

Softening Competition through Forward Trading

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Abstract

In the history of alleged manipulations on forward markets, it has been observed that high prices resulted from a cartel's long positions. The present paper addresses this issue in a simple model of price setting duopolists. We show that forward trading results in producers buying forward their own production, so that equilibrium prices are increased compared to the case without forward trading. This result contrasts with the social desirability of forward markets emphasized by the academic literature.

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1 Introduction

There is a widespread presumption among economists that forward trading is socially beneficial. Many theoretical arguments indeed support this view (see [3] for a review). Based upon the Keynes-Hicks theory of speculation, the most common argument is that forward trading allows producers to shift risks towards less risk-averse traders ([15], [2]). Besides improving insurance, forward trading may also improve information sharing as claimed in [11] or [13]. Moreover, forward trading still improves welfare when a producer has some market power. [4] shows that a risk-averse monopolist facing competitive traders tackles the same problem of hedging as in the competitive case and so is better off selling futures, with the result of increased production and lower cash prices. Considering the market of a good produced both by a dominant, risk-neutral firm and a competitive, risk-averse fringe, [14] reaches the conclusion that hedging is still the crucial force that leads risk-averse producers to sell futures. Finally, [1] claims that forward trading raises welfare even in the absence of any risk. They investigate the case of Cournot duopolists and characterize an equilibrium outcome with greater outputs, hence a lower spot price, compared to the case without forward trading. The point is that selling forward allows each duopolist to commit to a Stackelberg level of output on the spot market but, since both do so in equilibrium, no one succeeds in acquiring a leader advantage. As a result, competition is tougher on the spot market and welfare is improved, compared to a situation without forward trading.

This study fully concurs with [1] that the oligopolistic paradigm is appropriate for investigating competition between producers on forward markets. In a paper written before the development of oil markets in the eighties, [14] indeed quoted maize, wool, rubber among nine commodities, all with active futures markets and single country shares above 25 percent in world trade. Furthermore, there is historical evidence that forward trading is vulnerable to market power (see [10], [7] and [3] for reviews). According to the U.S. Commodity Exchange Act of 1936, forward markets might be manipulated by large traders (see also [16]). More striking is the fact that powerful producers tend to *buy* their own production forward, seemingly in order to sustain prices¹. The fact that producers may hold long positions is at odds to the theoretical arguments sketched above.

The challenging question we address here is whether oligopolistic producers may benefit from softening competition by taking long positions. In a model of price-setting duopolists

¹[10] recalls how major exporting countries coordinated on world coffee prices via long positions, in 1977-1978. Other examples of long positions and high prices are given in [7] and [3].

with differentiated products, we show that the answer is positive. The opening of forward markets then raises equilibrium prices, which is detrimental to welfare. This result calls for qualifying the admitted benefits of forward trading.

The intuition underlying these conclusions echoes the analysis in [9] and [5] of the effect of pre-commitment on competition. In the present framework, *buying* forward (rather than selling) commits a producer to set a higher spot price in order to increase the value of his position. Due to Bertrand competition on the spot market, the other producer reacts by raising his price, which increases the profit of the first producer. This effect proves to be quite strong in our analysis since buying forward appears as a strictly dominant strategy. In equilibrium, both producers buy forward and spot prices are raised above the levels reached in the absence of forward trading.

2 The Model

Before introducing forward trading, let us first consider a standard static duopoly model. Two producers, $i = A, B$, offer substitute goods on different spot markets. They compete *à la* Bertrand by simultaneously setting spot prices (p^i, p^j) ($j \neq i$). Producer i thus gets a spot profit equal to

$$\pi^i(p^i, p^j) = p^i D^i(p^i, p^j) - C^i(D^i(p^i, p^j))$$

where D^i is the demand function for good i . For $i = A, B$, we assume²

Assumption 1 *The cost function C^i and the demand function D^i are almost everywhere twice differentiable, with bounded derivatives. Moreover:*

- $C^i(D)$ is non-decreasing on $[0, +\infty)$.
- If $D^i(p^i, p^j) > 0$, then $D_1^i(p^i, p^j) < 0$ and $D_2^i(p^i, p^j) > 0$.
- $D^i(0, 0) > 0$.

This assumption essentially specifies that goods are substitutes ($D_2 > 0$) while ensuring that a producer cannot be excluded from spot markets ($D(0, 0) > 0$). Our second assumption introduces more structure on profit functions:

Assumption 2 *If $D^i(p^i, p^j) > 0$, then $\pi_{12}^i(p^i, p^j) > 0$ and $\pi_{11}^i(p^i, p^j) + \pi_{12}^i(p^i, p^j) < 0$.*

²Subscripts denote partial derivatives.

This assumption simplifies the study of the static Bertrand equilibrium without forward trading. In this simple game, the best response of producer i to p^j is defined by the maximization of $\pi^i(p^i, p^j)$. The strategic complementarity introduced in the first part of the assumption implies that best responses are increasing with p^j . The second part of the assumption implies that the slope of best response functions is between zero and one, which ensures the unicity of the static Bertrand equilibrium (see [8] and [6]).

Let us now consider that the Bertrand duopolists have the opportunity to trade forward. Suppose that, at date 0, each good can be traded on a competitive forward market before the opening of spot markets at date 1. Traders take positions on forward markets at date 0 and behave competitively. It follows that arbitrage profits are zero under perfect foresight³. As is customary (see [4]), we assume that forward contracts mature at the time spot markets meet so that forward positions are settled at spot prices⁴. By considering that payment occurs at date 1, we can ignore discount factors.

Consider the case of a producer buying forward his own production. At date 1, he might want to choose an arbitrarily high spot price in order to squeeze the traders holding the corresponding short positions. The possibility of squeezing thus threatens the efficiency of forward markets. Consequently actual forward markets have designed various devices to deter squeezing. Regulatory institutions often rely on legal requirements which specifically prohibit the setting of “artificial” prices⁵. Forward contracts also typically specify a default-settlement procedure, that is, for example, a maximum penalty if the short trader defaults. Contracts often include the right to close a short position either by delivering the same quantity of a similar good (second-sourcing) plus a monetary penalty (also called a delivery adjustment), or by calling for a cash payment indexed on the price of a similar good⁶. Following [16], the opportunity to bring additional supplies to the delivery market at some

³We assume away any risk in order to focus on strategic effects. Note also that consumers could intervene on the forward market at date 0 without any change in our results.

⁴To fix ideas we focus on the case of a forward market without physical delivery. This fits the functioning of most actual forward markets. Hence, positions are settled without physical delivery through an equivalent monetary payment. Nevertheless, our framework allows for both cases.

⁵Prices may be considered artificial when the spot prices of goods that are deliverable under the terms of the contract are high relative to the spot price of nondeliverable goods. According to the United States law, setting artificial prices is a criminal offense. The Commodity Futures Trading Commission is given the authority “to maintain or restore orderly trading in, or liquidation of, any futures contract” whenever it suspects “threatened or actual market manipulations and corners”.

⁶See [4]: “In reality, the payoff in a futures contract is bounded above even if the cash price is not because a short position holder has the option of defaulting on the contract. In such case, an exchange will follow a default settlement procedure that typically calls for a cash payment which reflects normal equilibrium price under the circumstance. For example, the Maine potatoes default of 1976 was settled this way.”

cost limits squeezing since it places an upper bound upon the price a manipulator can extract.

We specify here a default-settlement procedure which encompasses a variety of cases. In our model, a short trader on good i is allowed to settle his position either by paying the price p^i or by paying a penalty $\bar{p}^i(p^j)$ which may depend on the price of the substitute good $j \neq i$. For the sake of simplicity, we assume

Assumption 3 *For $i = A, B$, $\bar{p}^i(p^j)$ is positive and differentiable, with a slope strictly between 0 and 1. Moreover, for any p_j we have*

$$D^i(\bar{p}^i(p^j), p^j) > 0 \quad \pi_1^i(\bar{p}^i(p^j), p^j) < 0.$$

The constraint on the slope of the default price \bar{p} still allows for the different types of default-settlement procedures just described. The second part of the assumption excludes prices so high that demand vanishes. The last part requires that the default price be high enough, so that it does not constrain the best response functions in the Bertrand game without forward trading⁷.

We study the following game. At date 0, both producers simultaneously and publicly⁸ propose quantities (x^i, x^j) of their good to traders bidding competitively. This results in forward prices (f^i, f^j) . We adopt the following convention: $x^i > 0$ if the producer sells forward, and $x^i < 0$ if the producer buys forward. At date 1, producers simultaneously set spot prices (p^i, p^j) and produce up to the demand they respectively face on each spot market. Agents (traders or producers) then close their forward positions. An agent holding a short position on good i may either pay p^i or $\bar{p}^i(p^j)$ to agents with a long position. Clearly, he will choose to pay the lowest of these two prices.

Given forward quantities (x) , forward prices (f) and spot prices (p) , producer i gets a total profit which can be decomposed into a spot profit and an arbitrage profit:

$$\pi^i(p^i, p^j) + [f^i - \min(p^i, \bar{p}^i(p^j))]x^i. \quad (1)$$

Because traders behave competitively, in equilibrium the arbitrage profit corresponding to the bracketted term must equal zero. Nevertheless this term plays an important role because it drives the choice of spot prices at date 1.

⁷Two remarks are noteworthy here. First, such a default price schedule can be shown to exist under various assumptions. An example is given in Section 5. Second, the strict inequalities in assumption 3 could be relaxed in order to include limiting cases, at the price of simplicity since multiple equilibria could then appear.

⁸The observability of producers' positions on the forward market is crucial for the existence of a commitment effect. Our model is thus best-fitted for producers taking large positions such as the marketing boards of major exporting countries.

In the following we characterize the pure-strategy, Nash-perfect equilibria of this two-stage game. Subgames beginning at date 1 are called forward-influenced subgames. They are studied in the next section.

3 The forward-influenced subgames

Given forward quantities (x) and prices (f), consider the Bertrand game defined by the profit functions (1). Notice that if producer i sets a price p^i above the default price $\bar{p}^i(p^j)$, then the second term becomes a constant and the first term is strictly decreasing with p^i from assumption 3. Therefore any best response of producer i to the price p^j chosen by producer j must be solution to the following program \mathcal{P}^i :

$$\begin{cases} \max_{p^i} \pi^i(p^i, p^j) - p^i x^i \\ 0 \leq p^i \leq \bar{p}^i(p^j) \end{cases}$$

Under our assumptions, this program displays strong regularity properties. The following result then easily obtains (all proofs are given in Appendix) :

Proposition 1 *There exists a unique equilibrium $(P^A(x^A, x^B), P^B(x^B, x^A))$ in each forward-influenced subgame. These equilibrium spot prices are non-increasing with x^A and x^B .*

This result illustrates the main strategic effect. From the profit function in \mathcal{P}^i , we see that any increase in forward sales leads to choosing a lower spot price at date 1 because the producer wants to close his position at the lowest cost. This effect is quite general and also holds in [1], when Cournot competition prevails on the spot market⁹. Here this property of reaction functions translates into lower equilibrium prices, a result which also holds under simple models of Cournot competition. In fact, the key difference between these two frameworks only becomes apparent when producers choose their forward positions at date 0, as we shall see now.

4 The full game

In any Nash-perfect equilibrium of the full game, competition between traders must reduce any arbitrage profit to zero. Therefore, each producer's total profit (1) reduces to

$$\Pi^i(x^i, x^j) = \pi^i(P^i(x^i, x^j), P^j(x^j, x^i)). \quad (2)$$

⁹Under Cournot competition, producers still want to close their positions at the lowest price. This reduction in price is obtained through an increase in production.

As equilibrium spot prices are non-increasing with positions, intuitively each producer should reduce his position below the static level $x^i = 0$ in order to increase profits. This is indeed what happens and the result is strong enough to be stated in terms of dominated strategies :

Lemma 1 *For each producer i , selling forward ($x^i > 0$) is a strictly dominated strategy. Moreover, given that the other producer does not sell forward ($x^j \leq 0$), then not buying forward ($x^i \geq 0$) is a strictly dominated strategy.*

The intuition here is at odds with the intuition prevailing under Cournot competition. In both cases, at date 0 each producer realizes that selling less forward (or buying more forward) is a way to increase his spot price at date 1. However, due to the strategic complementarity inherent in Bertrand competition, an indirect effect is to lead the other producer to also increase its price, hence competition is relaxed and profits increase.

Note that two rounds of iterated elimination of strictly dominated strategies are needed here. This is simply because in our model, a producer is allowed to sell forward a quantity so high that he will be led to set a zero spot price at date 1. Clearly such a behaviour is dominated. Once it is excluded, an additional round is enough to get Lemma 1. Consequently :

Proposition 2 *Any equilibrium of the full game must be such that both producers buy their own production forward, with equilibrium prices higher than at the equilibrium without forward trading.*

The second part of the Proposition clearly follows from the first part, together with Proposition 1.

This competition-softening effect of long positions we exhibit here is far from being only of purely academic interest. For instance, the alleged coffee market manipulation in 1977 began with long positions held by Brazil and El Salvador, two of the world's largest producers, in the Coffee "C" futures contracts on the New York Coffee and Sugar Exchange. [10] reports that "35 percent of the long contracts on the New York exchange on November 15, 1978 were held by traders from producing countries", just before the Bogota Group was charged with agreeing on collusive coffee prices. In the well-known cases of the wheat futures contract in March 1979 at the Chicago Board of Trade and the May 1976 Maine potato contract at the New York Mercantile Exchange, long open interest was concentrated in a few hands and even exceeded the deliverable supply (see [3]).

Still Proposition 2 does not exclude cornering, in the sense that short traders are led to exercise the default-settlement procedure by paying the default price \bar{p} for one good or both. Intuitively cornering should not happen if default prices are high enough. The following result indeed supports this intuition:

Lemma 2 *Suppose that default prices verify*

$$\forall i, j \neq i \quad \forall p^j \quad (\pi_1^i + \pi_2^i)(\bar{p}^i(p^j), p^j) < 0. \quad (3)$$

Then for producer i , choosing to buy forward a quantity x^i such that $P^i(x^i, x^j) = \bar{p}^i(P^j(x^i, x^j))$ is a strictly dominated strategy.

Recall that, from assumption 2, $\pi_1^i(p, p') + \pi_2^i(p, p')$ is decreasing with p , for any p' . Hence (3) indeed requires that the default price be high enough. We immediately obtain:

Proposition 3 *Suppose that default prices are high enough to verify (3). Then in any equilibrium of the full game, prices are higher than in the equilibrium without forward trading, but are below default prices.*

This result shows that our competition-softening effect holds in a smooth manner. Indeed equilibrium prices and quantities do not depend anymore on default prices, as long as default prices verify (3).

5 Some comparative statics

Proposition 3 offers a condition under which none of the constraints in \mathcal{P}^i is binding in equilibrium. We can therefore define the price reaction function $R^j(p^i, x^j)$ of producer j as

$$\pi_1^j(R^j, p^i) = x^j.$$

The following expression plays an important role hereafter :

$$\theta^i \equiv \frac{D_2^i}{-D_1^i}(p^i, p^j) \frac{\partial R^j}{\partial p^i}(p^i, x^j).$$

The Appendix shows that $0 < \theta^i < 1$ when this expression is computed at the equilibrium outcome. As in [17], θ^i can be thought of as an index of both the substitutability between goods (first ratio) and the degree of competition (the derivative of R^j). The Appendix then shows that equilibrium prices and quantities must satisfy

$$\frac{p^i - C^{i'}}{p^i} = \frac{1}{\varepsilon^i} \frac{1}{1 - \theta^i} \quad \frac{-x^i}{D^i} = \frac{\theta^i}{1 - \theta^i} \quad (4)$$

where ε^i is good i 's demand elasticity at equilibrium prices. These conditions relate the Lerner index at equilibrium to the size of forward trade (x/D). Intuitively making goods more substitutable (increasing θ^i) increases the equilibrium price and the quantity traded forward. Hence the competition-softening effect of long positions has the largest impact when competition is toughest. Also, this contradicts the view in [1] that the opening of forward markets makes competition tougher.

This intuition may be verified in a simple linear demand/linear cost setting:

$$\forall i = A, B \quad D^i(p^i, p^j) = a^i - b^i p^i + d^i p^j \quad C^i(q) = c^i q.$$

Assume that these parameters are positive, with the usual conditions that $b^i > d^i$ and $a^i > b^i c^i$.¹⁰ Assumptions 1 and 2 are easily verified. Assumption 3 requires

$$\frac{a^i + b^i c^i + d^i p^j}{2b^i} < \bar{p}^i(p^j) < \frac{a^i + d^i p^j}{b^i}$$

with a slope of the default price between 0 and 1. Condition (3) in Proposition 3 reduces this interval to

$$\frac{a^i + (b^i - d^i)c^i + d^i p^j}{2b^i - d^i} < \bar{p}^i(p^j) < \frac{a^i + d^i p^j}{b^i}.$$

Under this assumption, any equilibrium must verify (4). Note that in this linear setting, we have

$$\theta^i = \frac{d^i}{b^i} \frac{d^j}{2b^j} = \theta^j \equiv \theta < \frac{1}{2}.$$

Interestingly we get from (4) that equilibrium quantities bought forward are proportional to final demands, even though firms are not symmetric:

$$\frac{-x^i}{D^i} = \frac{\theta}{1 - \theta} = \frac{-x^j}{D^j}$$

and this ratio is less than one because θ is less than one-half.

Finally the linearity of demands and costs ensures that the global profit in (2) is concave. Therefore the unique solution (x^A, x^B, p^A, p^B) to (4) indeed forms an equilibrium outcome of the game. It can be verified that these four quantities are increasing with θ .

¹⁰These conditions ensure that i) demand decreases when both prices are increased by the same amount ii) demand is positive when price equals marginal cost, even when $p^j = 0$.

6 Conclusion

In the history of alleged manipulations on futures markets, it has often been observed that unreasonable prices resulted from long positions held by a cartel. To address this issue, we have investigated a model in which duopolists producing two differentiated goods may trade forward before competing *à la* Bertrand on spot markets. As a result, in equilibrium producers buy forward their own production and prices are raised above the level reached without forward trading. Thus profits are increased and the traditional assertion that forward trading improves welfare turns out to be false.

The argument of this article is not that forward trading is harmful but rather that it is not as virtuous as the academic literature suggests. More research should be devoted to the study of forward trading as a collusive tool in a cartel's hand. As a suggestion for further research, the present model could be extended to allow for demand or price uncertainty on spot markets. There is obviously a conflict between the competition-softening effect of long positions emphasized here and the traditional risk-hedging motive that induces risk-averse producers to sell their output forward, insofar as they are more risk-averse than traders. It would be instructive to determine which motive would dominate and see to what extent the folklore that "forward trading causes output to rise and price to fall" is a good approximation.

References

1. B. Allaz and J-L. Vila, Cournot Competition, Forward Markets and Efficiency, *J. Econ. Theory* **59** (1993), 1-16.
2. R.W. Anderson and J. P. Danthine, Hedger Diversity in Futures Markets, *Econ. J.* **93** (1983), 370-389.
3. R.W. Anderson, The Industrial Organization of Futures Markets : A Survey, *in* "The Industrial Organization of Futures Markets" (R. Anderson, Ed.), Lexington Books, Lexington, MA, 1984.
4. R.W. Anderson and M. Sundaresan, Futures Markets and Monopoly, *in* "The Industrial Organization of Futures Markets" (R. Anderson, Ed.), Lexington Books, Lexington, MA, 1984.
5. J.I. Bulow, J.D. Geanakoplos and P.D. Klemperer, Multimarket Oligopoly: Strategic Substitutes and Complements, *J. Polit. Economy* **93** (1985), 488-511.
6. T.E. Cooper, Most-Favored Customer Pricing and Tacit Collusion, *RAND J. Econ.* **17** (1986), 377-388.
7. J.C. Edmunds, A Comment on Greenstone's 'The Coffee Cartel: Manipulation in the Public Interest', *J. Futures Markets* **2** (1982), 1, 19-24.
8. J.W. Friedman, Oligopoly and the Theory of Games, Amsterdam, North Holland, 1977.
9. D. Fudenberg and J. Tirole, The Fat Cat Effect, the Puppy Dog Ploy and the Lean and Hungry Look', *Amer. Econ. Rev., Papers and Proceedings* **74** (1984), 361-368.
10. W.D. Greenstone, The Coffee Cartel: Manipulation in the Public Interest, *J. Futures Markets* **1** (1981), 1, 3-16.
11. S.J. Grossman, The Existence of Futures Markets, Noisy Rational Expectations and

Informational Externalities, *Rev. Econ. Stud.* **44** (1977), 431-449.

12. H. Hotelling, Stability in Competition, *Econ. J.*, **39** (1929), 41-57.

13. P. Mahenc and V. Meunier, Forward Market and Signal of Quality, forthcoming in *RAND J. of Econ.*, 2003.

14. D.M.G. Newbery, The Manipulation of Futures Markets by a Dominant Producer, in "The Industrial Organization of Futures Markets" (R. Anderson, Ed.), Lexington Books, Lexington, MA, 1984.

15. D.M.G. Newbery and J.E. Stiglitz, The Theory of Commodity Price Stabilization, Oxford University Press, 1981.

16. S.C. Pirrong, Mixed Manipulation Strategies in Commodity Futures Markets, *J. Futures Markets* **15** (1992), 1, 13-38.

17. N. Singh and X. Vives, Price and Quantity Competition in a Differentiated Duopoly, *RAND J. Econ.* **15** (1984), 546-554.

Proof Appendix:

Proof of Proposition 1: under our assumptions, each program \mathcal{P}^i admits a unique, continuous solution $R^i(p^j, x^i)$, non-decreasing with respect to p^j , and non-increasing with respect to x^i . R^i may be equal to zero, or to $\bar{p}^i(p^j)$, or in the interior case be defined by

$$\pi_1^i(R^i, p^j) = x^i. \quad (5)$$

In each case, under assumptions 2 and 3 the slope of R^i with respect to p^j is non-negative and strictly less than 1. By symmetry the same result holds for the best response function of seller j . Existence and unicity of a Nash equilibrium then follow¹¹. Finally, any increase in x^i must reduce both equilibrium prices, since the best-response function R^i is non-increasing with x^i .

For further use, let us consider the properties of the best-response function $R^i(p, 0)$ in the absence of forward trading. Under assumption 1, $D^i(0, p)$ is positive for any p , and consequently $\pi_1^i(0, p) = D^i - C^{i'} D_1^i$ is also positive. Also, under assumption 3, $\pi_1^i(\bar{p}^i(p), p)$ is negative. Therefore the constraint in \mathcal{P}^i is not binding, and $R^i(p, 0)$ is defined by

$$\pi_1^i(R^i(p, 0), p) = 0. \quad (6)$$

Moreover, since

$$\pi_1^i = D^i + (p - C^{i'}) D_1^i \quad \pi_2^i = (p - C^{i'}) D_2^i \quad (7)$$

we get

$$\pi_2^i(R^i(p, 0), p) > 0. \quad (8)$$

Let us now explore more precisely the properties of the Nash correspondence $(P^A(x^A, x^B), P^B(x^B, x^A))$. Define

$$\bar{x}^i(x^j) = \pi_1^i(0, R^j(0, x^j)).$$

Notice that $\bar{x}^i(x^j) > 0$ since, as shown above, $\pi_1^i(0, p)$ is positive for any p . Now $(P^i = 0, P^j)$ are equilibrium prices for the subgame (x^i, x^j) if and only if the following best-response conditions are verified:

¹¹Strictly speaking, existence requires that the slopes of reaction functions be bounded away from 1. We neglect this point in the rest of the paper, so as to avoid cumbersome assumptions.

$$P^j = R^j(0, x^j) \quad \pi_1^i(0, P^j) \leq x^i$$

which holds if and only if $x^i \geq \bar{x}^i(x^j)$ and $P^j = R^j(0, x^j)$. Similarly, define $\hat{p}^j(x^j)$ as the unique solution in p to

$$p = R^j(\bar{p}^i(p), x^j) \tag{9}$$

and $\underline{x}^i(x^j)$ as

$$\underline{x}^i(x^j) \equiv \pi_1^i(\bar{p}^i(\hat{p}^j(x^j)), \hat{p}^j(x^j)) \tag{10}$$

which is negative from assumption 3. Then $(P^i = \bar{p}^i(P^j), P^j)$ are equilibrium prices for the subgame (x^i, x^j) if and only if the following best-response conditions are verified:

$$P^j = R^j(\bar{p}^i(P^j), x^j) \quad \pi_1^i(\bar{p}^i(P^j), P^j) \geq x^i$$

which holds if and only if $x^i \leq \underline{x}^i(x^j)$ and $P^j = \hat{p}^j(x^j)$.

To sum up:

Lemma 3 *Given i , there exist two thresholds $\underline{x}^i(x^j) < 0 < \bar{x}^i(x^j)$ such that:*

- *if $x^i \leq \underline{x}^i(x^j)$, then the equilibrium spot prices are $(\bar{p}^i(\hat{p}^j(x^j)), \hat{p}^j(x^j))$ and do not depend on x^i ;*
- *if $x^i \geq \bar{x}^i(x^j)$, then the equilibrium spot prices are $(0, R^j(0, x^j))$ and do not depend on x^i ;*
- *otherwise the equilibrium spot prices are such that*

$$\pi_1^i(P^i, P^j) = x^i. \tag{11}$$

Proof of Lemma 1 : First notice that from Lemma 3, Π^i does not depend on x^i for $x^i \geq \bar{x}^i(x^j)$. Let us now show that Π^i is decreasing with x^i when $0 < x^i < \bar{x}^i(x^j)$. By definition of the reaction function R^j , we can write

$$\Pi^i(x^i, x^j) = \pi^i(P^i(x^i, x^j), R^j(P^i(x^i, x^j), x^j)).$$

Since P^i and R^j are non-increasing with respect to x^i , we have almost everywhere (omitting the arguments for the sake of clarity)

$$\frac{\partial \Pi^i}{\partial x^i} = \frac{\partial P^i}{\partial x^i} [\pi_1^i + \pi_2^i \frac{\partial R^j}{\partial p^i}]. \quad (12)$$

From (11) and assumption 2, $\frac{\partial P^i}{\partial x^i} < 0$. There remains to study the bracketted term. We know that $\rho \equiv \frac{\partial R^j}{\partial p^i} \in [0, 1]$, and from assumption 2 $(\pi_1^i + \rho\pi_2^i)(p, p')$ is decreasing with p , for any p' . Moreover R^i is defined by (5) and is thus decreasing with x_i : for $x^i > 0$, we get

$$P^i = R^i(P^j, x^i) < R^i(P^j, 0).$$

This yields

$$(\pi_1^i + \rho\pi_2^i)(P^i, P^j) > (\pi_1^i + \rho\pi_2^i)(R^i(P^j, 0), P^j).$$

From (6) and (8), the right-hand-side is non-negative. This shows that Π^i is strictly decreasing with x^i , as announced. Therefore selling forward a positive quantity $x^i > 0$ is a strictly dominated strategy.

Now, if $x^j \leq 0$, then we know from Lemma 3 that either $P^j = \bar{p}^j(P^i)$ or P^j is interior. In both cases, $\frac{\partial R^j}{\partial p^i}$ is positive. Then using (6) and (8) we can compute (12) at $x^i = 0$, and show that it is negative. This concludes the proof. ■

Proof of Proposition 2: it follows from Lemma 1 that in any equilibrium one must have $x^i < 0$ and $x^j < 0$. Proposition 1 then yields the result. ■

Proof of Lemma 2 : from Lemma 3, it is sufficient to show that the right-derivative of (12) is positive at the right of $x^i = \underline{x}^i(x_j)$. For these values of x^i , P^i is given by (11) so that $\frac{\partial P^i}{\partial x^i} < 0$. Moreover, from (7) we obtain that $\pi_2^i > 0$ since $\pi_1^i = x^i < 0$. Therefore the bracketted term in (12) is below $\pi_1^i + \pi_2^i$, which is negative at $x^i = \underline{x}^i(x^j)$ from (3). This shows the domination result. ■

Proof of Proposition 3 : the result is a consequence of Lemma 2 and Proposition 2.

Section 5 : we now know that $x^i < 0$ and that (11) holds. Moreover the bracketted term in (12) must be zero. This yields

$$x^i = \pi_1^i = -\pi_2^i \frac{\partial R^j}{\partial p^i}. \quad (13)$$

Using (7), we get $\pi_2^i = (\pi_1^i - D^i)D_2^i/D_1^i$. Replacing in (13) we get

$$x^i = (x^i - D^i)\theta^i$$

which shows that $0 < \theta^i < 1$ since $x^i < 0$. The second part of (4) then follows. The first part is then a consequence of (13). ■