

# Green Paradox and Directed Technical Change: The Effects of Subsidies to Clean R&D\*

by

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## Abstract

The "green paradox" literature points out that environmental policies which are anticipated to become gradually more stringent over time may induce a more rapid extraction of fossil fuels, thus having a detrimental effect to the environment. The manifestation of such phenomena has been extensively studied in the case of taxes directly applied to the extraction of a polluting non-renewable resource and of subsidies applied to its non-polluting substitutes. This paper examines the effects of subsidies to "clean" R&D activities, aimed to improve the productivity of non-polluting substitutes. We borrow standard assumptions from the directed-technical-change literature to take a full account of the private incentives to perform R&D and of the patterns of complementarity/substitutability between dirty resource and clean non-resource sectors. We show that a gradual increase in relative subsidies to clean R&D activities does not have the adverse green paradox effect, which contradicts an earlier made conjecture. Instead, the presence of several R&D sectors implies arbitrages which give rise to other quite paradoxical results. However substitutable or complementary sectors are, and whatever the induced technological bias is, clean-R&D-support policies always enhance the long-run productivity of the resource and thus result in a less rapid extraction.

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## 1. INTRODUCTION

The so-called "green paradox" phenomenon refers to the fact that anticipated policies aimed to reduce the demand for an exhaustible resource result in this resource being exploited more rapidly. It is well known that when such resources are polluting – as are fossil fuels – free markets already tend to consume them too rapidly (Withagen, 1994). Hence, policies which entail a green paradox phenomenon further deteriorate the environment and are thus obviously sub-optimal.<sup>1</sup> Hans-Werner Sinn (2008) coined the expression meaning that "good intentions do not always breed good deeds" (p. 380).

Such phenomena may arise from several types of demand-reducing policies. In his formal analysis, Sinn (2008) has focused on taxation policies applied to a non-renewable resource when those policies leave the total amount to be ultimately extracted unaffected. In such Hotellian contexts, as he shows, tax instruments that are sufficiently rising over time – whether they are applied on cash flows or directly on resource quantities – induce a more rapid extraction.<sup>2</sup> Thus, an anticipated gradual introduction of environmental policies, or the anticipation that environmental policies will be implemented from some future dates on, might result in an undesired faster extraction.

Sinn (2008) anticipated the result to carry over to the case of any demand-reducing policies; in particular, to subsidies to resource substitutes and to technical improvements in the production of those substitutes. Most of following contributions have dealt

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<sup>1</sup>An exact evaluation of policies' environmental impacts requires an explicit assessment of their effects on pollution and on social welfare (e.g. Hoel, 2010; Gerlagh, 2011; van der Ploeg and Withagen, 2012). Following Sinn (2008) and Grafton, Kompas and Long (2012), a meaningful simplification consists in assessing whether policies enhance or reduce the speed at which the resource is extracted. Environmental policies which contribute to solving the environmental problem slow down the extraction of polluting resources; *vice versa* policies inducing a more rapid extraction are detrimental to the environment. This simple criterion has been considered to give a good intuition on whether policies are environmentally successful or not, contributing to the popularity of the green paradox problem as initially formulated by Sinn (2008).

<sup>2</sup>The reasoning goes as follows. A constant tax rate applied to cash flows is neutral because it amounts to a conventional profit tax on the total-discounted-profit objective. Absent any extraction cost, a constant-present-value levy on the resource is formally identical to a constant cash-flow tax. Even with non-zero extraction costs, there exists a continuum of neutral tax paths (Dasgupta *et al.*, 1981). All those extraction taxes are neutral for the same reason that they affect the equilibrium price in such a way that the equilibrium Hotelling rule remains satisfied without any further readjustments of quantities; in particular, they leave the producers' profit-maximizing extraction unchanged. Tax trajectories that are rising more rapidly than those neutral ones cause a more rapid extraction (see also the comprehensive analysis of Gaudet and Lasserre, 1990).

with subsidies to resources' clean substitutes while extending the traditional partial-equilibrium resource-depletion setting in several respects (e.g. Gerlagh, 2011; Grafton, Kompas and Long, 2012; van der Ploeg and Withagen, 2012; among others).<sup>3</sup>

There are three basic ways by which the demand for a commodity can be affected. The first one is to directly modify its price through taxation policies directly applied on its flows, in the spirit of Sinn (2008). Quite relatedly, the second one is to affect the price of its substitutes, in the spirit of Gerlagh (2011), Grafton *et al.* (2012) and van der Ploeg and Withagen (2012). Those two ways consist in modifying the price arguments of the demand function; using partial-equilibrium settings where the technology is given, they have been extensively analyzed by the above studies.

The third way to affect the demand for a commodity is the object of this paper: it consists in modifying the demand function itself instead of its price arguments. Macroeconomic demand drivers are arguably as strong as price incentives in the determination of the global demand for oil. Demand depends on the currently available technology; in a long-run perspective, the technology results from prior research and development (R&D) investments. The long-run dynamic mechanisms by which private research efforts respond to economic incentives have been highlighted by the economic growth literature. As this literature shows, such economic incentives do not affect all sectors in the same fashion (Acemoglu, 2002): research investments are directed to specific sectors; understanding the intricate process by which a particular sector is affected by endogenous R&D requires to assume that innovations are sector specific.

Acemoglu's concept of directed technical change has received a particular attention in resource economics. Indeed, directed-technical-change models disentangle the specific factors favoring the production of clean substitutes from those enhancing the productivity of dirty resources. Then, by refining the patterns of substitutability and complementarity between sectors, it permits to more precisely describe the role of R&D in the switch from the use of depletable resources to the use of producible (gross) substitutes. Major

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<sup>3</sup>Grafton *et al.* (2012) introduced decreasing-returns-to-scale in a substitute production, arising from land-supply limitations. Gerlagh (2011) and van der Ploeg and Withagen (2012) departed from the Hotelling assumption of resource homogeneity and examined the induced dynamics of pollution.

applications include, among others, André and Smulders (2004), Hart (2004), Grimaud and Rougé (2008), Di Maria and Valente (2008) and Pittel and Bretschger (2010). The most recent contribution on the issue is Acemoglu, Aghion, Bursztyn and Hemous' (2012) paper.

Subsidies to research investments are generally advocated by the economic growth literature<sup>4</sup>, yet irrespective of whether these investments aim to favor clean or dirty sectors. Subsidies to environmentally friendly research activities – enhancing productivity in non-polluting substitute sectors – affect the demand of polluting-resource inputs only indirectly; therefore, absent any constraints on the implementation of optimal environmental policies, they must not have any Pigovian dimension. In the presence of policy implementation difficulties however (perhaps related to green paradox phenomena), they may contribute to alleviate the environmental problem arising from polluting-resource use. On this ground, they can be advocated as not-too-bad substitutes to direct environmental policies (e.g. Grimaud, Lafforgue and Magné, 2011). In general, as Smulders and Di Maria (2012) recently pointed out, induced technical change interacts with pollution-generating inputs' demand in a very intricate way.<sup>5</sup>

Intellectual property rights confer sector-specific innovation activities the dimension of competing economic projects. The equilibrium allocation of efforts to polluting-resource-improving and to clean-substitute-improving R&D sectors is determined by no-arbitrage conditions involving the inclusive-of-subsidies returns to investments in both sectors. In this context, we show that the effects of relative subsidies to research efforts is not so intuitive. The equilibrium no-arbitrage requirement implies that any support to one specific R&D sector entails necessary compensations to the other sectors. As a result, depending

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<sup>4</sup>By providing incentives to increase otherwise sub-optimal R&D investments, they alleviate standard endogenous-growth distortions.

<sup>5</sup>As they show, the interaction between environmental policies and technological changes they induce leads to counter-intuitive effects even in models where there is a single aggregated R&D sector. Their analysis mainly focuses on the effects of environmental policies, taking into account endogenous technological adjustments. In contrast, we focus on policies directly supporting a specific R&D sector. In the standard two-sector CES-technology framework of the directed-technical-change literature, exogenous technical improvements in the clean sector can be shown to satisfy Smulders and Di Maria's definition of "brown" technologies regardless of sectors' substitutability/complementarity. In our intertemporal Hotelling model, extraction patterns are not only governed by the absolute effect of technological change on resource demand, but further depend on these effects at all dates relative to each other.

on whether production sectors are substitutable or complementary, supporting the clean-substitute R&D sector affects positively or negatively the relative contribution of the clean-substitute production sector. However, the overall effect of R&D-support policies on resource demand is irrespective of sectors' degree of substitutability/complementarity.

In any dynamic equilibrium, we find that a gradual, more-and-more stringent support to R&D activities aimed to improve productivity in clean substitutes sectors induces, among other effects, a less rapid resource extraction. Our result sharply contrasts with the commonly-made conjecture that technical improvements in the production of resource substitutes are tantamount to other policies aimed to reduce the demand for the resource.

Taking a full account of incentives to perform R&D requires to consider the endogenous process by which the productivity of both the resource and its substitutes is determined by specific R&D sectors. Our analysis suggests that the endogeneity of technical change is crucial to study the effects of supporting the development of resource and resource substitutes sectors. To set up our model, we borrow very standard assumptions from the recent literature on directed technical change and the environment. Our framework is largely inspired from Acemoglu *et al.*'s (2012) analysis.

We modify Acemoglu *et al.*'s (2012) model so as to examine the effects of specific R&D-support policies on non-renewable-resource extraction patterns in a single setup involving the minimal ingredients determining those effects: two production sectors combine to produce a unique final good; each of them is associated with a specific R&D sector; one of them (dirty) consumes flows of a non-renewable resource; the other one (clean) relies on flows of a renewable source of energy.

Unlike their model, the time dimension is continuous, which requires some minor adjustments which basically amount to a redefinition of the time scale. For simplicity, we assume away the allocation of unskilled labor to the two production sectors. The allocation of economic resources to these sectors is completely summarized by that of intermediate goods. Nevertheless, the allocation of research (labor) efforts to the two R&D sectors retains its central role identified by the directed-technical-change literature.

In general-equilibrium endogenous-growth models integrating an exhaustible-resource

sector, it is standard to assume that extraction costs are negligible.<sup>6</sup> With such a common approximation, traditional growth models still deliver good intuitions while preserving the regularity properties which are required in a long-run-growth perspective. In general, environmental policies affect the ultimately extracted resource quantity, either because some reserve units become uneconomic (e.g. Hoel, 2010; Gerlagh, 2011; van der Ploeg and Withagen, 2012) or because of lower exploration and development efforts (e.g. Daubanes and Lasserre, 2012).<sup>7</sup> For its purpose is to emphasize the role of R&D processes in a long-run-growth perspective, the present paper gives priority to the study of policies' effects on the speed of resource exploitation, at the expense of those on how much is ultimately depleted. As the above contributions show, a reduction of total extraction can easily be achieved with conventional environmental policies applied to the resource or to its substitutes directly.

The analysis is organized as follows. Section 2 exposes the setup. Section 3 analyzes the function by which resource demand is determined and identifies its macroeconomic drivers. Section 4 focuses on the effects of increased relative subsidies to clean R&D activities on the economy's structure. Section 5 combines the results of Sections 3 and 4 and integrates the intertemporal dimension of resource extraction to deliver the main message of the paper. Section 6 concludes.

## 2. THE MODEL

At each date  $t$  of the continuous set  $[0, +\infty)$ , the competitive final sector produces a quantity  $Y(t)$  of final good using a "clean" input  $Y_c(t)$  and a "dirty" input  $Y_d(t)$ , according to the CES aggregate production function

$$Y(t) = [Y_c(t)^{(\varepsilon-1)/\varepsilon} + Y_d(t)^{(\varepsilon-1)/\varepsilon}]^{\varepsilon/(\varepsilon-1)}, \quad (1)$$

where  $\varepsilon \in (0, +\infty)$  is the elasticity of substitution between the clean and dirty production sectors.

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<sup>6</sup>A notable exception is André and Smulders (2004). Still, their specification does not make the total cumulative extraction responsive to policies since the entire stock is always exhausted over the horizon.

<sup>7</sup>In either case, this amounts to take the heterogeneity of deposits into account.

The clean sector competitively produces the output  $Y_c(t)$  from a flow  $Q(t)$  of renewable energy supposed to be non polluting (e.g. solar, wind energies). For simplicity, we assume this flow  $Q(t)$  to be constant, equal to  $Q$ , as if it was produced from a constant flow of the archetypical renewable labor energy.<sup>8</sup>

The dirty sector's output  $Y_d(t)$  is competitively produced from a flow  $R(t)$  of non-renewable resource supposed to be polluting (e.g. fossil fuels).<sup>9</sup> This flow is costlessly extracted from a fixed stock  $S_0$  of Hotelling reserves:

$$\dot{S}(t) = -R(t), \quad (2)$$

where  $S(t)$  is the remaining stock of reserves to be exploited at date  $t \geq 0$  and where, as in the rest of the paper, a dot on top of a variable means that this variable is differentiated with respect to time.

Precisely, the clean and dirty sectors respectively produce  $Y_c(t)$  and  $Y_d(t)$  according to the production functions

$$Y_c(t) = Q(t)^{1-\alpha} \int_0^1 A_{ci}(t)^{1-\alpha} x_{ci}(t)^\alpha di, \quad (3)$$

$$Y_d(t) = R(t)^{1-\alpha} \int_0^1 A_{di}(t)^{1-\alpha} x_{di}(t)^\alpha di, \quad (4)$$

with  $0 < \alpha < 1$ . For each sector  $j = c, d$  there is a continuum of sector-specific intermediate goods indexed by  $i \in [0, 1]$ :  $x_{ji}(t)$  denotes the quantity of the intermediate good  $i$  used in sector  $j$ . Moreover,  $A_{ji}(t)$  denotes the contemporaneous quality level of this intermediate good.

Technical change is directed in the sense that there are two R&D sectors, one "clean" and one "dirty", respectively associated with the clean and the dirty production sectors,

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<sup>8</sup>For instance,  $Q(t)$  is produced from the labor quantity  $L_Q(t)$  only, according to the constant-return-to-scale production function  $Q(t) = \beta L_Q(t)$ , with  $L_Q(t) = L_Q$ . Also for simplicity, the use of the resource input  $R(t)$  in the other (dirty) sector will not require any labor energy. Had we assumed a flow of homogeneous labor to be allocated to the clean and dirty sectors, as in Acemoglu *et al.* (2012), our results would not have changed in any fundamental manner. Indeed, what ultimately matters for the equilibrium allocation to exhibit a trade-off between the two sectors is that some inputs – at least one – are used in both of them. As will be clear later on, these inputs are the intermediate ones.

<sup>9</sup>The analysis focuses on the policy-induced changes in the extraction pattern, in equilibrium outcomes where pollution is not internalized and in absence of direct externality-corrective policies. Thus, the polluting character of the resource need not be explicitly modeled. All along this character will remain implicit.



and respectively aimed to improve the quality level  $A_{ji}(t)$  of intermediate goods specific to those sectors.

In each R&D sector  $j = c, d$  a number  $L_{ji}(t)$  of atomistic scientists are dedicated to improving the quality level  $A_{ji}(t)$  of the intermediate good  $i \in [0, 1]$ . Each scientist in the R&D sector  $j$  has an instantaneous time-invariant and sector-specific probability  $\eta_j \in (0, 1)$  of being the successful innovator. In case such a success occurs in sector  $j$  on the part of one of the  $L_{ji}(t)$  scientists, the quality level  $A_{ji}(t)$  rises by  $\gamma A_{ji}(t)$  with  $\gamma > 0$ , which means that the new version of the associated intermediate good is more productive; otherwise, that is absent any such success,  $A_{ji}(t)$  remains unchanged. Thus, at any date  $t \geq 0$ , given the contemporaneous quality level  $A_{ji}(t)$  and the number of scientists  $L_{ji}(t)$ , it can be established that the expected instantaneous rise in  $A_{ji}(t)$  is given by the standard law of motion

$$\dot{A}_{ji}(t) = \gamma A_{ji}(t) \eta_j L_{ji}(t), \quad \forall j = c, d, \quad \forall i \in [0, 1]. \quad (5)$$

The arrival of innovation-generated intermediate-good versions raises the issue of how property rights are defined; an issue at the core of modern economic theories accounting for private incentives to perform R&D. Acemoglu *et al.* (2012) assume that any successful scientist is given a temporary monopoly right over the benefit derived from sales of the intermediate-good version generated by her innovation. In their discrete-time framework, they assume such patents are only enforced over the smallest definable unit of time, that is a period. In the long-run perspective of growth theory, the normalization is meaningful.<sup>10</sup> Most importantly, it is particularly convenient as it rules out the possibility, technically unmanageable in such models, that an innovation occurs in one sector while rights over the last intermediate-good version are still being enforced.

The assumption has a clear counterpart in our continuous-time model, implying that *any patent is only enforced at the very date when the innovation occurs*. Even when time is continuous, the crucial existence of quasi-rents which motivate R&D investments is compatible with the normalization. As we shall see, the assumption thus reproduces the

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<sup>10</sup>Since in practice intellectual property rights are enforced for a finite-time duration, the assumption is arguably stronger than the often-made alternative one that rights last forever.

standard structure of endogenous innovation processes in the simplest possible manner. On the other hand, the innovation process need not be continuous. To avoid discontinuity in intermediate goods pricing, however, we strictly follow Acemoglu *et al.* (2012): when no scientist is successful in one sector, the monopoly right survives and is randomly allocated to any potential entrepreneur, who then exploits the last intermediate-good version.

In either case, at each date  $t \geq 0$ , there is always a single intermediate good  $x_{ji}(t)$  of quality level  $A_{ji}(t)$ , which is produced by a monopoly, according to the linear production function

$$x_{ji}(t) = \frac{1}{\psi} y_{ji}(t), \quad j = c, d, \quad i \in [0, 1], \quad (6)$$

with  $\psi > 0$  and where  $y_{ji}(t)$  is an amount of final good.

The preferences of the representative, infinitely-lived household, are represented by the intertemporal utility function

$$U = \int_0^{+\infty} \ln(C(t)) e^{-\rho t} dt, \quad (7)$$

where  $\rho$  is a constant discount rate.<sup>11</sup> For our purpose, the intertemporal elasticity of substitution does not play any crucial role; the logarithmic form of the felicity function thus normalizes this elasticity to unity for simplicity.

Households are endowed with the constant flow  $L > 0$  of labor energy; each unit of labor is competitively supplied by one scientist to the R&D sectors. Normalizing the mass of scientists to unity, it must be that

$$\int_0^1 L_{ci}(t) di + \int_0^1 L_{di}(t) di = 1, \quad \forall t \geq 0. \quad (8)$$

Last, the final good produced at each date  $t \geq 0$  is either used for consumption or for the production of clean and dirty intermediate goods:

$$Y(t) = C(t) + \int_0^1 y_{ci}(t) + \int_0^1 y_{di}(t). \quad (9)$$

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<sup>11</sup>An explicit modeling of environmental damages would have raised the issue of how households are affected by those damages. The rest of the analysis would have remained unchanged with separable damages, under which the marginal utility solely depends on consumption.

### 3. INPUT DEMANDS BY PRODUCTION SECTORS

As is usual, we choose the final good as the numeraire good; its price is normalized to unity. In the rest of the paper,  $p_{ji}(t)$ ,  $p_Q(t)$ ,  $p_R(t)$  will respectively denote the price of the intermediate good  $i \in [0, 1]$  used in sector  $j = c, d$ , the price of the clean renewable resource and the price of the dirty non-renewable resource. Because they are competitive, the final sector and the clean and dirty production sectors can be aggregated without loss of generality so that their joint problem simply consists in the maximization of their total profit

$$\begin{aligned} \pi_Y(t) &= [Y_c(t)^{(\varepsilon-1)/\varepsilon} + Y_d(t)^{(\varepsilon-1)/\varepsilon}]^{\varepsilon/(\varepsilon-1)} \\ &\quad - \sum_{j=c,d} \int_0^1 p_{ji}(t)x_{ji}(t) di - p_Q(t)Q(t) - p_R(t)R(t), \end{aligned} \tag{10}$$

where prices are taken as parameters.

#### 3.1 Resource Demand

As one can anticipate in light of Sinn's (2008) analysis, resource demand will turn out to play the most fundamental role in the determination of the speed at which the resource is extracted.

The first-order condition for the choice of resource input  $R(t)$  writes  $p_R(t) = (1 - \alpha)Y(t)^{1/\varepsilon}Y_d(t)^{(\varepsilon-1)/\varepsilon}/R(t)$ , which equalizes the marginal productivity of the resource input to its price. Using (1), the marginal productivity of the resource can be expressed in such a way that

$$p_R(t) = \frac{(1 - \alpha)Y(t)}{R(t)} \frac{1}{(Y_c(t)/Y_d(t))^{(\varepsilon-1)/\varepsilon} + 1}, \tag{11}$$

which must be interpreted using the concept of conditional factor demand; precisely, the resource productivity as given by the right-hand side of (11) is the inverse resource demand function, conditional upon the amount of final output  $Y(t)$ .

As (11) shows, the conditional resource demand function only depends on the relative contribution  $Y_c(t)/Y_d(t)$  of sectors  $c$  and  $d$  to the economy, yet in a way that involves sectors' degree of substitutability/complementarity  $\varepsilon$ . Smulders and Di Maria (2012) recently pointed at the crucial and intricate channels by which technological change de-

termines inputs demand. In a multi-sector context, formula (11) further shows the role of the relative contribution of sectors: for a given output level, the relative contribution of sectors completely summarizes the determinants of resource marginal productivity and thus of total resource demand. The rest of the analysis will examine how the relative sectors' contribution is affected by R&D policies, thus making the following proposition a central result of the paper.

**Proposition 1** *A rise in the relative contribution of the clean sector to the economy causes the conditional resource demand function*

- i) to decrease if the clean and dirty sectors are (gross) substitutes ( $\varepsilon > 1$ ) or*
- ii) to increase if these sectors are (gross) complements ( $\varepsilon < 1$ ).*

### 3.2 Intermediate Good Quantities

On the one hand, the first-order conditions for the choice of quantities  $x_{ci}(t)$  and  $x_{di}(t)$  of clean and dirty intermediate goods maximizing (10) are

$$p_{ci}(t) = \alpha Y(t)^{1/\varepsilon} Y_c(t)^{-1/\varepsilon} \left[ \frac{Q(t)A_{ci}(t)}{x_{ci}(t)} \right]^{1-\alpha} \quad \text{and} \quad (12)$$

$$p_{di}(t) = \alpha Y(t)^{1/\varepsilon} Y_d(t)^{-1/\varepsilon} \left[ \frac{R(t)A_{di}(t)}{x_{di}(t)} \right]^{1-\alpha}, \quad (13)$$

which give the production sectors' inverse demands for intermediate goods.

On the other hand, our assumptions imply the standard property that at each date  $t \geq 0$ , all intermediate goods  $x_{ji}(t)$ ,  $j = c, d$ ,  $i \in [0, 1]$ , are monopolistically supplied. By (6), producing a quantity  $x_{ji}(t)$  of intermediate good requires an amount  $\psi x_{ji}(t)$  of final good. The profit derived from this activity thus writes

$$\pi_{ji}(t) = x_{ji}(t) [p_{ji}(t) - \psi], \quad (14)$$

where  $p_{ji}(t)$  is given by (12) and (13) because monopolies integrate the sensitiveness of the demand they face. In this context, monopoly prices  $p_{ji}(t)$  exhibit a mark-up above the marginal cost  $\psi$ ,

$$p_{ji}(t) = \frac{\psi}{\alpha}, \quad \forall j = c, d, \quad \forall i \in [0, 1], \quad (15)$$

and turn out to be time-invariant as well as independent of the sector  $j = c, d$  to which they are dedicated and of the type of intermediate good  $i \in [0, 1]$ .

As a result, inverse demand functions (12) and (13) imply that the equilibrium quantity of intermediate good  $x_{ji}(t)$  proportionally depends on the quality level  $A_{ji}(t)$  in a way that is independent of the type of intermediate good  $i \in [0, 1]$ . Making use of this property, Appendix A combines the relative contribution of the clean sector  $Y_c(t)/Y_d(t)$  obtained from production functions (3) and (4) with the relative marginal productivity derived from (12) and (13) and shows the following identity:

$$\frac{Y_c(t)}{Y_d(t)} = \left[ \frac{x_c(t)}{x_d(t)} \right]^{\varepsilon/(\varepsilon-1)}, \quad (16)$$

where  $x_c(t) \equiv \int_0^1 x_{ci}(t) di$  and  $x_d(t) \equiv \int_0^1 x_{di}(t) di$  denote the average quantities of intermediate goods respectively used in the clean and dirty production sectors.<sup>12</sup>

As (16) shows, at the production sector's optimum, changes in the relative contribution of the clean sector are completely summarized by changes in the relative use of intermediate goods by this sector. This is the message of the following lemma which will later turn out to be particularly useful.

**Lemma 1** *In equilibrium, the relative contribution of the clean sector to the economy*  
*i) increases with the relative use of clean intermediate goods if the clean and dirty sectors*  
*are (gross) substitutes ( $\varepsilon > 1$ ) and*  
*ii) decreases with it if these sectors are (gross) complements ( $\varepsilon < 1$ ).*

#### 4. R&D-SUPPORT POLICIES AND DIRECTED TECHNICAL CHANGE

This section investigates how R&D and intermediate activities are affected by R&D-support policies. For that purpose, we first establish how economic resources are allocated to competing R&D sectors in equilibrium.

When an innovation occurs at date  $t \geq 0$ , giving rise to a new, more productive type of intermediate good  $x_{ji}(t)$ ,  $j = c, d$ ,  $i \in [0, 1]$ , the innovator is entitled with an

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<sup>12</sup>The notations are introduced here for simplicity. It will shortly turn out to be true that  $x_j(t) = x_{ji}(t)$ , for all  $j = c, d$ , for all  $i \in [0, 1]$ .

exclusive right over the profits  $\pi_{ji}(t)$  immediately derived from the sales of the new intermediate good. The most basic and meaningful way to support R&D activities is to subsidize innovators' profits. Let  $\lambda_j(t) \geq 1$  be the time-varying subsidy factor applied to any innovator's profit in sector  $j = c, d$ . Hence, the inclusive-of-subsidy benefit from innovating is<sup>13</sup>

$$V_{ji}(t) = \lambda_j(t)\pi_{ji}(t), \quad (17)$$

where it follows from the analysis of the last section (formula (14) with equilibrium price (15)) that  $\pi_{ji}(t)$  can be expressed as a function of  $x_{ji}(t)$  only:

$$\pi_{ji}(t) = \frac{(1-\alpha)\psi}{\alpha}x_{ji}(t). \quad (18)$$

At each date  $t \geq 0$ ,  $L_{ji}(t)$  scientists are dedicated to improving the quality level of intermediate good  $i \in [0, 1]$  used in sector  $j = c, d$ , each having the instantaneous probability  $\eta_j$  of being the successful innovator. The total profit of this R&D sector writes

$$\pi_{R\&Dji}(t) = \eta_j L_{ji}(t)V_{ji}(t) - w(t)L_{ji}(t), \quad (19)$$

where  $w(t)$  is the wage rate and  $V_{ji}(t)$  is given by (17).

As is well known, it is theoretically possible in directed-technical-change models that  $\pi_{R\&Dji}(t)$  is strictly negative for any strictly positive  $L_{ji}(t)$ , so that innovations do not occur in all sectors (e.g. Acemoglu, 2008, Ch. 15). In such contexts, R&D-support policies can only induce innovations in all sectors when they are stringent enough and may be neutral otherwise. As a matter of fact, specific R&D activities currently take

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<sup>13</sup>Our modeling choice to follow Acemoglu *et al.* (2012) in assuming the duration of patents to be normalized to the smallest definable unit of time makes expression (17) much simpler than in traditional treatments where the benefit from innovating not only involves contemporaneous monopoly profits, but also the discounted stream of future expected ones. As explained in the introduction, the simplification does not imply any departure from the regular endogenous-growth mechanism where prospects of quasi-rents motivate R&D investments. In fact, the rest of the analysis would have remained formally the same if patents were assumed to be enforced for a long time, while the rate at which profits are discounted were taken as given. The proof goes as follows. Let parameter  $\theta(t)$  be this given rate of discount, which consists of the interest rate and possibly of an instantaneous risk of patents' erosion in the form described by Barro and Sala-i-Martin (1995, Ch. 6). Then, expression (17) would be  $V_{ji}(t) = \int_t^{+\infty} \lambda_j(s)\pi_{ji}(s)e^{-\int_t^s \theta(u) du} ds$ . Differentiating with respect to time by Leibniz rule would give  $\dot{V}_{ji}(t)/V_{ji}(t) - \theta(t) = -(\lambda_j(t)\pi_{ji}(t))/V_{ji}(t)$ , for the two sectors  $j = c, d$ . However, the analysis would only change to this extent: the R&D profit (19) as well as the free-entry condition (20) would still apply to both sectors. Because the latter implies the growth rate of  $V_{ji}(t)$  to be the same in the two sectors, the fundamental no-arbitrage equation (21) as well as the rest of the analysis would then hold in the same manner as under our assumption.

place simultaneously in resource and non-resource sectors. Thus, two main reasons make it interesting to study cases where the allocation of labor resources to several R&D sectors is interior. First, they are empirically relevant. Second, from a theoretical perspective, only those cases deliver a clear message on the direction in which policies can distort the economy.

In the rest of the analysis, we assume away the possibility of a corner allocation of labor to R&D sectors and so restrict attention to situations where this allocation is interior. Appendix B derives the underlying conditions on parameters.<sup>14</sup> Thus, labor entry is profitable in the two sectors; in any such equilibria, arbitrage possibilities result in the standard free-entry condition which applies to the clean and to the dirty sectors in a similar manner. From (19), it must be that

$$\eta_c V_{ci}(t) = \eta_d V_{di}(t) = w(t), \quad \forall i \in [0, 1], \quad (20)$$

which means that the marginal productivity of scientists is equalized across R&D sectors.

The above free-entry condition has two main implications. The first one amounts to a simplification. The equilibrium allocation of labor as per (20) implies that the net benefit of innovating  $V_{ji}(t)$ , while varying across sectors  $j = c, d$  according to the probability parameters  $\eta_c$  and  $\eta_d$ , does not depend on the type of intermediate good  $i \in [0, 1]$  the associated innovation improves. By (17) and (18), the same property applies to profits  $\pi_{ji}(t)$  and to intermediate good quantities  $x_{ji}(t)$ . In the rest of the analysis, we will make use of the notations  $V_j(t) = V_{ji}(t)$  and  $\pi_j(t) = \pi_{ji}(t)$ , while our earlier definition  $x_j(t) \equiv \int_0^1 x_{ji}(t) di$  now becomes the equilibrium identity  $x_j(t) = x_{ji}(t)$ , for any  $j = c, d$  and  $i \in [0, 1]$ .

The second implication of (20) is an essential piece of the paper's demonstration. The free-entry condition tells that the relative inclusive-of-subsidy benefit of innovating  $V_c(t)/V_d(t)$  is determined in a way that is irrespective of subsidy rates  $\lambda_c(t)$  and  $\lambda_d(t)$ . Recalling that by (17), the net value  $V_j(t)$  only depends on the subsidy rate  $\lambda_j(t)$  and on

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<sup>14</sup>In the *laissez-faire* equilibrium without R&D-support policies, the dirty R&D sector is always active. For the clean R&D sector to be active as well, expected prospects of improvements, which is given by the product of the quality improvement rate with the innovation probability, must be sufficiently large. As policies are introduced, the conditions for the allocation to be interior further depend on them.

$\pi_j(t)$ , we obtain the more enlightening condition

$$\eta_c \lambda_c(t) \pi_c(t) = \eta_d \lambda_d(t) \pi_d(t), \quad (21)$$

which shows that in absence of arbitrage, any support to the clean sector by say an increase in the relative subsidy  $\lambda_c(t)/\lambda_d(t)$  is necessarily compensated by a change in relative profits  $\pi_c(t)/\pi_d(t)$  of opposite direction.

No-arbitrage conditions are the keystone of equilibria in economies with competing investment possibilities. While absent in one-sector endogenous-growth models, directed-technical-change models reveal that such conditions must prevail at the R&D stage, that returns to investing in R&D must equalize across all R&D sectors. Thus, taking a full account of the process by which specific R&D investments are implemented as a response to economic incentives, clearly yields the above result that is otherwise not so intuitive. Another surprising aspect of the demonstration may be that the result holds regardless of the effect of R&D-support subsidies on the technological bias.

Keeping in mind from (18) that profits of the intermediate sectors can be expressed as functions of intermediate good quantities only, we immediately obtain the following lemma.

**Lemma 2** *In any interior equilibrium allocation, the relative use of clean intermediate goods always decreases in the relative subsidy to clean R&D activities.*

Lemma 2 provides an essential result of the paper which is not the most intuitive one. Shortly below, it will nicely combine with Lemma 1 and with Proposition 1 so as to determine the effect of R&D-support policies on resource demand.

## 5. R&D-SUPPORT POLICIES AND THE GREEN PARADOX

### 5.1 Transmission Channel

The first part of this section aims at summarizing the effect of R&D-support policies on resource demand. For that, we can rely on the results established earlier. On the one hand, Lemma 2 states the effect of R&D-support policies on the relative use of intermediate goods by the clean and dirty sectors. On the other hand, Lemma 1 tells us that



the relative use of intermediate goods completely summarizes the relative contribution of the clean and dirty sectors. The combination of Lemma 1 with Lemma 2 immediately yields the following proposition on the effect of R&D-support policies on the relative contribution of the clean and dirty sectors.

**Proposition 2** *A rise in the relative subsidy to clean R&D activities causes the equilibrium relative contribution of the clean sector to the economy*

*i) to decrease if the clean and dirty sectors are (gross) substitutes ( $\varepsilon > 1$ ) and*

*ii) to increase if these sectors are (gross) complements ( $\varepsilon < 1$ ).*

Formally speaking, Proposition 2 results from the combination of identity (16) that  $Y_c(t)/Y_d(t) = [x_c(t)/x_d(t)]^{\varepsilon/(\varepsilon-1)}$ , with condition (21) that  $\eta_c\lambda_c(t)\pi_c(t) = \eta_d\lambda_d(t)\pi_d(t)$ , making use of (18), which implies that relative profits  $\pi_c(t)/\pi_d(t)$  equal relative intermediate quantities  $x_c(t)/x_d(t)$ . Hence, in any interior equilibrium, we have

$$\frac{Y_c(t)}{Y_d(t)} = \left[ \frac{\lambda_d(t)\eta_d}{\lambda_c(t)\eta_c} \right]^{\varepsilon/(\varepsilon-1)}. \quad (22)$$

Let us now come back to resource demand. Substituting (22) in expression (11), gives the conditional inverse resource demand

$$p_R(t) = \frac{(1-\alpha)Y(t)}{R(t)}\Lambda(t), \quad (23)$$

where, for notational simplicity, we make use of the policy index  $\Lambda(t)$  defined as

$$\Lambda(t) \equiv \frac{1}{1 + \frac{\lambda_d(t)\eta_d}{\lambda_c(t)\eta_c}}. \quad (24)$$

The index  $\Lambda(t)$  is a basic measure of the relative support to clean R&D activities. For what follows, it only matters that  $\Lambda(t)$  is monotonically increasing in the relative subsidy to clean R&D  $\lambda_c(t)/\lambda_d(t)$ . Thus, equation (23) expresses the conditional resource demand, no longer as a function of  $Y_c(t)/Y_d(t)$  as in (11), but directly as a function of R&D-support policies.

Neither formula (23) nor definition (24) involves the elasticity of substitution  $\varepsilon$ . Thus it turns out that, taking the output-level condition  $Y(t)$  as given in formula (23), a rise in the relative clean-R&D subsidy causes resource demand to increase, regardless of the

pattern of substitutability/complementarity between the sectors. Indeed, although the role of the sectoral elasticity of substitution  $\varepsilon$  appears crucial in Proposition 2 (relation between R&D-support policies and sectors' contribution) and in Proposition 1 (relation between sectors' contribution and resource demand), this role completely vanishes when the results combine to determine the overall effect of policies on resource demand. To sum up, the final message arising from Propositions 1 and 2 is independent of sectors' degree of substitutability/complementarity: *a rise in the relative clean-R&D subsidy always causes resource demand to increase.*

So far, results have been established as static effects holding at each date  $t$  of the time set  $[0, +\infty)$ . Since the resource is non-renewable, its extraction pattern cannot be directly deduced from the above static analysis; similarly, the effects of R&D-support policies on the extraction speed cannot be deduced from the above static result arising from Propositions 1 and 2. The next and last section extends the above result from the static frame of the previous sections to its dynamic counterpart.

## 5.2 Impact of R&D-Support Policies on the Resource Extraction Pattern

Extending the above static results summarized by formula (23) so as to derive the effects of R&D-support policies on the speed of resource extraction further requires taking into account intertemporal decisions which determine the dynamic equilibrium.

On the one hand, households' consumption/saving arbitrage determines the growth rate of final output  $Y(t)$  in (23). The maximization of the intertemporal utility objective (7), subject to any intertemporal budget constraint arising under a perfect financial market, yields the standard Ramsey-Keynes condition:  $g_C(t) = r(t) - \rho$ , where  $r(t)$  is the rate of interest endogenously determined on the financial market, and where, as in the rest of the paper, the symbol  $g$  with a variable subscript denotes the growth rate of this variable. Appendix C shows that, although consumption is not the exclusive use of final output, the Ramsey-Keynes condition applying to the former also dictates the law of motion of the latter. Formally,

$$g_Y(t) = r(t) - \rho, \quad \forall t \geq 0, \quad (25)$$

must hold in any dynamic equilibrium.

On the other hand, the same assumption that there exists a perfect financial market implies that the standard rule of Hotelling must hold, meaning that any unit of reserves must fetch the same revenue in present value. Under our simplifying, although conventional in growth models, assumption that extraction is costless, the rule writes

$$g_{p_R}(t) = r(t), \quad \forall t \geq 0, \quad (26)$$

which must be satisfied in any dynamic equilibrium.

Let us now come back to the conditional expression of resource demand (23) and to its dynamic implications. Log-differentiating both sides and making use of the Keynes-Ramsey rule (25) and of the Hotelling rule (26), the following expression of the speed of extraction immediately follows

$$g_R(t) = g_\Lambda(t) - \rho, \quad (27)$$

where the effects of the interest rate on  $g_Y(t)$  and  $g_{p_R}(t)$  have canceled out.

We are left with the following statement which concludes our analysis.

**Proposition 3** *Under the assumptions of this paper, in any dynamic equilibrium, a gradual rise of the relative subsidy to clean R&D activities over time induces resource extraction to be less rapid, irrespective of the pattern of substitutability/complementarity between the clean and dirty production sectors.*

This final result should be interpreted through the lens of the preceding static messages of Propositions 1 and 2. Their combination and their dynamic extension tells the following. A gradual rise of the relative subsidy to clean R&D, whatever its other dynamic effects on the economy's structure, will always contribute to gradually increasing the demand for the resource. The rest is standard: the resulting rise in future demand relative to early demand leads market forces to exploit less resource at early dates and so more at distant ones.

## 6. CONCLUSION

Technical improvements in the production of clean substitutes to dirty non-renewable resources have been considered tantamount to policies that reduce the demand for those

resources (e.g. Sinn, 2008). In this context, such technical improvements, whether expected to take place in the long-term future, or gradually induced by more-and-more stringent R&D-support policies, would cause an undesirable green paradox phenomenon. The pessimistic argument has been made in parsimonious models, either adopting a partial-equilibrium approach taking the technology as exogenous, or assuming a single resource-consuming sector.

This paper shows that the conjecture is wrong. Borrowing standard assumptions from the modern-economic-growth literature, the analysis takes a full account of the endogenous directed R&D process by which productivity is specifically enhanced in clean renewable-resource and dirty non-renewable-resource sectors, and of the policy instruments by which such R&D activities can be promoted. We find that a gradual support to R&D activities aimed to improve productivity in clean sectors increases, among other effects, the long-run productivity and thus the demand for the non-renewable resource. From a policy perspective, the message delivered by the analysis is more optimistic than the aforementioned conjecture. Supporting clean R&D sectors does not cause a green paradox phenomenon.

## APPENDICES

### A PROOF OF IDENTITY (16)

Substituting  $p_{ci}(t) = p_{di}(t) = \psi/\alpha$  from (15) into equalities (12) and (13) and dividing the left and right sides of the latter by both sides of the former yields the following expression which gives the relative contribution of sectors

$$\left(\frac{Y_c(t)}{Y_d(t)}\right)^{1/\varepsilon} = \left(\frac{Q(t)}{R(t)}\right)^{1-\alpha} \left(\frac{A_{ci}(t)/x_{ci}(t)}{A_{di}(t)/x_{di}(t)}\right)^{1-\alpha}. \quad (\text{A.1})$$

In this identity, the ratios  $A_{ji}(t)/x_{ji}(t)$  are independent of  $i \in [0, 1]$  for each  $j = c, d$  as shown by (12) and (13) after the substitution of  $p_{ji}(t)$  from (15).

The later property implies that (3) and (4) can be rewritten with  $\int_0^1 A_{ji}(t)^{1-\alpha} x_{ji}(t)^\alpha di = (A_{ji}(t)/x_{ji}(t))^{1-\alpha} \int_0^1 x_{ji}(t) di$  where by definition  $\int_0^1 x_{ji}(t) di \equiv x_j(t)$ . Dividing the obtained expressions of equalities (3) and (4) by each other yields

$$\frac{Y_c(t)}{Y_d(t)} = \left(\frac{Q(t)}{R(t)}\right)^{1-\alpha} \left(\frac{A_{ci}(t)/x_{ci}(t)}{A_{di}(t)/x_{di}(t)}\right)^{1-\alpha} \frac{x_c(t)}{x_d(t)}. \quad (\text{A.2})$$

Finally, dividing (A.1) by (A.2) immediately gives identity (16).

### B ALLOCATION OF SCIENTISTS TO THE CLEAN AND DIRTY R&D SECTORS

The property shown in Appendix A that ratios  $A_{ji}(t)/x_{ji}(t)$  are independent of  $i \in [0, 1]$  for each  $j = c, d$ , combined with the result of Section 4 that  $x_{ji}(t) = x_j(t)$ , for all  $i \in [0, 1]$  and each  $j = c, d$ , implies that all  $A_{ci}(t)$  and all  $A_{di}(t)$ , for any  $i \in [0, 1]$ , must always follow the same average trajectory. In light of (5), this also implies that scientists in each R&D sector  $j = c, d$  are evenly allocated over the range  $i \in [0, 1]$ ; we can define  $L_j(t) = L_{ji}(t)$ , which is also the total number of scientists  $\int_0^1 L_{ji}(t) di$  employed in the R&D sector  $j = c, d$ . In this context, summing over the range  $i \in [0, 1]$  all equations (5) and making use of the variable  $A_j(t) \equiv \int_0^1 A_{ji}(t) di$  to denote the average quality level in each sector  $j = c, d$ , we obtain the average quality levels' laws of motion

$$g_{A_j}(t) = \gamma\eta_j L_j(t), \quad j = c, d, \quad (\text{B.1})$$

where the symbol  $g$  with a variable subscript denotes the growth rate of the variable. We are going to make use of this equation shortly below.

Moreover, proceeding the same way as Appendix A with the above mentioned property that ratios  $A_{ji}(t)/x_{ji}(t)$  are independent of  $i \in [0, 1]$  for each  $j = c, d$ , (3) and (4) can respectively be expressed as follows:

$$Y_c(t) = Q(t)^{1-\alpha} A_c(t)^{1-\alpha} x_c(t)^\alpha, \quad (\text{B.2})$$

$$Y_d(t) = R(t)^{1-\alpha} A_d(t)^{1-\alpha} x_d(t)^\alpha. \quad (\text{B.3})$$

Log-differentiating these two equations and simplifying with (B.1) and (27), we obtain

$$g_{Y_c}(t) = (1 - \alpha)\gamma\eta_c L_c(t) + \alpha g_{x_c}(t), \quad (\text{B.4})$$

$$g_{Y_d}(t) = (1 - \alpha)\gamma\eta_d L_d(t) + \alpha g_{x_d}(t) + (1 - \alpha)(g_\Lambda(t) - \rho). \quad (\text{B.5})$$

From (22), it follows that  $g_{Y_c}(t) - g_{Y_d}(t) = (g_{\lambda_d}(t) - g_{\lambda_c}(t))\varepsilon/(\varepsilon - 1)$ . Substituting the above expressions (B.4) and (B.5), simplifying by using the fact that  $g_{x_c}(t) - g_{x_d}(t) = g_{\lambda_d}(t) - g_{\lambda_c}(t)$  because  $x_c(t)/x_d(t) = \lambda_d(t)/\lambda_c(t)$  from (22) and (16), and rearranging, we obtain

$$(1 - \alpha)\gamma(\eta_c L_c(t) - \eta_d L_d(t)) = \left(\frac{\varepsilon}{\varepsilon - 1} - \alpha\right) (g_{\lambda_d}(t) - g_{\lambda_c}(t)) + (1 - \alpha)(g_\Lambda(t) - \rho). \quad (\text{B.6})$$

Using that  $L_d(t) = 1 - L_c(t)$  from (8), and rearranging adequately give the following expression of the labor quantity employed in the clean R&D sector in equilibrium:

$$L_c(t) = \frac{1}{(1 - \alpha)\gamma(\eta_c + \eta_d)} \left[ \left(\frac{\varepsilon}{\varepsilon - 1} - \alpha\right) (g_{\lambda_d}(t) - g_{\lambda_c}(t)) + (1 - \alpha)(g_\Lambda(t) - \rho) + (1 - \alpha)\gamma\eta_d \right]. \quad (\text{B.7})$$

The formula shows that R&D-support policies affect the equilibrium allocation of scientists in a complex manner. Absent any R&D-support policies,  $g_{\lambda_c}(t) = g_{\lambda_d}(t) = g_\Lambda(t) = 0$ . In that case, the formula simply becomes

$$L_c(t) = \frac{(\gamma\eta_d - \rho)}{\gamma\eta_c + \gamma\eta_d}. \quad (\text{B.8})$$

Still in this context, it is easy to see that  $L_c(t) > 1$ , and thus  $L_d(t) > 0$ , are always valid. To guarantee that the allocation of scientists is interior so that  $L_c(t) > 0$ , we further need the sufficient condition that  $\gamma\eta_d > \rho$ .

## C PROOF OF (25)

The proof given in the main text relies on the equality  $g_Y(t) = g_C(t)$ , which can be obtained as follows.

Substituting the price  $p_{ci}(t) = \psi/\alpha$  from (15) into (12), and using the property that the ratios  $A_{ci}(t)/x_{ci}(t)$ , for all  $i \in [0, 1]$  do not depend on  $i$  (see Appendix A), so that they can be replaced by the ratio  $A_c(t)/x_c(t)$ , yield the relation  $(\psi/\alpha^2)Y(t)^{-1/\varepsilon} = Y_c(t)^{-1/\varepsilon} (Q(t)A_c(t)/x_c(t))^{1-\alpha}$ . Using the simplified expression of sector  $c$ 's production function (B.2), the term  $(Q(t)A_c(t)/x_c(t))^{1-\alpha}$  can be replaced by  $Y_c(t)/x_c(t)$ . Replacing and rearranging, one immediately obtains the following expression:

$$Y_c(t)^{(\varepsilon-1)/\varepsilon} = \frac{\psi}{\alpha^2} \frac{x_c(t)}{Y(t)^{1/\varepsilon}}. \quad (\text{C.1})$$

Proceeding in the exact same way with variables relative to sector  $d$  instead of  $c$ , by using (13) and (B.3), instead of (12) and (B.2), one obtains the symmetric formula

$$Y_d(t)^{(\varepsilon-1)/\varepsilon} = \frac{\psi}{\alpha^2} \frac{x_d(t)}{Y(t)^{1/\varepsilon}}. \quad (\text{C.2})$$

Introducing the expressions of  $Y_c(t)^{(\varepsilon-1)/\varepsilon}$  and  $Y_d(t)^{(\varepsilon-1)/\varepsilon}$  given by (C.1) and (C.2) into the production function (1), isolating the factor  $((\psi/\alpha^2)Y(t)^{-1/\varepsilon})^{\varepsilon/(\varepsilon-1)}$  and rearranging, all powers involving  $\varepsilon$  cancel out to yield the expression

$$Y(t) = \frac{\psi}{\alpha^2} (x_c(t) + x_d(t)), \quad (\text{C.3})$$

which we will use shortly below.

Finally, taking Equation (9), substituting  $y_{ji}(t) = \psi x_{ji}(t)$  from (6), and making use of the notations  $x_c(t) = x_{ci}(t)$  and  $x_d(t) = x_{di}(t)$ , for all  $i \in [0, 1]$  (see in Section 4 the text immediately following Equation (20)), one obtains the following relation:

$$C(t) = Y(t) - \psi (x_c(t) + x_d(t)). \quad (\text{C.4})$$

By substitution of  $\psi (x_c(t) + x_d(t))$  from (C.3), we obtain the linear relation between  $C(t)$  and  $Y(t)$

$$C(t) = Y(t)(1 - \alpha^2), \quad (\text{C.5})$$

which implies that the growth rates of the two variables are identical in any equilibrium.

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