On the Path of an Oil Pigovian Tax

Antoine Belgodere[1]

UMR LISA Université de Corse - Campus Caraman - 20250 CORTE CORSICA (FRANCE)

Abstract

This paper studies optimal climate policy in the presence of oil rents. Several authors have found that, according to Hotelling's rule, in the long run, the optimal *ad-valorem* tax must decrease. However, if the pollution is a stock and if environmental concerns impose stopping the resource extraction before its exhaustion, we show that an *ad-valorem* tax defined over the rent can not decentralize the optimum. In this case, an increasing per-unit tax can decentralize the optimum. Such a tax implies the disappearance of the Hotelling rent. Thus, the extraction problem reduces to a pollution-control problem.

Key words:
Hotelling rent/rule, Global warming, Pigovian tax, Dynamic control

1 Introduction

Fossil fuels are of interest to researchers in Environmental economics for at least two reasons. First, they are nonrenewable resources and second, they are polluting assets. While these aspects are often treated separately in the literature, there is a line of research that deals explicitly with their interaction.

Schulze (1973), Hoel (1978), Forster (1980), Krautkraemer (1985), Krautkraemer (1986) , Withagen (1994) and Tahvonen (1997) all focus on the optimal extraction paths for nonrenewable and polluting resources. These studies consider a pollutant that accumulates into a stock. They are unanimous that, in

[1] The author is grateful to Flora Bellone, Dominique Prunetti and both anonymous referees for their precious comments.

these conditions, optimal consumption usually involves lower extraction in the first periods and a slower decrease of extraction in the latter periods [2].

Several papers focused on the way a government can implement an optimal extraction path in a decentralized economy. A contribution by Sinclair (1992) provides the counter-intuitive result that a *decreasing* Pigovian *ad-valorem* tax -rather than an increasing one- is required in the case of a polluting non-renewable resource; moreover, it is not the level of the tax, but only its rate of growth that is economically relevant. These results rely on the presence of a Hotelling rent in the resource sector. Indeed, in a model *à la* Hotelling, the initial price is set such that the resource is asymptotically exhausted. Given the impact of the tax on demand, any increase in the tax level is offset by a fall in the price. Moreover, the decreasing shape of the *ad-valorem* tax incites the private agents to consume more resource in the future, and, consequently, less in the present.

Another contribution by Schou (2002) goes even further in showing that to decentralize the optimal extraction path of a polluting nonrenewable resource it is not necessary to impose a tax if there are incentives to invest in technological progress. This author shows that the positive externality of research compensates for the negative externality of pollution. This result is obtained in a model where the pollution is modeled as a flow, and not a stock. Grimaud and Rougé (2005) generalize Schou's model and show that his result only holds for a given specification. They also confirm, in a general equilibrium context with endogenous growth, Sinclair's finding that a decreasing *ad-valorem* tax is required to postpone the extraction.

As a counter argument to Sinclair's view, Ulph and Ulph (1994) argue that the scarcity effect may be negligible compared to the polluting effect. In these circumstances, at least at the beginning of the planning horizon, the tax would be increasing [3]. Farzin and Tahvonen (1996) model a stock-dependent extraction cost *à la* Heal (1976), together with a complex carbon decay process. They show that several shapes can be obtained for the tax, including an increasing one. However, they don't explicitly model the decentralized economy, and they don't model an *ad-valorem* tax.

This present paper adds to this literature by describing a regime in which climate policy causes the Hotelling rent to disappear. As a feedback, in the absence of a Hotelling rent, oil scarcity don't interact anymore with the climate policy. In this context, an increasing Pigovian tax is needed. We also

[2] Optimal paths nonetheless may differ widely depending on the context. In an important contribution, Tahvonen (1997) establishes that no less than 11 different paths can be identified depending on the assumptions made about the use of a backstop technology and on the initial value of the stocks.

[3] Sinclair (1994) acknowledges this argument to be valid.

show that an *ad-valorem* tax does not decentralize the optimum, because the transversality condition of the government does not correspond to those of the equilibrium conditions of the fossil fuel market. A per-unit tax appears to be a more effective tool to decentralize the optimum. In this case, the stock of fossil fuel is common property, in equilibrium. If the policy tool employed is a subsidy based on the remaining stock of fossil fuel, rather than a tax, then the optimum is decentralized and the stock is privately owned.

The argument that climate policy can nullify the effect of a Hotelling rent is not totally new. It was intuited in the paper by Berck and Roberts (1996) (p68). The current paper adds to the knowledge provided by this paper in presenting a complete macroeconomic framework, close to that proposed by Krautkraemer (1986).

The paper is organized as follows: section 2 describes the model and the optimal path, section 3 investigates how the social optimum can be decentralized, section 4 discusses some empirical implications of the model and section 5 concludes.

2 The model and analysis of the optimal path

2.1 The model

We model an economy with a representative consumer endowed with an instantaneous utility function which depends on the consumption of energy $N(t)$, on the stock of pollution $M(t)$ generated by this resource, and on a non-energy good $X(t)$. In order to simplify the analysis, we assume that X enters linearly in the utility function [4]. So the utility is $X + V(N, M)$. We assume [5] $\frac{\partial V}{\partial X} > 0$, $\frac{\partial V}{\partial N} > 0$ and $\frac{\partial V}{\partial M} < 0$, as consumption impacts positively the consumer utility while pollution harms it.

The stock of pollution M is assumed to behave as

$$\dot{M} = \zeta E$$

where E is the rate of resource extraction and $\zeta \in [0; 1]$ is the natural absorp-

[4] This is the simplest way to provide a general equilibrium framework. Such a specification does not allow a fine analysis of the substitution between energy and non-energy goods. However, this paper focuses on the substitution between clean and dirty energy.

[5] Where no confusion could arise, time references are removed.

tion rate of pollution by the ecosystem. Note that the stock of pollution and the stock of resource are expressed in the same units. In the case of global warming, this can be thought of as the tons of carbon. Every ton of carbon extracted will be released into the environment. A fraction $(1-\zeta)$ is absorbed by the ecosystem (afforestation, oceans,...), and a fraction ζ is added to the cumulative pollution stock.

An important feature of this specification is that no decay process is assumed. This extreme position is adopted here because the more conventional simplifying assumption of constant decay is not suited to our setting. Indeed, with a linear decay function, the stock of pollution of a nonrenewable resource is always asymptotically nil. Consequently, the asymptotic behavior of the climatic system always induces full depletion of the resource [6]. As our interest here is in describing optimal consumption paths characterized by a partial depletion of the resource, this asymptotic behavior is obviously an undesirable feature. We prefer the alternative assumption of no decay process. The main drawback to this option is that it is unrealistic. However,we will argue that this is not as extreme as it may appear at first glance. For instance, a constant rate of decay clearly over estimates the ability of the terrestrial and oceanic sinks to dissolve the carbon in the long run. Schultz and Kasting (1997) criticizes Nordhaus (1994) on this ground. In response, Nordhaus and Boyer (2000) suggests modeling a decay process in which the oceans remove some carbon from the atmosphere without dissolving it so that a part of it is returned to the atmosphere. More complex decay processes, such as the ones modeled in Forster (1975) and Withagen and Toman (1998), become inactive once a given threshold of pollution is reached. In Farzin and Tahvonen (1996), carbon accumulates into four stocks, in which three have a strictly positive linear rate of decay, and no decay is assumed for the fourth. All these refinements account for the limits to the absorptive capacity of the environment in the long run. In all cases, on the optimal path the stock of pollution satisfies, $\lim_{t\to\infty} M(t) > 0$ instead of limiting it, $\lim_{t\to\infty} M(t) = 0$ as in the case of a constant decay process. In retaining, for the sake of simplicity, the assumption of no decay, the model is therefore unrealistic, but not unsatisfactory in so far as it deals with the asymptotic behavior of the climatic system.

The stock of nonrenewable resource evolves according to:

$$\dot{S} = -E \tag{1}$$

With this specification, for M(0) and S(0) given, M(t) can be expressed as a

[6] Only the presence of market imperfections, which would imply an excessive private cost of extraction, could explain, in this setting, a partial resource depletion in the long run.

function of S(t).

$$M(t) = M(0) + \zeta \left[S(0) - S(t) \right]$$

It follows that the utility can be expressed as a function of (X, N, S) instead of (X, N, M) [7]. If we define $U(N, S)$ as $U(N, S) \equiv V\{N, [M(0) + \zeta (S(0) - S)]\}$, we have $\frac{\partial U}{\partial S} > 0$ and $\frac{\partial^2 U}{\partial S^2} < 0$. U is assumed to be strictly concave. The sign of $\frac{\partial^2 U}{\partial N \partial S}$ is indetermined. In this formulation, the model becomes a *cake eating* model with a stock effect, *à la* Krautkraemer (1985) and Krautkraemer (1986).

The non energy-good is produced with labour with a one-for-one production function:

$$X = L_X$$

where L_X is the quantity of labour used in production.

The energy good $N(t)$ can stem from two different energy sources. The first is the polluting nonrenewable resource E(t). The second is an infinitely available but costly non-polluting resource A(t).

$$N(t) = E(t) + A(t)$$

The cost of producing the non-polluting resource, $q(A)$, expressed in terms of labour, is assumed to be an increasing convex function, with $q'(0) = 0$.

The resource constraint over the labour is:

$$L = q(A) + X$$

where L is the labour force. L is assumed to be sufficiently large so that X is always positive.

Under these specifications, energy sources as physical entities are perfect substitute, i.e. the joules from oil are exactly the same entities as the joules derived from wind. However, as economic entities, the elasticity of substitution between both types of energy inputs is neither infinite nor constant. With an increasing and convex production cost of the renewable energy, both sources will be used simultaneously if the price of the polluting nonrenewable resource, lets say oil, is low enough. However, in the case of a high oil price, only the

[7] Similar modelings are used in Schulze (1973), Hoel (1978), Forster (1980) and Sinclair (1994).

non-polluting source will be used[8]. This outcome seems realistic, but it could not be obtained with a constant elasticity of substitution[9].

This specification takes account of the fact that joules of energy may differ from one another both in terms of complementary services, such as storage and transportation required to "qualify" it as an input, and in terms of its production costs. For instance, the joule derived from oil and the joule derived from wind are not perfect substitutes for the motorist because transportation and storage are much more costly for the wind joules than for the joules from oil. To become available for use, the former type of energy must be transformed into hydrogen, which involves a very costly process. Also, the production costs of any non-polluting resource vary a great deal, depending, for instance, on the wind speed or the strength of the sun. Cost minimization implies that non-polluting energy inputs are allocated first to uses that are less demanding in terms of transportation and storage. More generally, the first joules produced from renewable energy inputs are the cheapest. However, as the price of the nonrenewable polluting resource increases, it becomes profitable to produce more expensive joules from renewable energy sources.

2.2 Social optimum

This section describes the optimal resource extraction path from a social point of view.

2.2.1 Characterization of the optimal path

The social planner maximizes $\int_0^\infty (X + U)e^{-\rho t}dt$, where ρ is the social discount rate, under the constraints:

$$X = L - q(A)$$

and

$$\dot{S} = -E \tag{2}$$

With $E, A, S \geq 0$ and $S(0) = S_0$ given.

[8] If, instead, the non polluting energy is at constant cost, the cost minimizing choice of input would be a *bang-bang* choice, i.e. only one energy source would be exploited, depending on the relative price of the energy inputs.
[9] This is the reason why we don't use a CES function, that is often used in the literature about the substitution between both kinds of energy.

The current-value Hamiltonian associated with this problem is:

$$H = L - q(A) + U - \lambda E$$

where λ is the co-state variable associated with S. The maximum principle[10] gives the following conditions[11] :

$$U_N \leq \lambda \; E \geq 0 \quad E\,(U_N - \lambda) = 0 \tag{3}$$
$$U_N = q'(A) \tag{4}$$
$$\dot{\lambda} = \rho\lambda - U_S \tag{5}$$
$$\lim_{t \to \infty} e^{-\rho t} \lambda S = 0 \tag{6}$$

The assumptions made about U_N and $q(A)$ allow us to look for an interior solution for A, but not for E. Equation (3), when $U_N = \lambda$[12], states that the marginal utility of the extractions must equate with the shadow price of the resource stock. Equation (4) states that the marginal productivity of the renewable resource must equate with its marginal cost. Equation (5) represents a modified Hotelling's rule. It can be interpreted as a non-arbitrage condition. The yield of an asset whose value is λ, in terms of utility and from a social point of view, will be the same as if this value were placed at a rate ρ. What it yields is U_S, the marginal utility of the resource stock, which corresponds to the avoided pollution, plus $\dot{\lambda}$, the gain in capital. Equation (6) is the transversality condition.

Eliminating λ in equations (3)-(5) gives the following equation, which describes, together with equation 1, the dynamics of the system along the optimal path:

$$\widehat{U_N} = \rho - \frac{U_S}{U_N} \tag{7}$$

where letters with a hat denote the growth rates of the variables. Equation (7) is a benchmark for comparison with the equilibrium path.

2.2.2 Variables on the long run

Given that the resource is nonrenewable, its use must be nil in the long run. Thus, we look for a stationary point in which $\dot{\lambda}* = \dot{S}* = 0$. A letter with an

[10] These conditions are both necessary and sufficient thanks to the conditions on U.
[11] The letter in index represents the partial derivative.
[12] That is to say when the non-negativity constraint on E is not binding.

asterisk denotes the long-run value of the variable. $\dot{S}* = 0$ implies :

$$E* = 0$$

There are two possibilities: the first is that the use of the resource stops in finite time, and the second is that it becomes asymptotically nil.

However, for the purposes of this paper, whether E becomes nil in finite time or asymptotically is not relevant because in either case, if the resource stock is not exhausted, the Hotelling rent disappears. Thus, for simplicity - and without weakening the main results of the model - in the remainder of this paper we focus the case where use becomes asymptotically nil. What matters is to know whether, in the long run, S stabilizes at a strictly positive level or at zero.

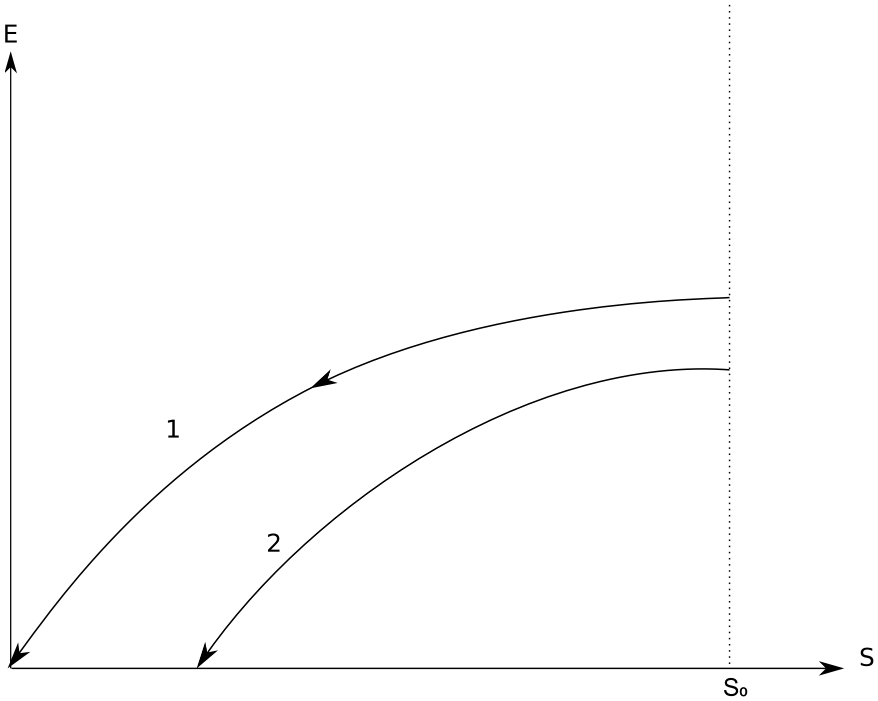

Fig. 1. Two paths for E and S

Figure 1 depicts two possible paths for (S, E). In both cases, the flow of resource use becomes nil. In path 1, the resource is fully exhausted, while in case 2, use is discontinued before complete exhaustion.

In a stationary point, just as $E* = 0$, equation 4 implies a relation between $A*$ and $S*$ if $U_{NS} \neq 0$ or directly gives $A*$ if $U_{NS} = 0$. As $\lambda* = 0$, equations 5 implies another relation between $S*$ and $A*$.

Figure 2 presents both relations. There are two cases that can arise.

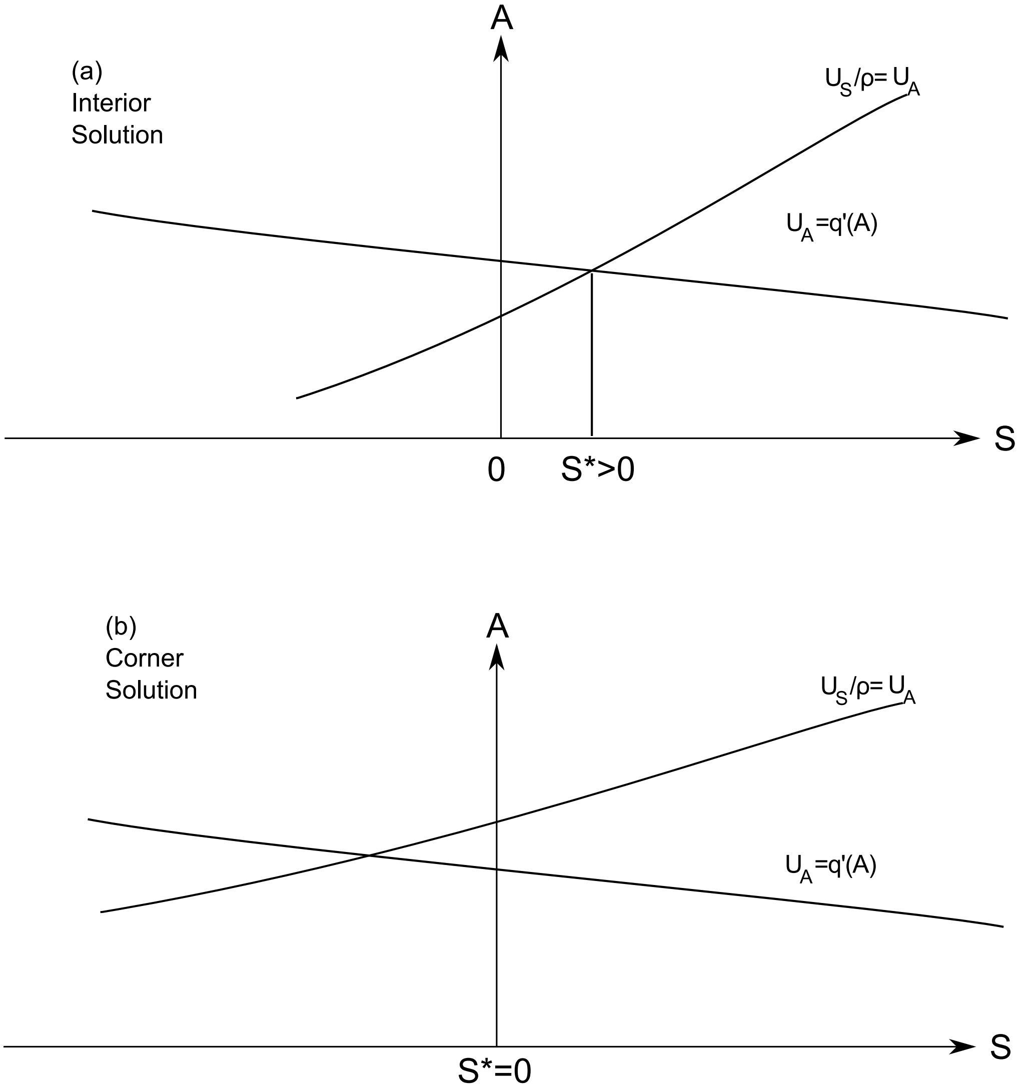

Fig. 2. Solutions of $S*$

- If the value of S such that $U_N = \frac{U_S}{\rho}$ is strictly positive, then the steady state is an interior solution, and the resource is not fully exhausted. This case corresponds to (a) in figure 2, and to path (2) in figure 1.
- If this value is negative, then the steady state is a corner solution, and the resource is fully exhausted, either in finite time or asymptotically. This case corresponds to (b) in figure 2, and to path (1) in figure 1.

If the solution is interior, the appendix shows that the system has a saddle path stability, provided:

$$U_{NS} > \frac{U_{SS}}{\rho}\left(1 - \frac{U_{NN}}{q''(A)}\right)(< 0)$$

We assume this inequality holds.

The stationary point can be determined with the following illustrating specifications:

$$U(N,S) = \frac{(A+E)^{1-\sigma}}{1-\sigma} - \frac{\epsilon}{2}(\Lambda - S)^2$$

$$q(A) = \frac{\beta}{2}A^2$$

Where $\sigma > 0$, $\epsilon > 0$, $\Lambda > S_0$ [13] and $\beta > 0$ are parameters. Notice that these specifications imply $U_{NS} = 0$, which ensures the saddle path stability if the solution is interior. Simple calculations give:

$$A* = \beta^{-\frac{1}{1+\sigma}}$$

$$\lambda* = \beta^{\frac{\sigma}{1+\sigma}}$$

$$S* = max\left[0, \Lambda - \frac{\rho}{\epsilon}\beta^{\frac{\sigma}{1+\sigma}}\right]$$

With this specification, full exhaustion of the resource is not warranted. More precisely, the resource will not be fully exhausted if:

$$\Lambda > \frac{\rho}{\epsilon}\beta^{\frac{\sigma}{1+\sigma}}$$

The likelihood of exhaustion depends positively on ρ and β, negatively on Λ and ϵ, and ambiguously on σ [14] : the less a society cares for the future, and the more expensive is the substitute for oil, the likelier is the full exhaustion of the resource. On the other hand, both ϵ and Λ account for the weight of the environment in the utility function, and decrease the likelihood of exhaustion.

Moreover, the transversality condition (6) is respected because λ and S tend toward a constant value.

3 Implementation of the optimum

This section identifies a way for the government to implement the optimal extraction path of the resource in a decentralized economy.

[13] This condition guaranties that the marginal utility of S is always positive.
[14] $\frac{\partial S*}{\partial \sigma}$ has the sign of $-ln(\beta)$.

Here, we are dealing with an economy composed of a government and four representative agents: a representative household, a representative firm in the non energy good sector, a representative firm in the renewable resource sector, and a representative firm that manages the stock of the nonrenewable resource.

The household maximizes his utility by choosing his consumption of X, E and A. The price of X is normalized to unity, P is the price of the nonrenewable resource and ω is the price of the renewable one. An *ad-valorem* tax $\tau - 1$ is raised over the nonrenewable resource. If R is the income of the household, then:

$$U_N = P\tau \tag{8}$$

$$U_N = \omega \tag{9}$$

$$X = R - P\tau E - \omega A$$

Perfect competition is assumed on every market. On the X market, the price equates the marginal cost and is the numeraire, so the wage rate is also unitary. On the renewable market, we have:

$$\omega = q'(A) \tag{10}$$

The convexity of $q()$ implies that the profit is not nil on this sector. The classical phenomenon of entry of new firms in the sector that lowers the profit is irrelevant here, because the convexity of $q()$ comes from the fact that the costliest units are produced first. The owners of the windiest units of production of wind energy sell their energy at the same price as the other producers, who face a higher production cost. So, the profit on this sector can be thought of as a Ricardian rent over a natural resource. It is given by $A\omega - q(A)$ and distributed to the household.

On the nonrenewable resource market, the firm maximizes $\int_0^\infty PEe^{-\rho t}dt$ under (1). It results in the classical hotelling tarification:

$$\dot{P} = \rho P \tag{11}$$

$$\lim_{t\to\infty} e^{-\rho t}P(t)S(t) = 0 \tag{12}$$

At each point of time, the profit is given by PE, which is distributed to the household. The government receipt is $PE(\tau - 1)$, which is distributed to the household as a lump-sum transfert T[15].

[15] Actually, T can be negative, and thus become a lump-sum tax, if $\tau < 1$, i.e. if

To complete the model, we can give the value of the household income:

$$R = L + T + A\omega - q(A) + PE$$

3.2 Optimal policy tool

Having described the equilibrium behavior of the agents, we look for the optimal policy from the government's point of view. We began with the *ad-valorem* tax already introduced, but the difficulties involved in decentralizing the optimum using this tool motivates the use of two alternative instruments: a per-unit tax and a subsidy.

3.2.1 The optimal ad-valorem tax

The possibility to decentralize the social optimal with the *ad-valorem* tax depends on the long term value of S, as we see in the following two propositions.

Proposition 1:

If $S* = 0$ on the optimal path, then the optimal path can be decentralized by any *ad-valorem* tax following:

$$\hat{\tau} = -\frac{U_S}{U_N} \tag{13}$$

Proof:

Equations 9 and 10 are equivalent to the equation 4. Equation 8 together with equation 11 imply $\hat{U_N} = \rho + \hat{\tau}$, which is equivalent to the benchmark equation 7. Moreover, the long-run condition $S* = 0$ is warranted by the equation 12 which rewrites, thanks to equation 11 $P(0) \lim_{t\to\infty} S_t = 0$. ■

In the case concerned by the proposition 1, we find the same result as Sinclair (1992) and Grimaud and Rougé (2005) of a decreasing tax with an arbitrary initial value [16] . But things change if $S* > 0$, as we state in proposition 2:

Proposition 2:

the environmental tax is in fact a subsidy.

[16] In the case of Grimaud and Rougé (2005), the result was found in a flow pollution framework.

If $S* > 0$ on the optimal path, then the optimal path can not be decentralized by an *ad-valorem* tax.

Proof:

If such a tax existed, then it would follow $\hat{\tau} = -\frac{U_S}{U_N}$. As we saw previously, such a tax enables to implement the optimal rate of growth for U_N. However, at the market equilibrium, the equations 11 and 12 imply, again, $P(0) \lim_{t \to \infty} S_t = 0$, which, this time, is contradictory with the optimum long run level of S. ■

The explanation for proposition 2 is that, on equilibrium, the initial price $P(0)$ is set so the resource is fully extracted on the long run. The reason why the whole resource is not sold, at a point of time, whereas the price is greater than the marginal cost, is because the remaining stock will yield to its owner a rate of return of ρ by being sold later. But if a fraction of the stock is never sold, then there is no incentive to save it: it is worth selling this fraction at the current price. This mechanism lowers the current price, until the demand is high enough to ensure the full resource exhaustion in the long run.

With the specifications introduce on the previous section, the demand function for E is given by:

$$E = (P\tau)^{-\frac{1}{\sigma}} - \frac{P\tau}{\beta}$$

By integrating, one gets:

$$\int_0^\infty E(t)dt = [P(0)\tau(0)]^{-\frac{1}{\sigma}} \int_0^\infty e^{-\frac{\rho - U_S/U_N}{\sigma}t} dt - \frac{P(0)\tau(0)}{\beta} \int_0^\infty e^{\rho - U_S/U_N t} dt$$

The cumulative extractions depend negatively on $P(0)$. If the owners of the resource stock anticipate $\int_0^\infty E(t)dt < S(0)$ for a given $\tau(0)$, then they increase their supply until $P(0)$ is low enough so that $\int_0^\infty E(t)dt = S(0)$. If the government reacts by increasing the initial level of the tax, then the same mechanism will apply: the stock owners will react by increasing their supply until the price fall again. If this process continues endlessly, then the price converges toward 0, and the price paid by the household becomes $0 * \tau = 0$, which clearly can not decentralize the optimum.

It follows that, along the equilibrium path, despite the tax, U_N has the good rate of growth but starts too low, compared to the optimal path. Stopping extraction before the resource exhaustion would require the price paid by the household (including the tax) to be high enough. But as the tax is ad-valorem, a fall in the price also implies a fall in the tax. However, this problem does not arise if the tax is a per-unit tax instead of an *ad-valorem* tax.

3.2.2 The optimal per-unit tax

With a per-unit tax, the government directly controls the level of the tax, whatever the market price. If θ is the *per-unit* tax, then the price paid by the consumer is $E(P+\theta)$. Thus, the equation 8 is replaced by the following:

$$U_N = P + \theta \tag{14}$$

The optimal taxation path seems harder to identify with this setting. The tax must be such as $\frac{\dot{P}+\dot{\theta}}{P+\theta} = \rho - U_S$. But when $S* = 0$ on the optimal path, then the *ad-valorem* tax can be used, and when $S* > 0$, the problem simplifies greatly, as is stated in the proposition 3 bellow. Before stating it, we introduce the following lemma:

lemma: A per-unit tax can decentralize $\lim_{t\to\infty} S(t) > 0$.

proof: According to equations 10 and 14, the demand function of nonrenewable resource is[17] :

$$E = U_N^{-1}(P+\theta) - q'^{-1}(P+\theta)$$

which is an increasing function of P and θ. The smallest value of P being 0 at the market equilibrium, the smallest value of $\lim_{t\to\infty} S(t)$ is

$$S_0 - \int_0^\infty \left[U_N^{-1}(\theta) - q'^{-1}(\theta)\right] dt$$

If the sequence of tax $[\theta(t)]_0^\infty$ is such as $\int_0^\infty \left[U_N^{-1}(\theta) - q'^{-1}(\theta)\right] dt < S_0$, then the long run value of S is strictly positive. $\blacksquare$

Proposition 3:

If $S* > 0$ on the optimal path, then:

a) The optimal sequence of $[\theta(t)]_0^\infty$ implies $P(t) = 0 \forall t$, and

b) $\theta = \lambda$ enables to decentralize the optimal path.

Proof:

[17] the exponent $^{-1}$ denotes the inverse function.

We start by proving the a). If $[\theta(t)]_0^\infty$ is optimal, then it must decentralize $\lim_{t\to\infty} S(t) > 0$, which is possible according to the lemma. The two equilibrium conditions on the nonrenewable market given by equations 11 and 12, together with $\lim_{t\to\infty} S(t) > 0$, imply:

$$P(0) \lim_{t\to\infty} S(t) = 0 \Leftrightarrow P(0) = 0$$

Thus, the hotelling rule 11 implies:

$$P(t) = P(0)e^{\rho t} = 0$$

which proves the point a).

Now, if $P = 0$, then equation 14 is identical to the equation 3 if $\theta = \lambda$. It follows that the set of equations describing the equilibrium are identical to the set describing the optimal path. ■

The interpretation of this taxation scheme is that the tax must equate the sum of the discounted flows of marginal damage caused by pollution. It can be seen by integrating equation 5 [18]:

$$\lambda(t) = \lambda(0) + \int_0^t [\rho\lambda(l) - U_S(l)]\, dl = e^{\rho t}\left(\lambda(0) - \int_0^t U_S e^{-\rho l} dl\right)$$

If $S* > 0$, then the transversality condition 6 implies $\lambda(0) = \int_0^\infty U_S e^{-\rho t} dt$ and thus $\theta = \lambda = \int_t^\infty U_S e^{\rho(t-l)} dl$.

The striking result is that, in this case, the optimal tax scheme leads the Hotelling rent to disappear. The price paid by the household is only the tax. It follows that we can't translate this tax into an ad-valorem tax, because it would imply a division by 0. However, if we introduced a constant marginal cost of extraction, that would be paid by the household, then the rate of growth of the optimal ad-valorem tax would be the same as θ's.

This rate is positive, because E is a decreasing function of λ [19], and must decrease toward 0.

Another consequence of the disappearance of the rent is that the resource stock can not be privately owned. Private agents have no incentive to invest in an asset with no private economic value. However, the government has to

[18] The author thanks one of the anonymous referees who provided this calculus.

[19] As long as $E > 0$, equations 3 and 4 imply $E = U_N^{-1}(\lambda) - q'^{-1}(\lambda)$, which is clearly a decreasing relation. In the specified version, it writes $E = \lambda^{-\frac{1}{\sigma}} - \frac{\lambda}{\beta}$.

own the stock, even the part of it that will never be depleted, in order to impose the tax to the consumers.

Remark: In the model presented here, we assume that decisions regarding the extraction path and the environmental policy are taken simultaneously in time 0. Now, assume that, for political reason [20], there exists a delay between the initial period and the period (t') where the tax is adopted. In the meanwhile, there is no tax, but the private agents anticipate that this tax will be adopted, and that it will lead to stop using the polluting resource before its exhaustion. In this conditions, this anticipation would lead to the disappearence of the rent from the initial period. So, between time 0 and time t', there would be neither rent nor tax to mitigate the extractions. So, the rate of extraction in this meanwhile would clearly be not only greater than the optimum level, but even greater than it would be without the anticipation of the future adoption of the tax.

3.2.3 The optimal subsidy

The representative household assumption does not directly allow us to adopt a distributive point of view, as Amundsen and Schob (1999) do in an international trade framework. However, one can guess that if the owners of the resource stock are different from the rest of the population, the distributive impact of the tax is important. This is especially true if, as is the case with the OPEC countries, a substantial part of these owners' income derives from exploitation of the resource [21]. In Amundsen and Schobs's paper, the rent is partly transferred to the other countries by means of an environmental policy. In the framework of the present model, this rent would be fully transferred. However, another policy tool could be used to implement the optimum, which would have the opposite distributive effect: a subsidy on the remaining resource stock.

A subsidy s is paid at each point of time to the firm in the resource sector for each unit of the resource stock. Its profit becomes, at a point of time, $PE+sS$. On the other hand, the price paid by the household for E reduces to P. The subsidy is financed by a lump-sum tax paid by the household, so $sS = -T$. In this context, the optimal subsidy is given in proposition 4.

Proposition 4: The optimum is decentralized if $s = U_S$.

[20] Like bargaining time.
[21] According to EIA (2005), OPEC countries such as Algeria, Iran, Kuwait, Libya, Nigeria, Qatar and Saudi Arabia earn thanks to oil rent from 70 % to 90 % of their government spending.

Proof: the equilibrium conditions in the resource market become:

$$\hat{P} = \rho - \frac{s}{P} \tag{15}$$

and (12), which is unchanged.

The utility maximisation of the household implies $U_N = P \Leftrightarrow \hat{U_N} = \hat{P} = \rho - \frac{s}{U_N}$, which is identical to the benchmark 7 if $s = U_S$. ■

This possibility to subsidy the owner of the resource stock recalls that the former are not necessarily the losers of an environmental policy. It is often acknowledged by policy analyst that a transfert scheme between winners and losers of a public policy may be required in order to implement the policy. In this context, the transfert scheme could just be a switch form an optimal tax to an optimal subsidy. With this policy, the resource stock would remain privately owned, because the private owners would have an incentive not to exhaust it.

On the other hand, such a subsidy would create the incentive to find new resource stocks, which is not taken into account in the present framework, where the size of the stock is exogenously given.

4 Empirical implication

If the Hotelling rule is understood in the strict sense of an increase in the resource price at the interest rate, then this rule is not compatible with the observed long run trends in oil prices. Empirical tests of the nonrenewable resources theory actually rely on more complex models [22] than the genuine Hotelling model. These models, by taking account of the presence of extraction costs and technical progress, make a non-increasing price trend compatible with the arbitrage assumption that underlies the 'r-rule' theory.

However, these tests are not consensual about the validity of the Hotelling 'r-rule'. One could suggest that this rule holds for a given size of the known stocks. As new stocks are discovered, a new path is initiated for the rent, which is still increasing, but starts from a lower level. However, this explanation is far from convincing as it assumes very myopic expectations.

The model presented in this paper provides a potential explanation for the non-increasing trend in oil rents. If the owners of the oil stocks anticipate that, in the long run, it will be socially optimal not to deplete these stocks,

[22] See Chermak and Patrick (2002) for a classification.

and that some political agreement will implement this optimum, then the market equilibrium implies a non-growing price. Note that the reason for these expectations is not necessarily related to global warming; such concerns were not paramount 25 years ago. There are many reasons that would result in the same effect, such as, for instance, the political wish of the countries to be energy independent.

While there is some doubt about the existence of an increasing scarcity rent, some oil rents do exist, and are shared by the oil-producing firms and the oil-exporting countries. The existence of these rents seems to contradict the model discussed here. But the presence of oil rents does not imply that they are Hotelling rents. They may simply be oligopoly rents. Indeed, oil extraction requires high investment, and involves high sunk costs, which advocate for an imperfectly competitive market.

Moreover, as extraction capacities, at a given point in time, are limited and take time to expend, booms in oil demand have an impact on price. For technical reasons, these short-run effects can last for a considerable time. Thus, big profits in the oil sector are not incompatible with a model that predicts no Hotelling rent.

5 Conclusion

This paper contributes to the debate about the interaction between oil depletion and climate change by countering the optimism about the long run evolution of an optimal carbon tax. According to this view, within the private sector, in the long run, the need for a carbon tax *will be replaced with* a scarcity rent. This view relies on a pessimistic assumption about the availability of renewable energy sources, and an optimistic assumption about the capacity of the environment to decay atmospheric pollution. We show that if one assumes that renewables can, in the long run, replace fossil fuels and that there are limits to the decay process in relation to pollution, outcomes may change dramatically. The optimal tax will make the Hotelling rent disappear, and the stock of hydrocarbon will become a public asset. In this context, the tax must increase, as in a standard Plourde model.

Appendix

The saddle path stability of the stationary point requires the jacobian matrix of the system to have a positive and a negative eigenvalue.

First, let $E(\lambda, S)$ and $A(\lambda, S)$ be the values of E and A that respect equations 3 and 4. By totally differentiating both equations, one gets:

$$\begin{pmatrix} U_{NN} & U_{NN} \\ 0 & q''(A) \end{pmatrix} \begin{pmatrix} dE \\ dA \end{pmatrix} = \begin{pmatrix} 1 & -U_{NS} \\ 1 & 0 \end{pmatrix} \begin{pmatrix} d\lambda \\ dS \end{pmatrix}$$
$$\Leftrightarrow \begin{pmatrix} dE \\ dA \end{pmatrix} = \begin{pmatrix} \frac{1}{U_{NN}} - \frac{1}{q''(A)} & -\frac{U_{NS}}{U_{NN}} \\ \frac{1}{q''(A)} & 0 \end{pmatrix} \begin{pmatrix} d\lambda \\ dS \end{pmatrix}$$

Thus

$$\frac{\partial E}{\partial \lambda} = \frac{1}{U_{NN}} - \frac{1}{q''(A)}$$

and

$$\frac{\partial E}{\partial S} = -\frac{U_{NS}}{U_{NN}}$$

λ and S follow the system given by:

$$\dot{\lambda} = \rho\lambda - U_S$$
$$\dot{S} = -E(\lambda, S)$$

Let J be the jacobian matrix of this system:

$$J = \begin{pmatrix} \rho & -U_{SS} \\ \frac{1}{q''(A)} - \frac{1}{U_{NN}} & \frac{U_{NS}}{U_{NN}} \end{pmatrix}$$

The eigenvalues of J are:

$$\mu_+ = \frac{\left(\rho + \frac{U_{NS}}{U_{NN}}\right) + \sqrt{\left(\rho - \frac{U_{NS}}{U_{NN}}\right)^2 + 4\left(\frac{U_{SS}}{U_{NN}} - \frac{U_{SS}}{q''(A)}\right)}}{2}$$
$$\mu_- = \frac{\left(\rho + \frac{U_{NS}}{U_{NN}}\right) - \sqrt{\left(\rho - \frac{U_{NS}}{U_{NN}}\right)^2 + 4\left(\frac{U_{SS}}{U_{NN}} - \frac{U_{SS}}{q''(A)}\right)}}{2}$$

Notice that the term under the square root in both expressions is positive, so we are dealing with real numbers. It can be checked that, whatever the sign of U_{NS}, μ_+ is strictly positive. Indeed, if $\left(\rho + \frac{U_{NS}}{U_{NN}}\right) \geq 0$, it is obvious. If $\left(\rho + \frac{U_{NS}}{U_{NN}}\right) < 0$, then $\mu_+ > 0$ implies:

$$
\begin{aligned}
&\left(\rho+\tfrac{U_{NS}}{U_{NN}}\right) > -\sqrt{\left(\rho-\tfrac{U_{NS}}{U_{NN}}\right)^2+4\left(\tfrac{U_{SS}}{U_{NN}}-\tfrac{U_{SS}}{q''(A)}\right)}\\
&\Leftrightarrow \left(\rho+\tfrac{U_{NS}}{U_{NN}}\right)^2 < \left(\rho-\tfrac{U_{NS}}{U_{NN}}\right)^2+4\left(\tfrac{U_{SS}}{U_{NN}}-\tfrac{U_{SS}}{q''(A)}\right)\\
&\Leftrightarrow \tfrac{U_{SS}}{U_{NN}}-\rho\tfrac{U_{NS}}{U_{NN}}-\tfrac{U_{SS}}{q''(A)}>0
\end{aligned}
$$

which is always true, because $\left(\rho+\frac{U_{NS}}{U_{NN}}\right)<0$ implies $\frac{U_{NS}}{U_{NN}}<0$.

Now, if $\left(\rho+\frac{U_{NS}}{U_{NN}}\right)\leq 0$, it is obvious that $\mu_- < 0$, so, in this case, the stationary point has a saddle path stability. If $\left(\rho+\frac{U_{NS}}{U_{NN}}\right)>0$, then $\mu_-<0$ implies, again:

$$\frac{U_{SS}}{U_{NN}}-\rho\frac{U_{NS}}{U_{NN}}-\frac{U_{SS}}{q''(A)}>0$$

But this time, it is not always true, because we are in the case where $\left(\rho+\frac{U_{NS}}{U_{NN}}\right)>0$.

To sum up, the condition to have a saddle path is that, at the stationary point:

$$U_{NS} > \frac{U_{SS}}{\rho}\left(1-\frac{U_{NN}}{q''(A)}\right)(<0)$$

Thus, U_{NS} can be negative, but not too much.

References

Amundsen, E., Schob, R., Jun. 1999. Environmental taxes on exhaustible resources. European Journal of Political Economy 15 (2), 311–329.

Berck, P., Roberts, M., 1996. Natural resource prices: Will they ever turn up? Journal of Environmental Economics and Management 31, 65–78.

Chermak, J., Patrick, R., 2002. Comparing tests of the theory of exhaustible resources. Resource and Energy Economics 24, 301–325.

EIA, Jun. 2005. Opec revenues: Country details. Tech. rep., available at http://www.eia.doe.gov/emeu/cabs/orevcoun.html.

Farzin, Y., Tahvonen, O., 1996. Global carbon cycle and the optimal path of a carbon tax. Oxford Economic Papers 48, 515–536.

Forster, B., 1975. Optimal pollution control with a nonconstant exponential rate of decay. Journal of Environmental Economics and Management 2, 1–6.

Forster, B., 1980. Optimal energy use in a polluted environment. Journal of Environmental Economics and Management 7, 321–333.

Grimaud, A., Rougé, L., Jun. 2005. Polluting non-renewable resources, innovation and growth: Welfare and environmental policy. Resource and Energy Economics 27 (2), 109–129.

Heal, G., 1976. The relationship between price and extraction cost for a resource with a backstop technology. The Bell Journal of Economics 2 (7), 371–378.

Hoel, M., 1978. Resource extraction and recycling with environmental costs. Journal of Environmental Economics and Management 5, 220–235.

Krautkraemer, J., 1985. Optimal growth, resource amenities, and the preservation of natural environments. Review of Economic Studies 52, 153–170.

Krautkraemer, J. A., June 1986. Optimal depletion with resource amenities and a backstop technology. Resources and Energy 8 (2), 133–149.

Nordhaus, W. D., 1994. Managing the Global Commons. MIT Press.

Nordhaus, W. D., Boyer, J., 2000. Warming the World: Economic Models of Global Warming. MIT Press.

Schou, P., 2002. When environmental policy is superfluous : Growth and polluting resources. Scandinavian Journal of Economics 104 (4), 605–620.

Schultz, P. A., Kasting, J. F., 1997. Optimal reductions in co2 emissions. Energy Policy 25, 491–500.

Schulze, W., 1973. The optimal use of nonrenewable resources: the theory of extraction. Journal of Environmental Economics and Management 1, 53–73.

Sinclair, P., 1992. High does nothing and rising is worse: Carbon taxes should be kept declining to cut harmful emissions. Manchester School 60, 41–52.

Sinclair, P., 1994. On the optimum trend of fossil fuel taxation. Oxford Economic Papers 46, 869–877.

Tahvonen, O., 1997. Fossil fuels, stock externalities and backstop technology. Canadian Journal of Economics 30, 865–874.

Ulph, A., Ulph, D., 1994. The optimal path of a carbon tax. Oxford Economic Papers 46, 857–868.

Withagen, C., August 1994. Pollution and exhaustibility of fossil fuels. Resource and Energy Economics 16 (3), 235–242.

Withagen, C., Toman, M., Jul. 1998. Accumulative pollution, "clean technology" and policy design. Resource for the Future 98–43.