\} \\
&\quad - [(p_A^* - c)\, q_{A1}^* + (p_B^* - c)\, q_{B1}^*] \\
&= \max_{q_{A1}} \{[P(q_{A1}, q_{B2}^*) - c]\, q_{A1}\} - [P(q_{A1}^*, q_{B1}^* + q_{B2}^*) - c]\, q_{A1}^* \\
&\quad + \max_{q_{B1}} \{[P(q_{B1} + q_{B2}^*, 0) - c]\, q_{B1}\} - [P(q_{B1}^* + q_{B2}^*, q_{A1}^*) - c]\, q_{B1}^* \\
&> \max_{q_{A1}} \{[P(q_{A1}, q_{B2}^*) - c]\, q_{A1}\} - [P(q_{A1}^*, q_{B2}^*) - c]\, q_{A1}^* \\
&\quad + \max_{q_{B1}} \{[P(q_{B1} + q_{B2}^*, 0) - c]\, q_{B1}\} - [P(q_{B1}^* + q_{B2}^*, 0) - c]\, q_{B1}^* \\
&\geq 0.
\end{aligned}$$

Thus, if these contracts are offered, it is a continuation equilibrium for retailers to accept all offers, and then to sell the equilibrium quantities identified above. We now show that manufacturers cannot profitably deviate from this candidate equilibrium.

We first note that the above tariffs are profitable for the manufacturers, as each manufacturer contributes positively to the profits generated by the retailers.[20] It follows that a deviation cannot be profitable if it is not accepted by the retailer. But then, M_B cannot profitably deviate in its offer to R_2, as it already appropriates all the profit that R_2 can generate. Likewise, no M_i can profitably deviate in its dealing with R_1, as: (i) M_i and R_1 cannot increase their joint profit above the equilibrium level, as M_j does not obtain more than its contribution to the profit generated by R_1; and (ii) following a deviation by M_i, R_1 can still secure its equilibrium profit by accepting only M_j's offer.

F Proof of Proposition 6

We first prove the second part of the Proposition, before establishing the existence of an equilibrium based on two-part tariffs.

[20] For instance, $\Delta_{A,1}^*$ can be expressed as

$$\begin{aligned}
\Delta_{A,1}^* &= \{[P(q_{A1}^*, q_{B1}^* + q_{B2}^*) - c]\, q_{A1}^* + [P(q_{B1}^* + q_{B2}^*, 0) - c]\, q_{B1}^*\} - \max_{q_{B1}} \{[P(q_{B1} + q_{B2}^*, 0) - c]\, q_{B1}\} \\
&= \max_{q_{A1}, q_{B1}} \{[P(q_{A1}, q_{B1} + q_{B2}^*) - c]\, q_{A1} + [P(q_{B1} + q_{B2}^*, 0) - c]\, q_{B1}\} - \max_{q_{B1}} \{[P(q_{B1} + q_{B2}^*, 0) - c]\, q_{B1}\},
\end{aligned}$$

which is positive as $q_{A1}^* > 0$.

In the second part of the Proposition, the assertion on cost-based contracts is an immediate implication of Lemma 1. And as retailers would obtain zero profit by rejecting the offers made by the manufacturers, in equilibrium the manufacturers fully appropriate the profit generated by their goods.

To see that (A.3) implies that the equilibrium quantities are unique and symmetric, note that the first-order condition of the retailer carrying manufacturer M_i's good is

$$\Psi(Q_i, Q_j) \equiv P(Q_i, Q_j) - c + \partial_1 P(Q_i, Q_j) Q_i = 0.$$

We have $\Psi(0, Q_j) = P(0, Q_j) - c$, $\partial_1 \Psi(Q_i, Q_j) = 2\partial_1 P(Q_i, Q_j) + Q_i \partial_{11}^2 P(Q_i, Q_j) \leq 0$ (with strict inequality if $P(Q_i, Q_j) > 0$) by (A.2), and $\Psi(Q_i, Q_j) < 0$ for Q_i sufficiently large by (A.1). Hence, the best-response to M_j selling Q_j units of output, $\chi^E(Q_j)$, is given by $\chi^E(Q_j) = 0$ if $P(0, Q_j) < c$, and by the unique solution to $\Psi(\chi^E(Q_j), Q_j) = 0$ otherwise. For Q_j such that $P(0, Q_j) \geq c$, we have

$$\frac{d\chi^E}{dQ_j}(Q_j) = -\frac{\partial_2 \Psi(Q_i, Q_j)}{\partial_1 \Psi(Q_i, Q_j)} = -\frac{\partial_2 P(Q_i, Q_j) + Q_i \partial_{12}^2 P(Q_i, Q_j)}{2\partial_1 P(Q_i, Q_j) + Q_i \partial_{11}^2 P(Q_i, Q_j)}.$$

(A.3) implies that $-1 < d\chi^E / dQ_j < 0$, which in turn implies that an equilibrium, if it exists, is unique and symmetric. To establish existence, it suffices to note that $\Psi(Q, Q)$ is continuous in Q, and satisfies $\Psi(0, 0) > 0$ and, from (A.1), $\Psi(Q, Q) < 0$ for Q sufficiently large. Hence, there exists Q^{**} such that $\Psi(Q^{**}, Q^{**}) = 0$.

We now turn to the first part of the Proposition, and consider a candidate equilibrium in which both manufacturers offer the cost-based two-part tariff $(w^{**}, F^{**}) = (c, \Pi_R^{**})$, where

$$\Pi_R^{**} = \left[P\left(Q^{**}, Q^{**}\right) - c\right] Q^{**}$$

denotes the profit generated by a retailer. The retailers are willing to accept those contracts, in which case they each put Q^{**} on the market and break even. Furthermore, each manufacturer obtains all the profits generated by its good, which is moreover maximal given the output level Q^{**} of the other good; it follows that there is no profitable deviation.

G Sufficient Conditions for Properties (P.1)-(P.3)

We provide sufficient assumptions on the inverse demand which, together with (A.1) and (A.2), yield Properties (P.1)-(P.3). In particular, throughout this section we will rely on the following assumption:

(B.1) For any $Q_i, Q_j \geq 0$ such that $P(Q_i, Q_j) > 0$, and for any $(q_i, q_j) \in [0, Q_i] \times [0, Q_j]$, we have:

$$\begin{aligned} & 2\partial_1 P(Q_i, Q_j) + q_i \partial_{11}^2 P(Q_i, Q_j) + q_j \partial_{22}^2 P(Q_j, Q_i) \\ < \ & \partial_2 P(Q_i, Q_j) + q_i \partial_{12}^2 P(Q_i, Q_j) + q_j \partial_{12}^2 P(Q_j, Q_i) \\ < \ & 0. \end{aligned}$$

In the case of linear demand, (B.1) simplifies to $2\partial_1 P < \partial_2 P < 0$, and is thus implied by (A.2).

G.1 Property (P.1)

We first show that the above assumption yields existence and uniqueness:

Proposition 12 *Under Assumption (B.1), the game Γ_{1-1} has a unique Nash equilibrium $(\tilde{q}_{A1}, \tilde{q}_{B2})$.*

Proof. The derivative of firm i's profit, $\hat{\Pi}_i$, with respect to q_{ih} is (for $ih \neq jk \in \{A1, B2\}$):

$$\Phi(q_{ih}; q_{jk}, \hat{q}_{jh}, \hat{q}_{ik}) \equiv P(q_{ih} + \hat{q}_{ik}, \hat{q}_{jh} + q_{jk}) - c + q_{ih}\partial_1 P(q_{ih} + \hat{q}_{ik}, \hat{q}_{jh} + q_{jk}) + \hat{q}_{jh}\partial_2 P(\hat{q}_{jh} + q_{jk}, q_{ih} + \hat{q}_{ik}),$$

and is thus such that:

$$\frac{d\Phi}{dq_{ih}}(q_{ih}; q_{jk}, \hat{q}_{jh}, \hat{q}_{ik}) = 2\partial_1 P(q_{ik} + \hat{q}_{il}, \hat{q}_{jk} + q_{jl}) + q_{ik}\partial_{11}^2 P(q_{ik} + \hat{q}_{il}, \hat{q}_{jk} + q_{jl}) + \hat{q}_{jk}\partial_{22}^2 P(\hat{q}_{jk} + q_{jl}, q_{ik} + \hat{q}_{il}).$$

From (A.2), $\Phi(q_{ih}; q_{jk}, \hat{q}_{jh}, \hat{q}_{ik}) = 0$ implies $P(q_{ih} + \hat{q}_{ik}, \hat{q}_{jh} + q_{jk}) \geq c\,(> 0)$, and thus, from (B.1), $\frac{d\Phi}{dq_{ih}}(q_{ih}; q_{jk}, \hat{q}_{jh}, \hat{q}_{ik}) < 0$; it follows that firm i's best-response

$$r(q_{jk}; \hat{q}_{B1}, \hat{q}_{A2}) = \arg\max_{q_{ih}} \hat{\Pi}_i(q_{A1}, q_{B2}; \hat{q}_{B1}, \hat{q}_{A2})$$

is single-valued. It is moreover positive whenever

$$P(\hat{q}_{ik}, \hat{q}_{jh} + q_{jk}) > c - \hat{q}_{jh}\partial_2 P(\hat{q}_{jh} + q_{jk}, \hat{q}_{ik}),$$

in which case it is characterized by the first-order condition $\Phi(q_{ih}; q_{jk}, \hat{q}_{jh}, \hat{q}_{ik}) = 0$. Differentiating this first-order condition with respect to q_{ih} and q_{jk} then yields:

$$\frac{dr}{dq_{jk}}(q_{jk}; \hat{q}_{jh}, \hat{q}_{ik}) = -\frac{\lambda_i}{\mu_i}, \tag{19}$$

where

$$\lambda_i \equiv \left. \frac{2\partial_1 P\left(q_{ih}+\hat{q}_{ik},\hat{q}_{jh}+q_{jk}\right)}{+q_{ih}\partial_{11}^2 P\left(q_{ih}+\hat{q}_{ik},\hat{q}_{jh}+q_{jk}\right)+\hat{q}_{jh}\partial_{22}^2 P\left(\hat{q}_{jh}+q_{jk},q_{ih}+\hat{q}_{ik}\right)} \right|_{q_{ih}=r(q_{jk};\hat{q}_{jh},\hat{q}_{ik})},$$

$$\mu_i \equiv \left. \frac{\partial_2 P\left(q_{ih}+\hat{q}_{ik},\hat{q}_{jh}+q_{jk}\right)}{+q_{ih}\partial_{12}^2 P\left(q_{ih}+\hat{q}_{ik},\hat{q}_{jh}+q_{jk}\right)+\hat{q}_{jh}\partial_{12}^2 P\left(\hat{q}_{jh}+q_{jk},q_{ih}+\hat{q}_{ik}\right)} \right|_{q_{ih}=r(q_{jk};\hat{q}_{jh},\hat{q}_{ik})}.$$

As$\lambda_i<\mu_i<0$ from (B.1), (19) implies

$$-1<\frac{dr}{dq_{jk}}<0.$$

Hence there exists a unique Nash equilibrium, which is moreover "stable" in the usual sense. ■

G.2 Property (P.2)

Let

$$Q^{**} \equiv \arg\max_{Q}\left[P(Q,Q^{**})-c\right]Q$$

denote the "duopoly" equilibrium output per good in game Γ_{1-1} when $\hat{q}_{A2}=\hat{q}_{B1}=0$. (From Proposition 12, we know that Q^{**} exists and is unique.)

We now provide an additional condition on demand that ensures that an increase in either $\hat{q}_{B1}$ or $\hat{q}_{A2}$ increases the total output of goods A and B. We then show that any such increase in aggregate output beyond that of the duopoly outcome (Q^{**},Q^{**}) reduces aggregate profit, implying (P.2).

(B.2) For any $Q_i,Q_j\geq 0$ such that $P\left(Q_i,Q_j\right)>0$ and $P\left(Q_j,Q_i\right)>0$, and for any $q_i\in[0,Q_i]$, we have

$$\begin{aligned} & 2\partial_1 P(Q_i,Q_j)+q_i\partial_{11}^2 P(Q_i,Q_j)+Q_j\partial_{22}^2 P\left(Q_j,Q_i\right) \\ < \; & \partial_2 P(Q_i,Q_j)+\partial_2 P(Q_j,Q_i)+q_i\partial_{12}^2 P(Q_i,Q_j)+Q_j\partial_{12}^2 P\left(Q_j,Q_i\right), \end{aligned} \quad \text{(B.2.a)}$$

and in addition, for any $q_j\in[0,Q_j]$:

$$\begin{aligned} & \partial_1 P\left(Q_i,Q_j\right)\begin{bmatrix} 2\partial_1 P\left(Q_j,Q_i\right)-\partial_2 P\left(Q_j,Q_i\right) \\ +q_j\left(\partial_{11}^2 P\left(Q_j,Q_i\right)-\partial_{12}^2 P\left(Q_j,Q_i\right)\right) \\ +q_i\left(\partial_{22}^2 P\left(Q_i,Q_j\right)-\partial_{12}^2 P\left(Q_i,Q_j\right)\right) \end{bmatrix} \\ > \; & \partial_2 P\left(Q_i,Q_j\right)\begin{bmatrix} 2\partial_1 P\left(Q_i,Q_j\right)-\partial_2 P\left(Q_i,Q_j\right) \\ +\left[Q_i-q_i\right]\left(\partial_{11}^2 P\left(Q_i,Q_j\right)-\partial_{12}^2 P\left(Q_i,Q_j\right)\right) \\ +\left[Q_j-q_j\right]\left(\partial_{22}^2 P\left(Q_j,Q_i\right)-\partial_{12}^2 P\left(Q_j,Q_i\right)\right) \end{bmatrix}. \end{aligned} \quad \text{(B.2.b)}$$

In the case of linear demand (B.2) simplifies to $\partial_1 P < \partial_2 P$ and $(\partial_1 P - \partial_2 P)(2\partial_1 - \partial_2 P) > 0$, and is thus implied by (A.2). We now show that (B.1) and (B.2) together imply that any increase in $\hat{q}_{il}$, for $il \in \{A2, B1\}$, increases total output:

Lemma 8 *Let $\tilde{Q}_A$ (resp., $\tilde{Q}_B$) denote the total equilibrium output of good A (resp., good B) in game Γ_{1-1}. If (B.1) and (B.2) hold, then an increase in either $\hat{q}_{A2}$ or $\hat{q}_{B1}$ leads to a strict increase in the total equilibrium output $\tilde{Q}_A + \tilde{Q}_B$.*

Proof. The claim is obvious (although in a weak sense, for the "decreasing" part) when $\tilde{q}_{A1} = \tilde{q}_{B2} = 0$, as then $\tilde{Q}_A = \hat{q}_{A2}$ and $\tilde{Q}_B = \hat{q}_{B1}$. Consider now the case where $\tilde{q}_{ih} > 0$ whereas $\tilde{q}_{jk} = 0$, for $ih \neq jk \in \{A1, B2\}$. We then have $\tilde{Q}_j = \hat{q}_{jh}$, and thus $\frac{\partial \tilde{Q}_j}{\partial \hat{q}_{ik}} = 0, \frac{\partial \tilde{Q}_j}{\partial \hat{q}_{jh}} = 1$. Turning to $\tilde{Q}_i = \tilde{q}_{ih} + \hat{q}_{ik}$, the first-order condition for $\tilde{q}_{ih}$ is:

$$P(\tilde{q}_{ih} + \hat{q}_{ik}, \hat{q}_{jh}) - c + \tilde{q}_{ih}\partial_1 P(\tilde{q}_{ih} + \hat{q}_{ik}, \hat{q}_{jh}) + \hat{q}_{jh}\partial_2 P(\hat{q}_{jh}, \tilde{q}_{ih} + \hat{q}_{ik}) = 0,$$

or:

$$P\left(\tilde{Q}_i, \hat{q}_{jh}\right) - c + \tilde{Q}_i \partial_1 P\left(\tilde{Q}_i, \hat{q}_{jh}\right) = \hat{q}_{ik}\partial_1 P\left(\tilde{Q}_i, \hat{q}_{jh}\right) - \hat{q}_{jh}\partial_2 P\left(\hat{q}_{jh}, \tilde{Q}_i\right).$$

Differentiating this equation with respect to $\tilde{Q}_i$ and $\hat{q}_{jk}$ yields:

$$\hat{\lambda}_i d\tilde{Q}_i = \partial_1 P\left(\tilde{Q}_i, \hat{q}_{jh}\right) d\hat{q}_{ik} - \left[\partial_2 P\left(\hat{q}_{jh}, \tilde{Q}_i\right) + \hat{\mu}_i\right] d\hat{q}_{jh},$$

where

$$\begin{aligned}
\hat{\lambda}_i &= 2\partial_1 P\left(\tilde{Q}_i, \hat{q}_{jh}\right) + \tilde{q}_{ih}\partial_{11}^2 P\left(\tilde{Q}_i, \hat{q}_{jh}\right) + \hat{q}_{jh}\partial_{22}^2 P\left(\hat{q}_{jh}, \tilde{Q}_i\right), \\
\hat{\mu}_i &= \partial_2 P\left(\tilde{Q}_i, \hat{q}_{jh}\right) + \tilde{q}_{ih}\partial_{12}^2 P\left(\tilde{Q}_i, \hat{q}_{jh}\right) + \hat{q}_{jh}\partial_{12}^2 P\left(\hat{q}_{jh}, \tilde{Q}_i\right).
\end{aligned}$$

Assumptions (B.1) yields $\hat{\lambda}_i < 0$, and thus:

$$\begin{aligned}
\frac{\partial \tilde{Q}_i}{\partial \hat{q}_{ik}} &= \frac{\partial_1 P\left(\tilde{Q}_i, \hat{q}_{jh}\right)}{\hat{\lambda}_i}, \\
\frac{\partial \tilde{Q}_i}{\partial \hat{q}_{jh}} &= -\frac{\partial_2 P\left(\tilde{Q}_j, \hat{q}_{jh}\right) + \tilde{\mu}_i}{\hat{\lambda}_i}.
\end{aligned} \tag{20}$$

Therefore, as $\tilde{Q}_j = \hat{q}_{jh}$:

$$\begin{aligned}
\frac{\partial\left(\tilde{Q}_A + \tilde{Q}_B\right)}{\partial \hat{q}_{ik}} &= \frac{\partial \tilde{Q}_i}{\partial \hat{q}_{ik}} = \frac{\partial_1 P\left(\tilde{Q}_i, \hat{q}_{jh}\right)}{\hat{\lambda}_i} > 0, \\
\frac{\partial\left(\tilde{Q}_A + \tilde{Q}_B\right)}{\partial \hat{q}_{jh}} &= 1 + \frac{\partial \tilde{Q}_i}{\partial \hat{q}_{jh}} = 1 - \frac{\partial_2 P\left(\hat{q}_{jh}, \tilde{Q}_i\right) + \tilde{\mu}_i}{\hat{\lambda}_i} > 0,
\end{aligned}$$

where the inequalities stems from $\hat{\lambda}_i < 0$ and Assumptions (A.2) and (B.2.a), which yield $\partial_1 P\left(\tilde{Q}_i, \hat{q}_{jh}\right) < 0$ and $\hat{\lambda}_i < \partial_2 P\left(\hat{q}_{jh}, \tilde{Q}_i\right) + \hat{\mu}_i$.

Let us now consider the case where $\tilde{q}_{A1}, \tilde{q}_{B2} > 0$, and are thus characterized by the first-order conditions:

$$\begin{aligned}
P(\tilde{q}_{A1} + \hat{q}_{A2}, \hat{q}_{B1} + \tilde{q}_{B2}) - c + \tilde{q}_{A1}\partial_1 P(\tilde{q}_{A1} + \hat{q}_{A2}, \hat{q}_{B1} + \tilde{q}_{B2}) + \hat{q}_{B1}\partial_2 P(\hat{q}_{B1} + \tilde{q}_{B2}, \tilde{q}_{A1} + \hat{q}_{A2}) &= 0, \\
P(\hat{q}_{B1} + \tilde{q}_{B2}, \tilde{q}_{A1} + \hat{q}_{A2}) - c + \tilde{q}_{B2}\partial_1 P(\hat{q}_{B1} + \tilde{q}_{B2}, \tilde{q}_{A1} + \hat{q}_{A2}) + \hat{q}_{A2}\partial_2 P(\tilde{q}_{A1} + \hat{q}_{A2}, \hat{q}_{B1} + \tilde{q}_{B2}) &= 0,
\end{aligned}$$

or, in terms of total equilibrium outputs $\tilde{Q}_A = \tilde{q}_{A1} + \hat{q}_{A2}$ and $\tilde{Q}_B = \hat{q}_{B1} + \tilde{q}_{B2}$:

$$\begin{aligned}
P\left(\tilde{Q}_A, \tilde{Q}_B\right) - c + \tilde{Q}_A \partial_1 P\left(\tilde{Q}_A, \tilde{Q}_B\right) &= \hat{q}_{A2}\partial_1 P\left(\tilde{Q}_A, \tilde{Q}_B\right) - \hat{q}_{B1}\partial_2 P\left(\tilde{Q}_B, \tilde{Q}_A\right), \\
P\left(\tilde{Q}_B, \tilde{Q}_A\right) - c + \tilde{Q}_B \partial_1 P\left(\tilde{Q}_B, \tilde{Q}_A\right) &= \hat{q}_{B1}\partial_1 P\left(\tilde{Q}_B, \tilde{Q}_A\right) - \hat{q}_{A2}\partial_2 P\left(\tilde{Q}_A, \tilde{Q}_B\right).
\end{aligned}$$

Differentiating these equations with respect to $\left(\tilde{Q}_A, \tilde{Q}_B\right)$ and $(\hat{q}_{A2}, \hat{q}_{B1})$ yields:

$$\begin{aligned}
\tilde{\lambda}_A d\tilde{Q}_A + \tilde{\mu}_A d\tilde{Q}_B &= \partial_1 P\left(\tilde{Q}_A, \tilde{Q}_B\right) d\hat{q}_{A2} - \partial_2 P\left(\tilde{Q}_B, \tilde{Q}_A\right) d\hat{q}_{B1}, \\
\tilde{\mu}_B d\tilde{Q}_A + \tilde{\lambda}_B d\tilde{Q}_B &= -\partial_2 P\left(\tilde{Q}_A, \tilde{Q}_B\right) d\hat{q}_{A2} + \partial_1 P\left(\tilde{Q}_B, \tilde{Q}_A\right) d\hat{q}_{B1},
\end{aligned}$$

where, for $ih \neq jk \in \{A1, B1\}$

$$\begin{aligned}
\tilde{\lambda}_i &= 2\partial_1 P\left(\tilde{Q}_i, \tilde{Q}_j\right) + \tilde{q}_{ih}\partial_{11}^2 P\left(\tilde{Q}_i, \tilde{Q}_j\right) + \hat{q}_{jh}\partial_{22}^2 P\left(\tilde{Q}_j, \tilde{Q}_i\right), \\
\tilde{\mu}_i &= \partial_2 P\left(\tilde{Q}_i, \tilde{Q}_j\right) + \tilde{q}_{ih}\partial_{12}^2 P\left(\tilde{Q}_i, \tilde{Q}_j\right) + \hat{q}_{jh}\partial_{12}^2 P\left(\tilde{Q}_j, \tilde{Q}_i\right).
\end{aligned}$$

Under Assumption (B.1), these coefficients satisfy $\tilde{\lambda}_i < \tilde{\mu}_i$; the determinant $D = \tilde{\lambda}_A\tilde{\lambda}_B - \tilde{\mu}_A\tilde{\mu}_B$ is therefore positive, and thus:

$$\begin{aligned}
\frac{\partial \tilde{Q}_i}{\partial \hat{q}_{ik}} &= \frac{\tilde{\lambda}_j \partial_1 P\left(\tilde{Q}_i, \tilde{Q}_j\right) + \tilde{\mu}_i \partial_2 P\left(\tilde{Q}_i, \tilde{Q}_j\right)}{D}, \\
\frac{\partial \tilde{Q}_j}{\partial \hat{q}_{ik}} &= -\frac{\tilde{\mu}_j \partial_1 P\left(\tilde{Q}_i, \tilde{Q}_j\right) + \tilde{\lambda}_i \partial_2 P\left(\tilde{Q}_i, \tilde{Q}_l\right)}{D}.
\end{aligned} \tag{21}$$

Therefore, we have:

$$\frac{\partial\left(\tilde{Q}_A + \tilde{Q}_B\right)}{\partial \hat{q}_{ik}} = \frac{\left(\tilde{\lambda}_j - \tilde{\mu}_j\right)\partial_1 P\left(\tilde{Q}_i, \tilde{Q}_j\right) - \left(\tilde{\lambda}_i - \tilde{\mu}_i\right)\partial_2 P\left(\tilde{Q}_i, \tilde{Q}_j\right)}{D}.$$

Assumption (B.2.b) ensures that the numerator, too, is positive, which concludes the proof. ■

Next we show that, in symmetric outcomes, increasing the output of each good beyond the "duopoly" output Q^{**} reduces aggregate profit:

Lemma 9 *Suppose that (B.1) holds, and consider a market outcome in game* Γ_{1-1} *where the total output of each good is equal to* Q*. Then, the aggregate profit* $\Pi(Q,Q) = 2[P(Q,Q) - c]Q$ *is strictly decreasing in* Q *for all* $Q \geq Q^{**}$*.*

Proof. This is obvious when Q is so large that $P(Q,Q) = 0$. When instead $P(Q,Q) > 0$, then the derivative of the aggregate profit with respect to per-good output Q is

$$\frac{d\Pi(Q,Q)}{dQ} = 2\left[P(Q,Q) - c + Q\partial_1 P(Q,Q) + Q\partial_2 P(Q,Q)\right].$$

We have:

$$\left.\frac{d\Pi(Q,Q)}{dQ}\right|_{Q=Q^{**}} = 2Q^{**}\partial_2 P(Q^{**},Q^{**}) < 0,$$

where the inequality stems from (A.2). In addition:

$$\begin{aligned}
\frac{1}{2}\frac{d^2\Pi(Q,Q)}{dQ^2} &= \frac{d}{dQ}\left[P(Q,Q) - c + Q\partial_1 P(Q,Q) + Q\partial_2 P(Q,Q)\right] \\
&= \left[2\partial_1 P(Q,Q) + Q\partial_{11} P(Q,Q) + Q\partial_{22} P(Q,Q)\right] \\
&\quad + \left[\partial_2 P(Q,Q) + Q\left(\partial_{12}^2 P(Q,Q) + \partial_{12}^2 P(Q,Q)\right)\right] + \partial_2 P(Q,Q),
\end{aligned}$$

where the last term is negative from (A.2) and the two terms in brackets are negative from (B.1). Hence $d^2\Pi(Q,Q)/dQ^2 < 0$, and thus $d\Pi(Q,Q)/dQ < 0$ for $Q \geq Q^{**}$. ■

We now show that, under Assumption (B.2.a), aggregate profit decreases when total output becomes asymmetrically distributed:

Lemma 10 *Suppose (B.2.a) holds. Then, for a fixed level of aggregate output* $Q_A + Q_B = 2Q$*, the aggregate profit* $\Pi(Q_A,Q_B) = [P(Q_A,Q_B) - c]Q_A + [P(Q_B,Q_A) - c]Q_B$ *is maximal for* $Q_A = Q_B = Q$*.*

Proof. Let us fix the total output $Q_A + Q_B = 2Q$, and consider the impact of a variation in Q_i (thus compensated by a mirror variation in Q_j, for $i \neq j \in \{A,B\}$). The aggregate profit being symmetric in Q_A and Q_B, its derivative with respect to Q_i, holding $Q_A + Q_B$ fixed, can be expressed as

$$\left.\frac{d\Pi(Q_A,Q_B)}{dQ_i}\right|_{Q_A+Q_B=2Q} = \Psi(Q_i,Q_j) - \Psi(Q_j,Q_i), \tag{22}$$

where

$$\Psi(Q_i,Q_j) \equiv \frac{\partial\Pi(Q_i,Q_j)}{\partial Q_i} = P(Q_i,Q_j) - c + Q_i\partial_1 P(Q_i,Q_j) + Q_j\partial_2 P(Q_j,Q_i).$$

The RHS of (22) is equal to zero when $Q_A = Q_B = Q$; we now show that it is negative whenever $Q_i > Q_j$. To see this, consider the derivative of Ψ with respect to Q_i, holding $Q_A + Q_B$ fixed:

$$\begin{aligned}
\left.\frac{d\Psi(Q_i,Q_j)}{dQ_i}\right|_{Q_A+Q_B=2Q} &= \frac{\partial\Psi(Q_i,Q_j)}{\partial Q_i} - \frac{\partial\Psi(Q_i,Q_j)}{\partial Q_j} \\
&= 2\partial_1 P(Q_i,Q_j) + Q_i\partial_{11}^2 P(Q_i,Q_j) + Q_j^2\partial_{12}^2 P(Q_j,Q_i) \\
&\quad - \left[\partial_2 P(Q_i,Q_j) + \partial_2 P(Q_j,Q_i) + Q_i\partial_{12}^2 P(Q_i,Q_j) + Q_j\partial_{12}^2 P(Q_j,Q_i)\right] \\
&< 0,
\end{aligned}$$

where the inequality follows from (B.2.a). Hence, if $Q_i > Q = \frac{Q_A+Q_B}{2} > Q_j$, then $\Psi(Q_i,Q_j) < \Psi(Q,Q) < \Psi(Q_j,Q_i)$, implying that (22) is negative; it follows that, keeping total output $Q_A + Q_B = 2Q$ constant, the aggregate profit $\Pi(Q_A,Q_B)$ is maximal for $Q_A = Q_B = Q$. ■

Combining the above three lemmas yields:

Proposition 13 *Under Assumptions (B.1)-(B.2), the game Γ_{1-1} has property (P.2).*

Proof. Lemmas 9 and 10 together imply that $\Pi(Q_A,Q_B) < \Pi(Q^{**},Q^{**})$ whenever $Q_A + Q_B > 2Q^{**}$; the conclusion then follows from Lemma 8. ■

G.3 Property (P.3)

For Property (P.3), we require another condition on demand:

(B.3) For any $Q_i, Q_j \geq 0$ such that $P(Q_i,Q_j) > 0$ and $P(Q_j,Q_i) > 0$, and for any $q_i \in [0,Q_i]$, we have:

$$\partial_1 P(Q_i,Q_j) + q_i\partial_{11}^2 P(Q_i,Q_j) + Q_j\partial_{22}^2 P(Q_j,Q_i) < 0, \tag{B.3.a}$$

and in addition, for any $q_j \in [0,Q_j]$:

$$\begin{array}{ccc}
\left[\partial_2 P_i + \partial_2 P_j + (Q_i - q_i)\,\partial_{12}^2 P_i + q_j\partial_{12}^2 P_j\right] & & \left[2\partial_1 P_i + (Q_i - q_i)\,\partial_{11}^2 P_i + q_j\partial_{22}^2 P_j\right] \\
\times\left[\partial_2 P_j + (Q_j - q_j)\,\partial_{12}^2 P_j + q_i\partial_{12}^2 P_i\right] & < & \times\left[\partial_1 P_j + (Q_j - q_j)\,\partial_{11}^2 P_j + q_i\partial_{22}^2 P_i\right],
\end{array} \tag{B.3.b}$$

where $P_i \equiv P(Q_i,Q_j)$.

In the case of linear demand, (B.3) simplifies to $(\partial_2 P)^2 < (\partial_1 P)^2$, which holds by (A.2).

Proposition 14 *Assume (B.1) and (B.3) hold. Then, the game Γ_{1-1} has Property (P.3).*

Proof. By symmetry, it suffices to show that, say, $\partial \tilde{Q}_B / \partial \hat{q}_{B1} \leq 1$. This is obvious when $\tilde{q}_{B2} = 0$, as then $\tilde{Q}_B = \hat{q}_{B1}$. Consider now the case where $\tilde{q}_{B2} > 0$. If $\tilde{q}_{A1} = 0$, then from (20):

$$\frac{\partial \tilde{Q}_B}{\partial \hat{q}_{B1}} = \frac{\partial_1 P\left(\tilde{Q}_B, \hat{q}_{A2}\right)}{\hat{\lambda}_B}.$$

Assumptions (A.2), (B.1) and (B.3.a) together imply $\hat{\lambda}_B < \partial_1 P\left(\tilde{Q}_B, \hat{q}_{A2}\right) < 0$, and thus $\partial \tilde{Q}_B / \partial \hat{q}_{B1} < 1$.

When instead $\tilde{q}_{A1} > 0$, then from (21):

$$\frac{\partial \tilde{Q}_B}{\partial \hat{q}_{B1}} = \frac{\tilde{\lambda}_A \partial_1 P\left(\tilde{Q}_B, \tilde{Q}_A\right) + \tilde{\mu}_{BA} \partial_2 P\left(\tilde{Q}_B, \tilde{Q}_A\right)}{D},$$

where $D < 0$ under Assumption (B.1). Hence, this expression is less than one if and only if

$$\begin{aligned}
& \left[\partial_2 P_A + \partial_2 P_B + \left(\tilde{Q}_A - \hat{q}_{A2}\right) \partial^2_{12} P_A + \hat{q}_{B1} \partial^2_{12} P_B\right] \\
& \times \left[\partial_2 P_B + \left(\tilde{Q}_B - \hat{q}_{B1}\right) \partial^2_{12} P_B + \hat{q}_{A2} \partial^2_{12} P_A\right] \\
< \; & \left[2\partial_1 P_A + \left(\tilde{Q}_A - \hat{q}_{A2}\right) \partial^2_{11} P_A + \hat{q}_{B1} \partial^2_{22} P_B\right] \\
& \times \left[\partial_1 P_B + \left(\tilde{Q}_B - \hat{q}_{B1}\right) \partial^2_{11} P_B + \hat{q}_{A2} \partial^2_{22} P_A\right],
\end{aligned}$$

which holds under Assumption (B.3.b). ■

H Proof of Proposition 7

Let Π°, Π^{*}, and Π^{**} denote the equilibrium industry profit under no exclusive dealing, single exclusive dealing, and pairwise exclusive dealing, respectively. From (P.2), we know that $\Pi^{**} > \Pi^{*}, \Pi^{\circ}$. In the absence of exclusive dealing, at least one pair, say $M_A - R_1$, makes a weakly lower joint profit than the other pair, i.e., $\Pi^{\circ}_{M_A - R_1} \leq \Pi^{\circ}/2 \leq \Pi^{\circ}_{M_B - R_2}$. We show below that, no matter how profits are shared, the pair $M_A - R_1$ would benefit from M_A not dealing with R_2, and in response the other pair, $M_B - R_2$, would benefit from M_B not dealing with R_1; i.e.: $\Pi^{*}_{M_A - R_1} > \Pi^{\circ}_{M_A - R_1}$ and $\Pi^{**}_{M_B - R_2} > \Pi^{*}_{M_B - R_2}$, where $\Pi^{\circ}_{M_i - R_h}$, $\Pi^{*}_{M_i - R_h}$ and $\Pi^{**}_{M_i - R_h}$ respectively denote the equilibrium joint profit of the pair $M_i - R_h$ under no exclusivity, under single exclusive dealing where M_A does not deal with R_2, and under pairwise exclusive dealing.

We first note that, under single exclusivity, M_A and R_1 must obtain at least what they could get by deviating to pairwise exclusivity, that is:

$$\Pi^{*}_{M_A - R_1} \geq \max_{q_{A1}} \left[P\left(q_{A1}, q^{*}_{B2}\right) - c\right] q_{A1}. \tag{23}$$

Indeed, M_A could otherwise profitably deviate by offering a forcing contract $(\hat{q}_{A1}, \hat{T}_{A1})$, where $\hat{q}_{A1} \equiv \arg\max_{q_{A1}} \left[P\left(q_{A1}, q_{B2}^*\right) - c\right] q_{A1}$ and $\hat{T}_{A1} = \pi_1^* + \varepsilon$, where $\varepsilon > 0$ sufficiently small. Clearly, R_1 would find it profitable to accept this offer, proving the claim.[21]

This, in turn, implies that single exclusivity gives the pair $M_A - R_1$ more than half of the profit under pairwise exclusivity:

$$\begin{aligned} \Pi_{M_A-R_1}^* &\geq \max_{q_{A1}} \left[P\left(q_{A1}, q_{B2}^*\right) - c\right] q_{A1} \\ &> \max_{q_{A1}} \left[P\left(q_{A1}, q_{B2}^{**}\right) - c\right] q_{A1} = \Pi_{M_A-R_1}^{**}, \end{aligned}$$

where the second inequality follows from (P.3), which implies $q_{B2}^* < q_{B2}^{**}$, and (A.2).

Using (P.2), we thus have:

$$\Pi_{M_A-R_1}^* > \Pi_{M_A-R_1}^{**} = \frac{\Pi^{**}}{2} > \frac{\Pi^\circ}{2} \geq \Pi_{M_A-R_1}^\circ,$$

which thus implies that, starting from no exclusivity, M_A and R_1 have an incentive to engage in single exclusivity.

Furthermore, using again (P.2), we have:

$$\Pi_{M_B-R_2}^{**} = \frac{\Pi^{**}}{2} > \frac{\Pi^*}{2} > \Pi^* - \Pi_{M_A-R_1}^* = \Pi_{M_B-R_2}^*,$$

where the second inequality follows from

$$\Pi_{M_A-R_1}^* > \Pi_{M_A-R_1}^{**} = \frac{\Pi^{**}}{2} > \frac{\Pi^*}{2}.$$

It follows that, in response to M_A and R_1 opting for single exclusivity, M_B and R_2 have also an incentive to engage in exclusive dealing.

Note finally, by construction, the pair $M_A - R_1$ obtains a larger joint profit under pairwise exclusivity than in the absence of any exclusivity:

$$\Pi_{M_A-R_1}^\circ \leq \frac{\Pi^\circ}{2} < \frac{\Pi^{**}}{2} = \Pi_{M_A-R_1}^{**}.$$

Therefore, M_A and R_1 do have an incentive to opt for exclusivity, even if this induces M_B and R_2 to engage in exclusive dealing as well.

[21] R_1 may find it profitable to combine M_A's deviant offer with the equilibrium contract offered by M_B. If so, this can only increase R_1's incentive to accept M_A's deviant offer, without affecting M_A's deviation profit.

I Proof of Proposition 8

Using the same notation as in the proof of Proposition 5, consider the Cournot duopoly game where firm 1 produces both goods A and B at marginal cost c and firm 2 produces good A at marginal cost $\hat{c}$ and good B at marginal cost c, and the two firms compete in quantities. Let $\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right)$ denote the solution to:

$$\hat{Q}_A(\hat{c}) = \hat{R}\left(\hat{Q}_B(\hat{c}), \hat{c}\right), \tag{24}$$

$$\hat{Q}_B(\hat{c}) = R\left(\hat{Q}_A(\hat{c})\right), \tag{25}$$

where $\hat{R}$ and R are as defined in the proof of Proposition 5. Note that $(Q^\circ, Q^\circ) = \left(\hat{Q}_A(c), \hat{Q}_B(c)\right)$ and $(Q_A^*, Q_B^*) = \left(\hat{Q}_A(\hat{c}^*), \hat{Q}_B(\hat{c}^*)\right)$, where

$$\hat{c}^* \equiv P(Q_A^*, Q_B^*) + q_{B2}^* \partial_1 P(Q_A^*, Q_B^*).$$

We can thus interpret the move from (Q°, Q°) to (Q_A^*, Q_B^*) as the evolution of the equilibrium $\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right)$ as $\hat{c}$ increases from c to $\hat{c}^*$.

We first consider the effects on output. Differentiating (24) and (25) with respect to $\hat{Q}_A$, $\hat{Q}_B$ and $\hat{c}$ yields:

$$\begin{aligned} d\hat{Q}_A - \frac{\partial \hat{R}}{\partial Q}\left(\hat{Q}_B, \hat{c}\right) d\hat{Q}_B &= \frac{\partial \hat{R}}{\partial c}\left(\hat{Q}_B, \hat{c}\right), \\ d\hat{Q}_B &= R'\left(\hat{Q}_A\right) d\hat{Q}_A, \end{aligned}$$

and thus:

$$\begin{aligned} \hat{Q}_A'(\hat{c}) &= \frac{\frac{\partial \hat{R}}{\partial c}\left(\hat{Q}_B(\hat{c}), \hat{c}\right)}{1 - \frac{\partial \hat{R}}{\partial \hat{c}}\left(\hat{Q}_B(\hat{c}), \hat{c}\right) R'\left(\hat{Q}_A(\hat{c})\right)} < 0, \\ -\hat{Q}_A'(\hat{c}) &> \hat{Q}_B'(\hat{c}) = R'\left(\hat{Q}_A(\hat{c})\right) \hat{Q}_A'(\hat{c}) > 0. \end{aligned}$$

It follows that, under (A.4), introducing an exclusive dealing agreement on product A leads to a reduction in the output of A and, to a lesser extent, to an increase in the output of product B:

$$Q_A^* < \frac{Q_A^* + Q_B^*}{2} < Q^\circ < Q_B^*.$$

We now turn to the effects on social welfare. Recall that total welfare is equal to

$$W(Q_A, Q_B) = U(Q_A, Q_B) - cQ_A - cQ_B,$$

and thus

$$\frac{\partial W}{\partial Q_i}(Q_A, Q_B) = \frac{\partial U}{\partial Q_i}(Q_A, Q_B) - c = P(Q_i, Q_j) - c.$$

We now show that

$$\hat{W}(\hat{c}) \equiv W\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right)$$

decreases as $\hat{c}$ increases. We have:

$$\begin{aligned}
\hat{W}'(\hat{c}) &= \frac{\partial W}{\partial Q_A}\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right)\hat{Q}'_A(\hat{c}) + \frac{\partial W}{\partial Q_B}\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right)\hat{Q}'_B(\hat{c}) \\
&= \left[\frac{\partial U}{\partial Q_A}\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right) - c\right]\hat{Q}'_A(\hat{c}) + \left[\frac{\partial U}{\partial Q_B}\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right) - c\right]\hat{Q}'_B(\hat{c}) \\
&= \left[P\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right) - c\right]\hat{Q}'_A(\hat{c}) + \left[P\left(\hat{Q}_B(\hat{c}), \hat{Q}_A(\hat{c})\right) - c\right]R'\left(\hat{Q}_A(\hat{c})\right)\hat{Q}'_A(\hat{c}) \\
&\leq \left[P\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right) - P\left(\hat{Q}_B(\hat{c}), \hat{Q}_A(\hat{c})\right)\right]\hat{Q}'_A(\hat{c}),
\end{aligned}$$

where the inequality uses $\hat{P}_B > c$, $\hat{Q}'_A(\hat{c}) < 0$, and the fact that Assumptions (A.2) and (A.4) imply $R'(Q) > -1$. As $\hat{Q}'_A(\hat{c}) < 0$, to conclude the argument, it thus suffices to establish that $\hat{P}_A < \hat{P}_B$; we have:

$$\begin{aligned}
\frac{d}{d\hat{c}}\left(\hat{P}_A - \hat{P}_B\right) &= \left[\partial_1 P\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right) - \partial_2 P\left(\hat{Q}_B(\hat{c}), \hat{Q}_A(\hat{c})\right)\right]\hat{Q}'_A(\hat{c}) \\
&\quad - \left[\partial_1 P\left(\hat{Q}_B(\hat{c}), \hat{Q}_A(\hat{c})\right) - \partial_2 P\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right)\right]\hat{Q}'_B(\hat{c}) \\
&> 0,
\end{aligned}$$

where the inequality stems from Assumptions (A.2) – which, using symmetry, implies $\partial_1 P(Q_i, Q_j) < \partial_2 P(Q_j, Q_i) = \partial_2 P(Q_i, Q_j)$ (see footnote 2) – and (A.4), which imply $\hat{Q}'_B(\hat{c}) > 0 > \hat{Q}'_A(\hat{c})$). As $\hat{P}_A = \hat{P}_B = P^\circ$ for $\hat{c} = c$, it follows that $\hat{P}_A > \hat{P}_B$ for any $\hat{c} > c$.

Hence, we have that, under (A.4), introducing an exclusive dealing agreement decreases welfare.

We can use the same approach for consumer surplus. Using

$$S(Q_A, Q_B) = W(Q_A, Q_B) - \Pi(Q_A, Q_B),$$

we have:

$$\begin{aligned}
\frac{\partial S}{\partial Q_i}(Q_A, Q_B) &= \frac{\partial W}{\partial Q_i}(Q_A, Q_B) - \frac{\partial \Pi}{\partial Q_i}(Q_A, Q_B) \\
&= P(Q_i, Q_j) - c - \left[P(Q_i, Q_j) - c + \partial_1 P(Q_i, Q_j) Q_i + \partial_2 P(Q_j, Q_i) Q_j\right] \\
&= -\partial_1 P(Q_i, Q_j) Q_i - \partial_2 P(Q_j, Q_i) Q_j.
\end{aligned}$$

Letting

$$\hat{S}(\hat{c}) \equiv S\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right)$$

denote consumer surplus in the equilibrium of the duopoly game, we have:

$$\begin{aligned}
\hat{S}'(\hat{c}) &= \frac{\partial S}{\partial Q_A}\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right)\hat{Q}'_A(\hat{c}) + \frac{\partial S}{\partial Q_B}\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right)\hat{Q}'_B(\hat{c}) \\
&= -\left[\partial_1 P(Q_A, Q_B) Q_A + \partial_2 P(Q_B, Q_A) Q_B\right]\hat{Q}'_A(\hat{c}) - \left[\partial_1 P(Q_B, Q_A) Q_B + \partial_2 P(Q_A, Q_B) Q_A\right]\hat{Q}'_B(\hat{c}) \\
&= -\left\{\partial_1 P(Q_A, Q_B) Q_A + \partial_2 P(Q_B, Q_A) Q_B + \left[\partial_1 P(Q_B, Q_A) Q_B + \partial_2 P(Q_A, Q_B) Q_A\right] R'\left(\hat{Q}_A(\hat{c})\right)\right\}\hat{Q}'_A(\hat{c}) \\
&= -\left\{\begin{array}{l} \partial_1 P(Q_A, Q_B) Q_A\left[1 + \frac{\partial_2 P(Q_A,Q_B)}{\partial_1 P(Q_A,Q_B)} R'\left(\hat{Q}_A(\hat{c})\right)\right] \\ +\partial_1 P(Q_B, Q_A) Q_B\left[R'\left(\hat{Q}_A(\hat{c})\right) + \frac{\partial_2 P(Q_B,Q_A)}{\partial_1 P(Q_B,Q_A)}\right] \end{array}\right\}\hat{Q}'_A(\hat{c}).
\end{aligned}$$

As $\hat{Q}'_A(\hat{c}), \partial_1 P(.), \partial_2 P(.) < 0$ and $R'(.) > -1$, it follows that $\hat{S}'(\hat{c}) < 0$ if (A.5) holds.

We thus have that, under (A.4) and (A.5), introducing an exclusive dealing agreement decreases consumer surplus as well as social welfare.

We now turn to the effects on industry profit. For $\hat{c} = \hat{c}^*$, $q^*_{A2} = 0$ and thus the industry profit in the duopoly game coincides with the "true" industry profit, based on the actual cost c:

$$\Pi^* = (p^*_A - c) Q^*_A + (p^*_B - c) Q^*_B.$$

Therefore, to compare Π^* with Π°, it suffices to study how the industry profit, *based on true costs*, evolves with $\hat{c}$ in the duopoly game. Thus, let define:

$$\hat{\Pi}(\hat{c}) \equiv \left(P\left(\hat{Q}_A(\hat{c}), \hat{Q}_B(\hat{c})\right) - c\right)\hat{Q}_A(\hat{c}) + \left(P\left(\hat{Q}_B(\hat{c}), \hat{Q}_A(\hat{c})\right) - c\right)\hat{Q}_B(\hat{c}).$$

We have:

$$\begin{aligned}
\hat{\Pi}'(\hat{c}) &= \left[P\left(\hat{Q}_A, \hat{Q}_B\right) - c + \partial_1 P\left(\hat{Q}_A, \hat{Q}_B\right)\hat{Q}_A + \partial_2 P\left(\hat{Q}_B, \hat{Q}_A\right)\hat{Q}_B\right]\hat{Q}'_A \\
&\quad + \left[P\left(\hat{Q}_B, \hat{Q}_A\right) - c + \partial_1 P\left(\hat{Q}_B, \hat{Q}_A\right)\hat{Q}_B + \partial_2 P\left(\hat{Q}_A, \hat{Q}_B\right)\hat{Q}_A\right]\hat{Q}'_B,
\end{aligned}$$

which, using the FOCs for R_1's outputs $\hat{q}_{A1}$ and $\hat{q}_{B1}$:

$$P\left(\hat{Q}_B, \hat{Q}_A\right) - c + \partial_1 P\left(\hat{Q}_B, \hat{Q}_A\right)\hat{q}_{B1} + \partial_2 P\left(\hat{Q}_A, \hat{Q}_B\right)\hat{q}_{A1} = 0,$$

$$P\left(\hat{Q}_A, \hat{Q}_B\right) - c + \partial_1 P\left(\hat{Q}_A, \hat{Q}_B\right)\hat{q}_{A1} + \partial_2 P\left(\hat{Q}_B, \hat{Q}_A\right)\hat{q}_{B1} = 0,$$

can be written as:

$$\begin{aligned}
\hat{\Pi}'(\hat{c}) &= \left[\partial_1 P\left(\hat{Q}_A, \hat{Q}_B\right)\hat{q}_{A2} + \partial_2 P\left(\hat{Q}_B, \hat{Q}_A\right)\hat{q}_{B2}\right]\hat{Q}'_A + \left[\partial_1 P\left(\hat{Q}_B, \hat{Q}_A\right)\hat{q}_{B2} + \partial_2 P\left(\hat{Q}_A, \hat{Q}_B\right)\hat{q}_{A2}\right] R'\left(\hat{Q}_A\right)\hat{Q}'_A \\
&= \left\{\left[\partial_1 P\left(\hat{Q}_A, \hat{Q}_B\right) + \partial_2 P\left(\hat{Q}_A, \hat{Q}_B\right) R'\left(\hat{Q}_A\right)\right]\hat{q}_{A2} + \left[\partial_2 P\left(\hat{Q}_B, \hat{Q}_A\right) + \partial_1 P\left(\hat{Q}_B, \hat{Q}_A\right) R'\left(\hat{Q}_A\right)\right]\hat{q}_{B2}\right\}\hat{Q}'_A.
\end{aligned}$$

The first term within bracket is negative, as $\partial_1 P\left(\hat{Q}_A, \hat{Q}_B\right) < \partial_2 P\left(\hat{Q}_A, \hat{Q}_B\right) < 0$ and $R'\left(\hat{Q}_A\right) > -1$. As $\hat{Q}_A' < 0$, it follows that $\hat{\Pi}'(\hat{c}) > 0$ if the second term within brackets is non-positive, i.e., if:

$$\partial_2 P\left(\hat{Q}_B, \hat{Q}_A\right) + \partial_1 P\left(\hat{Q}_B, \hat{Q}_A\right) R'\left(\hat{Q}_A\right) \leq 0,$$

which amounts to Assumption (A.5). Hence, under (A.4) and (A.5), introducing an exclusive dealing agreement increases industry profit at the expense of consumer surplus and social welfare.

J Proof of Proposition 9

We first use a revealed preference argument to show that $Q^{**} \leq Q^{\circ}$. Recall that $Q^{**} = q^{**}$ and $Q^{\circ} = 2q^{\circ}$ are such that

$$\begin{aligned} Q^{**} &= \arg\max_q \left[P(q, Q^{**}) - c\right] q, \\ \frac{Q^{\circ}}{2} &= \arg\max_q \left[P\left(\frac{Q^{\circ}}{2} + q, Q^{\circ}\right) - c\right] q + \left[P\left(Q^{\circ}, \frac{Q^{\circ}}{2} + q\right) - c\right] \frac{Q^{\circ}}{2}. \end{aligned}$$

Hence, we have

$$\left[P(Q^{**}, Q^{**}) - c\right] Q^{**} \geq \left[P(Q^{\circ}, Q^{**}) - c\right] Q^{\circ} \tag{26}$$

and

$$\begin{aligned} \left[P\left(Q^{\circ}, Q^{\circ}\right) - c\right] Q^{\circ} &\geq \left[P\left(Q^{**}, Q^{\circ}\right) - c\right]\left(Q^{**} - \frac{Q^{\circ}}{2}\right) + \left[P\left(Q^{\circ}, Q^{**}\right) - c\right] \frac{Q^{\circ}}{2} \\ &= \left[P\left(Q^{**}, Q^{\circ}\right) - c\right] Q^{**} + \left[P\left(Q^{\circ}, Q^{**}\right) - P\left(Q^{**}, Q^{\circ}\right)\right] \frac{Q^{\circ}}{2}. \end{aligned}$$

If $Q^{**} > Q^{\circ}$, the last term on the RHS is positive from (A.2), implying

$$\left[P\left(Q^{\circ}, Q^{\circ}\right) - c\right] Q^{\circ} \geq \left[P\left(Q^{**}, Q^{\circ}\right) - c\right] Q^{**}. \tag{27}$$

Combining (26) and (27) yields

$$\left[P(Q^{**}, Q^{**}) - P\left(Q^{**}, Q^{\circ}\right)\right] Q^{**} \geq \left[P(Q^{\circ}, Q^{**}) - P\left(Q^{\circ}, Q^{\circ}\right)\right] Q^{\circ},$$

i.e.,

$$\int_{Q^{\circ}}^{Q^{**}} Q^{**} \partial_2 P(Q^{**}, Q) dQ \geq \int_{Q^{\circ}}^{Q^{**}} Q^{\circ} \partial_2 P(Q^{\circ}, Q) dQ,$$

which is equivalent to

$$\int_{Q^{\circ}}^{Q^{**}} \int_{Q^{\circ}}^{Q^{**}} \left[\partial_2 P(\tilde{Q}, Q) + \tilde{Q} \partial_{12}^2 P(\tilde{Q}, Q)\right] d\tilde{Q} dQ \geq 0.$$

(A.2) implies that the term in brackets is strictly negative, a contradiction. Hence, we must have $Q^{**} \leq Q^{\circ}$.

Next, suppose that $Q^{**} = Q^{\circ}$. The first-order conditions of the above maximization problems (for $q^{**} = Q^{**}$and $q^{\circ} = Q^{\circ}/2$) then yield:

$$P(Q^{\circ}, Q^{\circ}) - c = -\left[\partial_1 P(Q^{\circ}, Q^{\circ}) + \partial_2 P(Q^{\circ}, Q^{\circ})\right] \frac{Q^{\circ}}{2} = -\partial_1 P(Q^{\circ}, Q^{\circ}) Q^{\circ},$$

implying $\partial_1 P(Q^{\circ}, Q^{\circ}) = \partial_2 P(Q^{\circ}, Q^{\circ})$, and thus contradicting (A.2). Hence, we must have $Q^{**} < Q^{\circ}$.

It follows that consumer surplus is greater in the absence of exclusive dealing, as $S(Q,Q)$ increases with Q:

$$\frac{dS(Q,Q)}{dQ} = -2Q\left[\partial_1 P(Q,Q) + \partial_2 P(Q,Q)\right],$$

which is positive from (A.2).

Exclusive dealing also harms welfare, as $W(Q,Q)$ increases with Q as long as $P(Q,Q) > c$:

$$\begin{aligned} \frac{dW(Q,Q)}{dQ} &= P(Q,Q) - c + \int_0^Q \partial_2 P(q,Q) dq + P(Q,0) - c \\ &= P(Q,Q) - c + \int_0^Q \partial_2 P(Q,q) dq + P(Q,0) - c \\ &= 2\left[P(Q,Q) - c\right], \end{aligned}$$

where the second equality follows from the fact that demand symmetry implies that $\partial_2 P(q,Q) \equiv \partial_2 P(Q,q)$. To conclude the argument, it suffices to note that $P(Q,Q)$ is decreasing in Q from (A.2), and that the first-order condition for q° yields $P(Q^{\circ}, Q^{\circ}) > c$:

$$P(Q^{\circ}, Q^{\circ}) - c = -\left[\partial_1 P(Q^{\circ}, Q^{\circ}) + \partial_2 P(Q^{\circ}, Q^{\circ})\right] \frac{Q^{\circ}}{2}, \tag{28}$$

where (A.2) implies that the term in brackets is strictly negative, and thus the LHS is positive.

Finally, to show that $\Pi^{**} > \Pi^{\circ}$, it suffices to note that the industry-wide aggregate profit $\Pi(Q,Q)$ is concave in Q under (A.4), and maximal to $Q < Q^{**}$:

$$\begin{aligned} \frac{1}{2}\frac{d^2\Pi(Q,Q)}{dQ} &= \left[2\partial_1 P(Q,Q) + \partial_{11}^2 P(Q,Q)\,Q + \partial_{22}^2 P(Q,Q)\,Q\right] \\ &\quad + \left[2\partial_2 P(Q,Q) + 2\partial_{12}^2 P(Q,Q)\,Q\right], \end{aligned}$$

where both expressions in brackets on the RHS are negative under (A.4), and the first-order derivative, evaluated at Q^{**}, satisfies:

$$\left.\frac{d\Pi(Q,Q)}{dQ}\right|_{Q=Q^{**}} = 2Q^{**}\partial_2 P(Q^{**}, Q^{**}),$$

where the RHS is negative from (A.2).

K Proof of Proposition 10

K.1 Candidate equilibrium

We first characterize some of the properties of the candidate equilibrium described in Proposition 10.

K.1.1 Equilibrium quantities

The equilibrium output levels satisfy:

$$
\begin{aligned}
q_{A1}^*, q_{B1}^* &= \arg\max_{q_{A1}, q_{B1}} \left\{ \left[P\left(q_{A1}, q_{B1} + q_{B2}^*\right) - c \right] q_{A1} + \left[P\left(q_{B1} + q_{B2}^*, q_{A1}\right) - c \right] q_{B1} \right\}, \\
q_{B2}^* &= \arg\max_{q_{B2}} \left\{ \left[P\left(q_{B1}^* + q_{B2}, q_{A1}^*\right) - c \right] q_{B2} \right\}.
\end{aligned}
$$

The equilibrium profits are thus equal to:

$$
\begin{aligned}
\pi_{M_A - R_1}^* &= \left[P\left(q_{A1}^*, q_{B1}^* + q_{B2}^*\right) - c \right] q_{A1}^* + P\left(q_{B1}^* + q_{B2}^*, q_{A1}^*\right) q_{B1}^* - T_{B1}^*, \\
\pi_{R_2}^* &= \left[P\left(q_{B1}^* + q_{B2}^*, q_{A1}^*\right) - c \right] q_{B2}^*, \\
\pi_{M_B}^* &= T_{B1}^* - c q_{B1}^*.
\end{aligned}
$$

K.1.2 Equilibrium fees

Determination of T_{B1}^* In equilibrium, R_1 must be indifferent between rejecting or accepting M_B's offer:

- R_1 should not benefit from rejecting the offer, otherwise it would do so;
- conversely, if R_1 was strictly better off accepting the offer, then M_B could slightly increase its fee: with passive beliefs R_1 would then still accept the offer, making M_B's deviation profitable.

If R_1 rejects M_B's offer, it will sell $\tilde{q}_{A1}$ units of good A, where:

$$
\tilde{q}_{A1} \equiv \arg\max_{q_{A1}} \left[P\left(q_{A1}, q_{B2}^*\right) - c \right] q_{A1},
$$

and thus obtain a profit equal to:

$$
\tilde{\pi}_{M_A - R_1} \equiv \left[P\left(\tilde{q}_{A1}, q_{B2}^*\right) - c \right] \tilde{q}_{A1}.
$$

Therefore, the fee T_{B1}^* should be such that $\pi_{M_A - R_1}^* = \tilde{\pi}_{M_A - R_1}$, or:

$$
T_{B1}^* = \left[P\left(q_{A1}^*, q_{B1}^* + q_{B2}^*\right) - c \right] q_{A1}^* + P\left(q_{B1}^* + q_{B2}^*, q_{A1}^*\right) q_{B1}^* - \left[P\left(\tilde{q}_{A1}, q_{B2}^*\right) - c \right] \tilde{q}_{A1}.
$$

This in particular ensures that M_B's equilibrium profit is non-negative:

$$\begin{aligned}
\pi^*_{M_B} &= T^*_{B1} - cq^*_{B1} \\
&= \left[P\left(q^*_{A1}, q^*_{B1} + q^*_{B2}\right) - c\right] q^*_{A1} + \left[P\left(q^*_{B1} + q^*_{B2}, q^*_{A1}\right) - c\right] q^*_{B1} - \left[P\left(\tilde{q}_{A1}, q^*_{B2}\right) - c\right] \tilde{q}_{A1} \\
&= \max_{q_{A1}, q_{B1}} \left\{\left[P\left(q_{A1}, q_{B1} + q^*_{B2}\right) - c\right] q_{A1} + \left[P\left(q_{B1} + q^*_{B2}, q_{A1}\right) - c\right] q_{B1}\right\} - \max_{q_{A1}} \left[P\left(q_{A1}, q_{B1} + q^*_{B2}\right) - c\right] q_{A1} \\
&\geq 0. \qquad (29)
\end{aligned}$$

Determination of $\left(\hat{q}, \hat{T}\right)$ The described equilibrium is such that R_2 must obtain its equilibrium profit $\pi^*_{R_2}$ by accepting only M_A's contract $\left(\hat{q}, \hat{T}\right)$; conversely, R_2 should not obtain more profit by dealing with both manufacturers. As M_B offers to supply R_2 at cost, this in turn implies that $\hat{q}$ should be "large enough" to ensure that, conditional on selling $\hat{q}$ units of good A, R_2 does not want to sell any positive quantity of good B. Assumptions (A.1) and (A.2) ensure that such large values exist for $\hat{q}$; if R_2's profit is quasi-concave in q_{B2}, then a necessary and sufficient condition is that accepting M_A's contract lowers the marginal revenue for good B below its cost, i.e.

$$P\left(q^*_{B1}, q^*_{A1} + \hat{q}\right) + \partial_2 P\left(q^*_{A1} + \hat{q}, q^*_{B1}\right) \hat{q} \leq c. \qquad (30)$$

Remark 1 *We are considering "quantity forcing" contracts where the retailer commits itself to sell the agreed quantity. If R_2 only commits itself to buy the quantity $\hat{q}$, we would also need to check that it is willing to put all the quantity on the market; if R_2's profit is quasi-concave in q_{A2} and q_{B2}, then a necessary and sufficient condition is:*

$$P\left(q^*_{A1} + \hat{q}, q^*_{B1}\right) + \partial_1 P\left(q^*_{A1} + \hat{q}, q^*_{B1}\right) \hat{q} \geq 0. \qquad (31)$$

As R_2 could obtain $P\left(q^*_{A1} + \hat{q}, q^*_{B1}\right) \hat{q} - \hat{T}$ by deviating and selecting M_A's offer $\left(\hat{q}, \hat{T}\right)$ instead of M_B's offer, we must have:

$$\hat{T} = P\left(q^*_{A1} + \hat{q}, q^*_{B1}\right) \hat{q} - \left[P\left(q^*_{B1} + q^*_{B2}, q^*_{A1}\right) - c\right] q^*_{B2}.$$

Remark 2 *The payment $\hat{T}$ thus satisfies:*

$$\hat{T} = \max_{q_{B2}} \left\{P\left(q^*_{A1} + \hat{q}, q^*_{B1} + q_{B2}\right) \hat{q} + \left[P\left(q^*_{B1} + q_{B2}, q^*_{A1} + \hat{q}\right) - c\right] q_{B2}\right\} - \max_{q_{B2}} \left\{\left[P\left(q^*_{B1} + q_{B2}, q^*_{A1}\right) - c\right] q_{B2}\right\}.$$

Intuitively, this implies that $\hat{T} \geq 0$ if (31) holds. Indeed, when differentiating R_2' deviation profit with respect to $\hat{q}$, the envelope theorem yields:

$$\frac{d\pi_{R_2}}{d\hat{q}} = P\left(q^*_{A1} + \hat{q}, q^*_{B1} + \hat{q}_{B2}\right) + \partial_1 P\left(q^*_{A1} + \hat{q}, q^*_{B1} + \hat{q}_{B2}\right) \hat{q} + \partial_2 P\left(q^*_{B1} + \hat{q}_{B2}, q^*_{A1} + \hat{q}\right) \hat{q}_{B2},$$

where $\hat{q}_{B2}$ denotes R_2's best response output of good B when selling $\hat{q}$ units of good A (thus, for $\hat{q}$ large enough, $\hat{q}_{B2} = 0$). Assuming that this marginal profit decreases in $\hat{q}$, (31) *ensures that $\hat{T} \geq 0$.*

K.2 Deviations

The above characterization of equilibrium quantities and fees ensures that retailers have no profitable deviation, neither at the acceptance stage nor at the product market competition stage. We thus now focus on manufacturers' deviations at the offer stage.

K.2.1 Deviations by M_B

Given the passive beliefs assumption, it suffices to consider one-sided deviations. Also, by construction such a one-sided deviation cannot be profitable if it is not accepted, as in equilibrium M_B makes a non-negative profit with both R_1 and R_2.

Consider first a deviant offer to R_1. Such a deviation cannot reduce $M_A - R_1$'s payoff, which it can secure by rejecting M_B's offer. But it cannot increase the joint profit that M_B generates with $M_A - R_1$ through R_1's sales either, as the equilibrium contract offered to R_1 is bilaterally efficient.

Consider now a deviant offer to R_2. Again, such a deviation cannot reduce R_2's payoff, which it can secure by rejecting M_B's offer and accepting instead M_A's offer. And it cannot increase the joint profit that M_B generates with R_2 either, as the equilibrium contract offered to R_2 is bilaterally efficient, regardless of whether R_2 accepts or rejects M_A's offer.

K.2.2 Deviations by $M_A - R_1$

Consider first a deviant offer by $M_A - R_1$ that induces R_2 to reject it Suppose first that, in the continuation equilibrium, R_2 accepts M_B's offer. If R_1 also accepts M_B's offer, then the continuation equilibrium quantities are $(q^*_{A1}, q^*_{B1}, q^*_{B2})$; $M_A - R_1$ thus obtains its equilibrium profit, making the deviation unprofitable. If instead R_1 rejects M_B's offer then, from (P.3), in the continuation equilibrium M_B puts on the market a larger quantity $q_{B2} > q^*_{B2}$, and thus $M_A - R_1$ thus obtains less than its equilibrium profit, making again the deviation unprofitable.

Therefore, if such a deviation is profitable, it must induce R_2 to reject M_B's offer. We can distinguish two cases, depending on R_1's acceptance decision of M_B's offer:

- If R_1, too, rejects M_B's offer, then it will sell q_{A1} units of good A, so as to maximize

$$\left[P\left(q_{A1},0\right)-c\right]q_{A1},$$

and thus such that $p_A = P\left(q_{A1},0\right) > c$. But then, $p_B = P\left(0,q_{A1}\right) > P\left(q_{A1},0\right) > c$ by Assumption (A.2), which in turn implies that R_2 would rather accept M_B's offer and sell a positive quantity of good B, in contradiction with R_2's supposed rejection of M_B's offer.

- If instead R_1 accepts M_B's offer, then it will sell q_{B1}^* units of good B and q_{A1}^a units of good A, so as to maximize

$$\left[P\left(q_{A1},q_{B1}^*\right)-c\right]q_{A1}+P\left(q_{B1}^*,q_{A1}\right)q_{B1}^*-T_{B1}^*.$$

By revealed preference, this must exceed the profit it could achieved by rejecting M_B's offer and selling only good A, which implies:

$$\begin{aligned}
P\left(q_{B1}^*,q_{A1}^a\right)q_{B1}^* &\geq T_{B1}^*+\max_{q_{A1}}\left[P\left(q_{A1},0\right)-c\right]q_{A1}-\left[P\left(q_{A1}^a,q_{B1}^*\right)-c\right]q_{A1}^a\\
&> T_{B1}^*+\max_{q_{A1}}\left[P\left(q_{A1},0\right)-c\right]q_{A1}-\left[P\left(q_{A1}^a,0\right)-c\right]q_{A1}^a\\
&\geq cq_{B1}^*,
\end{aligned}$$

where the strict inequality follows from Assumption (A.2) and the last inequality stems from (29). Therefore, $P\left(q_{B1}^*,q_{A1}^a\right) > c$, which again implies that R_2 would rather accept M_B's offer and sell a positive quantity of good B, in contradiction with R_2's supposed rejection of M_B's offer.

Consider now a deviant offer by $M_A - R_1$ that is accepted by R_2 together with M_B's offer Let $\bar{q}$ denote the quantity of good A sold by R_2 in the continuation equilibrium.

Suppose first that, in the continuation equilibrium, R_1 also keeps accepting M_B's offer.

In such a continuation equilibrium:

- By Property (P.2), the aggregate profit cannot exceed that achieved in the candidate equilibrium.

- M_B obtains the same profit (namely, $T_{B1}^* - cq_{B1}^*$) as in the candidate equilibrium;

- R_2 gets at least its candidate equilibrium profit $\pi_{R_2}^*$; if that were not the case, R_2 could profitably deviate by rejecting M_A's deviant offer and deal instead only with M_B: denoting by q^b_{A1} R_1's output of good A in the continuation equilibrium, R_2 would obtain in this way:

$$\max_{q_{B2}} \left[P\left(q^*_{B1}+q_{B2},q^b_{A1}\right)-c\right]q_{B2} \geq \max_{q_{B2}}\left[P\left(q^*_{B1}+q_{B2},q^*_{A1}\right)-c\right]q_{B2} = \pi^*_{R_2},$$

where the inequality follows from (P.3) and the fact that R_2's profit decreases in q_{A1}.

Thus, the deviation cannot be profitable for $M_A - R_1$.

Remark 3 *Applying this reasoning to a deviant offer equal to the equilibrium shadow offer* $\left(\hat{q},\hat{T}\right)$ *shows that, in equilibrium,* $M_A - R_1$ *strictly prefers that* R_2 *rejects* M_A*'s offer.*

Remark 4 *The previous remark does not necessarily imply that offering* $\left(\hat{q},\hat{T}\right)$ *is a dominated strategy for* $M_A - R_1$*: this will indeed not be the case if there exists a* q_{B1} *(together with an appropriate best response* q_{A1}*) and a* q_{B2} *such that* $M_A - R_1$ *is better off when* R_2 *mistakenly accepts* $\left(\hat{q},\hat{T}\right)$*.*

Suppose now that, in the continuation equilibrium, R_1 *rejects* M_B*'s offer.*

Such a deviation yields quantities $q_{A1}=\tilde{q}_{A1}\left(0,\bar{q}\right), q_{B1}=0, q_{A2}=\bar{q}, q_{B2}=\tilde{q}_{B2}\left(0,\bar{q}\right))$, where $\left(\tilde{q}_{A1},\tilde{q}_{B2}\right)$ denote the equilibrium outputs of the game Γ_{1-1} for $\hat{q}_{B1}=0$ and $\hat{q}_{A2}=\bar{q}$.

We first show that the continuation equilibrium, following M_A's deviation (the "deviation equilibrium," referred to below with a superscript c), is less profitable for $M_A - R_1$ than the equilibrium of the game Γ_{1-1} for $\hat{q}_{A2}=\hat{q}_{B1}=0$ (the "alternative equilibrium," referred to below with a superscript d). To see this, note that:

- M_B makes the same profit in both equilibrium scenarios: $\pi^d_{M_B}=\pi^c_{M_B}=0$.
- R_2 cannot make more profit in the alternative equilibrium than in the deviation equilibrium, i.e.: $\pi^d_{R_2}\leq\pi^c_{R_2}$. To show this, consider a deviation from the "deviation equilibrium," in which R_2 only accepts M_B's offer; this deviation gives R_2 a profit

$$\pi^e_{R_2}\equiv\max_{q_{B2}}\left[P\left(q_{B2},q^c_{A1}\right)-c\right]q_{B2},$$

which (weakly) exceeds

$$\pi^d_{R_2}=\max_{q_{B2}}\left[P\left(q_{B2},q^d_{A1}\right)-c\right]q_{B2},$$

as $q^d_{A1}=\tilde{q}_{A1}\left(0,0\right)\geq q^c_{A1}=\tilde{q}_{A1}\left(0,\bar{q}\right)$, from Property (P.3). As the "deviation from the deviation equilibrium" has to be unprofitable, we have: $\pi^c_{R_2}\geq\pi^e_{R_2}\geq\pi^d_{R_2}$.

- Property (P.2) ensures that the aggregate profit is larger in the alternative equilibrium than in the deviation equilibrium: $\pi^d_{M_B} + \pi^d_{M_A-R_1} + \pi^d_{R_2} \geq \pi^c_{M_B} + \pi^c_{M_A-R_1} + \pi^c_{R_2}$.

It follows that the integrated firm makes more profit in the alternative equilibrium than in the deviation equilibrium:

$$\pi^d_{M_A-R_1} \geq \pi^c_{M_A-R_1}.$$

But $\pi^d_{M_A-R_1} < \tilde{\pi}_{M_A-R_1} = \max_{q_{A1}} \left[P\left(q_{A1}, q^*_{B2}\right) - c\right] q_{A1}$, as R_2 is more aggressive in the alternative equilibrium than in the "pseudo duopoly" scenario in which R_1 carries A only and R_2 carries B only, but R_2 anticipates that R_1 is also carrying B, and thus sells $q^*_{B2} = \tilde{q}_{B2}\left(q^*_{B1}, 0\right)$ rather than $q_{B2} = \tilde{q}_{B2}\left(0, 0\right)$. As $\tilde{\pi}_{M_A-R_1} = \pi^*_{M_A-R_1}$, we have:

$$\pi^c_{M_A-R_1} \leq \pi^d_{M_A-R_1} \leq \pi^*_{M_A-R_1}.$$

That is, $M_A - R_1$'s deviation is not profitable.

Finally, consider now a deviation by $M_A - R_1$ that induces R_2 to drop M_B's offer. Let $\left(q_{A2} = \bar{q} > 0, T_{A2} = \bar{T}\right)$ denote the deviant offer. As R_2 can costlessly accept M_B's offer to supply at cost, and then choose $q_{B2} = 0$, the above reasoning (for deviations inducing R_2 to accept the deviant offer by M_A as well as M_B's offer) still applies, which concludes the proof.

L Proof of Proposition 11

To show existence of the equilibrium, let

$$\Pi(Q_A, Q_B) \equiv \left[P(Q_A, Q_B) - c\right] Q_A + \left[P(Q_B, Q_A) - c\right] Q_B$$

denote aggregate output when $q_{A1} + q_{A2} = Q_A$ and $q_{B1} + q_{B2} = Q_B$. To support the vector $(q^{**}_{A1}, q^{**}_{A2}, q^{**}_{B1}, q^{**}_{B2}) = (Q^{**}, 0, 0, Q^{**})$ as an equilibrium outcome, we suppose that the two integrated firms do not offer contracts to each other, i.e., $\tau^{**}_{A2} = \varnothing$ and $\tau^{**}_{B1} = \varnothing$. To show that there is no profitable deviation, suppose that the integrated $M_i - R_h$ deviates from this candidate equilibrium by offering a contract $\widetilde{\tau}_{ik}\left(.\right)$ that induces a quantity $\hat{q}_{ik}$ through the channel $M_i - R_k$. By assumption, M_j does not offer any contract to R_h in the candidate equilibrium, and thus we still have $\hat{q}_{jh} = 0$, as in the candidate equilibrium. The resulting quantities $\tilde{q}_{ih}(\hat{q}_{ik}, 0)$ and $\tilde{q}_{jk}(\hat{q}_{ik}, 0)$ are the equilibrium quantities in game Γ_{2-1} when $\hat{q}_{jh} = 0$:

$$\tilde{q}_{ih}(\hat{q}_{ik}, 0) = \arg\max_{q_{ih}} \Pi_h\left(q_{ih}, \tilde{q}_{jk}(\hat{q}_{ik}, 0); \hat{q}_{ik}, 0\right), \quad (32)$$

$$\tilde{q}_{jk}(\hat{q}_{ik}, 0) = \arg\max_{q_{jk}} \Pi_k\left(\tilde{q}_{ih}(\hat{q}_{ik}, 0), q_{jk}; \hat{q}_{ik}, 0\right). \quad (33)$$

Now, note that each integrated firm $M_j - R_k$ can guarantee itself at least the candidate equilibrium profit $\Pi^{**}/2 \equiv \Pi(Q^{**}, Q^{**})/2$ by simply rejecting M_i's deviant offer; in this way, it would obtain:

$$\max_{q_{jk}} \Pi_k\left(\tilde{q}_{ih}(\hat{q}_{ik}, 0), q_{jk}; 0, 0\right).$$

But (P.3) implies $\tilde{q}_{ih}(\hat{q}_{ik}, 0) \leq \tilde{q}_{ih}(0, 0) = q_{ih}^{**}$; as the profit of $M_j - R_k$ decreases in q_{ih}, the above profit is at least equal to:

$$\max_{q_{jk}} \Pi_k\left(q_{ih}^{**}, q_{jk}; 0, 0\right) = \Pi^{**}/2.$$

Therefore, in order to be profitable, the deviation must increase the total profits of the two integrated firms:

$$\Pi(\tilde{q}_{ih}(\hat{q}_{ik}, 0) + \hat{q}_{ik}, \tilde{q}_{jk}(\hat{q}_{ik}, 0)) > \Pi(Q^{**}, Q^{**}).$$

But this contradicts (P.2).

To show uniqueness of equilibrium, suppose instead that there exists another equilibrium $(q_{A1}^{**}, q_{A2}^{**}, q_{B1}^{**}, q_{B2}^{**}) \neq (Q^{**}, 0, 0, Q^{**})$. This implies, in particular, that $q_{A2}^{**} > 0$ or $q_{B1}^{**} > 0$. The induced aggregate profit is $\Pi(Q_A^{**}, Q_B^{**})$, where $Q_A^{**} \equiv q_{A1}^{**} + q_{A2}^{**}$ and $Q_B^{**} \equiv q_{B1}^{**} + q_{B2}^{**}$. By (P.2), we have $\Pi(Q_A^{**}, Q_B^{**}) < \Pi\left(Q^{**}, Q^{**}\right)$. The equilibrium profit of at least one of the two integrated firms, say $M_A - R_1$, must therefore be strictly less than $\Pi\left(Q^{**}, Q^{**}\right)/2$. Consider the following deviation by $M_A - R_1$: it does not offer any contract to the rival retailer R_2 nor does it accept any contract from the rival manufacturer M_B. The deviation at the offer stage induces a continuation equilibrium in which R_2 expects R_1 to put some quantity q_{B1} of good B on the market, and therefore chooses a quantity $\tilde{q}_{B2}(q_{B1}, 0) \geq 0$. Property (P.3) then implies $\tilde{q}_{B2}(q_{B1}, 0) \leq \tilde{q}_{B2}(0, 0) = Q^{**}$. As $M_A - R_1$'s deviation profit decreases with q_{B2}, it is bounded from below by $\Pi\left(Q^{**}, Q^{**}\right)/2$, a contradiction.

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