International coordination over emissions and R&D expenditures in the context of oil scarcity

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1 Introduction

In a recent report (IEA (2006)), the International Energy Agency (IEA) emphasised the huge increase in oil prices and CO2 concentrations. Both have increased by more than 20 percent in the last decade. The report concludes by recommending policies aimed at the promotion of energy savings and the use of low-carbon technology. Those policies will be reliant on the allocation of R&D budgets adequate to enable technological progress in areas such as hydrogen and fuel cells, advanced renewable energies, next-generation biofuels and energy storage.

The interaction between climate policy and endogenous technological change has been referred to in several papers (see Golombek, R., Hoel, M. (2005) and Golombek, R., Hoel, M. (2006) for overviews). However, none of these studies has focused on the interaction between oil scarcity, technological progress and greenhouse gas emissions.

The contribution of this paper is in studying a process of international negotiation over global warming, involving three variables: pollution, the marginal extraction costs of the resource, and the level of knowledge in the renewable, non polluting resource sector.

Our approach uses a differential game model close to van der Ploeg, F., de Zeeuw, A. (1994) who compared centralized and decentralized solutions to a global pollution problem with investment in clean technology[1][2].

Our model takes account of two asymmetric players, similar to List, J. A., Mason, C. F. (2001), which can be thought of as two groups of nations:

[1] Van der Ploeg and de Zeeuw's study, however, does not link these issues to the problem of oil depletion .

[2] By computing the decentralized equilibrium, we rely on the simplifying assumption that, with the appropriate tools, each state can decentralize its own policy . This assumption allows us to abstract from the complex issues of what determines the choice of policy instruments.

rich and poor. We think that assumption of asymmetry is more realistic than assuming symmetry in the consideration of climate change problems.

The paper is organized as follows. In Section 2, we develop the general structure of the model. In Section 3, we derive the cooperative and non-cooperative equilibria. In Section 4, we implement a Monte Carlo procedure enabling us to numerically solve the model. The results are presented in Section 5 and Section 6 concludes.

2 The model

We consider a world with two players; indexed as $i = 1, 2$; corresponding to two asymmetric countries, which differ in terms of both wealth and awareness of the environment. Both countries use oil, i.e. a non renewable polluting energy, to produce a homogeneous consumption good.

2.1 Scarcity and pollution

Oil extraction has two harmful effects. First, it reduces the available oil stocks for the future. In this paper, we do not model oil as a finite-sized stock. We assume that the oil stock is infinite, but that the marginal extraction cost is an increasing function of the cumulated extractions[3]. Moreover, the marginal extraction cost does not depend on the instantaneous rate of extraction (in other words, the extraction cost is a linear function of the rate of extraction). It follows that the evolution of the marginal extraction cost of the resource (denoted by P) can be expressed by:

$$\dot{P} = \sum_{i=1}^{2} \zeta E_i \tag{1}$$

where E_i is the rate of resource extraction by country i, and ζ is a parameter. ζ denotes the *importance* of the scarcity. For $\zeta = 0$, the resource is infinitely available at a constant marginal cost; for $\zeta \to \infty$, the marginal cost of extraction increases so fast that extraction becomes no longer profitable.

The second harmful effect of oil extraction is pollution. In this paper, we model oil pollution as a cumulative process. The stock of pollution follows:

$$\dot{M} = \sum_{i=1}^{2} E_i - \delta M \tag{2}$$

[3] Heal, G. (1976)

where δ is a constant rate of decay of pollution.

Under this specification, pollution generates an external cost given by $\alpha_i M^2$, where $\alpha_i > 0$ measures the degree of sensitivity to pollution in country i.

2.2 The resource sector

The resource is used as an input to produce an aggregate good Q_i, together with a renewable non polluting energy. *Ceteris paribus*, an improvement in the knowledge about the backstop technology makes it profitable to shift from a hydrocarbon to a clean energy for a number of economic activities. Thus, for a given E_i, this improvement generates an increase in the opportunity cost of E_i. This effect is depicted in figure 1[4].

Let A be a renewable non polluting resource and X a coefficient denoting the level of knowledge in the renewable energy sector. For a given X, the optimal level of production is Q, and the optimal combination of oil and clean energy is given by (E, A). So, what happens if X increases, inducing a fall in the cost of the clean energy? First, the relative price of clean energy to oil falls. Would this change in the price ratio not have affected the quantity of output produced, the new optimal combination of input would have been (E', A'), with more clean energy and less oil, compared to the previous equilibrium. But, as the fall in the clean energy price also induces a fall in the aggregate energy price, this creates incentives to produce a greater level of final output. Let the new optimal output level be given by Q'. In this situation, the new optimal combination of input is (E'', A''), with both more clean energy and more oil compared to (E', A'). Actually, whether E'' is greater or smaller than E is not unambiguous. It depends on both the elasticity of production with respect to the energy price, and the elasticity of the substitution between oil and clean energy. Here, however, we assume that different energy sources are strong enough substitutes to ensure that a fall in the clean energy price always results in a fall in oil consumption [5].

Within this setting, the two consequences of an increase in X are, first, an increase in production, and second, a fall in the use of oil. The net production

[4]Technological change is presented, for illustrative purpose, at a point in time, to represent the effects within a two-dimensional figure. In a dynamic framework, the effects of the changes are, in fact, integrated over time.

[5]The joules from oil are perfect substitutes for the joules from any other energy source. Only storage and transportation costs differ from one source to another.

Figure 1: Substitution between oil and clean energy

Figure 2: Production function net of the oil cost

function of country i can then be written as[6]:

$$Q_i = (\beta_{i,1} - X)\, E_i - \beta_2 E_i^2 + \eta_{i,1} X - \eta_2 X^2$$

where $(\beta_{i,1}, \beta_2, \eta_{i,1}, \eta_2) > 0$ are parameters. $\beta_{i,1}$ and $\eta_{i,1}$ are respectively the emissions productivity and the productivity in clean energy sector of country i.

Figure 2 displays the production function, net of the oil cost.

The lower curve represents the net production as a function of the oil use before any increase in the knowledge stock. As soon as X increases, the net

[6] An alternative way to model these effects might be to introduce the clean energy A as a control variable in the model. Our current formulation is simpler and makes the model more tractable.

production switches to the second upper curve. As expected, the new level of production is higher, while the new level of oil use is lower.

2.3 The research sector

Economic activity in the research sector results in an increase in X. X is a pure public good. The motivations to invest in research are twofold: first, R&D investment lowers the economic impact of the increasing scarcity of the non-renewable resource; second, R&D investment lowers the abatement cost of an environmental policy that aims at substituting non-polluting for polluting energy. As long as country i invests I_i in research, X follows:

$$\dot{X} = \sigma \left(I_1 + I_2\right) - \epsilon X \tag{3}$$

where σ and ϵ are positive parameters.

This knowledge production function has two main features. First, with no investment in research, X decreases by ϵX per unit of time. This feature accounts for the necessity to maintain some minimum level of research activity to transmit the knowledge from the present generation to future generations. Second, research productivity is constant[7].

Finally, it is assumed that each country faces an investment cost given by γI_i^2, where γ is a constant parameter.

2.4 Welfare functions

The welfare function of country i is given by:

$$W_i = \int_0^\infty e^{-\rho t} \left[\left(\beta_{i,1} - P - X\right) E_i - \beta_2 E_i^2 + \eta_{i,1} X - \eta_2 X^2 - \alpha_i M^2 - \gamma I_i^2\right] dt$$

[7] This assumption is different from the one frequently adopted in the endogenous growth literature, such as in Romer, P. M. (1990), in which research productivity is linear with knowledge stock. However, it should be noted that modelling the evolution of an aggregate stock of knowledge is not the same as modelling the evolution of a sectoral stock of knowledge. As pointed out by Aghion, P., Howitt, P. (1998), the number of new ideas in any sector that remain undiscovered should not be thought of as an infinite stock. The linear modelling in macroeconomic models accounts both for the knowledge increase in each sector (quality innovations) and for the increase in the number of sectors (variety innovations). Within a given sector, the best way to model innovation would probably be logistic function. This would account for the 'giants' shoulders' effect when the stock of knowledge is low, and for the rarefaction of the remaining undiscovered ideas when the stock of knowledge is high. To keep some connection with the linear quadratic formulation, the simplest specification we can use is the constant productivity assumption in equation 3

where ρ is the discount rate.
Both countries differ with respect to their concern over the environment and over their wealth. The higher α_i, the higher the sensitiveness to the environment by country i. The higher are $\beta_{i,1}$ and $\eta_{i,1}$, the richer is country i. A country's wealth comes from its capital accumulation. Capital accumulation makes energy (both clean and polluting) more productive.

Countries are supposed to be asymmetric. Let country 1 be the richer one. We pose:

$$\beta_{1,1} > \beta_{2,1}$$

and

$$\eta_{1,1} > \eta_{2,1}$$

Should we also assume that country 1 is more sensitive to the environment? Following the Environmental Kuznets Curve, which involves an inverted U relationship between environmental pressure and per capita income, such an assumption would appear reasonable.

However, as pointed out by the recent IPCC report (IPCC (2007)), the harmful effects of global warming will be more severe for developing countries than for rich ones. Consequently, it is not obvious whether α_1 should be superior or inferior to α_2.

We then decided to investigate the two possible cases: In the first, the rich country is supposed to be more sensitive to environmental damage than the poor country ($\alpha_1 > \alpha_2$); In the second case, the reverse holds ($\alpha_1 < \alpha_2$). The simulations in section 5 are run under both those alternative specifications.

3 Cooperative and non-cooperative equilibria

3.1 Cooperative equilibrium

In this section, we characterize the optimal path, which allows to maximize the sum W of both national objectives.

$$W \equiv W_1 + W_2$$

subject to 1, 2 and 3. This path characterizes the shape of an international agreement between the two countries.

The cooperative problem can be restated as the minimization of

$$W = \int_0^\infty \left(y'^c Q^c y + v'^c R^c v\right) dt$$

subject to

$$\dot{y} = Ay + B^c v$$

with

$$y(t) \equiv e^{-\frac{1}{2}\rho t} \begin{pmatrix} P \\ M \\ X \\ 1 \end{pmatrix} ;\ v(t) \equiv e^{-\frac{1}{2}\rho t} \begin{pmatrix} \frac{X-\beta_{1,1}+P}{2\sqrt{\beta_2}} + \sqrt{\beta_2}E_1 \\ \frac{X-\beta_{2,1}+P}{2\sqrt{\beta_2}} + \sqrt{\beta_2}E_2 \\ I_1 \\ I_2 \end{pmatrix}$$

$$A \equiv \begin{pmatrix} -\frac{\rho}{2} - \frac{\zeta}{\beta_2} & 0 & -\frac{\zeta}{\beta_2} & \frac{\beta_{1,1}\,\zeta+\beta_{2,1}\,\zeta}{2\beta_2} \\ -\frac{1}{\beta_2} & -\frac{\rho}{2} - \delta & -\frac{1}{\beta_2} & \frac{\beta_{1,1}+\beta_{2,1}}{2\beta_2} \\ 0 & 0 & \frac{-\rho}{2} - \epsilon & 0 \\ 0 & 0 & 0 & -\frac{\rho}{2} \end{pmatrix}$$

$$B^c \equiv \begin{pmatrix} \frac{\zeta}{\sqrt{\beta_2}} & \frac{\zeta}{\sqrt{\beta_2}} & 0 & 0 \\ \frac{1}{\sqrt{\beta_2}} & \frac{1}{\sqrt{\beta_2}} & 0 & 0 \\ 0 & 0 & \sigma & \sigma \\ 0 & 0 & 0 & 0 \end{pmatrix} ;\ R^c \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & \gamma \end{pmatrix}$$

$$Q^c \equiv \begin{pmatrix} -\frac{1}{2\beta_2} & 0 & -\frac{1}{2\beta_2} & \frac{\beta_{1,1}+\beta_{2,1}}{4\beta_2} \\ 0 & \alpha_1 + \alpha_2 & 0 & 0 \\ -\frac{1}{2\beta_2} & 0 & -\frac{1}{2\beta_2} + 2\eta_2 & \frac{\beta_{1,1}+\beta_{2,1}}{4\beta_2} - \frac{1}{\eta_2} \\ \frac{\beta_{1,1}+\beta_{2,1}}{4\beta_2} & 0 & \frac{\beta_{1,1}+\beta_{2,1}}{4\beta_2} - \frac{1}{\eta_2} & -\frac{\beta_{1,1}^2}{4\beta_2} - \frac{\beta_{2,1}^2}{4\beta_2} \end{pmatrix}$$

$$S^c \equiv B^c {R^c}^{-1} B^{c'}$$

The optimal linear strategy is given by:

$$v^c = C^c y$$

where $C^c = -{R^c}^{-1} B^{c'} K^c$ and K^c is the symmetric stabilizing solution of the following algebraic Riccati equation:

$$A'^c + K^c A - K^c S^c K^c + Q^c = 0$$

One of the purposes of this paper is to analyze the equilibrium strategies of the players. However, C^c cannot be analyzed in isolation because of the matrix transformations required to solve the model. We have to define the following two transformation matrices:

$$TR_1 = \begin{pmatrix} \frac{0.5}{\sqrt{\beta_2}} & 0 & \frac{0.5}{\sqrt{\beta_2}} & -\frac{0.5\beta_{1,1}}{\sqrt{\beta_2}} \\ \frac{0.5}{\sqrt{\beta_2}} & 0 & \frac{0.5}{\sqrt{\beta_2}} & -\frac{0.5\beta_{2,1}}{\sqrt{\beta_2}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; TR_2 = \begin{pmatrix} \frac{0.5}{\sqrt{\beta_2}} & 0 & 0 & 0 \\ 0 & \frac{0.5}{\sqrt{\beta_2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

which enable us to compute the following matrix Z^c:

$$Z^c = TR_2 \left(C^c - TR_1 \right) = \begin{pmatrix} z^c_{1,1} & z^c_{1,2} & z^c_{1,3} & z^c_{1,4} \\ z^c_{2,1} & z^c_{2,2} & z^c_{2,3} & z^c_{2,4} \\ z^c_{3,1} & z^c_{3,2} & z^c_{3,3} & z^c_{3,4} \\ z^c_{4,1} & z^c_{4,2} & z^c_{4,3} & z^c_{4,4} \end{pmatrix}$$

The optimal strategies for each of our two players are then given by:

$$\begin{pmatrix} E^c_1 \\ E^c_2 \\ I^c_1 \\ I^c_2 \end{pmatrix} = Z^c \begin{pmatrix} P \\ M \\ X \\ 1 \end{pmatrix}$$

3.2 Closed-loop differential game

In this section, we look for a closed-loop Nash equilibrium[8] . We want to characterize the behaviour of both countries when they act in isolation[9]. Each country seeks to minimize:

$$\int_0^\infty \left(y' Q^m_i y + v'_i R^m v_i \right) dt$$

subject to:

$$\dot{y} = Ay + B^m_i v_i + B^m_j v_j; \; j \neq i$$

with

[8] Open-loop strategies, as in van der Ploeg, F., de Zeeuw, A. (1994), could also be envisaged. However, the differential games literature assumes that "open-loop" strategies produce smaller payoffs than"closed-loop" strategies. The principal reason is that "closed-loop" strategies, such as the linear Markov perfect strategy, in contrast to open-loop strategies (Cf., e.g., Fudenberg, D., Tirole, J. (1992), p.74-77).

[9] The only cooperation assumed here concerns the choice of stabilizing strategies.

$$v_i(t) \equiv e^{-\frac{1}{2}\rho t} \begin{pmatrix} \frac{X-\beta_{i,1}+P}{2\sqrt{\beta_2}} + \sqrt{\beta_2} E_i \\ I_i \end{pmatrix}$$

$$B_i^m \equiv \begin{pmatrix} \frac{\zeta}{\sqrt{\beta_2}} & 0 \\ \frac{1}{\sqrt{\beta_2}} & 0 \\ 0 & \sigma \\ 0 & 0 \end{pmatrix} ; \ R^m \equiv \begin{pmatrix} 1 & 0 \\ 0 & \gamma \end{pmatrix} ; \ S_i^m \equiv B_i^m R^{m\,-1} B_i^{m'}$$

$$Q_i^m \equiv \begin{pmatrix} -\frac{1}{4\beta_2} & 0 & -\frac{1}{4\beta_2} & \frac{\beta_{i,1}}{4\beta_2} \\ 0 & \alpha_i & 0 & 0 \\ -\frac{1}{4\beta_2} & 0 & -\frac{1}{4\beta_2} + \eta_2 & \frac{\beta_{i,1}}{4\beta_2} - \frac{1}{2\eta_2} \\ \frac{\beta_{i,1}}{4\beta_2} & 0 & \frac{\beta_{i,1}}{4\beta_2} - \frac{1}{2\eta_2} & -\frac{\beta_{i,1}^2}{4\beta_2} \end{pmatrix}$$

In this setting, as shown in Engwerda, J. C. (2005), the Markovian linear strategy, for player i, is given by:

$$v_i^m = C_i^m y$$

where $C_i^m = -R^{m\,-1} B_i^{m'} K_i^m = \begin{pmatrix} c_i^m(1,1) & c_i^m(1,2) & c_i^m(1,3) & c_i^m(1,4) \\ c_i^m(2,1) & c_i^m(2,2) & c_i^m(2,3) & c_i^m(2,4) \end{pmatrix}$ and K_i^m, $i = 1, 2$ are the symmetric stabilizing solutions of the following system of algebraic Riccati equations[10]:

$$\begin{aligned} (A - S_2^m K_2^m)' K_1^m + K_1^m (A - S_2^m K_2^m) - K_1^m S_1^m K_1^m + Q_1^m + K_2^m S_2^m K_2^m = 0 \\ (A - S_1^m K_1^m)' K_2^m + K_2^m (A - S_1^m K_1^m) - K_2^m S_2^m K_2^m + Q_2^m + K_1^m S_1^m K_1^m = 0 \end{aligned} \tag{4}$$

Let us define C^m and v^m as:

$$C^m = \begin{pmatrix} c_1^m(1,1) & c_1^m(1,2) & c_1^m(1,3) & c_1^m(1,4) \\ c_2^m(1,1) & c_2^m(1,2) & c_2^m(1,3) & c_2^m(1,4) \\ c_1^m(2,1) & c_1^m(2,2) & c_1^m(2,3) & c_1^m(2,4) \\ c_2^m(2,1) & c_2^m(2,2) & c_2^m(2,3) & c_2^m(2,4) \end{pmatrix} ; \ v^m = C^m y$$

As in the previous section, a transformation is necessary in order to interpret the results. Z^m is defined as:

$$Z^m = TR_2 (C^m - TR_1) = \begin{pmatrix} z_{1,1}^m & z_{1,2}^m & z_{1,3}^m & z_{1,4}^m \\ z_{2,1}^m & z_{2,2}^m & z_{2,3}^m & z_{2,4}^m \\ z_{3,1}^m & z_{3,2}^m & z_{3,3}^m & z_{3,4}^m \\ z_{4,1}^m & z_{4,2}^m & z_{4,3}^m & z_{4,4}^m \end{pmatrix}$$

[10]The algorithm used to solve this system is described in ANNEX 1.

such that the Markovian strategies are given by:

$$\begin{pmatrix} E_1^m \\ E_2^m \\ I_1^m \\ I_2^m \end{pmatrix} = Z^m \begin{pmatrix} P \\ M \\ X \\ 1 \end{pmatrix}$$

4 Monte Carlo procedure

A complete solution of the model would express each endogenous variable as a function of the set of parameters. Unfortunately, such a solution is very difficult, if not impossible, to compute. Let $f_i(\pi)$ be the function that gives the endogenous variable ϕ_i and π the set of the N exogenous parameters indexed by k. A Monte Carlo procedure enables us to provide a first-order Taylor approximation of f_i for a range of parameter values. Indeed, for a given π, simple algorithms compute the particular solutions of the model. We ran 1,000 simulations, choosing randomly π_j for each iteration j. Let us call $\bar{\phi}_i$ the average value of ϕ_i in the sample. Then, we can compute the OLS (Ordinary Least Squares) estimators $\hat{\psi}$ for the set of parameters ψ in the following linear function:

$$\phi_i = \psi_{0,i} + \sum_{k=1}^{N} \psi_{(k,i)} \pi_k + e_i \tag{5}$$

where e is an error term[11].

For each iteration, the parameters are chosen via a homogeneously distributed density function defined between $0.5\mu_{\pi_i}$ and $1.5\mu_{\pi_i}$ where μ_{π_i} is the mean value of π_i, except for $\beta_{i,1}$ and $\eta_{i,1}$ for which it is between $0.8\mu_{\pi_i}$ and $1.2\mu_{\pi_i}$.

The endogenous variables are the 16 components of the matrix Z^c and Z^m, the state of the stocks after 10 periods (which represents a relevant future), the state of the stocks after 600 periods (which represents the steady state of the model), and the intertemporal welfare computed over the 10 first periods, both for the cooperative equilibrium and the Markovian equilibrium.

The mean values of the parameters are given in table 1.

[11] Notice that the error term is treated as random, even if it is not: it is the difference between the genuine deterministic function and its Taylor approximation.

α_1	$0,1$
α_2	$0,01$
$\beta_{1,1}$	5000
$\beta_{2,1}$	3000
β_2	200
$\eta_{1,1}$	200
$\eta_{2,1}$	150
η_2	2
γ	2
δ	$0,01$
ϵ	$0,01$
σ	1
ζ	2
ρ	$0,03$

Table 1: Mean values of the parameters

5 Results

5.1 General Results

As noted above, we differentiated between two different study-cases depending on the relative country's sensitiveness to environmental damage. In the first case (referred to as Case 1), the poor country is supposed to display the greatest sensitiveness to environmental damage ($\alpha_1 < \alpha_2$). In the second case (referred to as Case 2), the rich country is the one that is most aware of environmental damage ($\alpha_1 > \alpha_2$).

The results are presented in Tables 1 and 2[12].

Five notable model features emerge from these simulations.

First, any increase in the marginal extraction cost of oil, the pollution stock and the stock of knowledge induces lower emissions. This result holds for 100% of the simulations. It simply reflects the fact that the three stocks increase the costs of using oil: the direct private cost (marginal extraction

[12] See tables in annex A. Table 1 displays the Cooperative equilibrium and the equilibrium issues of the Markovian game, for Case 1 and Case 2 respectively. Figures in brackets represent the percentage of iterations where the absolute value of the corresponding component in the Markovian strategy is greater than the absolute value of the same component in the Cooperative equilibrium. Table 2 presents data for the two cases on the welfare of each country (designed by $W_i(10), i = 1, 2$) and the aggregated welfare after 10 periods, in Cooperative and Markovian equilibria, and the value of the three stocks after 10 and 600 periods.

cost), the external cost (pollution) and the opportunity cost (knowledge). This effect is reinforced when the richest country is the more sensitive to environmental damage[13].

Second, in the Cooperative equilibrium, the same level of R&D investment is required in the poor and in the rich countries: from this point of view, the solution is egalitarian rather than equitable. However, this result is based on the assumption that both countries have the same costs of investment in research activity (γ is identical for both countries), and otherwise would not hold.

Third, in the Cooperative equilibrium, for both Case 1 and Case 2, the most important emissions come from the more technologically efficient country, i.e. Country 1, but in terms of marginal impact for the three stocks, this kind of game still lead to an egalitarian solution: the marginal impact of the three stocks on the emissions is the same for both countries, whatever the relative environmental concerns. Obviously, this result can be explained by the fact that, in the Cooperative equilibrium, the objective is to maximize the aggregated welfare, yet this objective involves different emission levels in each country (a higher level for the more technologically efficient country, which proceed from the emissions' constant), but in terms of marginal impacts the Cooperative equilibrium only requires an increase in one of the three stocks (therefore the emission level is integrating the difference in technological efficiency) to produce a decrease in total emissions shared equally by both countries. This is explained by three features of the model: (i) the damage caused by the emissions of the two countries is the same; (ii) the choice of a linear-quadratic functions leads to linear emissions paths; and (iii) the same weighting is given to the welfare of both countries.

Fourth, comparison of the welfare gains after 10 periods yields an interesting result. As expected, the aggregated welfare is always greater in the Cooperative outcome than the Markovian one. However, the Cooperative equilibrium is preferred by both nations only in Case 2, namely where the richest country is also the most sensitive to environmental damage. In this case, the adoption of free-riding strategies always leads to a decrease in welfare for both countries. In Case 1, however, cooperation produces advantages only for the poor country. The welfare of Country 1 increases in the Markovian outcome compared to the Cooperative one. This result explains why some industrialized countries are reluctant to ratify international agreements on Climate Change, such as the Kyoto Protocol.

A closer examination of the impacts of the three stocks on oil consumption

[13] This can be seen by comparing the marginal impacts from the three stocks across Case 1 and Case 2.

helps to explain why the adoption of free riding strategies in the Markovian game is likely. First, note that in both Case 1 and Case 2, the variations are small (in absolute values) for the country that is the most sensitive to environmental damage, and large for the other country[14]. In both cases, the rich country is responsible for the greater part of the pollution. In Case 1, it is the poor country that suffers the most from the pollution caused by the rich country. This is why cooperation in Case 1 implies greater emissions abatement for the rich country, compared to the Markovian equilibrium, which does not take into account the welfare of the poor country.

Second, the negative impact of pollution on emissions is stronger in the cooperative game than in the Markovian game. It should be pointed out that the difference is rather important for the country that is less sensitive to environmental consequences. This supports the idea that the adoption of free riding strategies in the Markovian game primarily comes from the behaviour of the less sensitive country[15].

Third, a similar argument holds concerning the impact of the knowledge stock over the emissions. This impact is weaker for the less environmentally sensitive country (in 98% of the simulations in Case 1 and in 98,4% of the simulations in Case 2) and is stronger for the more environmentally sensitive country (in 100% of the simulations for the two cases). Moreover, the amplitude of the variations is lower for the more sensitive country compared to the less sensitive one. Finally, the impact of the marginal oil extraction cost on oil consumption is stronger in the Markovian game than in the Cooperative game for both countries, with the most important variation for the less sensitive environmental country. To understand the underlying reason for this over-reaction to the marginal oil extraction cost on oil consumption, one needs to take account of the fifth main result of our research.

The fifth result is that the aggregate level of R&D expenditures is lower, in most simulations, under the cooperative outcome than the non-cooperative one. In other words, an agreement based on both R&D and emissions cutting, reduces aggregate cumulative R&D expenditure. We call this effect the "paradox of knowledge". On the one hand, the public good nature of the knowledge implies that, *ceteris paribus*, the aggregate R&D expenditure is

[14]Indeed, the Cooperative equilibrium is closer to the individual preferences of the most environmental sensitive country.

[15]Indeed, $\frac{\bar{z}^c_{1,2}}{\bar{z}^m_{1,2}} = 9,86$ in Case 1 and $\frac{\bar{z}^c_{2,2}}{\bar{z}^m_{2,2}} = 10,03$ in Case 2. $z^c_{i,2} > z^m_{i,2}$ is true in 100% of the simulations, for i=1 in Case 1 and i=2 in Case 2. Moreover, the difference is unimportant for the country that is more sensitive to environmental issues ($\frac{\bar{z}^c_{i,2}}{\bar{z}^m_{i,2}} = 1,03$ for i=2 in Case 1 and i=1 in Case 2 and $z^c_{i,2} > z^m_{i,2}$ is true for only 74,6% of the simulations for i=2 in Case 1 and for 71,5% of the simulations for i=1 in Case 2).

higher in the cooperative case than in the non cooperative case. But, on the other hand, in the cooperative case, both the stock of pollution and the marginal oil extraction costs are small compared to the non-cooperative case (Cf. Table 2). The private incentives to invest in research are higher, then, in the non-cooperative equilibrium, as knowledge improvements can be used to offset higher marginal oil extraction costs and higher levels of pollution. Of course, the cooperative outcome remains a better outcome even if it implies less research. Finally, this "paradox of knowledge" also explains, at least in part, why oil consumption over reacts to marginal oil extraction costs in the Markovian game. Indeed, the players know that a Markovian game leads to an over-investment in knowledge, which allows for more productive output from research sector to substitute for emissions in the presence of an increase in the marginal oil extraction cost.

This result has strong normative implications. It indeed offers a counter-argument to the view that increasing the amounts of R&D spending on low-carbon technology should be considered as a key criterion in any future agreements. From this perspective, our result echoes the results in van der Ploeg, F., de Zeeuw, A. (1994), that, in the absence of international coordination over pollution control, levels of clean technology stocks are too excessive. This also holds in our setting. Let us examine the impacts of the three stocks on the level of R&D investment. First, in Case 1 and Case 2, the investments made by both countries are less sensitive to the stock of knowledge. Second, in both cases, the less environmentally sensitive country knows that the other player has to invest more in case of an increase of the pollution stock. As a consequence, it can invest less in the face of this increase: in fact, the impact of the stock pollution becomes negative for the less environmentally sensitive country in the Markovian game. For a symetric reason, the reactions to environmental damage of the most environmentally sensitive country increase to compensate for the behaviour of the other country: it can be seen, that in both cases, the impact of the pollution stock on investment is, in absolute value, larger in the Markovian game than in the Cooperative equilibrium, for the most environmentally sensitive country. The opposite argument, in terms of countries, explains the fact that the sign of the impact of the oil marginal extraction cost on investment becomes negative for the most environmentally sensitive country and increases in absolute value (for the majority of the simulations) for the less environmentally sensitive country.

In order to anayze this more deeply, we move to some comparative statics. We investigated the impact of changing the value of key parameters on the coefficients of the regressions explained in section 4[16]. The large number of coefficients, however, does not allow us to study each of them in a systematic way. We instead selected the most salient effects.

5.2 On Chinese growth

The non-inclusion of emerging countries, such as China, is a major criticism of the Kyoto protocol, levelled by the United States. In this subsection, we study the impact of an increase in the wealth of the poor country through an increase of $\beta_{2,1}$ and $\eta_{2,1}$ on the relevant variables. Indeed, if one acknowledges that economic growth produces an increase in capital stock which increases energy productivity, then this growth must result in an increase in both $\beta_{2,1}$ and $\eta_{2,1}$. The results are presented in Tables 9 to 12.

Table 10 show that when growth in the poor country is due to an increase in its emissions (through parameter $\beta_{2,1}$), it always induces a significant welfare loss for the rich country. This welfare loss is larger when the rich country is also the most sensitive to environmental damage (Case 2). However, the welfare of the poor country does not necessarily increase. Table 10 clearly shows that an increase in $\beta_{2,1}$ leads to an increase in the welfare of Country 2 only when this country is less sensitive to environmental damage (Case 2). In the opposite case, i.e when the poor country is very sensitive to environmental damage (Case 1), an increase in its wealth through $\beta_{2,1}$ paradoxically lowers the welfare of both economies.

At the aggregated level, it is noticeable that an increase in the wealth of the poor economy through $\beta_{2,1}$, does not necessarily lead to a significant increase in aggregated emissions, and consequently, to a significant increase in the global pollution stock. (see in particular. the impact of the knowledge stock on emissions for the two types of equilibria and the impact of the marginal oil extraction cost for Country 1 in the Markovian equilibrium for Case 1, and the impact of the pollution stock on emissions of Country 2 in the Markovian equilibrium for Case 2). This effect is partially compensated by an increase in the emission constant of the poor country. But, at the same time, it reduces the emission constant of the rich country, which may lead to a reduction in its emissions.

In sum, the rich country always suffers from an increase in the wealth of the poor economy, driven by improvements in its emission. Moreover, the rich country loses more in the Cooperative equilibrium when the poor country is also more sensitive to environmental damage (Case 1) and in the

[16]The P-value associated with the corresponding variation are presented in Tables 3 to 8 (Cf. A.2). In order to have a P-value that is never greater than 15% we only comment on absolute values above 1,5.

Markovian game when the rich country is more sensitive to environmental damage (Case 2).

Let us now turn to the impact of an increase in the other parameters reflecting the level of development of the poor country: $\eta_{2,1}$. In this case, things change substantially. Specifically, the rich country is less likely to suffer from the wealth improvement of the poor economy while welfare gains are guaranteed for the poor country. This result can be explained as follows. First, as shown in Table 11, an increase of $\eta_{2,1}$ leads to an increase in the knowledge stock[17]. It also leads to a reduction, in the Cooperative equilibrium, of the impact of the pollution stock on emissions when the poor country is the most environmentally sensitive and to an increase in the impact of the marginal oil extraction cost on the emissions when the poor country is the least environmentally sensitive. In terms of welfare, the poor country always gains while the rich country loses only in Case 1 for the Markovian equilibrium. This result advocates for policies that promote clean technology, such as the Clean Development Mechanism included in the Kyoto Protocol.

5.3 On inequality growth

In the previous section, we looked at the impact of a reduction in the wealth gap between the poor and the rich countries. In this section, we investigate the opposite case, i.e. the case of a widening of the wealth gap between the two groups of countries[18]. To investigate this issue, we consider how an increase in the wealth of the rich country through an increase of $\beta_{1,1}$ and $\eta_{1,1}$ affects the relevant variables. Our main results are presented in Tables 13 to 16.

The impact of an increase in $\beta_{1,1}$ is fairly easy to understand. Whatever the relative sensitiveness of the countries to the environment, an increase in $\beta_{1,1}$ leads to an increase in the emissions of Country 1 and to a decrease in the emissions of Country 2 (through its impact on the emissions constant). At an aggregate level, this leads to an increase in emissions demonstrated by the increase in the marginal oil extraction cost and the pollution stock after 10 and 600 periods.

Moreover, an increase in $\beta_{1,1}$ has no significant impact on the knowledge stock, whatever the type of equilibrium, and whatever the case.

[17]This result is due to an increase in the investment constant for both countries in the Cooperative equilibrium, and to an increase in the investment constant for Country 2, which overcompensates the decrease in the investment constant for Country 1 in the Markovian game.

[18]One can think, for instance, of the relative positions of some African countries compared to the industrialized world.

Finally, table 14 shows the impacts in terms of welfare. An increase in $\beta_{1,1}$ always leads to welfare gains for the rich country and to welfare losses for the poor country. This result is amplified when the poor country is the most sensitive to environmental damage.

The impact of an increase of $\eta_{1,1}$ is presented in tables 15 and 16. Unexpectedly, an increase in $\eta_{1,1}$ does not have any significant impact on the emissions, on the oil marginal extraction cost or on the pollution stock. However, this result rests strongly on the specification of the production function. Nevertheless, in the Cooperative equilibrium, both increase their R&D investments[19]. Moreover[20], in line with free riding strategies, Country 2 decreases its R&D investment. Indeed, Country 2 knows that Country 1 has to increase its investment, and Country 1 increases its investment more strongly than in the Cooperative equilibrium, because it knows that Country 2 has to decrease its investment. These effects lead to an increase in the knowledge stock[21].

In term of welfare, an increase in $\eta_{1,1}$ is more equitable than an increase in $\beta_{1,1}$ because it improves the welfare of the rich country without harming the poor country. Actually, the poor country experiences welfare gains in all cases except one, i.e. in the Markovian equilibrium when the rich country is the most sensitive to environmental damage.

5.4 The Impacts of an increase in environmental concerns

We now want to study the impacts of an increase in environmental concern. We distinguish two possible cases: an increase in the environmental concern of the rich country (increase of α_1) and an increase in the environmental concern of the poor country (increase of α_2). The results are summarized in tables 17 to 20.

As expected, in the Cooperative equilibrium, an increase in environmental concern, in whatever country, always leads to an increase, in absolute value, in the impact of the pollution stock on the emissions. In some cases, in the case of the Cooperative equilibrium, this increase, which reflects an increase in external costs (pollution) is partly compensated by a significant decrease in the impact of the marginal oil extraction cost on the emission.

An increase in environmental concern in one of the countries leads to a decrease in the constant of emissions of both countries when this country

[19] As shown by the investment constants.

[20] In both Case 1 and Case 2, and in Markovian game.

[21] Except for the first periods under Case 1.

is the more environmentally sensitive. When this country is the less environmentally sensitive, it leads to a decrease in the emissions constant in the poor Country in both cases,.

In terms of investment in the knowledge stock, in the Cooperative equilibrium, there is also always an increase of the impact of the pollution stock on the investment in response to an increase in concern for the environment [22].

In the Markovian game, the issue is more complex because of the free riding strategies. In terms of emissions, a unilateral increase in environmental concerns always results in an increase in the impact of the pollution stock on the country's own emissions. When the country is the more sensitive to environmental damage, this increase is levelled out by a decrease in the impact of the marginal oil extraction cost and the knowledge stock on its emissions. The other country can respond by implementing one of three alternative strategies. It can lower the impact of the pollution stock on the emissions[23]. Alternatively, it can lower the impact of the marginal oil extraction cost on the emissions[24]. Finally, it can lower the impact of the two stocks[25]. In terms of the emissions constant, the rich country lowers it when environmental concerns are greater[26]. The poor country lowers it after an increase of α_2 in both cases.

In terms of investments, in the Markovian game and in Case 1, the increase in the environmental concern of the rich country results in a more virtuous issue since a decrease in the impact of the marginal oil extraction cost on investment and an increase in the impact of the pollution stock on investment occur in both countries. Indeed, in this case, there is a partial reconciliation between the concerns of the two countries: the more technologically efficient country is the richer one, and the more environmentally sensitive is the poorer one. Moreover, this mechanism is necessary as a partial counterpart to the excessive presence of non-socially efficient strategies as shown by an examination of the evolution of stocks. In the three other cases there is a reinforcement of the free riding strategies: the negative or positive impacts of both the marginal oil extraction cost and of the pollution stock on investment, always increase.

[22] In terms of the emissions, this increase is partially compensated by a significant decrease in the impact of the marginal oil extraction cost on the emissions, in all situations except for the case of an increase in environmental awareness in the richest country in Case 1.

[23] Rich country strategies in response to an increase of α_2.

[24] Poor country strategies when they are more sensitive to environmental damage.

[25] Poor country strategies in the face of an increase in α_1 in Case 2.

[26] After an increase of α_1 in Case 2 and after an increase of α_2 in case 1.

5.5 The impacts of an increase in the rate of pollution decay

It has been well-known since the seminal work of Forster [27], that an increase in the rate of pollution decay does not unambiguously affect the pollution stock. Rather it has two opposite effects. On the one side, it reduces the pollution stock as it increases the free cleaning contribution of the environment. On the other side, it increases the pollution stock as the additional free cleaning contribution reduces the environmental concerns related to the presence of the pollution stock in the environment, which leads to higher production and additional waste. The pollution stock can therefore be increased or decreased depending on the relative weight of the two effects.

The presence of these counterbalancing forces can be checked in Tables 21 and 22. In particular, we show that an increase in the rate of pollution decay induces a direct increase in emissions in all situations. Consequently, the marginal oil extraction cost increases in all cases after 10 and 600 periods. Similar mechanisms lead to a decrease in the knowledge stock when the latter impact is significant.

Our main result however is that the second effect is overwhelmed by the first for all situations. Consequently, an increase in the rate of pollution decay in our model unambiguously improves the welfare of both countries and reduces the pollution stock after 10 and 600 periods.

5.6 The impacts of an increase in the discount rate

The use of discount rates in long-run environmental and natural resource use problems is a controversial issue. The basic concern is that too high discount rates can lead to intergenerational conflicts. In particular, they may over-weight the welfare of the current generations to the detriment of future generations (see e.g. Howarth, R. (1996)).

Moreover, in the context of the present research, the discount rate tends to increase the opportunity cost of the knowledge cost (which require actual investment for future use) with regard to the use of the "dirty" production technology. These intuitions are confirmed by the results in tables 23 and 24.

As shown by these tables, an increase in the discount rate leads to:

(i) an increase in the emissions constant for both countries,

(ii) an increase in the impact of the variable reflecting the direct private cost of the use of the "dirty technology" (marginal oil extraction cost) on

[27] Cf. Forster, B. (1975) and Forster, B. (1977).

emissions,

(iii) a decrease in the impact of the pollution stock on the level of emissions,

(iv) a decrease in the impact of the knowledge stock on emissions (only in Case 2).

In terms of R&D investment, an increase in the discount rate leads to an increase in the impact of marginal oil extraction cost on R&D investment in the Cooperative equilibrium and to a decrease in this impact in the Markovian game. It also leads to a decrease in the impact of the pollution stock on R&D investment in both cases, to a decrease in the investment constants in a majority of situations, and to a decrease in the impact of the knowledge stock on R&D investment.

As a result, there is an increase in the marginal oil extraction cost after 10 and 600 periods for all situations, an increase in the pollution stock after 10 periods, and a decrease after 600 periods for Case 2 (in Case 1 the pollution stock is not significantly affected after 600 periods) and a decrease in the stock of knowledge after 10 periods.

The discount rate seems also to be important in terms of geographic transfers. Table 24 shows that an increase in the discount rate leads to an increase in the welfare for both countries only in Case 2, i.e. when the rich country is also the most sensitive to environmental damage. In the alternative Case 1, the welfare of the rich country improves while the poor country experiences a worsening in its welfare. These results hold both for the Cooperative equilibrium and for the Markovian game.

6 Conclusion

In this paper, we studied the problem of international coordination in climate policy using three state-variables (oil marginal extraction cost, pollution and knowledge), two asymmetric countries (a rich one and a poor one) and a differential game with Markov-linear strategies. We used a Monte Carlo procedure to obtain an insight into the behaviour of the model. This allowed us to make a study in terms of emission and investment strategies, of the state of the three stocks and the welfare of the two countries. We also provided some comparative statics for the more relevant parameters. We distinguished two cases regarding the relative sensitiveness to environmental damages: in the first case, the poor country is supposed to be the more environmentally sensitive, in the second case the rich country is assumed to be the more environmentally sensitive.

This study highlights the “paradox of knowledge”: while knowledge is a public good, the non-cooperative equilibrium displays a higher level of R&D expenditures than the optimal path in most simulations This over-investment in R&D is a reaction to the increase in the marginal extraction cost of oil, which is faster in the non-cooperation equilibrium than along the optimal path. This paradox questions the assertion that R&D spending on low-carbon technology should be a key feature of any future agreements.

The results of our comparative statics also call into question several points regarding the current debates on climate change, such as the non-inclusion of emerging countries (e.g. China) in the Kyoto protocol. We find that an increase in a poor country's wealth by the mean of an increase in its dirty energy productivity always leads to a welfare loss for the rich country, sometimes to a welfare loss for the poor country (when it is more sensitive to environmental damage) and does not necessary lead to a significant increase in the global pollution stock. By comparison, if the increase in the wealth of the poor country is based on its increased use of clean technology capabilities (the goal of the clean development mechanisms promoted by the Kyoto protocol), the rich country is less likely to suffer as a result of wealth improvement in the poor economy, and welfare gains are warranted for the poor country.

A Tables

A.1 Mean values of the endogenous variables

Table 1: Equilibrium Strategies

Table 2: Various value after 10 and 600 years

A.2 T-statistics for the impact of different parameters

Table 3: T-statistics for the Cooperative Equilibrium, Case 1 (1/3)

Table 4: T-statistics for the Markovian Equilibrium, Case 1 (2/3)

Table 5: T-statistics for Case 1 (3/3)

Table 6: T-statistics for the Cooperative Equilibrium, Case 2 (1/3)

Table 7: T-statistics for the Markovian Equilibrium, Case 2 (2/3)

Table 8: T-statistics for Case 2 (3/3)

A.3 Impact of the growth in the poor country

Table 9: Impact of $\beta_{2,1}$ on Equilibrium Strategies

Table 10: Impact of $\beta_{2,1}$ on different values

Table 11: Impact of $\eta_{2,1}$ on Equilibrium Strategies

Table 12: Impact of $\eta_{2,1}$ on different values

A.4 Impact of the growth in the rich country

Table 13: Impact of $\beta_{1,1}$ on Equilibrium Strategies

Table 14: Impact of $\beta_{1,1}$ on different values

Table 15: Impact of $\eta_{1,1}$ on Equilibrium Strategies

Table 16: Impact of $\eta_{1,1}$ on different values

A.5 Impact of an increase in environmental concerns

Table 17: Impact of α_1 on Equilibrium Strategies

Table 18: Impact of α_1 on different values

Table 19: Impact of α_2 on Equilibrium Strategies

Table 20: Impact of α_2 on different values

A.6 Impact of the rate of pollution decay

Table 21: Impact of δ on Equilibrium Strategies

Table 22: Impact of δ on different values

A.7 Impact of the social discount rate

Table 23: Impact of ρ on Equilibrium Strategies

Table 24: Impact of ρ on different values

B Algorithm for the system of coupled algebraic Riccati equations

The algorithm used to compute the solutions of system 4 is taken from Freiling, G., Jank, G., Abou-Kandil, H. (1996).

1. We compute $K_1^m(0)$ and $K_2^m(0)$ the stabilizing symmetric solutions of the following autonomous algebraic Riccati equations:

$$\begin{array}{c} A'K_1^m + K_1^m A + Q_1 - K_1^m S_1^m K_1^m = 0 \\ A'K_2^m + K_2^m A + Q_2 - K_2^m S_2^m K_2^m = 0 \end{array}$$

2. We compute the following discrete dynamical system, by taking $K_1^m(0)$ and $K_2^m(0)$ as initial conditions:

$$\begin{array}{l} K_1^m(i+1)\left[A - S_2^m K_2^m(i)\right] + \left[A - S_2^m K_2^m(i)\right]' K_1^m(i+1) + Q_1 \\ -K_1^m(i+1) S_1^m K_1^m(i+1) = 0 \end{array}$$

$$\begin{array}{l} K_2^m(i+1)\left[A - S_1^m K_1^m(i)\right] + \left[A - S_1^m K_1^m(i)\right]' K_2^m(i+1) + Q_2 \\ -K_2^m(i+1) S_2^m K_2^m(i+1) = 0 \end{array}$$

   Where i is the number of iterations

3. We stop after i^*, where i^* is such as:

$$\begin{array}{l} \left|K_1^m(i^*)\left[A - S_2^m K_2^m(i^*)\right] + \left[A - S_2^m K_2^m(i^*)\right]' K_1^m(i^*) + Q_1 - K_1^m(i^*) S_1^m K_1^m(i^*)\right| \\ + \left|K_2^m(i^*)\left[A - S_1^m K_1^m(i^*)\right] + \left[A - S_1^m K_1^m(i^*)\right]' K_2^m(i^*) + Q_2 - K_2^m(i^*) S_2^m K_2^m(i^*)\right| < \varepsilon \end{array}$$

   Where ε is a small number, set equal to 10^{-8} in the current simulations.

4. $K_1^m(i^*)$ and $K_2^m(i^*)$ are the solutions of system 4.

Notice that there exist no proof of convergence for this algorithm. However, in the simulations made for this paper, it always converged.

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